$K_L \to \gamma\gamma$: Theory and phenomenology

Ernest Ma and A. Pramudita
Department of Physics, University of Wisconsin at Madison, Madison, Wisconsin 53706
and Department of Physics and Astronomy, University of Hawaii at Manoa, Honolulu, Hawaii 96822
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We compute the one-loop quark-diagram contribution to $d\bar{s}\to\gamma\gamma$ in the standard model as a function of m_q where $q = u, c, t$, etc. We find it to be bounded, and such that the process $K_L\to\gamma\gamma$ is likely to be dominated by low-energy contributions. We then use a phenomenological pole-dominance model to obtain $|A_{\text{theory}}/A_{\text{experiment}}| = 1.0 \pm 0.3$.

I. INTRODUCTION

The rare decay process $K_L\to\gamma\gamma$ is a flavor-changing radiative transition, which naively is of the same order in weak and electromagnetic couplings as $K_L\to\mu^+\mu^-$; yet the branching ratio[1] of the former is $(4.9\pm 0.5)\times 10^{-4}$, whereas that[1] of the latter is only $(9.1\pm 1.9)\times 10^{-9}$. The strong suppression of $K_L\to\mu^+\mu^-$ versus $K^+\to\mu^+\nu$, for example, is now understood as being due to the Glashow-Iliopoulos-Maiani (GIM) mechanism,[2] which not only makes it natural to have the absence of flavor-changing neutral currents in the standard electroweak gauge model[3] at the tree level, but also suppresses the induced one-loop effect by factors proportional to m_q^2/M_W^2, where m_q refers to the mass of the $u, c, t, \ldots$ quarks in this case, and M_W is the mass of the charged intermediate vector boson. On the other hand, $K_L\to\gamma\gamma$ does not appear to be suppressed at all by the GIM mechanism at the one-loop level. In fact, if we use $\Gamma(K_L\to\mu^+\mu^-)/\Gamma(K_L\to\gamma\gamma)$ as a very crude estimate of $(m_c^2/M_W^2)^2$, we find a rather reasonable value of 5 GeV for m_c. Early, but more detailed, analyses of this kind[4,5] have yielded essentially the same result. Specifically, it was shown[5] that in the quark-model calculation of $K_L\to\gamma\gamma$, the contribution of each internal quark is proportional to a function of m_q^2/m_K^2, which goes rapidly to zero as the argument increases beyond unity. Hence the dominant contribution is due to the u quark, and it is not suppressed by m_q^2/M_W^2. However, the calculation was made under the assumption $m_q \ll M_W$, and it may not be correct if the yet-to-be-discovered t quark turns out to be very heavy. In fact, for $K_L\to\mu^+\mu^-$, we find[6] that the correct expression for arbitrary m_q grows with m_q, which implies that heavy-quark contributions are in principle not negligible for that process. Similar statements are true for the K_L-K_S mass difference.[7]

In this paper we present[8] a detailed quark-model calculation of the effective $d\bar{s}\to\gamma\gamma$ transition, and show that, in contrast to $d\bar{s}\to\mu^+\mu^-$ and $d\bar{s}\to s\bar{d}$, the heavy-quark contribution is always bounded. Our result then modifies that of Gaillard and Lee,[5] but still shares with it the problem that the total contribution is numerically only a fraction of the experimentally measured amplitude. However, this is to be expected, since the quark-model result is in some sense only an average over the entire hadronic spectrum relevant for that process. What our calculation shows is that for $K_L\to\gamma\gamma$, the low-energy $u\bar{u}$ contributions are likely to be dominant; but since these are represented mainly by individual particles with large-mass splittings, i.e., π^0, η, and η', the true average is sensitive to the detailed dynamical mechanism involved in the formation of hadrons, which is of course largely unknown. Hence we also use a phenomenological pole model,[9] where the $K_L\to\gamma\gamma$ amplitude is assumed to be dominated by the intermediate states π^0, η, and η', for comparison with data. With various experimental inputs and theoretical assumptions, but otherwise no adjustable parameter, we find the ratio of the theoretical to the experimental amplitudes to be 1.0 ± 0.3. Details are given in Sec. III.

