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Nucleon two-body decays in SU(5)

William B. Rolnick
Department of Physics, Wayne State University, Detroit, Michigan 48202
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Proton and neutron decay to meson and antilepton is studied in SU(5) as a function of the momentum of the antiquark produced; two different Higgs structures are considered. Branching ratios which are independent of the motion of the antiquark are displayed, as well as branching ratios which probe the antiquark's motion. The partial-rate predictions for Cabibbo-suppressed modes of the two different Higgs structures are compared. Tables of reduced partial rates are presented and their connection to the absolute partial rates is displayed. We expect the dominant part of the nucleon's width to result from two-body decay (including η, ρ, and ω production). We find that it varies by a factor of 3 depending upon how the motion of the antiquark at production is treated. Momentum suppression due to "spectator" recoil and muon phase space is also incorporated, and tables of absolute partial rates and proton and neutron lifetime are included.

I. INTRODUCTION

The simplest, most restrictive, as well as minimal, grand unified theory (GUT) is the SU(5) theory proposed by Georgi and Glashow.[1] Its many attractive features have been elaborated upon by a number of authors.[2] The unification of the electroweak and strong forces is expected to occur at energies near 10^{14} GeV, which is certainly inaccessible to any forseeable experiment. The most dramatic ramification of GUT's at the low energies and masses available to us is the prediction of nucleon decay to mesons and leptons, which violates baryon-number conservation. Although the lifetime predicted ($\tau \sim 10^{30}$ yr) is much longer than the age of the Universe, the decay rate for a sample containing kilotons of matter will be measurable and many large-scale experiments are in progress to find such decays.

Within the framework of an SU(5) GUT there are features which are not completely determined. Spontaneous symmetry breakdown is supposed to be due to the existence of Higgs bosons; however, the behavior of these particles under group operations (i.e., the representation to which they belong), as well as their Yukawa couplings (which give mass to the fermions in the theory), are not determined by the group structure. Different choices can lead to different generation mixing when the resulting fermion mass matrices are diagonalized.

In order to confront experimental results, a theory must make predictions for the matrix elements of the interaction between physical states. The states which concern us are the nucleon and meson states as well as those of leptons. Although the leptons are elementary particles (as far as we know), the nucleon and mesons are bound-quark systems. There are no reliable quantum-chromodynamics (QCD) calculations of these bound states, and various simple assumptions appear in the literature on nucleon decay.[3] The purpose of this paper is to add to the knowledge of the hadron matrix element for SU(5); similar studies will be undertaken in the future for other models.

In this paper we explore the SU(5) predictions for the decay of the proton or neutron into a meson and an antilepton. These are expected to be the predominant decay modes of the nucleon when pion resonances are included. (Three-body decays have been treated elsewhere.[4]) For definiteness, we employ SU(6) wave functions for the nucleon and meson, as has been done by many authors.[2,5,6] (Small corrections are expected due to the hyperfine interaction in QCD.[7]) As usual, we assume that the fundamental process is the fusion of two quarks to produce an antilepton and an antiquark, the latter then combining with the third (spectator) quark to produce a meson. There have been various assumptions for the momentum of the antiquark produced (zero, relativistic, bag model, etc.) yielding different partial rates and branching ratios.[6] In order to test the validity of the SU(5)

GUT we wish to avoid this ambiguity. In this paper we find the partial rates as a function of the momentum of the outgoing antiquark, and look at branching ratios which are independent of that momentum (ratios of rates with the same functional dependence). Furthermore, by comparing rates which have very different functional behavior we can determine the "effective" momentum of the antiquark.

The usual Higgs-boson coupling scheme employed in SU(5) GUT's (called[8] F mass) produces a nondiagonal mass matrix (between the first two generations) for the up-type quarks which leads to a simple Cabibbo rotation. Another coupling scheme (J mass) has been suggested[8] in the context of SO(10) which involves Higgs bosons belonging to a complex $\underline{10}$ and a $\underline{126}$ representation. This leads to nondiagonal mass matrices (between the first two generations) for three of the fermion types appearing (up, down, and charged lepton). The transformations which diagonalize these are more complicated and involve many parameters (which may be determined from observed masses and the Cabibbo angle). In this paper we display the two sets of partial widths which arise from these two coupling schemes. We note the many Cabibbo-suppressed modes whose branching ratios are considerably different in the two schemes. The $K^\circ e^+$ mode is especially enhanced in the J-mass scheme, as has been noted before.[8] [If we have SO(10) symmetry, with the J-mass scheme, which breaks to SU(5) above 10^{14} GeV we would have remnants of that structure in the subsequent generation mixing.] We discuss, as well, the one undetermined parameter in the J-mass scheme. Finally, we indicate the factors necessary to obtain absolute partial rates from our tables, or conversely, the factors which should be removed from the observed rates in order to obtain reduced partial rates; relationships among the latter are predicted from (the two coupling schemes of) SU(5) in this paper. We expect two-body decays, including the resonances η, ρ, and ω, to dominate the decay of the proton. Therefore, we calculate an upper limit to the lifetime of the proton by summing the partial decay rates for all the two-body modes, with the expectation that $\tau_{\text{actual}} \approx \tau_{2\,\text{body}}$. This paper verifies the results previously obtained in Refs. 2, 5, and 6 extended as explained above. Our absolute rates and lifetimes, for the no-recoil case, differ from those of Ref. 5 due to our use of a different value[2] for the probability of the two quarks in the nucleon being close enough to fuse, and momentum suppression factors.

II. ANTIQUARK DYNAMICS

The amplitudes for nucleon decay into an antilepton and a meson are calculated from an SU(5) grand unified theory. The fundamental process is the fusion of two quarks to form a heavy gauge boson (diquark), which subsequently decays (as a leptoquark) to form an antiquark and an antilepton. We assume SU(6) wave functions for the initial nucleon (taken at rest) and the final meson. We neglect any quark motion within the nucleon, but consider that the final antiquark may recoil in a direction opposite to the momentum of the antilepton ($+z$ direction); the mass of the antilepton is neglected in the spinors. We also assume that one of the three quarks in the nucleon behaves as a "spectator" during the fundamental decay, and only subsequently interacts to form a meson. A correction (momentum suppression) factor will be discussed later. We define

$$\begin{aligned}\epsilon &= p_z(E+m)^{-1}\\ &= -p(E+m)^{-1} \quad \text{for the antiquark}\,,\end{aligned} \tag{1}$$

where p is the momentum and E is the energy of the antiquark, and we neglect any motion in the x-y plane. The amplitudes for the allowed nucleon decay channels are found to be proportional to one of the following functions:

$$\begin{aligned}&A_1 \equiv [(1+\epsilon)-4(1-\epsilon)]D/3,\quad A_2 \equiv (1+\epsilon)D\,,\\ &A_3 \equiv [(1+\epsilon)-2(1-\epsilon)]D\,,\\ &A_4 \equiv [2(1+\epsilon)-3(1-\epsilon)]D,\quad A_5 \equiv (1-\epsilon)D\,,\\ &A_6 \equiv \epsilon D,\quad A_7 \equiv [2(1-\epsilon)+(1+\epsilon)]D/3\,,\\ &A_8 = = [2(1+\epsilon)+(1-\epsilon)]D/3\,,\end{aligned} \tag{2}$$

where $D \equiv [2(1+\epsilon^2)]^{-1/2}$. [Note that $A_i(0)=2^{-1/2}$ for $i \neq 6$.] Experiments can measure only the squares of these: $R_i \equiv A_i^2$; we shall call R_i the "reduced rates". Since the R_i are of the form $[a+b\epsilon+c\epsilon^2]D^2$, only three are independent. We select as independent amplitudes R_2, R_3, and R_5, and find

$$\begin{aligned}&9R_1 = -R_2+2R_3+8R_5\,,\\ &R_4 = R_2+3R_3-3R_5\,,\\ &9R_7 = 2R_2-R_3+8R_5\,,\\ &9R_8 = 5R_2-R_3+5R_5\,,\\ &8R_6 = R_2+R_3-2R_5\,.\end{aligned} \tag{3}$$

