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Short-distance contribution to $K \to \pi\pi\gamma$ and the $\Delta I = 1/2$ rule

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We analyze $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$ and $K^{0}_{L,S} \to \pi^{+}\pi^{-}\gamma$ decays in connection with the explanation of the $\Delta I = 1/2$ rule proposed by Shifman, Vainshtein, and Zakharov. We ask whether the mechanism which enhances the $\Delta I = 1/2$ transition can enhance $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$, thus contradicting the experimental observation $\Gamma(K^{\pm} \to \pi^{\pm}\pi^{0}\gamma) \sim (\alpha/\pi)\Gamma(K^{\pm} \to \pi^{\pm}\pi^{0})$. We show, within the same approximation scheme used in the $\Delta I = 1/2$ rule analysis, that the $\Delta I = 1/2$ rule enhancement is absent in the decay $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$.

I. INTRODUCTION

In 1969 Wilson[1] proposed a mechanism to explain the $\Delta I = \frac{1}{2}$ rule which is observed in the weak nonleptonic decay of strange particles. The basic hypothesis used by Wilson are the following.

(a) Operator-product expansion. At short distances the product of currents can be expressed in terms of a set of operators $\mathcal{O}_n$ constrained by symmetry laws.

(b) Existence of anomalous dimension. The dimension of fields (and operators) in mass units are modified by the strong interactions.

Gaillard and Lee, and Altarelli and Maiani[2] developed the method proposed by Wilson in the standard model, i.e., using the Glashow-Weinberg-Salam (GWS) model to describe electroweak interactions and quantum chromodynamics (QCD) for strong interactions. The results obtained by these authors show that the short-distance contribution of the strong-interaction corrections only modify the weight of the operators already existing in the free theory (GWS). In fact strong interactions enhance the $\Delta I = \frac{1}{2}$ and suppress the $\Delta I = \frac{3}{2}$ operators; however these corrections are far from reproducing the experimental data.

Recently Shifman, Vainshtein, and Zakharov[3] (SVZ) have reconsidered the problem. The point remarked by these authors is that the calculations by Gaillard and Lee, and Altarelli and Maiani are only valid if the quark masses are negligible with respect to the mass of the vector boson W as well as the other mass scale which comes into the problem, the square of the momentum transferred by the gluon q^2. If quark masses are negligible with respect to q^2 the strong-interaction corrections coming from the penguin diagram (Fig. 1) vanish. On the other hand, once the penguin-diagram contributions are taken into account, strong interactions not only modify the weight of the operators already existing in the free theory but also introduce new operators. This means that the calculations by Gaillard and Lee, and Altarelli and Maiani are only able to take into account strong-interaction corrections coming from gluon exchange whose momentum transfer q^2 is larger than the heaviest quark mass squared. The new operators, called penguin operators, have the following characteristics.

(a) They satisfy the $\Delta I = \frac{1}{2}$ rule; in other words they only induce $\Delta I = \frac{1}{2}$ transitions.

(b) The matrix elements of these operators are strongly enhanced with respect to the matrix elements of the operators existing in the free theory.

Using this scheme SVZ have studied the decays

$$K^{\pm} \to \pi^{\pm}\pi^{0}, \quad K^{0}_{L,S} \to \pi^{+}\pi^{-}, \tag{1}$$

which are commonly used to exemplify the $\Delta I = \frac{1}{2}$ rule. The results obtained by SVZ are in good agreement with the experimental data once the coefficients of the different operators are fitted using experimental data from other nonleptonic decays.[4] Furthermore, several authors have applied the same approach to hyperon[3,4] and charmed-particle decays[5] with analogous results. Thus, it seems that the standard model (GWS + QCD) provides a natural explanation of the $\Delta I = \frac{1}{2}$ rule.[6]

The purpose of this paper is to extend the application of this scheme to the radiative nonleptonic decays associated with (1):

$$K^{\pm} \to \pi^{\pm}\pi^{0}\gamma, \quad K^{0}_{L,S} \to \pi^{+}\pi^{-}\gamma. \tag{2}$$

These processes are interesting by themselves since they provide a further test of the structure of the weak Hamiltonian. If the penguin diagram

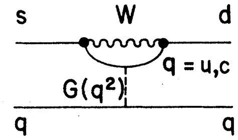

FIG. 1. Penguin-diagram contribution to H_{eff}.

