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Models for polarization asymmetry in inclusive hadron production

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The parton-recombination model plus SU(6) symmetry is used to relate polarizations in inclusive baryon production to those of the underlying constituent subprocesses. A description of the origin of the polarization asymmetry in terms of Thomas precession of the quarks' spins in the recombination process accounts for all of the qualitative features of the baryon and antibaryon polarization data.

I. INTRODUCTION

The unexpected discovery[1] of large polarization effects in inclusive Λ^0 production by unpolarized protons at high energy and small transverse momentum has shown that important spin effects exist in high-energy collisions. In this paper we study the quark dynamics underlying polarizations within the framework of the parton-recombination model.[2] Assuming minimal complexity for the quark transitions and using SU(6) symmetry we relate the polarizations of numerous baryon-to-baryon transitions. These relations test the basic recombination picture but are independent of the dynamical origin of the polarization on the quark level. We then propose a semiclassical model in which this polarization is due to a Thomas precession effect in the quark-recombination process. Our model suggests that polarization of leading particles in multiparticle jets should be seen not only in jets initiated by hadronic fragmentation but also in quark jets produced in e^+e^- annihilation or deep-inelastic lepton-hadron scattering.[3]

The data which we wish to explain show[4] that Λ^0's produced in pp and p-nucleus scattering are polarized transversely to the production plane and preferentially along the $(\vec{p}_\Lambda \times \vec{p}_p)$ axis. This polarization is independent of the beam energy and weakly dependent on x_F, the fraction of the proton's longitudinal momentum carried off by the Λ^0. It increases in magnitude roughly linearly with the transverse momentum of the Λ^0. Finally, $\bar{\Lambda}^0$'s produced in pp and pA collisions are *not* polarized. Recent measurements of polarizations of Ξ^0, Ξ^-, and Σ^+ hyperons produced by a proton beam[5] suggest that the Ξ^0 and Ξ^- have the *same* polarization as the Λ^0, whereas the Σ^+ and the Λ^0 polarizations have *the same size but opposite signs*.

We choose to analyze this problem using the parton-recombination model, which has been applied with good success to many small-transverse-momentum fragmentation processes. In this model, a proton in the infinite-momentum frame is built up of three valence quarks plus a large number of sea partons; in the collision the slow or "wee" partons interact with the target, destroying the coherence of the wave function, which then decays into a many-hadron final state.[6] The partons-into-hadrons transformation takes place semilocally in rapidity and is pictured as proceeding via quark recombination: $q\bar{q}$ pairs and qqq triplets form mesons and baryons, respectively. The fastest particles are formed by the recombination of the beam's valence quarks with other valence quarks or sea partons. For example, fast Λ^0's (Σ^+'s) are produced by the recombination of a valence ud (uu) pair with an s quark from the sea of the proton (VVS recombinations). In processes in which the minimal number of exchanged quarks is two, such as $p \to \Xi^0$ and $p \to \Xi^-$, VVS recombination is not possible and the production of fast particles proceeds through VSS recombination. Finally, if the fragmenting and the secondary particles have no common valence quarks, only SSS recombination is possible. In this case cross sections are expected to be small and to have a steeply falling x_F spectrum, in accord with experiment.

Our discussion of polarization asymmetry will use only the most general results of the parton-recombination model. Only near the end of this paper, when we discuss details of the x_F and $P_\perp$ dependence of polarization, will we refer to specifics. Our analysis will proceed in two stages. First, we shall formulate a rule which will allow us to relate the polarizations of various hadron-to-hadron transitions in terms of the spins of the quarks participating in the reactions. All of these predictions will be independent of any assumptions about dynamics. Then we will describe a simple semiclassical model for the origin of the quark polarization; we ascribe it to the effects of Thomas precession on the quarks' spins as they move under the influence of the (nearly) spin-independent confining force. While we cannot make a detailed calculation of a polarization asymmetry, our analysis makes plausible the origin and systematics of the asymmetry as arising from a combination of simple kinematics and strong momentum ordering in the beam particle's infinite-momentum-frame wave function, where on the average valence quarks are much faster than sea quarks.

II. AN "ALGEBRAIC" MODEL FOR POLARIZATION ASYMMETRY

We begin our discussion of polarization asymmetry by trying to find a simple set of rules which will reproduce the known features of the data, and allow us to derive relationships between various reactions. Our model is as follows.

Production of fast hadrons involves recombination of a maximum number of valence quarks in the beam projectile with a minimum number of sea partons.

We assume that all baryons may be described by minimal three-quark SU(6) wave functions. This is clearly an obvious description for the outgoing baryon but requires some additional comment for the beam particle where it is equivalent to a factorization assumption for the infinite-momentum-frame multiquark wave function into separate valence and sea pieces. In order to justify this description we must boost ourselves into the rest frame of the proton, where all of the sea resides in the target. In this frame $p\rightarrow\Lambda$ is the "pickup" reaction $p+s\rightarrow\Lambda+u$, and the wave function does factorize.