TABLE I. Reduced partial amplitudes and rates for nucleon decay. Polarizations all refer to the direction of motion of the antilepton. All neutron decay modes other than those listed may be obtained from corresponding proton decay modes and are displayed in the text.

Mode	Partial amplitude: F mass (Cabibbo)	Partial amplitude: J mass	Partial rate: F mass	Partial rate: J mass
$p_\downarrow \rightarrow$				
$e^+_\downarrow \pi^0$	0.974 } $\times 4\sqrt{3}A_1$	$0.766 \mp 1.76i$ } $\times 2\sqrt{3}A_1$	0.95 } $\times 48R_1$	0.92 } $\times 48R_1$
$\mu^+_\downarrow \pi^0$	0.112	$0.0648 \mp 0.3i$	0.01	0.024
$e^+_\downarrow \eta^0$	0.974 } $\times 4A_2$	$0.766 \mp 1.76i$ } $\times 2A_2$	0.95 } $\times 16R_2$	0.92 } $\times 16R_2$
$\mu^+_\downarrow \eta^0$	0.112	$0.0648 \mp 0.3i$	0.01	0.024
$e^+_\downarrow \rho^0_\rightarrow$	0.974 } $\times \frac{4}{\sqrt{3}}A_3$	$0.766 \mp 1.76i$ } $\times \frac{2}{\sqrt{3}}A_3$	0.95 } $\times \frac{16}{3}R_3$	0.92 } $\times \frac{16}{3}R_3$
$\mu^+_\downarrow \rho^0_\rightarrow$	0.112	$0.0648 \mp 0.3i$	0.01	0.024
$e^+_\downarrow \omega^0_\rightarrow$	0.974 } $\times \frac{12}{\sqrt{3}}A_3$	$0.766 \mp 1.76i$ } $\times \frac{6}{\sqrt{3}}A_3$	0.95 } $\times 48R_3$	0.92 } $\times 48R_3$
$\mu^+_\downarrow \omega^0_\rightarrow$	0.112	$0.0648 \mp 0.3i$	0.01	0.024
$e^+_\uparrow \rho^0_\downarrow$	1 } $\times \frac{(-2\sqrt{2})}{\sqrt{3}}A_4$	$0.386 \mp 0.875i$ } $\times \frac{-2\sqrt{2}}{\sqrt{3}}A_4$	1 } $\times \frac{8}{3}R_4$	0.91 } $\times \frac{8}{3}R_4$
$\mu^+_\uparrow \rho^0_\downarrow$	0	$0.106 \mp 0.240i$	0	0.069
$e^+_\uparrow \omega^0_\downarrow$	1 } $\times 2\sqrt{6}A_5$	$0.386 \mp 0.875i$ } $\times 2\sqrt{6}A_5$	1 } $\times 24R_5$	0.91 } $\times 24R_5$
$\mu^+_\uparrow \omega^0_\downarrow$	0	$0.106 \mp 0.240i$	0	0.069
$\nu^c_e \rho^+_\downarrow$	0.974 } $\times \frac{4}{\sqrt{3}}A_4$	$0.370 \mp 0.879i$ } $\times \frac{4}{\sqrt{3}}A_5$	0.95 } $\times \frac{16}{3}R_4$	0.91 } $\times \frac{16}{3}R_4$
$\nu^c_\mu \rho^+_\downarrow$	0	$0.0744 \mp 0.177i$	0	0.037
$\mu^+_\downarrow K^0$	1.05 } $\times 2\sqrt{6}A_7$	$0.391 \mp 0.858i$ } $\times 2\sqrt{6}A_7$	1.11 } $\times 24R_7$	0.89 } $\times 24R_7$
$e^+_\downarrow K^0$	0.224	$0.252 \mp 0.419i$	0.05	0.24
$e^+_\uparrow \pi^0$	1 } $\times 2\sqrt{3}A_1$	$0.386 \mp 0.875i$ } $\times 2\sqrt{3}A_1$	1 } $\times 12R_1$	0.91 } $\times 12R_1$
$\mu^+_\uparrow \pi^0$	0	$0.106 \mp 0.240i$	0	0.069
$e^+_\uparrow \eta^0$	1 } $\times 2A_2$	$0.386 \mp 0.875i$ } $\times 2A_2$	1 } $\times 4R_2$	0.91 } $\times 4R_2$
$\mu^+_\uparrow \eta^0$	0	$0.106 \mp 0.240i$	0	0.069
$e^+_\uparrow \rho^0_\rightarrow$	1 } $\times \left[\frac{-2}{\sqrt{3}}\right]A_3$	$0.386 \mp 0.875i$ } $\times \left[\frac{-2}{\sqrt{3}}\right]A_3$	1 } $\times \frac{4}{3}R_3$	0.91 } $\times \frac{4}{3}R_3$
$\mu^+_\uparrow \rho^0_\rightarrow$	0	$0.106 \mp 0.240i$	0	0.069
$e^+_\uparrow \omega^0_\rightarrow$	1 } $\times(-2\sqrt{3})A_3$	$0.386 \mp 0.875i$ } $\times(-2\sqrt{3})A_3$	1 } $\times 12R_3$	0.91 } $\times 12R_3$
$\mu^+_\uparrow \omega^0_\rightarrow$	0	$0.106 \mp 0.240i$	0	0.069

TABLE I. (*Continued.*)