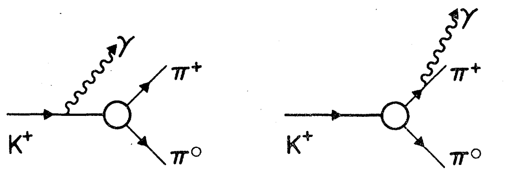

FIG. 2. Inner-bremsstrahlung contribution to $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$.

contributed to $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$, which is not necessarily a $\Delta I = \frac{3}{2}$ transition, we might expect

$$\Gamma(K^{\pm} \to \pi^{\pm}\pi^{0}\gamma) \gg \frac{\alpha}{\pi}\Gamma(K^{\pm} \to \pi^{\pm}\pi^{0})$$

leading to a contradiction of the experimental observation.

In general, processes of type (2) have two contributions.

Inner bremsstrahlung (IB); Fig. 2. This term can be understood independently of any model. It necessarily exists as a consequence of the decay $K \to \pi\pi$; its contribution can be directly calculated once the amplitude for $K \to \pi\pi$ is known.

Direct-emission contribution (DE); Fig. 3. In order to understand this term we have to consider a particular model, given that here the photon is emitted by some intermediate states.

It is therefore interesting to ask whether or not the direct-emission contribution is directly related to the mechanism that produces the $\Delta I = \frac{1}{2}$ rule, or in other words if the penguin diagram is the dominant intermediate state of the direct-emission contribution.

Our plan is as follows: Section II contains the most general form of the effective Hamiltonian, as well as a brief discussion of the main points in evaluating the matrix elements. In Sec. III we present the experimental data and compare with our results. Finally, Sec. IV contains the conclusions.

II. DESCRIPTION OF THE MODEL

Let us consider the decay

$$K^{+}(K) \to \pi^{+}(p_{+}) + \pi^{0}(p) + \gamma(q), \tag{3}$$

where $K, p_{+}, \ldots$ denotes the four-momentum associated with each particle. Because of the weak nature of the decay, the amplitude M_{μ} for this process contains in general two contributions: a vector part M_{μ}^{-}, which is parity violating, and an axial-vector parity-conserving party M_{μ}^{+}.

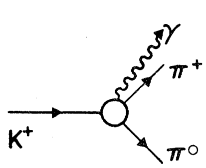

FIG. 3. Direct-emission contribution to $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$.

Using Lorentz and gauge invariance, the most general form of these amplitudes are

$$\begin{aligned} M_{\mu}^{-} &= ieA_{0}\left(\frac{p_{+\mu}}{p_{+}\cdot q} - \frac{k_{\mu}}{k\cdot q}\right) - i\tilde{\gamma}(K\cdot q p_{+\mu} - p_{+}\cdot q K_{\mu}), \\ M_{\mu}^{+} &= \tilde{\beta}\epsilon_{\mu\nu\rho\sigma}K^{\nu}p_{+}^{\rho}q^{\sigma} \end{aligned} \tag{4}$$

in the expression $A_{0} = A_{0}(K^{\pm} \to \pi^{\pm}\pi^{0})$ denotes the on-shell nonradiative amplitude.[7] On the other hand $\tilde{\gamma}, \tilde{\beta}$ are two arbitrary form factors which depend in general on two independent variables to be chosen. In discussing the DE contribution, it is convenient to write the invariant amplitude in the form

$$M \equiv \epsilon^{\mu}M_{\mu} \equiv \epsilon^{\mu}(M_{\mu}^{-} + M_{\mu}^{+}) = M_{\mathrm{IB}} + \gamma + \beta, \tag{5}$$

where ϵ_{μ} is the photon polarization vector[8]

$$\begin{aligned} M_{\mathrm{IB}} &\equiv -ieA_{0}\left(\frac{p_{+}\cdot\epsilon}{p_{+}\cdot q} - \frac{K\cdot\epsilon}{K\cdot q}\right), \\ \gamma &\equiv -i\tilde{\gamma}(K\cdot q p_{+}\cdot\epsilon - K\cdot\epsilon p_{+}\cdot q), \\ \beta &\equiv \tilde{\beta}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\mu}K^{\nu}p_{+}^{\rho}q^{\sigma}. \end{aligned} \tag{6}$$

In this way we can identify M_{IB} as the inner-bremsstrahlung contribution, whereas γ and β are, respectively, the electric (vector) and magnetic (axial-vector) contributions to the DE term.[9]