We assume that all sea quarks are initially unpolarized. The recombination mechanism generates an asymmetry by enhancing recombination in one spin state over another.

For the purpose of discussion we will treat baryons as bound states of a quark and a diquark, and identify the diquark with the two quarks whose wave functions are most similar: the two valence quarks in VVS recombination and the two sea quarks in VSS recombination. This identification is a realization of dynamics in the recombination process: It is a statement that the effect of recombination on the partons in the proton as they are transferred into the outgoing hadron may be different depending on whether they are accelerated (as are the slow sea partons) or decelerated (as are the fast valence partons). It is also a statement that two partons with similar wave functions in the proton may interact with themselves differently than they interact with a parton whose wave function is different.

Finally we assume that the transverse momentum of the outgoing hadron and the transverse momentum of each of its constituents are more or less parallel. This feature is shared by all models in which recombination is short range in transverse momentum. Thus hadron production is a trigger which selects quarks or diquarks whose transverse momentum lies in the scattering plane directed toward the outgoing hadron. A polarization asymmetry will arise only if there is a correlation between the spin and the transverse momentum of the recombining quark.

So we describe baryon production in terms of two amplitudes for the odd quark, $A_\uparrow$ and $A_\downarrow$, which parametrize its recombination with spin up or down in the scattering plane, and four amplitudes $A_{j,m}$ which parametrize the recombination of the diquark in the states $j=0$, $m=0$ or $j=1$, $m=\pm1,0$, again measured with the quantization axis chosen to lie along the normal to the scattering plane. The amplitude for the production of a baryon B' in spin state s' from a baryon B in spin state s is then

$$\langle B'_{s'}|B_s\rangle=\sum_{j=0,1}(\alpha_j A_\uparrow A_{j,s'-1/2}+\beta_j A_\downarrow A_{j,s'+1/2}) \tag{2.1}$$

with α and β SU(6) Clebsch-Gordan coefficients.

When we square (2.1) and combine terms to form the polarization asymmetry we neglect interference terms. This is a usual parton-model assumption which can be justified on the quark level by noting (see Fig. 1) that the spectator valence quarks in the terms proportional to $A_\uparrow$ and $A_\downarrow$ in (2.1) lie in different spin states, so that their overlap vanishes. For example, in the reaction $p\rightarrow\Sigma^+$, the spectator d quark is spin up in the former amplitude and spin down in the latter. Recombination of the d quark with the remaining partons into hadrons may give rise to nonvanishing overlap—for instance, the $d_\uparrow$ and $d_\downarrow$ may both be members of a π. However, we choose to ignore this possibility in the interest of simplicity.

We must now parametrize the asymmetry in terms of the A_{jm}'s. It is easiest to proceed by example.

Consider first $p\rightarrow\Lambda$. We find an asymmetry

$$P(p\rightarrow\Lambda)=\frac{|A_\uparrow|^2-|A_\downarrow|^2}{|A_\uparrow|^2+|A_\downarrow|^2}\,. \tag{2.2}$$

Since the ud diquark in the Λ had $j=0$, the spin of the Λ is the spin of its strange quark. The data can be parametrized by taking $|A_\uparrow|^2=A(1-\epsilon)$, $|A_\downarrow|^2=A(1+\epsilon)$, where ϵ is small, linear in $p_\perp$,

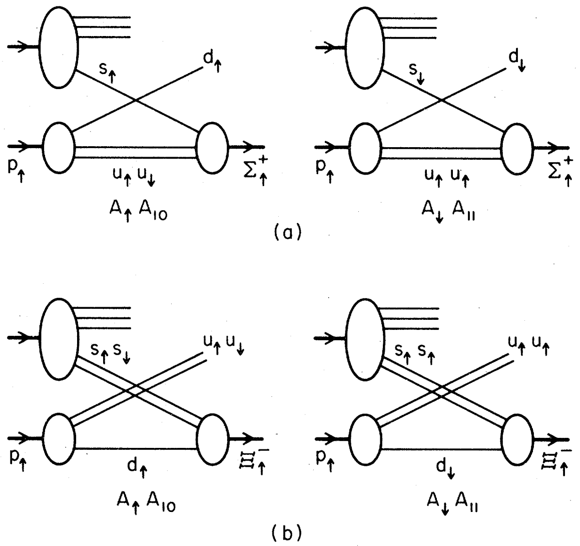

FIG. 1. (a) The reaction $p\uparrow \to \Sigma^+_\uparrow$ in the recombination model, showing the quarks and their spins together with the amplitudes $A_{j\,m}$ of Eq. (2. 1). (b) The reaction $p_\uparrow \to \Xi^-_\uparrow$.

and weakly x_F dependent. Thus, to lowest order in ϵ,

$$P(p\to\Lambda) = -\epsilon\,. \tag{2.3}$$

The data show us that the slow sea quark preferentially recombines with its spin down in the scattering plane.