Mode	Partial amplitude: F mass (Cabibbo)	Partial amplitude: J mass	Partial rate: F mass	Partial rate: J mass
$e^+_\downarrow \rho^0_\uparrow$ $\mu^+_\downarrow \rho^0_\uparrow$	$\left.\begin{matrix}0.974\\0.112\end{matrix}\right\}\times\frac{4\sqrt{2}}{\sqrt{3}}A_4$	$\left.\begin{matrix}0.766\ \mp 1.76i\\0.0648\mp 0.3i\end{matrix}\right\}\times\frac{2\sqrt{2}}{\sqrt{3}}A_4$	$\left.\begin{matrix}0.95\\0.01\end{matrix}\right\}\times\frac{32}{3}R_4$	$\left.\begin{matrix}0.92\\0.024\end{matrix}\right\}\times\frac{32}{3}R_4$
$e^+_\downarrow \omega^0_\uparrow$ $\mu^+_\downarrow \omega^0_\uparrow$	$\left.\begin{matrix}0.974\\0.112\end{matrix}\right\}\times(-4\sqrt{6})A_5$	$\left.\begin{matrix}0.766\ \mp 1.76i\\0.0648\mp 0.3i\end{matrix}\right\}\times(-2\sqrt{6})A_5$	$\left.\begin{matrix}0.95\\0.01\end{matrix}\right\}\times 96R_5$	$\left.\begin{matrix}0.92\\0.024\end{matrix}\right\}\times 96R_5$
$\nu^c_e \pi^+$ $\nu^c_\mu \pi^+$	$\left.\begin{matrix}0.974\\0\end{matrix}\right\}\times 2\sqrt{6}A_1$	$\left.\begin{matrix}0.370\ \mp 0.879i\\0.0744\mp 0.177i\end{matrix}\right\}\times 2\sqrt{6}A_1$	$\left.\begin{matrix}0.95\\0\end{matrix}\right\}\times 24R_1$	$\left.\begin{matrix}0.91\\0.037\end{matrix}\right\}\times 24R_1$
$\nu^c_e \rho^+_\rightarrow$ $\nu^c_\mu \rho^+_\rightarrow$	$\left.\begin{matrix}0.974\\0\end{matrix}\right\}\times\frac{2\sqrt{2}}{\sqrt{3}}A_3$	$\left.\begin{matrix}0.370\ \pm 0.879i\\0.0744\mp 0.177i\end{matrix}\right\}\times\frac{2\sqrt{2}}{\sqrt{3}}A_3$	$\left.\begin{matrix}0.95\\0\end{matrix}\right\}\times\frac{8}{3}R_3$	$\begin{matrix}0.91\\0.037\end{matrix}\ \times\frac{8}{3}R_3$
$\nu^c_\mu K^+$	$0.974\ \frac{4\sqrt{2}}{\sqrt{3}}A_6$	$(12A_{\mu,e}A_6-18B_{\mu,e}A_5)\frac{\sqrt{2}}{3\sqrt{3}}$	$10.13\ R_6$	a
$\nu^c_e K^+$	$-1.38\ \frac{\sqrt{2}}{\sqrt{3}}A_5$		$1.27\ R_5$	
$\mu^+_\uparrow K^0$ $e^+_\uparrow K^0$	$\left.\begin{matrix}1\\0\end{matrix}\right\}\times 2\sqrt{6}A_7$	$\left.\begin{matrix}-0.386\ \pm 0.875i\\0.106\ \mp 0.24i\end{matrix}\right\}\times 2\sqrt{6}A_7$	$\left.\begin{matrix}1\\0\end{matrix}\right\}\times 24R_7$	$\left.\begin{matrix}0.91\\0.069\end{matrix}\right\}\times 24R_7$
$n_\downarrow \rightarrow$				
$\nu^c_e \omega^0_\downarrow$ $\nu^c_\mu \omega^0_\downarrow$	$\begin{matrix}0.974\\0\end{matrix}\ \times 2\sqrt{6}A_5$	$\left.\begin{matrix}0.370\ \mp 0.879i\\0.0744\mp 0.177i\end{matrix}\right\}\times 2\sqrt{6}A_5$	$\left.\begin{matrix}0.95\\0\end{matrix}\right\}\times 24R_5$	$\left.\begin{matrix}0.91\\0.037\end{matrix}\right\}\times 24R_5$
$n_\uparrow \rightarrow$				
$\nu^c_\mu K^0$	$(0.974)\times 2\sqrt{6}A_8$	$(18A_{\mu,e}A_8-18B_{\mu,e}A_5)\frac{\sqrt{2}}{3\sqrt{3}}$	$22.8R_8$	a
$\nu^c_e K^0$	same as $p\rightarrow\nu^c_e K^+$		$1.27R_5$	a
$\nu^c_e \eta^0$ $\nu^c_\mu \eta^0$	$\left.\begin{matrix}0.974\\0\end{matrix}\right\}\times 2A_2$	$\left.\begin{matrix}0.370\ \mp 0.879i\\0.0744\mp 0.177i\end{matrix}\right\}\times 2\sqrt{6}A_5$	$\left.\begin{matrix}0.95\\0\end{matrix}\right\}\times 4R_2$	$\left.\begin{matrix}0.91\\0.037\end{matrix}\right\}\times 4R_2$

$A_e=0.704\mp 0.177i$ $\quad A_\mu=0.370\mp 0.879i$ $\quad B_e=0.143\mp 0.177i$ $\quad B_\mu=-0.0288\pm 0.0355i$

[a]See Table II for νK modes.

TABLE II. Reduced partial rates for decay to $K\nu^c$.

	$\epsilon=0$ (nonrelativistic)		$\epsilon=-\frac{1}{3}$ (recoil)		$\epsilon=-1$ (relativistic)	
	J mass	F mass	J mass	F mass	J mass	F mass
$R(p\to\nu_\mu^c K^+)$	0.025	0	0.629	0.507	4.21	2.53
$R(p\to\nu_e^c K^+)$	0.621	0.64	1.28	1.02	2.01	1.27
$R(n\to\nu_\mu^c K^0)$	13.1	11.4	9.88	8.11	3.71	2.53
$R(n\to\nu_e^c K^0)$	0.057	0.64	0.235	1.02	0.670	1.27

The dynamics of the antiquark in the meson are not understood and various models have been proposed,[5,6] e.g., antiquark produced at rest ($\epsilon=0$), having correct motion to conserve momentum ($\epsilon=-\frac{1}{3}$), produced relativistically ($\epsilon=-1$), or more complicated distributions in ϵ as in the bag model.[9,10] We shall find branching ratios which are independent of this question, by considering those channels which have the same ϵ dependence.

In Tables I, II, and III we display the partial amplitude and reduced partial decay rate for each mode of proton decay. (We display reduced rates for K^*, even though phase space is very small.) Amplitudes and rates for neutron decay modes may be obtained from those of proton decay as follows:

$$A(n_\downarrow\to l_\downarrow^+ M_{1\to}^-)=-\sqrt{2}A(p_\downarrow\to l_\downarrow^+ M_{1\to}^0)\ ,$$

$$A(n_\uparrow\to l_\uparrow^+ M_{1\to}^-)=\sqrt{2}A(p_\uparrow\to l_\uparrow^+ M_{1\to}^0)\ ,$$

$$A(n_\uparrow\to l_\downarrow^+ M_{1\uparrow}^-)=\sqrt{2}A(p_\uparrow\to l_\downarrow^+ M_{1\uparrow}^0)\ ,$$

$$A(n_\downarrow\to l_\uparrow^+ M_{1\downarrow}^-)=\sqrt{2}A(p_\downarrow\to l_\uparrow^+ M_{1\downarrow}^0)\ ,$$

TABLE III. Reduced partial rates for decays to K^*l. There is no phase space to $K^*\mu^+$. The constants R_l, C_{pl}, C_{nl} are the same mixing coefficients which appear in Table I for decay to K and the corresponding lepton. For the J-mass scheme, the detailed mixing to ν^c has been omitted.