Finally, we conclude in Sec. IV with some remarks on the largely unsuccessful applications of quantum chromodynamics (QCD) and the bag model in calculating the $K_L\to\pi^0$ amplitude for the purpose of the pole model.

II. QUARK-MODEL CALCULATION OF $K_L\to\gamma\gamma$

The basic process under study is $d\bar{s}\to\gamma\gamma$. In the R_ξ gauge,[10] there are 30 one-particle-irreducible (1PI) diagrams, and 36 one-particle-reducible (1PR) diagrams. We evaluate for each the contribution of an intermediate quark of arbitrary mass m_q, keeping only the approximation that all external momenta are small compared to M_W, which is certainly justified for $K_L\to\gamma\gamma$. In fact, because of gauge invariance, the lowest-order nonzero contribution to the $d\bar{s}\to\gamma\gamma$ amplitude turns out to be a third-rank tensor in external momenta, which makes the actual computation rather lengthy and time consuming. In the end, we find that if m_q is less than a few GeV, then only the two 1PI diagrams of Fig. 1 are non-negligible, as was pointed out in Ref. 5; but if m_q is much greater than a few GeV, then the dominant contributions are those of the 1PR diagrams. However, both the 1PI and 1PR amplitudes are still bounded in magnitude, and are very insensitive to large changes in m_t. Hence the theoretical prediction is relatively stable, unlike that for $K_L \to \mu^+\mu^-$ or the K_L-K_S mass difference, each of which depends, for a given set of nonzero mixing angles, sensitively on m_t. (An upper bound on m_t, subject to some theoretical assumptions, was recently obtained[11] by use of $K_L \to \mu^+\mu^-$ and Δm_K data.)

To a very good approximation, the $K_L \to \gamma\gamma$ amplitude is dominated by $K_2 \to \gamma\gamma$, where

$$K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0) \tag{2.1}$$

is a pure CP-odd state. The requirements of angular-momentum conservation, CP invariance, and gauge invariance then uniquely determine the form of the amplitude to be proportional to $\epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu\epsilon_2^\nu q_1^\alpha q_2^\beta$, where $\epsilon_{1,2}$ and $q_{1,2}$ are the polarization and momentum vectors of the two photons. In contrast, the quark-model calculation of $d\bar{s} \to \gamma\gamma$ (and $s\bar{d} \to \gamma\gamma$) involves many possible initial and final states. To single out the K^0 (and $\bar{K}^0$) contribution, we must also use

$$\langle 0 | \bar{s}\gamma^\mu\gamma_5 d | K^0 \rangle = i f_K p_K^\mu , \tag{2.2}$$

where f_K is the kaon decay constant and p_K its momentum, together with a few other relationships derivable from this one.

The result of the quark-model calculation is given by

$$A(K_L \to \gamma\gamma) = \frac{G_F \alpha}{\pi} i f_K \epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu\epsilon_2^\nu q_1^\alpha q_2^\beta \times \sum_{j=u,c,t} \mathrm{Re}(\lambda_{jd}\lambda_{js}^*)[A_j^{(i)} + A_j^{(r)}] , \tag{2.3}$$

where

$$A_j^{(i)} = (Q+1)^2 \left\{ 2 + \frac{4x_j}{x_K} \int_0^1 \frac{dy}{y} \ln\left[1 - y(1-y)\frac{x_K}{x_j}\right]\right\} \tag{2.4}$$

is the 1PI contribution of the jth quark, and

$$A_j^{(r)} = \xi \left\{ Q^2 \left[\frac{5}{3} + \frac{1 - 5x_j - 2x_j^2}{(1-x_j)^3} - \frac{6x_j^2 \ln x_j}{(1-x_j)^4}\right] + Q\left[3 + \frac{3 - 9x_j}{(1-x_j)^2} - \frac{6x_j^2 \ln x_j}{(1-x_j)^3}\right]\right\} \tag{2.5}$$

is the 1PR contribution. In the above, $Q = -\frac{1}{3}$ is the electric charge of the d and s quarks, $x_j = m_j^2/M_W^2$ with $j = u$, c, or t, $x_K = m_K^2/M_W^2$ and λ_{ij} is the charged-current mixing matrix for six quarks.[12] The parameter ξ in Eq. (2.5) is given by