Next, consider $p\to\Sigma^+$. Here we encounter $j=1$ diquarks. A general expression is unilluminating. If we, however, again assume that there is a small asymmetry in the diquark recombination amplitudes of the *opposite* sign to the sea-quark recombination amplitude

$$\begin{aligned} |A_{11}|^2 &= B(1+\delta)\,,\\ |A_{10}|^2 &= |A_{00}|^2 = B\,,\\ |A_{1-1}|^2 &= B(1-\delta)\,. \end{aligned} \tag{2.4}$$

We find, again to lowest order in ϵ and δ,

$$P(p\to\Sigma^+) = \tfrac{1}{3}\epsilon + \tfrac{2}{3}\delta \tag{2.5}$$

opposite in sign to $P(p\to\Lambda)$ and (if $\epsilon=\delta$) equal to it in magnitude, as the data require.

So the data for Λ and Σ^+ production seem to be explained by the simple rule:

Slow partons preferentially recombine with their spins down in the scattering plane while fast partons recombine with their spins up.

We may check this rule by calculating Ξ production. Here the ss diquark is slow, so it preferentially recombines with $m=-1$,

$$\begin{aligned} |A_{11}|^2 &= B(1-\delta)\,,\\ |A_{10}|^2 &= |A_{00}|^2 = B\,,\\ |A_{1-1}|^2 &= B(1+\delta)\,, \end{aligned} \tag{2.6}$$

while the u or d quark is fast, so it preferentially recombines with $S=+\frac{1}{2}$:

$$|A_\uparrow|^2 = A(1+\epsilon)\,,\quad |A_\downarrow|^2 = A(1-\epsilon)\,. \tag{2.7}$$

We find

$$P(p\to\Xi^0) = P(p\to\Xi^-) = -\tfrac{1}{3}\epsilon - \tfrac{2}{3}\delta \tag{2.8}$$

once again equal in magnitude and sign to $P(p\to\Lambda)$ if $\epsilon=\delta$, as experiment demands.

A large number of predictions of this model for different beams and final-state particles are summarized in Table I. Among them we note that the reaction $K^-\to\Lambda$ at fast x_F is a particularly important test since all the asymmetry resides in the leading s quark. We expect a positive asymmetry here.[7]

It is easy to extend these predictions to the use of polarized beams. Since it is unlikely that completely polarized beams can be obtained for arbitrary incident particle, the most convenient presentation of our results is in terms of cross sections for beams of one definite spin state. Then the asymmetry can be read off from

$$P(B\to B') = \frac{N(\uparrow)\Delta\sigma(B_\uparrow\to B') + N(\downarrow)\Delta\sigma(B_\downarrow\to B')}{N(\uparrow)\sum\sigma(B_\uparrow\to B') + N(\downarrow)\sum\sigma(B_\downarrow\to B')}$$

with $N(\uparrow)$ $[N(\downarrow)]$ the flux of spin-up (spin-down) incident particles, $\Delta\sigma = \sigma(B\to B'_\uparrow) - \sigma(B\to B'_\downarrow)$, and $\sum\sigma$ the sum of the cross sections. Table II displays σ's for some interesting reactions. The constant terms in the table are asymmetries which reflect the SU(6) nature of the wave func-

TABLE I. Polarizations for various transitions predicted by the leading-partons–trailing-partons model.

$B\to B'$ Transition	Polarization
$p\rightleftarrows n$, $\Sigma^-\rightleftarrows\Xi^-$, $\Sigma^+\rightleftarrows\Xi^0$, $\Sigma^+,\Sigma^-,\Xi^-,\Xi^0\rightleftarrows\Sigma^0$	$-\frac{20}{21}\epsilon + \frac{1}{42}\delta$
$p\rightleftarrows\Sigma^+$, $n\rightleftarrows\Sigma^-$, $\Xi^-\rightleftarrows\Xi^0$, $p,n\rightleftarrows\Sigma^0$	$\frac{1}{3}\epsilon + \frac{2}{3}\delta$
$p,n\rightleftarrows\Lambda^0$	$-\epsilon$
$\Sigma^+,\Sigma^-,\Xi^-,\Xi^0\rightleftarrows\Lambda^0$	$-\frac{2}{3}\epsilon + \frac{1}{6}\delta$
$p\rightleftarrows\Xi^0,\Xi^-,\Sigma^-$	$-(\frac{1}{3}\epsilon + \frac{2}{3}\delta)$
$n\rightleftarrows\Xi^0,\Xi^-,\Sigma^+$	$-(\frac{1}{3}\epsilon + \frac{2}{3}\delta)$
$\pi, K^+\to\Lambda$	$-\delta/2$
$K^-\to\Lambda$	ϵ

TABLE II. Cross sections for polarized beams to use in constructing asymmetries. $\sigma(\downarrow\rightarrow\downarrow)$ and $\sigma(\downarrow\rightarrow\uparrow)$ are obtained from $\sigma(\uparrow\rightarrow\uparrow)$ and $\sigma(\uparrow\rightarrow\downarrow)$ by changing the signs of ϵ and δ.