Mode	Amplitude	Rate
$p_\uparrow\to K_\to^{*+}\nu_\mu^c$	$R_\nu\left[\frac{-4\sqrt{2}}{\sqrt{3}}\right]D$	$R_\nu{}^2\frac{32}{3}D^2$
$p_\uparrow\to K_\to^{*+}\nu_e^c$	$(\frac{1}{3})$(ampl to $K^+\nu_e^c$)	$\frac{1}{9}$(rate to $K^+\nu_e^c$)
$p_\downarrow\to K_\to^{*0}e_\uparrow^+$	$(-\frac{1}{3})$(ampl to K^0e^+)	$\frac{1}{9}$(rate to K^0e^+)
$p_\downarrow\to K_\to^{*0}e_\downarrow^+$	$R_l\left[\frac{2\sqrt{2}}{\sqrt{3}}\right]\bar{A}_3$	$R_l{}^2\frac{8}{3}R_3$
$n_\uparrow\to K_\to^{*0}\nu_\mu^c$	$R_\nu\frac{2\sqrt{2}}{\sqrt{3}}A_8$	$R_\nu{}^2\frac{8}{3}R_8$
$n_\uparrow\to K_\to^{*0}\nu_e^c$	$(-\frac{1}{3})$(ampl to $K^0\nu_e^c$)	$\frac{1}{9}$(rate to $K^0\nu_e^c$)
$p_\uparrow\to K_\uparrow^{*0}e_\downarrow^+$	$R_l\frac{4}{\sqrt{3}}A_2$	$R_l{}^2\frac{16}{3}R_2$
$p_\downarrow\to K_\downarrow^{*0}e_\uparrow^+$	$R_l\frac{4}{\sqrt{3}}A_2$	$R_l{}^2\frac{16}{3}R_2$
$p_\downarrow\to K_\downarrow^{*+}\nu_{\mu\uparrow}^c$	$C_{p\mu}\frac{8}{\sqrt{3}}A_2$	$C_{p\mu}{}^2\frac{64}{3}R_2$
$p_\downarrow\to K_\downarrow^{*+}\nu_e^c$	$C_{pe}\frac{12}{\sqrt{3}}A_5$	$C_{pe}{}^2 48R_5$
$n_\downarrow\to K_\downarrow^{*0}\nu_\mu^c$	$C_{n\mu}\frac{4}{\sqrt{3}}A_2$	$C_{n\mu}{}^2\frac{16}{3}R_2$
$n_\downarrow\to K_\downarrow^{*0}\nu_e^c$	$C_{ne}\left[\frac{-12}{\sqrt{3}}A_5\right]$	$C_{ne}{}^2 48R_5$

which yield

$$R(n_\lambda \to l^+_{\lambda'} M^-_{1\lambda''}) = 2R(p_\lambda \to l^+_{\lambda'} M^0_{1\lambda''}) , \tag{4}$$

and

$$A(n_\uparrow \to \pi^0 \nu^c) = -\frac{1}{\sqrt{2}} A(p_\uparrow \to \pi^+ \nu^c) ,$$

$$\tfrac{1}{3} A(n_\uparrow \to \omega^0_\rightarrow \nu^c) = A(n_\uparrow \to \rho^0_\rightarrow \nu^c)$$

$$= \frac{1}{\sqrt{2}} A(p_\uparrow \to \rho^+_\rightarrow \nu^c) ,$$

$$A(n_\downarrow \to \rho^0_\downarrow \nu^c) = \frac{1}{\sqrt{2}} A(p_\downarrow \to \rho^+_\downarrow \nu^c) ,$$

which yield

$$R(n_\lambda \to \nu^c M^0_{1\lambda'}) = \tfrac{1}{2} R(p_\lambda \to \nu^c M^+_{1\lambda'}) , \tag{5}$$

and

$$R(n_\uparrow \to \omega^0_\rightarrow \nu^c) = 9R(n_\uparrow \to \rho^0_\rightarrow \nu^c) .$$

The relationship between neutron-to-neutrino decays and proton-to-positive-lepton decays found previously by several authors[11] are not precisely maintained because the terms with neutrinos mix differently than those which produce l^+'s. Therefore, the neutron decay modes (to neutrinos) for which there are no corresponding proton modes (to neutrinos) are explicitly displayed in Table I.

Before proceeding with this analysis, we note that the tables have entries for two different Higgs-boson coupling schemes (F mass and J mass). We shall investigate these two possible choices of Higgs-boson coupling (which lead to different mixing of the first two generations of fermions) in the next section. Here, we shall consider those amplitudes whose behavior with ϵ is independent of the coupling choice. (As can be seen from Table I, we must omit R_6 from consideration. It arises in decays to $K\nu^c$, which are very different in the two coupling schemes.) The nucleon decay experiments in progress at this time involve unpolarized nucleons. The helicity of the positrons will be unobservable as well. For the production of first-generation mesons, although the branching ratios for Cabibbo-suppressed (to allowed) transitions are different in the two Higgs-boson coupling schemes, the partial rates for unpolarized nucleon decay are approximately the same for the two schemes. We find that for each Cabibbo-allowed mode the unpolarized rates (obtained in the two schemes) agree with each other to about 10%, and for the sum of each suppressed mode to its corresponding allowed mode to about 6%. (They vary from their average by less than $\pm 5\%$ and $\pm 3\%$, respectively.) For production of second-generation mesons, the dominant modes contain μ^+ whose helicities are measurable. The two coupling schemes yield slightly different results. From Table I, we obtain a number of relations which are *independent of antiquark dynamics,*

$$\begin{aligned}
&R(p \to l^+\pi^0):R(p \to \pi^+\nu^c):R(n \to l^+\pi^-):R(n \to \pi^0\nu^c):(0.95)R_1/3 = 10:4:20:2:1 ,\\
&R(p \to l^+\eta^0):R(n \to \nu^c\eta^0):1.94R_2 = 5:1:1 ,\\
&R(p \to l^+\rho^0_\rightarrow):R(p \to l^+\omega^0_\rightarrow):R(p \to \rho^+_\rightarrow\nu^c):R(n \to l^+\rho^-_\rightarrow):R(n \to \nu^c\rho^0_\rightarrow): R(n \to \omega^0_\rightarrow\nu^c):\frac{1.94}{3}R_3 = 5:45:2:10:1:9:1 ,\\
&R(p \to l^+\rho^0_\uparrow):R(p \to l^+\rho^0_\downarrow):R(n \to l^+\rho^-_\uparrow):R(n \to l^+\rho^-_\downarrow):R(p \to \rho^+_\downarrow\nu^c): R(n \to \rho^0_\downarrow\nu^c):\frac{3.88}{4}R_4 = 4:1:8:2:2:1:1,\\
&R(p \to l^+\omega^0_\uparrow):R(p \to l^+\omega^0_\downarrow):R(n \to \omega^0_\downarrow\nu^c):12(0.97)R_5 = 4:1:1:1 ,\\
&R(p \to K^0\mu^+_\uparrow) \approx R(p \to K^0\mu^+_\downarrow) \approx 21.6R_7 \quad (J \text{ mass}) ,\\
&R(p \to K^0\mu^+_\uparrow):R(p \to K^0\mu^+_\downarrow):24(0.99)R_7 = 1:1:1 \quad (F \text{ mass}) ,
\end{aligned} \tag{6}$$