$$\xi = \frac{m_K^2}{16} \left\langle K_L \left| \frac{1}{q_1 \cdot p_1} + \frac{1}{q_2 \cdot p_2} + \frac{1}{q_1 \cdot p_2} + \frac{1}{q_2 \cdot p_1} \right| K_L \right\rangle , \tag{2.6}$$

where $p_{1,2}$ are the momenta of the valence quarks inside the kaon. We point out that $A_j^{(i)}$ coincides with the result of Ref. 5, and $A_j^{(r)}$ is simply related to the one-loop effective $\gamma d\bar{s}$ coupling.[13] We also note that both are bounded functions of x_j.

Before we indicate how the kinematical factor ξ can be computed, let us put in the explicit Kobayashi-Maskawa (KM) matrix[12] for λ_{ij}. Together with the approximation $x_u = 0$, we then obtain

$$\sum_{j=u,c,t} \mathrm{Re}(\lambda_{jd}\lambda_{js}^*)[A_j^{(i)} + A_j^{(r)}] = -s_1 \left\{ \frac{8}{9} c_1 c_3 + \frac{2x_K}{27x_c} c_2(c_1c_2c_3 + s_2s_3\cos\delta) - \xi\left[\frac{7x_t - 5x_t^2 - 8x_t^3}{9(1-x_t)^3} + \frac{2x_t^2(2-3x_t)\ln x_t}{3(1-x_t)^4}\right] s_2(c_1s_2c_3 - c_2s_3\cos\delta)\right\} , \tag{2.7}$$

where $s_{1,2,3}$ and $c_{1,2,3}$ are the sines and cosines of the canonical KM mixing angles, and δ the CP-nonconserving phase. Since the factor involving x_t in Eq. (2.7) is numerically between 0 and $\frac{8}{9}$, and all the mixing-angle factors are bounded by unity, the entire expression is certainly also bounded.

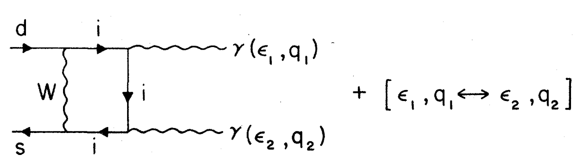

FIG. 1. Dominant one-particle-irreducible (1PI) diagrams for $d\bar{s} \to \gamma\gamma$; $i = u, c, t$, etc.

To evaluate ξ, let us consider the analogous case of $\pi^0 \to \gamma\gamma$. The quark-model calculation of the decay amplitude is given by

$$A(\pi^0 \to \gamma\gamma) = -\frac{8}{3} i\pi\alpha \frac{f_\pi \xi_\pi}{m_\pi^2} \epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu\epsilon_2^\nu q_1^\alpha q_2^\beta . \tag{2.8}$$

Using $c_1 f_\pi = 128$ MeV from $\pi^+ \to \mu^+\nu$ data, and the experimental $\pi^0 \to \gamma\gamma$ rate, we get $\xi_\pi = 5.9 \times 10^{-2} c_1$. Using the definition of ξ in Eq. (2.6) as a guide, we then estimate ξ_K to be roughly given by $(m_K^2/$

$m_\pi^2)\xi_\pi$, i.e.,

$$\xi_K \simeq 0.8 c_1 . \tag{2.9}$$

The above estimate is more or less equivalent to saying that the charge radii of π^0 and K_L are roughly equal, which is perhaps not too far from the truth. In any case, the result is quite reasonable, as we do expect ξ to be unity in the nonrelativistic approximation, and the kaon certainly comes much closer to such a description than the pion.