Reactions	Cross sections
$p\rightleftarrows n$, $\Sigma^-\rightleftarrows\Xi^-$, $\Sigma^+\rightleftarrows\Xi^0$, $\Sigma^+,\Sigma^0,\Xi^-,\Xi^0\rightleftarrows\Sigma^0$	$\sigma(\uparrow\rightarrow\uparrow)=4(1+\epsilon)(1+\delta)+82(1-\epsilon)$ $\sigma(\uparrow\rightarrow\downarrow)=82(1+\epsilon)$
$\Sigma^+,\Sigma^-,\Xi^-,\Xi^0\rightleftarrows\Lambda$	$\sigma(\uparrow\rightarrow\uparrow)=2(1+\epsilon)(1+\delta)+5(1-\epsilon)$ $\sigma(\uparrow\rightarrow\downarrow)=5(1+\epsilon)$
$p\rightleftarrows\Sigma^+$, $n\rightleftarrows\Sigma^-$, $\Xi^-\rightleftarrows\Xi^0$, $p,n\rightleftarrows\Sigma^0$	$\sigma(\uparrow\rightarrow\uparrow)=4(1+\epsilon)(1+\delta)+(1-\epsilon)$ $\sigma(\uparrow\rightarrow\downarrow)=(1+\epsilon)$
$p\rightleftarrows\Xi^0,\Xi^-,\Sigma^-$, $n\rightleftarrows\Xi^0,\Xi^-,\Sigma^+$	$\sigma(\uparrow\rightarrow\uparrow)=4(1-\epsilon)(1-\delta)+(1+\epsilon)$ $\sigma(\uparrow\rightarrow\downarrow)=2[1-\epsilon+(1+\epsilon)(1+\delta)]$

tions. They are independent of all other details of the model.

Miettinen and Markinen have calculated asymmetries for the production of vector mesons and decuplet baryons. Their results, given in terms of density matrices, are reported in Ref. 8.

In addition to asymmetries, the model also predicts ratios of inclusive cross sections. SU(6) breaking enters here, but ratios of cross sections involving the same number of strange sea quarks should be reliable. These predictions are for "prompt" production and ignore cascading completely. We find

$$\sigma(p\rightarrow\Sigma^+):\sigma(p\rightarrow\Sigma^0):\sigma(p\rightarrow\Lambda)=8:1:9\,;$$

$$\sigma(p\rightarrow\Xi^0):\sigma(p\rightarrow\Xi^-)=1:2\,.$$

Finally, just as leading baryons in baryon fragmentation and in meson fragmentation are polarized, so should leading baryons be in quark fragmentation. The relevant plane which defines the polarization axis is that defined by the jet and the produced baryon, where the jet axis is given by the photon or W-boson momentum in deep-inelastic scattering or by the opposite side jet in e^+e^- annihilation. (The former case is probably easier to measure; the normal $\hat{n}=\hat{P}_W\times\hat{P}_{B'}$.) The produced quarks are usually in helicity eigenstates, so their spin wave functions are equal combinations of $\vec{s}\cdot\hat{n}=\pm\frac{1}{2}$. This implies that jet asymmetries are equal to those of the appropriate pseudoscalar meson. Thus $P(u \text{ or } d\rightarrow\Lambda)=P(\pi\rightarrow\Lambda)$, $P(s\rightarrow\Lambda)=P(K^-\rightarrow\Lambda)$, etc. It would be most useful to look for these polarization effects in bubble-chamber data for $\nu N\rightarrow\mu^-\Lambda X$.[9]

As the P_T of the produced baryon becomes large we expect the dominant production mechanism will cease to be three-quark recombination. Rather, production will proceed by a hard scattering which produces a single quark at large P_T which then fragments the baryon. Kane, Pumplin, and Rebko[10] have argued that this subprocess can yield only unpolarized baryons. However, if one detects both the jet and the baryon, one should be able to see an asymmetry arising from soft hadronization: once information about the orientation of the jet-baryon scattering plane is lost the baryon polarization will average to zero.

III. DYNAMICAL MODELS FOR POLARIZATION ASYMMETRY

All of the results of the last section are independent of a dynamical origin for ϵ and δ. We would now like to propose simple semiclassical models to account for the slow-partons-spin-down–fast-partons-spin-up rule. Once again, it is easiest to restrict our attention to the case $p\rightarrow\Lambda$.

The fundamental observation underlying the model is that the s quark involved in the recombination resides in the sea of the proton and carries a very small fraction x_s (~0.1) of its momentum. However, it is a valence quark in the Λ and must carry a large fraction (~$\frac{1}{3}$) of the Λ's momentum. Since the Λ also carries a large fraction x_F of the proton's momentum, recombination induces a large increase in the longitudinal momentum of the s quark, from x_sP to $\frac{1}{3}x_FP$.

At the same time, the s quark carries transverse momentum: on the average, $P_T(s \text{ in } P) \sim P_T(s \text{ in } \Lambda)\sim\frac{1}{2}P_T(\Lambda)$. Therefore, the velocity vector of the s quark is not parallel to the change in momentum induced by recombination (see Fig. 2).