where l^+ means sum over e^+ and μ^+ production, ν^c means sum over ν_e^c and ν_μ^c, and we have assumed unpolarized initial nucleons. (The tables contain rates for each nucleon polarization separately, which will be useful when polarized nucleons are observed in future experiments.) The R's are reduced partial rates; see Sec. IV. (The proportionalities and relationships among the R_i's are also valid if one divides the observed partial rates solely by the phase-space and momentum suppression factors.) The relationships among the reduced partial rates in Eqs. (3) and (4) are tests of the SU(5) theory with SU(6) wave functions. Predictions concerning enhancement in the spin singlet due to hyperfine color interactions which modify SU(6) wave functions have been made and these may somewhat change the results above.[7]

In order to get some information concerning the motion of the antiquark at production we may look at branching ratios of processes in different lines of Eq. (4). (When we examine the behavior of the R_i's with ϵ it is clear that R_4 increases very rapidly, R_3 increases rapidly, R_2 decreases very rapidly, R_5 and R_6 change slowly, and R_7 only changes by 10%, when ϵ goes from 0 to -1.) Ratios of rapidly changing to slowly changing rates would give some indication of the effective ϵ of the produced antiquark. For example, $9(R_4/R_7)$ ranges from 1 to 20.2 as ϵ goes from 0 to -1, thus $(4.28)R(n\to e^+\rho_1^-)/R(p\to K^0\mu^+)$ has this range, where I have used the largest R_4-type channel and unpolarized muons. Rates R_3, R_4, and R_5 involve measurement of the polarization of the vector meson, and if one wishes to find ratios which do not involve such measurements one can utilize R_2 (which changes rapidly) with R_6 or R_7. Furthermore, utilizing Eq. (4) we find the *unpolarized* partial rates

$$
\begin{aligned}
R(p\to l^+\rho^0)&=\tfrac{10}{3}(0.95R_3+1.98R_4)\\
&=\tfrac{1}{2}R(n\to l^+\rho^-)=5R(n\to\nu^c\rho^0)\,,\\
R(p\to l^+\omega^0)&=30(0.95R_3+1.98R_5)\\
&=5R(n\to\nu^c\omega^0)\,.
\end{aligned}
\tag{7}
$$

(Note that $r^2+1=5$ here, in the notation of Wilczek and Zee.[11]) The behavior of $R_\rho\equiv0.95R_3+1.98R_4$ and $R_\omega\equiv0.95R_3+1.98R_5$ with ϵ is as follows: R_ρ ranges from 1.5 to 22 and R_ω ranges from 1.5 to 6, as ϵ goes from $0\to-1$. Thus, R_ρ/R_1 would give a very sensitive determination of the effective ϵ (or momentum) of the antiquark.

III. GENERATION MIXING

Calculations of the decay of the nucleon have usually ignored Cabibbo-suppressed modes due to the smallness of the Cabibbo angle. However, the generation mixing of the fermions is determined by the detailed structure of the Higgs-boson sector. The latter accounts for the masses of the observed fermions, and the diagonalization of the resulting mass matrices for the up quarks, down quarks, and leptons, respectively, determine the mixings. (Cabibbo universality implies only mixing of the first two generations.) De Rújula, Georgi, and Glashow[8] have pointed out that the mixing implicitly assumed by those using a simple Cabibbo angle (which they have called F mass) fails on the phenomenological level, whereas a more complicated scheme of Higgs-boson couplings (called J mass) is more successful. They show that the latter, due to the off-diagonal mass matrices (mixing the first two generations) of all three fundamental fermions, leads to four parameters. Using the measured lepton masses, the Cabibbo angle and other empirical information, the authors in Ref. 8 have determined the values of these parameters (except for the sign of the imaginary part of a phase η). They have shown how branching ratios for certain Cabibbo-suppressed modes of nucleon decay (in conjunction with neutrino branching ratios) may be used to determine which coupling scheme occurs.

Let us first consider the F-mass coupling scheme which is that of the ordinary Cabibbo rotation. It has been shown[12] that

$$
\begin{aligned}
\mathscr{L}=(2\sqrt{2}G)\epsilon_{ijk}\{\,&[\bar{u}^c_{kL}\gamma_\mu u_{jL}]\,((1+\cos^2\theta_C)\bar{e}^+_L+\sin\theta_C\cos\theta_C\bar{\mu}^+_L)\gamma^\mu d_{iL}+((1+\sin^2\theta_C)\bar{\mu}^+_L+\sin\theta_C\cos\theta_C\bar{e}^+_L)\gamma^\mu s_{iL}\\
&+\bar{e}^+_R\gamma^\mu d_{iR}+\bar{\mu}^+_R\gamma^\mu s_{iR}]\\
&+[\bar{u}^c_{kL}\gamma_\mu(d_{jL}\cos\theta_C+s_{jL}\sin\theta_c)][\bar{\nu}^c_{eR}\gamma^\mu d_{iR}+\bar{\nu}^c_{\mu R}\gamma^\mu s_{iR}]\,\}+\text{H.c.}
\end{aligned}
\tag{8}
$$

Notice that for decays to first-generation mesons (M_1), as well as decays to mesons with one second-generation quark (M_2) plus l^+, the Cabibbo-suppressed modes have the same structure (with different coefficients) as the corresponding favored modes. This is not true for decays to $M_2+\nu^c$, as can be seen from the last terms in $\mathscr{L}$. We find that the rate for decay to $M_2+\nu_e^c$ is proportional to $(\bar{u}^c_L\gamma^\mu s_L)(\bar{\nu}^c_{eR}\gamma_\mu d_R)$, whereas that for decay to $M_2+\nu_\mu^c$ is proportional to $(\bar{u}^c_L\gamma^\mu d_L)(\bar{\nu}^c_{\mu R}\gamma_\mu s_R)$. Thus the ϵ dependence will be different and we list these separately in Table II.

(It should be noted that one must be careful to perform the mass matrix rotations before any Fierz transformations are applied.)