The IB contribution to the decay $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$ and $K_{L}^{0} \to \pi^{+}\pi^{-}\gamma$ will be strongly suppressed and in fact will vanish if we consider the $\Delta I = \frac{1}{2}$ rule and CP conservation as exact.[9] This can be directly deduced from the fact that the IB contribution is proportional to the on-shell nonradiative amplitude A_{0}, and because of the $\Delta I = \frac{1}{2}$ rule and CP conservation:

$$\begin{aligned} A_{0}(K^{\pm} \to \pi^{\pm}\pi^{0}) &= 0, \\ A_{0}(K_{L}^{0} \to \pi^{+}\pi^{-}) &= 0. \end{aligned} \tag{7}$$

So we hope the DE term will be important both in $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$ and in $K_{L}^{0} \to \pi^{+}\pi^{-}\gamma$ but not in $K_{s}^{0} \to \pi^{+}\pi^{-}\gamma$ where there is nothing to suppress the IB contribution. In the following we will not discuss the IB contribution and we only give the results for it as obtained from the low-photon-energy theorem. This allows us to forget all q^{0} terms (where q is the photon energy) arising from the DE term, since these are exactly known and included through the low-energy theorem in the IB one.[7]

A. The effective Hamiltonian

Following SVZ, the basic idea in deriving the form factors $\tilde{\gamma}$ and $\tilde{\beta}$ is to construct an effective Hamiltonian, independent of the initial and final states, which describes $\Delta S = 1$ radiative decays. Once the Hamiltonian is constructed, we have to evaluate the matrix elements.

TABLE I. Value of the coefficients $c_1,\ldots,c_6$ [see Eq. (8)] in the GWS model and including QCD corrections.

	GWS	GWS+QCD
c_1	-1	-2.49
c_2	$\frac{1}{5}$	0.086
c_3	$\frac{2}{15}$	0.083
c_4	$\frac{2}{3}$	0.415
c_5	0	-0.064
c_6	0	-0.064

We can divide the construction of the effective Hamiltonian $H_{\rm eff}$ in two steps.

First we construct $H_{\rm eff}$, by using the Wilson expansion and the renormalization-group equation in the absence of electromagnetism. This is precisely the $H_{\rm eff}$ constructed by SVZ[3]:

$$H_{\rm eff}=-\sqrt{2}\,G_F\sin\theta_c\cos\theta_c\sum_{i=1}^{6}(c_i\mathcal{O}_i+\text{H.c.})\,, \tag{8}$$

where

$$\begin{aligned}
\mathcal{O}_1&=\bar u_L\gamma_\mu u_L\bar d_L\gamma^\mu s_L-\bar d_L\gamma_\mu u_L\bar u_L\gamma^\mu s_L\,,\\
\mathcal{O}_2&=\bar u_L\gamma_\mu u_L\bar d_L\gamma^\mu s_L+\bar d_L\gamma_\mu u_L\bar u_L\gamma^\mu s_L\\
&\quad+2\bar d_L\gamma_\mu d_L\bar d_L\gamma^\mu s_L+2\bar s_L\gamma_\mu s_L\bar d_L\gamma^\mu s_L\,,\\
\mathcal{O}_3&=\bar u_L\gamma_\mu u_L\bar d_L\gamma^\mu s_L+\bar d_L\gamma_\mu u_L\bar u_L\gamma^\mu s_L\\
&\quad+2\bar d_L\gamma_\mu d_L\bar d_L\gamma^\mu s_L-3\bar s_L\gamma_\mu s_L\bar d_L\gamma^\mu s_L\,,\\
\mathcal{O}_4&=\bar u_L\gamma_\mu u_L\bar d_L\gamma^\mu s_L+\bar d_L\gamma_\mu u_L\bar u_L\gamma^\mu s_L-\bar d_L\gamma_\mu d_L\bar d_L\gamma^\mu s_L\,,\\
\mathcal{O}_5&=\bar d_L\gamma_\mu\lambda_a s_L(\bar u_R\gamma^\mu\lambda^a u_R+\bar d_R\gamma^\mu\lambda^a d_R+\bar s\gamma^\mu\lambda^a s_R)\,,\\
\mathcal{O}_6&=\bar d_L\gamma_\mu s_L(\bar u_R\gamma^\mu u_R+\bar d_R\gamma^\mu d_R+\bar s_R\gamma^\mu s_R)\,.
\end{aligned} \tag{9}$$

The coefficients c_i are given in Table I, $q_{L,R}\equiv\frac{1}{2}(1\mp\gamma_5)q$, and the λ^a are the color-SU(3) matrices.