That being the case, the quark's spin will feel the effects of Thomas precession.[11] An additional term will appear in the effective Hamiltonian which describes the recombination process:

$$U=\vec{S}\cdot\vec{\omega}_T \tag{3.1}$$

with the Thomas frequency

$$\vec{\omega}_T=\frac{\gamma}{\gamma+1}\frac{\vec{F}}{m_s}\times\vec{V}\,, \tag{3.2}$$

where V is the strange quark's velocity, F the

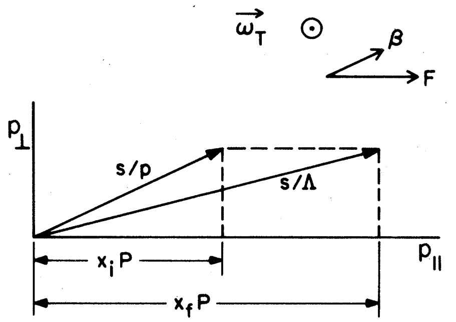

FIG. 2. Momentum vectors for the s quark in the scattering plane in the sea of the proton (labeled by subscripts s/p) and in the Λ (labeled by subscript s/Λ). The recombination force is along the beam direction and the Thomas frequency $\vec{\omega}_T$ is out of the scattering plane.

force, m_s the strange-quark mass, and $\gamma = (1-V^2)^{-1/2}$; $\gamma/(\gamma+1) \simeq 1$ in the infinite momentum frame.

Thomas precession is an essentially kinematic effect which arises from the fact that the product of noncollinear boosts is both a boost and a rotation. We imagine being able to describe the recombination process by equations of motion which may be valid provided that the evolution of the rest frame is described by infinitesimal boosts without rotations. The rotation induced by the noncollinear boosts leads to the extra term (3.1) which describes the precession of the quark spin. It will always be present no matter what other interactions are present as long as $\vec{F}\times\vec{V}\neq 0$.

In the example considered here, the force is essentially parallel to the beam direction and so $\vec{\omega}_T$ points up and out of the scattering plane [as shown in Fig. (2)]. Because the recombination Hamiltonian now contains the Thomas term Eq. (3.1), we expect that the scattering amplitude will depend on $\vec{S}\cdot\vec{\omega}_T$—that is, whether the spin of the s quark is up or down in the scattering plane. Because of the cross product we expect the symmetry to be (approximately) linear in $p_\perp$.

Leading partons are decelerated by the recombination process. For example, in $p \to \Sigma^+$ the uu diquark loses momentum from $\frac{2}{3}P$ to $\frac{2}{3}x_F P$ as it passes from the proton to the Σ^+. Thus the orientation of its $\vec{\omega}_T$ is opposite to that of the s quark's $\vec{\omega}_T$ so that the same spin forces which generate one sign of an asymmetry for the sea partons should generate an asymmetry of the opposite sign for leading partons. This is the dynamical origin of the fast-spins-up–slow-spins-down rule of the last section, provided, of course, that we can show that recombination is enhanced when slow partons' spins are down.

In this model a polarization asymmetry arises as the result of strong momentum ordering in the proton's wave function where the s quarks are wee and the u and d quarks fast. This ordering does not happen when recombination involves only sea quarks, all of whose wave functions are more or less similar, as will occur in antibaryon production from baryons. The average value of $\vec{\omega}_T$ vanishes so a zero polarization asymmetry is predicted—and seen—for $p \to \bar{\Lambda}$ at the same x_F and $P_\perp$'s where Λ asymmetry is nonzero.

Unfortunately, low-transverse-momentum recombination is a "soft" process (with an effective c.m. energy-squared $s \sim M^2/x_F$). This is the regime in which quarks and gluons interact strongly with one another and perturbation theory breaks down. We have not been able to carry out an honest calculation of the polarization asymmetry. We can, however, offer several simplified semiclassical pictures which suggest that the sign of Λ polarization is negative and hint at an origin of the observed x_F-$P_\perp$ structure.

Let us first consider the reaction $p+s \to \Lambda + u$ as the scattering of a spin-$\frac{1}{2}$ particle by an external potential in semiclassical approximation. The in state of the particle is the s quark, originally a wee parton in the wave function of the beam; the out state is the Λ. The normal to the p-Λ scattering plane is $\hat{n} = -\hat{p}_s \times \hat{p}_\Lambda$. The potential represents the attraction of the ud diquark for the s quark: at long distance it vanishes, screened by the presence of other partons which wish to recombine with the s quark. Thus $V(r)<0$ and $\lim_{r\to\infty} V(r) \to 0$.

In potential scattering

$$\vec{S}\cdot\vec{\omega}_T = -\frac{1}{r}\frac{\partial V}{\partial r}\vec{L}\cdot\vec{S}$$

with $\vec{L}$ the s quark's orbital angular momentum. As the potential is attractive, a semiclassical argument[12] shows $\vec{L}\cdot\hat{n}<0$ (see Fig. 3). If V is attractive, $-(1/r)\partial V/\partial r$ is also attractive, and the effective potential felt by the s quark,

$$V_{\rm eff} = V(r) - \frac{1}{2m^2}\frac{1}{r}\frac{\partial V}{\partial r}\vec{L}\cdot\vec{S}$$

is more attractive if $\vec{L}\cdot\vec{S}>0$. As there will be more scattering if $|V_{\rm eff}|$ is larger, and as that situation will occur if $\vec{L}\cdot\vec{S}>0$ and $\hat{n}\cdot\vec{S}<0$, the difference between scattering in the two spin states and hence the polarization asymmetry for the scattering $s\to\Lambda$ will be *negative* with respect to the p-Λ scattering plane.