The J-mass coupling scheme employs a complex $\underline{10}$ and a $\underline{126}$ of Higgs bosons in SO(10) to obtain more reasonable relations between quark and lepton mass ratios. The resulting fermion mass matrices are displayed in Ref. 8 and are of the form

$$\begin{bmatrix} 0 & 1 \\ 1 & a \end{bmatrix},$$

with a real for down quarks (D) and leptons (L) and equal to $\eta \mid a \mid$ for up quarks (U). Diagonalization by transformations of the form $U^{c\dagger}\mathscr{M}U$ are accomplished with[13]

$$L^- \to \begin{bmatrix} c_e & -s_e \\ s_e & c_e \end{bmatrix} \begin{bmatrix} e^- \\ \mu^- \end{bmatrix}, \quad L^+ \to \begin{bmatrix} c_e & s_e \\ s_e & -c_e \end{bmatrix} \begin{bmatrix} e^+ \\ \mu^+ \end{bmatrix},$$

$$D \to \begin{bmatrix} c_d & s_d \\ -s_d & c_d \end{bmatrix} \begin{bmatrix} d \\ s \end{bmatrix}, \quad D^c \to \begin{bmatrix} -c_d & s_d \\ s_d & c_d \end{bmatrix} \begin{bmatrix} d^c \\ s^c \end{bmatrix}, \qquad (9)$$

$$U \to \begin{bmatrix} c_u & \eta s_u \\ -\eta^* s_u & c_u \end{bmatrix} \begin{bmatrix} u \\ c \end{bmatrix}, \quad U^c \to \begin{bmatrix} -\eta^* c_u & s_u \\ s_u & c_u \end{bmatrix} \begin{bmatrix} u^c \\ c^c \end{bmatrix}.$$

The relevant field operators for proton decay [arising in SO(10)] are

$$(\bar{U}^c_\alpha \mathscr{O} U_\alpha)(\bar{L}^+_\beta \mathscr{O} D_\beta),\ (\bar{U}^c_\alpha \mathscr{O} D_\alpha)(\bar{L}^+_\beta \mathscr{O} U_\beta),$$

$$(\bar{U}^c_\alpha \mathscr{O} D_\alpha)(\bar{\nu}^c_\beta \mathscr{O} D_\beta),\ (\bar{D}^c_\alpha \mathscr{O} D_\alpha)(\bar{\nu}^c_\beta \mathscr{O} U_\beta),$$

$$(\bar{D}^c_\alpha \mathscr{O} U_\alpha)(\bar{\nu}^c_\beta \mathscr{O} D_\beta),\ (\bar{D}^c_\alpha \mathscr{O} U_\alpha)(\bar{L}^+_\beta \mathscr{O} U_\beta),$$

where the repeated Greek subscript means sum over generations and $\mathscr{O}$ represents the appropriate Dirac γ matrices in each case.

These authors have looked at nucleon decays to e^+K, μ^+K, $\bar{\nu}\pi^+$, $\bar{\nu}\pi^0$, and l^+K, as a function of the ratio of the masses of the two sets of exchanged gauge bosons involved. We shall investigate the implications for SU(5), where the only residual effect of the SO(10) is the Higgs-boson coupling scheme, which is equivalent to assuming that the extra set of exchanged gauge particles have masses which are much larger than those of the set in SU(5). (In the language of Ref. 8, we assume $M_A/M_B \approx 0$.) The parameters s_e, s_d, and Reη are determined from the known lepton masses and Cabibbo angle; a simple empirical assumption determines s_u.[8]

Tables I, II, and III show the partial rates to Cabibbo-suppressed and Cabibbo-allowed channels predicted by the two schemes of Higgs-boson couplings. The amplitudes in the J-mass scheme have the sign of their imaginary parts undetermined because the sign of the imaginary part of η is not known. The partial rates calculated here are independent of that sign. This is easily understood as follows: Since the amplitudes are real functions of η, the rates are of the form $R = A(\eta)[A(\eta)]^* = A(\eta)A(\eta^*)$ which is unchanged by the replacement $\eta \leftrightarrow \eta^*$ (equivalent to changing the sign of the imaginary part of η). In order to obtain information about the sign, we would need to include higher-order Feynman diagrams in our calculation (the reason being similar to the one that implies that a first Born calculation never produces a phase for the scattering amplitude).

Noteworthy observations from the table are the following.

(a) In the J-mass scheme the branching ratio $R(p \to e^+K^0)/R(p \to \mu^+K^0)$ is 26.9% for left-handed leptons, 7.5% for right-handed leptons, 17% for unpolarized leptons, and 35% if we use the rate to left-handed (or right-handed) muons with that of unpolarized electrons. The corresponding numbers for the F-mass scheme are 4%, 0, 2%, and 4%.

(b) $R(p \to l^+K^0)$ and $R(p \to l^+\omega^0)$ are very large. $R(p \to \mu^+K^0)$ has a 10% enhancement due to the Cabibbo rotation (F-mass scheme).

(c) $R(p \to \nu^c_\mu K^+) \neq 0$ as long as $\epsilon \neq 0$.

(d) $R(p \to l^+\eta) = 0$ if $\epsilon = -1$.

(e) Decays to a first-generation nonstrange meson $(M_1) + \nu^c_\mu$ have nonzero partial rates ($\sim$4% of the corresponding decay to ν^c_e) in the J-mass scheme, but zero in the F-mass (usual Cabibbo angle) scheme.

(f) Decays to $\mu^+_\uparrow + M_1$ have nonzero partial rates ($\sim$7.5%$\times$corresponding decay to $e^+_\uparrow$) in the J-mass scheme, but zero in the F-mass scheme. Georgi, Glashow, and Machacek[14] have shown that *muon-polarization* predictions are independent of phase space and insensitive to low-energy assumptions. Evaluating at $\epsilon = 0$ we obtain

$$\frac{L-R}{L+R} = \begin{cases} 1 & \text{for } F \text{ mass} \\ 0.16 & \text{for } J \text{ mass} \end{cases} \quad \text{for decay to } M_1 + \mu^+_\lambda,$$

and

$$\frac{L-R}{L+R} = \begin{cases} 0.05 & \text{for } F \text{ mass} \\ -0.01 & \text{for } J \text{ mass} \end{cases} \quad \text{for decay to } M_2 + \mu^+_\lambda$$

in agreement with their result. As they point out, these are indeed sensitive tests of the coupling scheme.

(g) The decays to $e^+ + M_2$ have similar properties as those to $\mu^+ + M_1$ but unless we can measure the polarization of the e^+ they will not be useful. (The decays to $e_\uparrow^+$ are zero in the F-mass scheme.) Summing the polarizations, both schemes yield a branching ratio $\sim 5-6\%$.

(h) Decays to $\nu^c + M_2$ are very different in the two schemes and these results are summarized in Table II. Large reduced rates $(8-10)$ imply $\epsilon \geq -\frac{1}{3}$, $(11-13)$ imply $\epsilon \approx 0$. A large branching ratio

$$0.2 < \frac{R(n \to \nu_e^c K^0)}{R(n \to \nu_\mu^c K^0)} \lesssim 0.5$$

would imply F coupling and $\epsilon \approx -1$. With other values of ϵ, determined from rates or as indicated in Sec. II, the J-mass scheme predicts that this branching ratio will be many times that of the F-mass scheme.

(i) If ϵ is zero, the F-mass scheme predicts no decay into $\nu_\mu^c K^+$, the J-mass scheme has its partial rate suppressed by a factor of 30 compared with that of the Cabibbo-suppressed mode.