The second step consists in investigating which are the modifications introduced by electromagnetism. Using Lorentz and gauge invariance, and given that we are interested only in the $\Delta S=1$ operators of dimension six or less, the possible operators introduced by electromagnetism are

$$\bar d(a+b\gamma_5)\sigma^{\mu\nu}sF_{\mu\nu}\,, \tag{10a}$$

$$\bar d\Gamma^\mu s\partial^\nu F_{\mu\nu}\,, \tag{10b}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor. For real photons $\partial^\nu F_{\mu\nu}=0$ and therefore the only possibility is (10a). At lowest order in electroweak interactions, this operator is generated by the diagrams shown in Fig. 4. However an explicit calculation of these diagrams lead us to the conclusion that a,b are of order $(m_c^2-m_u^2)/M_W^2$ and therefore negligible with respect to (9).[10,11]

Thus we can conclude that electromagnetism does not introduce any change in weak-interaction models where only left-handed currents appear when we are interested in describing real-photon processes. Therefore weak nonleptonic decays with only real photons are described by the matrix elements of the effective Hamiltonian (8).

B. Evaluation of the matrix elements

In order to evaluate the matrix elements we will assume that a good approximation is obtained factorizing the matrix elements, by inserting the vacuum in all possible ways between the product of currents which form the operators (9).[12]

First of all, let us consider the matrix elements of $\mathcal{O}_5$, $\mathcal{O}_6$, which are the operators directly related to the penguin diagrams:

$$\langle\pi^+(p_+)\pi^0(p)\gamma(q)\,|c_5\mathcal{O}_5+c_6\mathcal{O}_6|K^+(K)\rangle=-\tfrac{8}{9}(c_5+\tfrac{3}{16}c_6)(Tc)_s\,, \tag{11}$$

where

$$\begin{aligned}
(Tc)_s=&\langle\pi^0|\bar su|K^+\rangle\langle\pi^+\gamma|\bar u(1-\gamma_5)d|0\rangle+\langle\pi^+|\bar sd|K^+\rangle\langle\pi^0\gamma|\bar d(1-\gamma_5)d+\bar s(1-\gamma_5)s|0\rangle\\
&-\langle\pi^0\gamma|\bar s(1+\gamma_5)u|K^+\rangle\langle\pi^+|\bar u\gamma_5 d|0\rangle+\langle0|\bar s\gamma_5u|K^+\rangle\langle\pi^+\pi^0\gamma|\bar u(1-\gamma_5)d|0\rangle\,.
\end{aligned} \tag{12}$$

In this expression there are two typical classes of matrix elements.

The first class is

$$\text{(i)}\quad\langle\pi^0|\bar su|K^+\rangle\langle\pi^+\gamma|\bar u(1-\gamma_5)d|0\rangle\,. \tag{13}$$

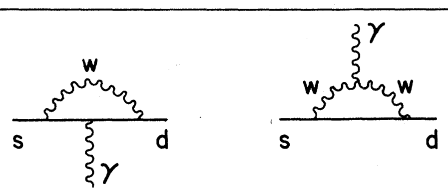

FIG. 4. Radiative decay of the strange quark. These diagrams lead to contributions of the type (10a).

Without loss of generality we can write

$$\begin{aligned}
&\langle\pi^0(p)|\bar su|K^+(K)\rangle=ap\cdot K\,,\\
&\langle\pi^+(p_+)\gamma(q,\epsilon)|\bar u(1-\gamma_5)d|0\rangle=b(p_+,q)p_+\cdot\epsilon\,.
\end{aligned} \tag{14}$$

However using these two expressions it is impossible to construct a contribution with the structure of β or γ given in Eq. (6) imposed by Lorentz and gauge invariances. Therefore

$$\langle \pi^0 | \bar{s}u | K^+\rangle \langle \pi^+\gamma | \bar{u}(1-\gamma_5) d | 0\rangle = 0 . \quad (15)$$

The second class is

(ii) $\langle \pi^0(p)\gamma(q) | \bar{s}(1+\gamma_5)u | K^+\rangle \langle \pi^+ | \bar{u}\gamma_5 d | 0\rangle$.