This argument is essentially geometric, so we expect that the sign of the asymmetry in any realistic model will be the same as we find here. An important caveat arises when we study the problem of a Dirac particle interacting with an external potential. If the potential is a world scalar

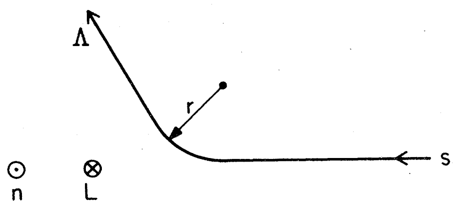

FIG. 3. Semiclassical trajectory of a particle in an attractive potential showing the distance of the orbit from the origin $\vec{r}$, and the orientations of the scattering plane $\vec{n}$ (up and out of the page) and of the particle's orbital angular momentum $\vec{L}$ (into the page).

then the only $\vec{L}\cdot\vec{S}$ term is the Thomas term. If the potential is the fourth component of a four vector (the familiar case of a Coulomb potential, for example) the sign of the $\vec{L}\cdot\vec{S}$ term—and of the asymmetry—is reversed. So ascribing the polarization asymmetry to Thomas precession is only valid if the long-distance confining potential is a scalar. Evidence already exists from charmonium that the confining potential is not the fourth component of a four-vector,[13] and that observation may be relevant to the present discussion.

We may schematically compute the x_F and P_T dependence of the polarization asymmetry arising from the Thomas precession a different way, using old-fashioned perturbation theory. The relevant diagram is shown in Fig. 4. The diquark carries fraction x_D of the proton's momentum; the s quark a fraction x_s, with the Λ carrying fraction $x_F = x_D + x_s$. The scattering amplitude for $pp \to \Lambda X$ is inversely proportional to the energy difference between intermediate and final states,

$$A_S \propto \frac{1}{\Delta E_0 + \vec{\omega}_T \cdot \vec{S}}\,. \tag{3.3}$$

ΔE_0 is the energy difference in the absence of spin effects,

$$\Delta E_0 = (P_D{}^2 + m_D{}^2)^{1/2} + (P_s{}^2 + m_s{}^2)^{1/2} - (P_\Lambda{}^2 + m_\Lambda{}^2)^{1/2}\,.$$

Choosing our axis of quantization along the normal to the scattering plane we have

$$A_\uparrow \propto \frac{1}{\Delta E_0 + \frac{1}{2}\omega_T}\,,$$

$$A_\downarrow \propto \frac{1}{\Delta E_0 - \frac{1}{2}\omega_T}\,.$$

To leading order in ω_T the polarization asymmetry is then

$$P(p \to \Lambda) = -\frac{\omega_T}{\Delta E_0}\,. \tag{3.4}$$

Let us evaluate these terms in the infinite-momentum frame.

First, ΔE_0. At fixed x_s, x_D, and P_T we find

$$\Delta E_0 = \frac{1}{2x_F P}\left[\frac{m_D{}^2 + P_{TD}{}^2}{1-\xi} + \frac{m_s{}^2 + P_{Ts}{}^2}{\xi} - m_\Lambda{}^2 - P_{T\Lambda}{}^2\right] \equiv \frac{M^2}{2x_F P} \tag{3.5}$$

with $\xi = x_s/x_F$.

Now we evaluate ω_T. By taking a time average

$$\vec{\omega}_T = \frac{1}{\Delta t}\int dt\,\frac{(\vec{F}\times\vec{V})}{m} \tag{3.6}$$

$$= \frac{\langle \sin\theta\rangle}{\Delta t}\,\frac{\Delta P}{m}\,\hat{n}\,, \tag{3.7}$$

where ΔP is the change in momentum of the s quark,

$$\Delta P \sim (\tfrac{1}{3}x_F - x_S)P\,,$$

$\langle \sin\theta\rangle = P_T/P_\parallel^{\text{ave}}$ with $P_\parallel^{\text{ave}} \sim \frac{1}{2}(\frac{1}{3}x_F + x_s)P$ and $P_T^s \sim \frac{1}{2}P_T^\Lambda$, and Δt is a characteristic (boosted) recombination time $\Delta t \sim (P_\parallel^{\text{ave}}/m)\Delta x_0$, where Δx_0 is a distance scale on the order of a proton radius. Therefore,

$$\omega_T = \frac{1}{x_F P}\,\frac{3}{2\Delta x_0}\,\frac{(1-3\xi)}{[(1+3\xi)/2]^2}\,P_T\,. \tag{3.8}$$