Using the experimental $K_L \to \gamma\gamma$ branching ratio of $(4.9 \pm 0.5) \times 10^{-4}$, and $s_1 c_3 f_K = 35.3$ MeV from $K^+ \to \mu^+\nu$ data, we find that the right-hand side of Eq. (2.7) has to be equal to $(3.4 \pm 0.2) s_1 c_3$. However, using $c_1 = 0.97$ and $s_3 \leq 0.5$ (Ref. 14), we find the numerical maximum of that expression to be only $(0.86 + 0.08\xi) s_1 c_3$, and since ξ is of order unity, the quark-model calculation definitely misses by a factor of 3 or 4. Therefore, our conclusion is the same as that of Ref. 5, except of course we have now proved that no amount of heavy-quark contribution is going to change the result. This is to be contrasted with the situation for $K_L \to \mu^+\mu^-$ and Δm_K, where a fourth generation will significantly affect the theoretical calculations.

Since s_2 is probably small, the right-hand side of Eq. (2.7) is likely to be dominated by the u-quark contribution; but since this is represented in reality by individual particles with masses comparable to m_K, but with large mass splittings among themselves, the quark-model result, which is in some sense an average, is certainly not too reliable. If the contribution were to come from heavy quarks, then the corresponding hadrons would have mass splittings much smaller than the difference between m_K and the mass of the heavy quark, hence the effective average of the quark-model calculation would be much more reliable. For $K_L \to \gamma\gamma$, we must now consider in detail the low-energy contributions in a more phenomenological approach.

III. PHENOMENOLOGICAL POLE-DOMINANCE MODEL FOR $K_L \to \gamma\gamma$

Assuming that $K_L \to \gamma\gamma$ is indeed dominated by low-energy contributions, we discuss in this section a very simple phenomenological model, which does give the correct experimental rate, with no adjustable parameter. We assume that the $K_L \to \gamma\gamma$ amplitude is well approximated by the sum of one-particle intermediate states, i.e.,

$$A(K_L \to \gamma\gamma) = \sum_i \frac{A(P_i \to \gamma\gamma)\langle P_i|\mathcal{H}_W|K_L\rangle}{m_K^2 - m_i^2} , \tag{3.1}$$

where the pseudoscalar mesons P_i refer to π^0, η, and η'. Since the masses and two-photon amplitudes of all these particles are now known,[1] what remain to be evaluated are the matrix elements for the $K_L \to \pi^0$, η, and η' transitions. Of the possible two-particle intermediate states, only $K_L \to 2\pi$ is potentially important, but because it is CP-nonconserving, its amplitude is strongly suppressed. In contrast, for $K_S \to \gamma\gamma$, the 2π intermediate state is expected to be dominant.[15]

First we evaluate $\langle \pi^0|\mathcal{H}_W|K_L\rangle$ by using the symmetric soft-pion reduction[16] of the $K \to 2\pi$ amplitudes, i.e.,

$$\begin{aligned}\langle \pi^0|\mathcal{H}_W|K_L\rangle &= -2f_\pi\langle \pi^0\pi^0|\mathcal{H}_W|K_S\rangle \\ &= -6.93 \times 10^{-2}\ \text{MeV}^2 .\end{aligned} \tag{3.2}$$

Then we note that the physical states η and η' are not pure states under SU(3), but are given by

$$\begin{aligned}\eta &= \eta_8 \cos\theta + \eta_1 \sin\theta , \\ \eta' &= -\eta_8 \sin\theta + \eta_1 \cos\theta ,\end{aligned} \tag{3.3}$$

where η_8 and η_1 are the SU(3)-octet and -singlet pseudoscalar-meson states, and $\theta = -10.6 \pm 0.5$ degrees.[17] The next step is to relate the $K_L \to \eta_8$ and η_1 amplitudes to $K_L \to \pi^0$. For $K_L \to \eta_8$, we observe that $\mathcal{H}_W$ is a combination of $\underline{8}$ and $\underline{27}$ operators under SU(3). For the $\underline{8}$ piece, it is easily shown that

$$\langle \eta_8|\mathcal{H}_W|K_L\rangle = \frac{1}{\sqrt{3}}\langle \pi^0|\mathcal{H}_W|K_L\rangle . \tag{3.4}$$

For the $\underline{27}$ piece, it turns out that because $\mathcal{H}_W$ has U spin equal to one, Eq. (3.4) also holds. Now we must deal with $K_L \to \eta_1$, which is of course outside the scope of SU(3). For this, we simply assume that $\mathcal{H}_W$ is a $\underline{15}$-dominant operator in SU(4), in analogy with $\underline{8}$ dominance in SU(3), and find

$$\langle \eta_1|\mathcal{H}_W|K_L\rangle = \frac{1}{\sqrt{6}}\langle \pi^0|\mathcal{H}_W|K_L\rangle , \tag{3.5}$$

where we have defined

$$\begin{aligned}\eta_1 &= -\frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s) , \\ \eta_8 &= \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s) , \\ \pi^0 &= \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d) .\end{aligned} \tag{3.6}$$

The unconventional minus sign for η_1 will be justified in the following.