Using in this case the soft-pion approximation and current algebra one obtains a relation for these matrix elements

$$\langle \pi^0\gamma | \bar{s}(1+\gamma_5)u | K^+\rangle \propto \langle \gamma | \bar{s}(1+\gamma_5)u | K^+\rangle ,$$

and then using the same argument as in (i), we conclude

$$\langle \pi^0\gamma | \bar{s}(1+\gamma_5)u | K^+\rangle \langle \pi^+ | \bar{u}\gamma_5 d | 0\rangle = 0 . \quad (16)$$

With similar approximation the matrix elements of the operators $\mathcal{O}_1, \ldots, \mathcal{O}_4$ can be expressed in terms of the form factor F_V, F_A. (See Appendix B for the definition of the four factors F_V, F_A.) In fact, given that the contributions coming from the penguin diagram vanish [Eqs. (12), (15), and (16)], the relation between these matrix elements and the invariant transition amplitude is

$$M_{\rm DE} = -\sqrt{2}\, G_F \sin\theta_c \cos\theta_c \sum_{i=1}^{4} c_i \langle \pi^+\pi^0\gamma | \mathcal{O}_i | K^+\rangle$$
$$= -\sqrt{2}\, G_F \sin\theta_c \cos\theta_c [e F_V B \epsilon_{\mu\nu\rho\sigma} \epsilon^\mu K^\nu p_+^\rho q^\sigma + ie F_A D (p_+ \cdot \epsilon K\cdot q - p_+\cdot q K\cdot \epsilon)] . \quad (17)$$

The value of the coefficients B, D are given in Table II for all three decays (2).

III. NUMERICAL RESULTS

The formulas for the partial and total decay rates, expressed in terms of the form factors $\tilde{\gamma}$ and $\tilde{\beta}$ which are directly deduced from Eq. (17), have been left for the Appendix A. To derive these equations we have used the following approximations:

(a) We have neglected all CP violation except that induced through the IB contribution.

(b) We have neglected the momentum-transfer dependence of the form factors F_V, F_A.

The predictions obtained under these assumptions for the different decay rates are given in Table III. In Appendix B we have estimated the error introduced by these approximations.

IV. CONCLUSIONS

In this paper we have extended the scheme of SVZ to the nonleptonic radiative decays (2).

If the $\Delta I = \frac{1}{2}$ rule is a consequence of some dynamical mechanism, instead of being an intrinsic property of weak interactions, then it is possible that this mechanism (in this case, the penguin diagrams) does not play any role in the radiative decays.

Using the soft-pion limit and current algebra we have shown that the contribution of the mechanism (penguin diagrams) proposed by SVZ to the direct-emission term of the process (2) vanishes. On the other hand, we have also analyzed (see Tables II and III) the importance of the short-distance contribution to the strong-interaction corrections in the description of (2).

Taking into account the approximations used in the calculations (see Sec. IV and Appendix B), the decay rates that we obtain both for the charged- and neutral-kaon decay are in reasonable agreement with the experimental data.

In conclusion the standard model (GWS+QCD) gives a satisfactory explanation of the decays $K \to \pi\pi\gamma$. Furthermore, the mechanism proposed by SVZ to explain the $\Delta I = \frac{1}{2}$ rule does not play an important role in the decay $K \to \pi\pi\gamma$ within the context of the vacuum-saturation approximation. Study of the correction to the vacuum-saturation approximation and how such a correction might influence $K \to \pi\pi\gamma$ is beyond the scope of this paper.

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APPENDIX A

The invariant transition amplitude for the decay $K^\pm(K) \to \pi^\pm(p_+) + \pi^0(p) + \gamma(q)$ is given in Eq. (5). In the frame where K^+ is at rest

$$K = (M, 0), \quad p_+ = (E, \vec{p}_+), \quad K^2 = M^2,$$
$$q = (\omega, \vec{q}), \quad p = (\xi, \vec{p}), \quad p_+^2 = p^2 = M, \quad (A1)$$

the differential decay rate is[9,11]

TABLE II. Value of the coefficients B and D in the GWS model and including QCD corrections.