IV. ABSOLUTE RATES

A. Renormalization-group analysis

It has been shown[11] that, using Fierz transformations, all (lowest-order) operators responsible for nucleon decay can be put into five distinct forms. The forms arising in SU(5) have been analyzed by Gavela *et al.*[5] (They used the definition $u_R^c = C\gamma^0 P_R u^*$, where $C = i\gamma^2\gamma^0$, which is equivalent to the spinor in Abbott and Wise[15] defined by $u_L^c = P_L C\gamma^0 u^*$. Thus we must translate the u^c's appearing in Ref. 15 before examining their behavior.) In the notation of Abbott and Wise,[15] the SU(5) operators (found in Ref. 5 to be) responsible for nucleon decay are $O(1)$ and $O(2)$ for l_R^+ and l_L^+ production, respectively. It is also shown there that for decay to nonstrange as well as strange channels the sole effect on these two operators is to produce a factor of about (3.4) with no operator mixing.

B. Phase space and nucleon properties

When the partial decay rate (w) is calculated at the grand unified mass (GUM) we obtain[16]

$$w = \frac{G^2}{8\pi} |\psi(0)|^2 R m_p^2 \left[1 - \frac{m^2}{m_p^2}\right]\left[1 - \frac{m^4}{m_p^4}\right]$$

$$= \frac{G^2}{\pi} |\psi(0)|^2 R m_p^2 \rho(m) , \tag{10}$$

where G is the coupling constant, $\psi(0)$ is the probability that the two initial quarks overlap, R is the reduced rate defined in Sec. II and displayed in Tables I, II, and III, m_p is the proton mass, m is the mass of the outgoing meson, and $\rho(m)$ is the phase-space factor $[\rho(k^2)]$ used in Ref. 5 and displayed here in Table IV. In the limit $(\epsilon \to 0)$ our unrotated results agree with those of Gavela *et al.*[5]

C. Net result

The result is

$$w = \frac{(3.4)^2 \pi \alpha_{\mathrm{GUM}}^2}{2} \frac{m_p^5}{M_X^4} S\, R\, \rho(m) , \tag{11}$$

where $S \equiv |\psi(0)|^2 m_p^{-3}$ is dimensionless. If we express M_X in 10^{14} GeV, we obtain

TABLE IV. Phase-space and suppression factors.

Meson	M (MeV)	$\rho(m)$	ρ suppression for μ^+	f=suppression due to spectator recoil
π^0	135	0.122	0.99	0.775
π^+	140	0.122	0.99	0.762
η	549	0.073	0.96	0.892
ρ	773	0.022	0.78	0.973
ω	783	0.019	0.77	0.976
K^+	494	0.083	0.98	0.871
K^0	498	0.083	0.98	0.872
K^*	892	0.0022	0	0.998

TABLE V. Branching ratios in percent. Ratios do not add to 100% because of rounding errors.

Mode	$\epsilon=0$	$\epsilon=-\frac{1}{3}$	$\epsilon=-1$
$p_{\downarrow}\rightarrow$			
$l^+_{\downarrow}\pi^0$	25	29	29
$l^+_{\downarrow}\eta^0$	6	1	0
$l_{\downarrow}\rho^0_{\rightarrow}$	0.7	1	1.7
$l^+_{\downarrow}\omega^0_{\rightarrow}$	5	9	13
$l^+_{\uparrow}\rho^0_{\downarrow}$	0.3	1	2
$l^+_{\uparrow}\omega^0_{\downarrow}$	3	2	1.7
$\nu^c\rho^+_{\downarrow}$	0.7	2	3.6
$l^+_{\downarrow}K^0$	11.5	7	3
$p_{\uparrow}\rightarrow$			
$l^+_{\uparrow}\pi^0$	6.5	8	7.6
$l^+_{\uparrow}\eta^0$	1.5	0.3	0
$l^+_{\uparrow}\rho^0_{\rightarrow}$	0.2	0.3	0.4
$l^+_{\uparrow}\omega^0_{\rightarrow}$	1.3	2.5	3
$l^+_{\downarrow}\rho^0_{\uparrow}$	1.3	4	7.4
$l^+_{\downarrow}\omega^0_{\uparrow}$	10	8	6.4
$\nu^c\pi^+$	12	14	14.5
$\nu^c\rho^+$	0.3	0.6	0.8
$\nu^c_{\mu}K^+$	0	0.2	0.7
$\nu^c_eK^+$	0.6	0.5	0.3
$l^+_{\uparrow}K^0$	10	6	3
l^cK^* modes	5	2	1
	Unpolarized results		
$p\rightarrow$			
$l^+\pi^0$	32	37	37
$l^+\eta^0$	8	1	0
$l^+\rho^0$	2.5	6	11
$l^+\omega^0$	19	21.5	24
$\nu^c\rho^+$	1	2.5	3.5
l^+K^0	21.5	13	6
$\nu^c\pi^+$	12	15	15
$\nu^c_{\mu}K^+$	0	0.2	0.7
$\nu^c_eK^+$	0.6	0.5	3
l^cK^* (total $Q=+1$)	5	2	1
$n_{\downarrow}\rightarrow$			
$l^+_{\downarrow}\pi^-$	55	55	51.6
$l^+_{\downarrow}\rho^-_{\rightarrow}$	1.4	2.3	2.9
$l^+_{\uparrow}\rho^-_{\downarrow}$	0.7	2.1	3.3
$\nu^c\rho^0_{\downarrow}$	0.3	1	1.6
$\nu^c\omega^0_{\downarrow}$	2.5	1.9	1.3
$\nu^c_{\mu}K^{*0}_{\downarrow}$	0.1	0	0
$\nu^c_eK^*_{\downarrow}$	1.3	0.9	0.6
$n_{\uparrow}\rightarrow$			
$l^+_{\uparrow}\pi^-$	14.4	14.3	13.5
$l^+_{\uparrow}\rho^-_{\rightarrow}$	0.4	0.6	0.7
$l^+_{\downarrow}\rho^-_{\uparrow}$	2.8	8.1	13.2
$\nu^c\pi^0$	6.8	6.8	6.4
$\nu^c\eta^0$	1.5	0.3	0
$\nu^c\rho^0_{\rightarrow}$	0.2	0.3	0.4
$\nu^c\omega^0_{\rightarrow}$	1.5	2.5	3.2
$\nu^c_{\mu}K^0$	10.4	3.4	0.6

TABLE V. (*Continued.*)

Mode	$\epsilon=0$	$\epsilon=-\frac{1}{3}$	$\epsilon=-1$
$n_{\downarrow}\to$			
$\nu_e^c K^0$	0.6	0.4	0.3
νK^{*0}	0	0	0
	Unpolarized results		
$n\to$			
$l^+\pi^-$	69	69	65
$l^+\rho^-$	5.3	13	20
$\nu^c\rho^0$	0.5	1	2
$\nu^c\omega^0$	4	4.5	4.5
l^cK^* (total $Q=0$)	1.4	1	0.6
$\nu^c\pi^0$	6.8	6.8	6.4
$\nu^c\eta^0$	1.6	0.3	0
ν^cK^0	11	3.4	0.9

express M_X in 10^{14} GeV, we obtain

$$w=\frac{(3.4)^2\pi}{2}\alpha_{\rm GUM}{}^2\left[\frac{10^{14}\text{ GeV}}{M_X}\right]^4 S\,R\,\rho(m)\times 3.47\times 10^{-25}\text{ yr}^{-1}\,. \tag{12}$$