In the 2γ decays of π^0, η, and η', if we assume U-spin invariance, then

$$A(\eta_8 \to \gamma\gamma) = \frac{1}{\sqrt{3}} A(\pi^0 \to \gamma\gamma) . \tag{3.7}$$

Now since η_8 is a mixture of η and η', we have

$$\cos\theta A(\eta\to\gamma\gamma) - \sin\theta A(\eta'\to\gamma\gamma) = \frac{1}{\sqrt{3}} A(\pi^0\to\gamma\gamma) . \quad (3.8)$$

From the experimental decay rates,[1] we can determine the magnitude of each 2γ amplitude, but not its sign. Using the commonly accepted value[17] of -10.6 ± 0.5 degrees for θ, we find that Eq. (3.8) can only be satisfied with the opposite sign for the η' amplitude relative to the π^0 and η amplitudes. Specifically, we have

$$\begin{aligned} A(\pi^0\to\gamma\gamma) &= 2.535\times10^{-5}\ \text{MeV}^{-1}, \\ A(\eta\to\gamma\gamma) &= 1.98(\pm0.14)\times10^{-5}\ \text{MeV}^{-1}, \quad (3.9) \\ A(\eta'\to\gamma\gamma) &= -3.5(\pm0.6)\times10^{-5}\ \text{MeV}^{-1}. \end{aligned}$$

From the quark-model point of view, the assignments of Eq. (3.6) are then quite reasonable.

All the ingredients for evaluating $A(K_L\to\gamma\gamma)$ are now assembled, and we just plug in the numbers. Using Eqs. (3.1) to (3.5), as well as Eq. (3.9), we find

$$A(K_L\to\gamma\gamma)_{\text{theory}} = 3.13(\pm0.95)\times10^{-12}\ \text{MeV}^{-1}, \quad (3.10)$$

with relative contributions of -7.66, $+12.63(\pm0.89)$, and $-1.84(\pm0.34)$ from π^0, η, and η', respectively. Using the experimental rate,[1] we find

$$A(K_L\to\gamma\gamma)_{\text{experiment}} = 3.19(\pm0.33)\times10^{-12}\ \text{MeV}^{-1}. \quad (3.11)$$

Obviously, Eqs. (3.10) and (3.11) are in excellent agreement with each other, and we find

$$\left|\frac{A_{\text{theory}}}{A_{\text{experiment}}}\right| = 1.0\pm0.3, \quad (3.12)$$

with most of the uncertainties coming from the poorly measured widths of η and η'. Therefore, it appears that our simple phenomenological model is able to account for the data quite well. It also shows that there is indeed a large cancellation among the π^0, η, and η' contributions, in support of our expectation from the quark-model calculation in Sec. II.

IV. CONCLUDING REMARKS

As an alternative to using the symmetric soft-pion reduction for obtaining $\langle\pi^0|\mathcal{H}_W|K_L\rangle$, we can try using QCD together with the bag model. The procedure is to write[18,19]

$$\mathcal{H}_W = \sum_i c_i O_i, \quad (4.1)$$

where O_i are operators with definite SU(3) transformation properties, and c_i are coefficient functions obtainable in QCD. The matrix elements $\langle\pi^0|O_i|K_L\rangle$ are then evaluated[20] by using the bag model. The coefficients c_i $(i=1,\ldots,7)$ for the complete set of operators O_i have been calculated in QCD, but different authors[19] use somewhat different methods and their results do not all agree. However, in no case is there any agreement with the numerical result of Eq. (2.3) for $\langle\pi^0|\mathcal{H}_W|K_L\rangle$. This is of course to be expected, since the QCD calculations cannot explain fully the $\Delta I=\frac{1}{2}$ enhancement for weak nonleptonic processes, whereas Eq. (3.2) uses the $K_S\to\pi^0\pi^0$ amplitude directly so that the enhancement is automatically taken into account.