	GWS		GWS+QCD	
	B	D	B	D
$K^\pm \to \pi^\pm\pi^0\gamma$	−0.169	−1.65	3.277	−2.017
$K_L^Q \to \pi^+\pi^-\gamma$	6.63	0	9.27	0
$K_S^Q \to \pi^+\pi^-\gamma$	0	−2.04	0	−4.166

$$\frac{1}{W_0}\frac{dW}{dE}=B+2\frac{\tilde{\gamma}}{e|A_0|}I\cos(\delta_{11}-\delta_{20}+\phi)+\left(\left|\frac{\tilde{\gamma}}{eA_0}\right|^2+\left|\frac{\tilde{\beta}}{eA_0}\right|^2\right)D, \qquad (A2)$$

where

$$W_0=\frac{\beta_0}{16\pi M}|A_0(K^\pm\to\pi^\pm\pi^0)|^2 \qquad (A3)$$

is the total decay rate for the nonradiative process $K^\pm\to\pi^\pm\pi^0$ and

$$\beta_0=\left(1-\frac{4m^2}{M^2}\right)^{1/2} \qquad (A4)$$

and

$$B=\frac{\alpha}{\pi\beta_0}\left[\frac{8|\vec{p}_+|}{M(2E-M)}+\left(\frac{2}{M}-\frac{1}{2E-M}\right)\ln\frac{E-|\vec{p}_+|}{E+|\vec{p}_+|}\right],$$

$$I=\frac{\alpha(2E-M)}{\pi\beta_0}\left[M|\vec{p}_+|-\frac{M^2}{2}\ln\frac{M-E+|\vec{p}_+|}{M-E-|\vec{p}_+|}+\frac{M^2}{2}\ln\frac{M(E+|\vec{p}_+|)-M^2}{M(E-|\vec{p}_+|)-M^2}\right], \qquad (A5)$$

$$D=-\frac{\alpha M|\vec{p}_+|^3}{6\pi\beta_0}\frac{(2E-M)^3}{(1+m^2/M^2-2E/M)^4},$$

$$\alpha=\frac{e^2}{4\pi}.$$

Let us remark that B, I, and D are, respectively, the inner-bremsstrahlung, interference, and direct-emission contribution to the total decay rate. The phases δ_{IJ}, where I and J denote respectively the isospin and angular momentum, take into account the rescattering of the particles in the final state, whereas ϕ is introduced to parametrize a possible CP violation.[13]

In order to obtain the total decay rate we have to integrate Eq. (A5) over the pion energy according to the experimental conditions. Here we give the numerical results for the experimental data of Ref. 14.

$$\frac{W_{\rm IB}(K^\pm\to\pi^\pm\pi^0\gamma)}{W(K^\pm\to{\rm all})}=2.61\times10^{-9},$$

$$\frac{W_{\rm int}(K^\pm\to\pi^\pm\pi^0\gamma)}{W(K^\pm\to{\rm all})}=4.28\times10^{9}\left\{2\frac{\tilde{\gamma}}{e}[\cos\phi-\sin\phi\tan(\delta_{11}-\delta_{20})]\right\}, \qquad (A6)$$

$$\frac{W_{\rm DE}(K^\pm\to\pi^\pm\pi^0\gamma)}{W(K^\pm\to{\rm all})}=3.03\times10^{24}\left(\left|\frac{\tilde{\gamma}}{e}\right|^2+\left|\frac{\tilde{\beta}}{e}\right|^2\right).$$

In the case of the neutral-kaon decay we begin by considering the photon energy spectrum

$$\frac{1}{W_0}\frac{dW}{d\omega^*}=\frac{\alpha}{\pi\beta_0}\left[M_{\rm IB}+\frac{2\tilde{\gamma}_i}{|eA_0|}M_{\rm int}+\left(\left|\frac{\tilde{\gamma}_i}{eA_0}\right|^2+\left|\frac{\tilde{\beta}_i}{eA_0}\right|^2\right)M_{\rm DE}\right]. \qquad (A7)$$

TABLE III. Summary of the theoretical and experimental results for the decays $K^\pm\to\pi^\pm\pi^0\gamma$ and $K^0_{L,S}\to\pi^+\pi^-\gamma$.