It is believed that

$$\alpha_{\rm GUM}\approx\tfrac{1}{42}$$

and it has been shown[2] that

$$S\approx 2.42\times 10^{-3}\,.$$

Previous authors[5,6] have made an estimate of the effect of the recoil of the "spectator" quark (using harmonic-oscillator wave functions) and we incorporate that factor (f) using the parameters in Ref. 5. Combining the above we obtain

$$w\approx\left[\frac{10^{14}\text{ GeV}}{M_X}\right]^4 R\rho(m)f\times 8.67\times 10^{-30}\text{ yr}^{-1}\,. \tag{13}$$

In Table V we display actual branching ratios to two-body modes including phase and suppression factors. We include K^* modes which would occur at rates $\lesssim 5\%$ of the total and $\sim 9\%$ to 18% of the strangeness-changing decays. (Even though its phase space is about 3% that for K production, it is a vector particle with three components, whereas the K is a pseudoscalar.) We notice that among the dominant modes, $l^+\eta^0$, $l^+\rho^0$, l^+K^0, $l^+\rho^-$, and ν^cK^0 are very sensitive to the value of ϵ. We also verify that the branching ratio to l^+K^0 is indeed large ($\sim 22\%$), as previously pointed out by Gavela *et al.*,[5] at $\epsilon\approx 0$.

From Eq. (13) we can find the reduced partial rates

$$R=w\left[\frac{M_X}{10^{14}\text{ GeV}}\right]^4\rho^{-1}(m)f^{-1}\times 1.15\times 10^{29}\text{ yr} \tag{14}$$

whose properties have been investigated in the previous sections.

D. Nucleon lifetime

The total rate of decay into two-body modes accounts for most of the decay of the nucleon in

TABLE VI. Lifetime from two-body decays.

M_X (GeV) \ ϵ	τ_p (yr) 0	τ_p (yr) $-\frac{1}{3}$	τ_p (yr) -1	τ_n (yr) 0	τ_n (yr) $-\frac{1}{3}$	τ_n (yr) -1
10^{14}	2.6×10^{28}	1.4×10^{28}	8.7×10^{27}	2.9×10^{28}	1.3×10^{28}	7.7×10^{27}
3×10^{14}	2.1×10^{30}	1.1×10^{30}	7.0×10^{29}	2.4×10^{30}	1.1×10^{30}	6.2×10^{29}
3.8×10^{14}	5.4×10^{30}	2.9×10^{30}	1.8×10^{30}	6.1×10^{30}	2.8×10^{30}	1.6×10^{30}
4.6×10^{14}	1.2×10^{31}	6.3×10^{30}	3.9×10^{30}	1.3×10^{31}	6.0×10^{30}	3.4×10^{30}
6×10^{14}	3.4×10^{31}	1.8×10^{31}	1.1×10^{31}	3.8×10^{31}	1.7×10^{31}	1.0×10^{31}
10^{15}	2.6×10^{32}	1.4×10^{32}	8.7×10^{31}	2.9×10^{32}	1.3×10^{32}	7.7×10^{31}

SU(5) GUT when η, ρ, and ω are included. The rate we obtain is less than, but of the order of, the total rate.

The inverse of that rate gives us an upper bound on the lifetime of the proton. We list in Table VI the lifetime so obtained, as a function of M_X and ϵ. This is approximately the same as previous authors have obtained. Note that as ϵ ranges from 0 to $-\frac{1}{3}$ the lifetime drops by a factor of 2, and as ϵ goes to -1 the lifetime drops to $\frac{1}{3}$ of the value at $\epsilon=0$. The lifetime is very sensitive to the value of M_X, varying as $M_X{}^4$, so we would have to determine M_X to about 20% before we could make any estimate of ϵ. If we used branching ratios, as suggested in Sec. II, to find ϵ, lifetime measurements would provide an excellent determination of M_X.

The lifetime results for $\epsilon=0$ agree well with Gavela *et al.*[5] (after readjustment[2] of $|A_3|^2S$ to 0.0117) but are slightly modified due to our inclusion of suppression factors and K^* modes. Bag-model calculations,[9,10] employing large suppression factors (which are $\sim 2 \rightarrow 3$ for pion modes) and a renormalization group factor of 3.7 (instead of 3.4) yield $\tau_p=1.8\times 10^{31}$ yr for $M_X=4.6\times 10^{14}$ GeV, and $\tau_p=8\times 10^{30}$ yr for $M_X=3.8\times 10^{14}$ GeV. The suppression and renormalization-group factors employed are the correct size to account for the discrepancy (by a factor of 4) of those results with the corresponding lifetimes calculated here assuming SU(6) wave functions, smaller suppression, a smaller renormalization-group factor, and $\epsilon\approx 1$.

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[1]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[2]See P. Langacker, Phys. Rep. (to be published).

[3]Basically, there are two different approaches: SU(6) wave functions (modified) or bag-model calculations. See, also, Y. Tomozawa, Phys. Rev. Lett. 46, 463 (1981).

[4]M. B. Wise, R. Blankenbecler, and L. F. Abbott, Phys. Rev. D 23, 1591 (1981).

[5]M. B. Gavela, A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, report (unpublished); Phys. Rev. D 23, 1591 (1981); Phys. Lett. 98B, 51 (1981).

[6]See G. Karl and G. L. Kane, Phys. Rev. D 22, 2808 (1980); M. Machacek, Nucl. Phys. B159, 37 (1979), and Ref. 2.

[7]G. Karl and H. J. Lipkin, Phys. Rev. Lett. 45, 1223 (1980).

[8]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. 45, 413 (1980); H. Georgi and D. V. Nanopoulos, Nucl. Phys. B159, 16 (1979); H. Georgi and C. Jarlskog, Phys. Lett. 86B, 297 (1979). Generation-mixing effects on proton decay, in SU(5) models with simple Higgs structures, have been discussed by R. Mohapatra, Phys. Rev. Lett. 43, 893 (1979).

[9]E. Golowich, Phys. Rev. D 22, 1148 (1980).

[10]J. F. Donoghue, Phys. Lett. 92B, 99 (1980).

[11]S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979); F. Wilczek and A. Zee, *ibid.* 43, 1571 (1979).

[12]J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Phys. Lett. 88B, 320 (1979).

[13]Our result for U, obtained by requiring $U^{c\dagger}\mathscr{M}U=\mathrm{diag}(m_u,m_c)$, differs from theirs by an asterisk which we assume was a misprint.

[14]H. Georgi, S. L. Glashow, and M. Machacek, Phys. Rev. D 23, 783 (1981).

[15]L. F. Abbott and M. B. Wise, Phys. Rev. D 22, 2208 (1980).

[16]The notation employed is that of J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, Reading, Mass., 1967).