To understand Eq. (3.4) in terms of O_i, we note that the only $\underline{27}$ operators are O_3 and O_4, corresponding to the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ components, respectively. However, because $\mathcal{H}_W$ has U spin equal to one, the two must only appear in the combination $\frac{1}{5}O_3+O_4$, i.e., $c_3=\frac{1}{5}c_4$, independent of QCD corrections.

To check further into the validity of the $\Delta I=\frac{1}{2}$ rule in the QCD and bag calculations, we have considered the ratio of the $I=\frac{3}{2}$ to $I=\frac{1}{2}$ pieces of $\langle\pi^0|\mathcal{H}_W|K_L\rangle$, and we find that it is indeed never much less than one, so that it cannot account for the experimentally observed value of about $\frac{1}{20}$. In addition, the prediction for $A(K_L\to\gamma\gamma)$ so obtained is always much smaller than the experimental value. Therefore, we must conclude that this application of QCD and the bag model is largely unsuccessful.

In summary, we have shown in Sec. II, that $K_L\to\gamma\gamma$ is not further suppressed at the one-loop level by the GIM mechanism, and that it is likely to be dominated by low-energy contributions. We have also shown in Sec. III that a phenomenological pole-dominance model, with π^0, η, and η' as the intermediate states, and with no adjustable parameter, agrees well with the data. Since the $K_S\to\pi^0\pi^0$ amplitude is used as an input, it appears that some hadronic enhancement for the $\Delta I=\frac{1}{2}$ amplitude is required to reproduce the data. This helps to explain why the quark-model calculation falls short of the experimental amplitude by a factor of 3 or 4.

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[1]Particle Data Group, Rev. Mod. Phys. 52, S1 (1980).

[2]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[3]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367; S. L. Glashow, Nucl. Phys. 22, 579 (1961).

[4]E. Ma, Phys. Rev. D 9, 3103 (1974).

[5]M. K. Gaillard and B. W. Lee, Phys. Rev. D 10, 897 (1974).

[6]E. Ma and A. Pramudita, Phys. Rev. D 22, 214 (1980).

[7]T. Inami and C. S. Lim, Prog. Theor. Phys. 65, 297 (1981).

[8]Preliminary results were reported by E. Ma and A. Pramudita, in Proceedings of the Workshop on Nuclear and Particle Physics at Energies up to 31 GeV, Los Alamos, 1981, edited by J. D. Bowman *et al.*, LASL Report No. LA-8775-C (unpublished), p. 92.

[9]Our treatment of the pole model is rather different from that of Ref. 5.

[10]K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972).

[11]A. Buras, Phys. Rev. Lett. 46, 1354 (1981).

[12]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[13]E. Ma and A. Pramudita, Phys. Rev. D 24, 1410 (1981), and references therein.

[14]R. E. Shrock and L.-L. Wang, Phys. Rev. Lett. 41, 1692 (1978).

[15]N. Cabibbo and E. Ferrari, Nuovo Cimento 18, 928 (1960); J. Dreitlein and H. Primakoff, Phys. Rev. 124, 268 (1961); V. Barger, Nuovo Cimento 32, 127 (1964).

[16]M. Suzuki, Phys. Rev. 144, 1154 (1966).

[17]N. Isgur, Phys. Rev. D 12, 3770 (1975), and references therein.

[18]M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974).

[19]M. A. Shifman, A. I. Vaǐnshteǐn, and V. I. Zakharov, Nucl. Phys. B120, 316 (1977); F. J. Gilman and M. B. Wise, Phys. Rev. D 20, 2392 (1979); C. T. Hill and G. G. Ross, Nucl. Phys. B171, 141 (1980).

[20]J. F. Donoghue, E. Golowich, B. R. Holstein, and W. A. Ponce, Phys. Rev. D 23, 1213 (1981), and references therein.