	Expt.	Ref.	Expt.	Ref.	Theor ($r=-0.44$)	Theor. ($r=-2.36$)
$\frac{W_{\rm IB}(K^\pm\to\pi^\pm\pi^0\gamma)}{W(K^\pm\to{\rm all})}$	$(2.55\pm0.17)\times10^{-4}$	14	$(2.87\pm0.32)\times10^{-4}$	18	2.61×10^{-4}	
$\frac{W_{\rm DE}(K^\pm\to\pi^\pm\pi^0\gamma)}{W(K^\pm\to{\rm all})}$	$(1.56\pm0.35)\times10^{-5}$	14	$(2.3\ \pm0.32)\times10^{-5}$	18	0.52×10^{-5}	1.5×10^{-5}
$\frac{W_{\rm int}(K^\pm\to\pi^\pm\pi^0\gamma)}{W(K^\pm\to{\rm all})}$	0		0		0.29×10^{-5}	1.5×10^{-5}
$\frac{W_{\rm IB}(K^0_L\to\pi^+\pi^-\gamma)}{W(K^0_L\to{\rm all})}$	$(1.56\pm0.16)\times10^{-5}$	19			1.41×10^{-5}	
$\frac{W_{\rm DE}(K^0_L\to\pi^+\pi^-\gamma)}{W(K^0_L\to{\rm all})}$	$(2.89\pm0.28)\times10^{-5}$	19			1.73×10^{-5}	
$\frac{W_{\rm int}(K^0_L\to\pi^+\pi^-\gamma)}{W(K^0_L\to{\rm all})}$	0	19			0	0
$\frac{W(K^0_S\to\pi^+\pi^-\gamma)}{W(K^0_S\to\pi^+\pi^-)}$	$(2.8\ \pm0.6)\ \times10^{-3}$	20	$(2.68\pm0.15)\times10^{-3}$	21		
$\frac{W_{\rm IB}(K^0_S\to\pi^+\pi^-\gamma)}{W(K^0_S\to\pi^+\pi^-)}$						2.55×10^{-3}
$\frac{W_{\rm DE}(K^0_S\to\pi^+\pi^-\gamma)}{W(K^0_S\to\pi^+\pi^-)}$	$<6\times10^{-5}$	20			0.16×10^{-8}	4.69×10^{-8}
$\frac{W_{\rm int}(K^0_S\to\pi^+\pi^-\gamma)}{W(K^0_S\to\pi^+\pi^-)}$	$<9\times10^{-5}$	20			0.33×10^{-5}	1.77×10^{-5}

Here $W_0 = W_0(K_i^0 \to \pi^+\pi^-)$ is the total decay rate for the nonradiative process; the index $i = L, S$ refers to the "long-lived" and "short-lived" neutral kaon:

$$M_{\rm IB} = \left(1 - \frac{2\omega^*}{M}\right)\frac{1}{\omega^*}\left((1+v^2)\ln\frac{1+v}{1-v} - 2v\right),$$

$$M_{\rm int} = \frac{1}{6}\left(1 - \frac{2\omega^*}{M}\right)M^2\omega^*\left(v - \frac{1-v^2}{2}\ln\frac{1+v}{1-v}\right), \quad \text{(A8)}$$

$$M_{\rm DE} = \frac{1}{6}\left(1 - \frac{2\omega^*}{M}\right)M^4\omega^{*3}v^3 .$$

Here v is the pion velocity in the c.m. frame of the two pions and it is related to the photon energy ω^* in the rest frame of the decaying particle by the equation

$$v^2 = 1 - \frac{4m^2}{M^2 - 2M\omega^*} . \quad \text{(A9)}$$

In this case we will give the numerical results for the total decay rate for two experimental conditions.[15]

(a) $\omega^* > 20$ MeV:

$$\frac{W_{\rm IB}(K_L^0 \to \pi^+\pi^-\gamma)}{W(K_L^0 \to \text{all})} = 1.41\times 10^{-5},$$

$$\frac{W_{\rm int}(K_L^0 \to \pi^+\pi^-\gamma)}{W(K_L^0 \to \text{all})} = 2\frac{\tilde{\gamma}_0^L}{e}(2.65\times 10^9), \quad \text{(A10)}$$

$$\frac{W_{\rm DE}(K_L^0 \to \pi^+\pi^-\gamma)}{W(K_L^0 \to \text{all})} = \left(\left|\frac{\tilde{\gamma}_0^L}{e}\right|^2 + \left|\frac{\tilde{\beta}_0^L}{e}\right|^2\right)1.31\times 10^{24} .$$