To calculate an asymmetry both ω_T and ΔE_0 must be averaged in x_0 and P_T over the appropriate parton distributions,

$$P(p \to \Lambda) = -\left\langle \frac{\omega_T(\xi, x_F, P_T^s)}{\Delta E(\xi, x_F, P_T^s)}\right\rangle_{x_s, P_T^s \ldots}\,.$$

However, to carry out that average presupposes considerable knowledge about the parton wave functions. At the slender level of rigor at which we are working it suffices merely to calculate average values of x_S or P_T^s as functions of x_F and P_T, and replace ξ, P_T^s, and P_T^D in Eqs. (3.5) and (3.8) by these averages. When we do that, we see

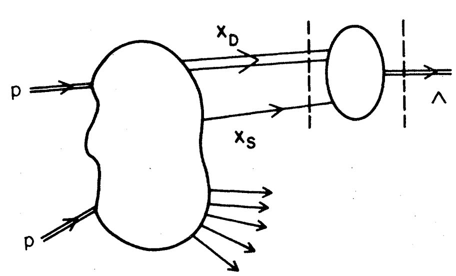

FIG. 4. Amplitude for the reaction $p+p \to \Lambda + x$. Sea quark and diquark carry momentum fractions x_s and x_D, respectively. The two dashed lines indicate E_i and E_f, the intermediate and final energies surrounding the confining term.

that the polarization asymmetry in this model for $p \to \Lambda$ is negative, approximately linear in P_T, and weakly dependent (through ξ) on x_F:

$$P(p \to \Lambda) = -A(P_T, x_F) P_T \,, \tag{3.9}$$

where the slope parameter is

$$A(P_T, x_F) = \frac{3}{\Delta x_0} \frac{(1-3\xi)}{[(1+3\xi)/2]^2} \frac{1}{M^2} \,. \tag{3.10}$$

We expect that the slope parameter will be larger at large x_F than at small x_F since the momentum change experienced by the s quark is $\frac{1}{3}x_F - x_s$, and x_s is cut off by the wave function of the strange sea in the proton. At large P_T, $M^2 \propto P_T^2$ and so the asymmetry flattens and eventually decreases.[14]

To quantify this statement we may do either of two things. We could build a model (along the lines of the recombination picture of Das and Hwa,[15] for example) and explicitly compute $\xi(x_F)$. This is a task which is much too involved for the discussion here. A better alternative is simply to guess a reasonable parametrization of $\xi(x_F)$. At x_F near 0, $\xi \sim \frac{1}{3}$ since the spectra of all flavors near $x \simeq 0$ is roughly equal in shape. For large x_F, ξ must be small—perhaps about 0.1—since x_S is cut off sharply by the proton's wave function. The simplest parametrization for ξ is one which linearly interpolates between these values: $\xi(x_F) \sim \frac{1}{3}(1-x_F) + 0.1x_F$. To complete our parametrization we choose $m_s = \frac{1}{2}$ GeV, $M_D = \frac{2}{3}$ GeV, and $\langle P_T^2 \rangle_{s \text{ or } D} = \frac{1}{4} P_{T\Lambda}^2 + \langle k_T^2 \rangle$ with $\langle k_T^2 \rangle = 0.25$ GeV^2. We show our predictions for the Λ asymmetry at fixed x_F versus P_T and at fixed P_T versus x_F along with the data of Ref. 5 in Figs. 5 and 6. The normalization of $A(x_F, P_T)$ requires $\Delta x_0 = 5$ GeV^{-1}, a reasonable hadronic size scale. (In fact, that is the radius of the proton in the bag model.[16]) Qualitative agreement is not bad. Given the crudeness of the model, it is probably not worthwhile trying to improve the fit.

In this picture the asymmetry for antibaryons is zero because the proton's sea wave function is symmetric between all the antiquarks. Thus, $\xi(x_F) = \frac{1}{3}$ for all x_F. This symmetry could be broken by quark-mass effects, but that seems not to be the case, at least for nonstrange versus strange masses.

Finally we should comment on the relative sizes of ϵ and δ, which were found to be equal in the model of Sec. II. In the Thomas-precession model their ratio is

$$\frac{\epsilon}{\delta} = \frac{J_s \omega_T(s)}{J_D \omega_T(D)}$$

which for $J_s = \frac{1}{2}$ and $J_D = 1$ is naively not unity. However, in VVS recombination we expect $\omega_T(D) < \omega_T(s)$ since the mean momentum of the diquark is greater than that of the quark. So it is not unreasonable that $\epsilon/\delta \sim 1$: numerical estimates for the model are probably reliable only as orders of magnitude. Similar considerations hold for the ϵ and δ of fast-quark–slow-diquark recombination.

IV. DISCUSSION

We have seen that all available data on polarization asymmetry can be accounted for by a simple quark model, and we have argued that the dynamics of the quark model are plausibly explained by the

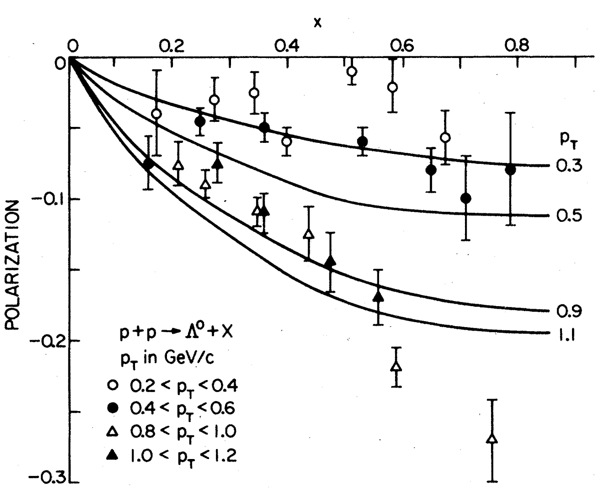

FIG. 5. Polarization asymmetry for $p \to \Lambda$ at fixed x_F vs $p_\perp$. Data from Ref. 5; rough fit from the model described in the text.

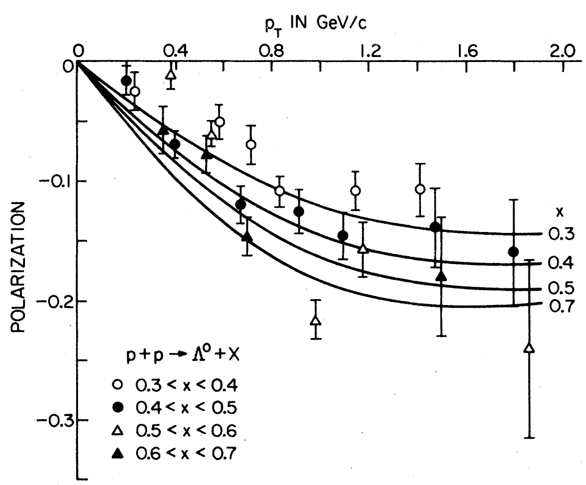

FIG. 6. Polarization asymmetry for $p \to \Lambda$ at fixed $p_\perp$ vs x_F. Data from Ref. 5; rough fit from the model described in the text.

effects of Thomas precession. While semiclassical arguments support this conclusion, the model should be considered preliminary until a more detailed calculation can be performed. In the meantime, several important experimental tests of the model may be easily carried out.

In our opinion the most important experiments involve measuring polarization asymmetries from antibaryon or meson beams. If the asymmetry is essentially kinematic at the quark level all antibaryon asymmetries should equal those of the corresponding baryons. Meson beams test the universality of the spin dependence of the recombination process: The reaction $K^- \to \Lambda$ is particularly important since all of the polarization asymmetry arises from the leading strange quark. If either of these predictions fail, the model is wrong.

The Thomas-precession model predicts P_T and x_F dependence of the slope parameter $A(x_F, P_x, P_T)$ which is in qualitative agreement with the trend of the data for $p \to \Lambda$. It will be useful to compare data for different final-state baryons to see whether the trends are similar and whether the x_F and P_T dependence of the slope is different in different reactions. Slight deviations from the predictions of Table I might be accounted for by the effects of different x dependences of different quark flavors, but the overall signs must not change. For example, the sea in mesons is thought to be flatter than in baryons, so $\xi(x_F)$ might not fall so quickly with increasing x_F. This would imply that the change in the slope parameter from small to large x_F might be smaller for mesons than for baryons.

Our model may provide an amusing test of a recently proposed model for hadronic charm production in which charm production proceeds out of a $(c\bar{c}uud)$ Fock state of the proton's wave function.[17] Since all quarks in the Fock state have the same velocity, the charm-quark distribution peaks at large x $[x_i \sim m_c/(2m_c + 2m_u + m_d) \sim 0.5]$ because its mass is so great. Recombination into a charmed baryon, a Λ_c, for example, at x_F, then involves deceleration of the charmed quark from x_i to Zx_F, where $Z \sim m_c/(m_c + m_u + m_d)$. Thus it preferentially recombines with its spin up, and the resulting polarization asymmetry is opposite to that of an ordinary Λ.

Finally, polarization asymmetry is an important source of information on the mechanism of hadronization. We have argued that the origin of the asymmetry is essentially kinematic. In addition, an asymmetry cannot occur without strong flavor ordering in rapidity in the evolving multiparton jet, and therefore it is a good probe of that flavor ordering. Small differences in the asymmetries of various final-state baryons may be important in establishing relative orderings of one or several sea partons or among several valence partons in the jet. In that respect measurements of polarization asymmetry can be as useful as measurements of correlations of particles in the jet. These same arguments suggest that polarization is a pervasive though subtle phenomenon in inclusive production, one that is present in a wide variety of reactions and experimental situations, and that its measurement can be used as a probe of the process of hadronization.

Note added in proof. A recent 70-GeV $K^+p \to \bar{\Lambda}X$ experiment reports $P(K^- \to \bar{\Lambda}) > 0$, but with an absolute magnitude much greater than $P(p \to \Lambda)$.[18]

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