(b) $\omega^* > 50$ MeV:

$$\frac{W_{\rm IB}(K_S^0 \to \pi^+\pi^-\gamma)}{W(K_S^0 \to \pi^+\pi^-)} = 2.55\times 10^{-3},$$

$$\frac{W_{\rm int}(K_S^0 \to \pi^+\pi^-\gamma)}{W(K_S^0 \to \pi^+\pi^-)} = 2\frac{\tilde{\gamma}_0^S}{e}(2.33\times 10^9), \quad \text{(A11)}$$

$$\frac{W_{\rm DE}(K_S^0 \to \pi^+\pi^-\gamma)}{W(K_S^0 \to \pi^+\pi^-)} = \left(\left|\frac{\tilde{\gamma}_0^S}{e}\right|^2 + \left|\frac{\tilde{\beta}_0^S}{e}\right|^2\right)3.23\times 10^{21} .$$

Finally we give the expressions of the form factors in terms of the constants B, D of Table II as

$$\tilde{\gamma} = \sqrt{2}\, eF_V rDG_F \sin\theta_C \cos\theta_C ,$$
$$\tilde{\beta} = -\sqrt{2}\, eF_V BG_F \sin\theta_C \cos\theta_C , \quad \text{(A12)}$$

where we have introduced[16]

$$r \equiv -\frac{F_V}{F_A} .$$

Currently there are two experimental values for this parameter.[17]

$$r = \begin{cases} -2.36 , \\ 0.44 . \end{cases}$$

APPENDIX B

In evaluating the results presented in Table III we have neglected in Eq. (A2) all CP violation except that induced through the IB, i.e., we have considered $\phi = \phi' = 0$. On the other hand, we have also neglected the momentum-transfer dependence of the form factors F_V, F_A. We can make an estimation of the modification introduced by this approximation in the following way. Let us consider

$$T = \langle \pi^0(p)\,|\,\bar{s}\gamma_\mu u\,|K^+(K)\rangle\langle \pi^+(p_+)\gamma(q)\,|\,\bar{u}\gamma^\mu d\,|0\rangle , \quad \text{(B1)}$$

using the definitions of the form factors F_V, F_A, f_+:[16]

$$\langle \pi^+(p_+)\gamma(q)\,|\,\bar{u}\gamma^\mu d\,|0\rangle = -e\epsilon_{\mu\nu\rho\sigma}\epsilon^\nu q^\rho p_+^\sigma F_V(p_+, q) ,$$

$$\langle \pi^+(p_+)\gamma(q)\,|\,\bar{u}\gamma_\mu\gamma_5 d\,|0\rangle = -ie\left[\sqrt{2}f_\pi \frac{p_+\cdot\epsilon}{p_+\cdot q}(q+p_+)_\mu - \sqrt{2}f_\pi\epsilon_\mu + (p_+\cdot q\epsilon_\mu - p_+\cdot\epsilon q_\mu)F_A(p_+, q)\right] , \quad \text{(B2)}$$

$$\langle \pi^0(p)\,|\,\bar{s}\gamma_\mu u\,|K^+\rangle = -\frac{1}{\sqrt{2}}\left[f_+(K+p)^\mu + f_-(K-p)^\mu\right] .$$

Then

$$T = \frac{ef_+F_V}{2}(K+p)^\mu\epsilon_{\mu\nu\rho\sigma}\epsilon^\nu q^\rho p_+^\sigma . \quad \text{(B3)}$$

Given that a possible approximation in parametrizing the momentum dependence of the form factors is

$$f_+(K,p) = \frac{m_{K^*}^2}{m_{K^*}^2 - (K-p)^2} f_+(0) ,$$
$$F_V(p_+, q) = \frac{m_\rho^2}{m_\rho^2 - (q+p_1)^2} F_V(0) , \quad \text{(B4)}$$

then

$$\frac{T(K, p_+, q)}{T(0)} = \frac{m_{K^*}^2}{m_{K^*}^2 - (K-p)^2}\,\frac{m_\rho^2}{m_\rho^2 - (q+p_+)^2} \leqq 1.5 . \quad \text{(B5)}$$

Therefore

$$T(K, p_+, q) \leqq \tfrac{3}{2}T(0) . \quad \text{(B6)}$$

Equation (B6) allows us to establish a limit on the possible modifications introduced once we consider the momentum-transfer dependence of the form factors F_V, F_A, and f_+.

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