Analysis of $\pi^- p \to \eta n$ scattering at high energies

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The differential cross sections for $\pi^- p \to \eta n$ including the most recent measurements at high energies and up to $-t \simeq 5$ $(\mathrm{GeV}/c)^2$ are fitted by using a Regge pole + cut model. It is also shown that the results obtained at Fermilab and Serpukhov are not mutually consistent.

The process $\pi^- p \to \eta n$ owing to its simplicity is considered to be very important theoretically. Although the Regge trajectories A_2 and $\delta(980)$ are exchanged in this process, the A_2 trajectory being higher is supposed to dominate at higher energies. The differential cross sections $d\sigma/dt$ for this reaction were measured in 1973 by Bolotov *et al.*[1] from 20 to 50 GeV/c. The four-momentum-transfer-squared interval was $0 \leq -t \leq 3$ $(\mathrm{GeV}/c)^2$ at 32.5 and 48 GeV/c and $0 \leq -t \leq 1.5$ $(\mathrm{GeV}/c)^2$ for the other incident momenta. In 1976 measurements on this reaction were carried out at Fermilab by Dahl *et al.*[2] from 20 to 200 GeV/c and for momentum-transfer squared in the range $0.004 \leq -t \leq 1.1$ $(\mathrm{GeV}/c)^2$. Recently Apel *et al.*[3] have reported measurements of the production cross section of η mesons in $\pi^- p$ collisions,

$$\pi^- p \to \eta n \quad , \qquad \eta \searrow 2\gamma$$

carried out at the 70-GeV/c Serpukhov accelerator for five values of the incident pion momentum in the 15–40 GeV/c range and up to $-t \leq 5$ $(\mathrm{GeV}/c)^2$. The cross sections for this reaction had already been measured for momentum-transfer squared $-t \leq 1$ $(\mathrm{GeV}/c)^2$. Progress into the range of larger momentum transfer was prevented by lack of experiments with sufficient statistical significance: 1000–5000 events were reported in some papers, about 30 000 in another paper. In Ref. 3, more than 300 000 events were recorded. This made it possible to extend the measurement of the reaction $\pi^- p \to \eta n$ to $-t \leq 5$ $(\mathrm{GeV}/c)^2$ and also to make a very detailed study of the region of small momentum transfer, where a marked drop is noted in the t dependence of the cross section when $t \to 0$. In fact, in early experiments where the statistics were limited to about a thousand events, it was shown that at small $-t$ the increase in the differential cross section with decreasing $-t$ is slowed down and the cross section levels off to a plateau. However, with the increase in statistical accuracy, a considerable reduction in cross section was revealed at small $-t > 0$. This marked drop in cross section at small $-t$ has been observed at all momenta. There are no irregularities in the behavior of the cross section in this region. Any spikes in the individual points of the previous experiments may be ascribed to statistical fluctuations.

The main characteristics of this high-energy angular distribution are the following.

(1) The differential cross section has a turnover near the forward direction.

(2) The differential cross section decreases with an increase in energy.

(3) For $0.1 \leq -t < 1$ $(\mathrm{GeV}/c)^2$, $d\sigma/dt$ decreases exponentially with a slope of about 8 $(\mathrm{GeV}/c)^{-2}$.

(4) The behavior of $d\sigma/dt$ changes quite sharply near $-t = 1$ $(\mathrm{GeV}/c)^2$. In fact for $-t \simeq 1$ $(\mathrm{GeV}/c)^2$, a kink is observed in the differential cross section which then falls much more slowly with a slope ~ 2 $(\mathrm{GeV}/c)^{-2}$ at $p_{\mathrm{lab}} = 40$ GeV/c and 1.5 $(\mathrm{GeV}/c)^{-2}$ at $p_{\mathrm{lab}} = 25$ GeV/c.

We will show that all these characteristics can be described by using a pole + cut model with phenomenological residue functions. The fact that the energy dependence of $d\sigma/dt$ in the range $0 \leq -t < 1$ $(\mathrm{GeV}/c)^2$ is found to be different from that observed in the range after break near $-t = 1$ $(\mathrm{GeV}/c)^2$ makes the problem very interesting as the A_2 exchange trajectory cannot explain the energy dependence throughout the measured range. This variation in energy dependence for different domains of momentum transfer makes it imperative to assume that either another trajectory or a cut is also contributing significantly in this reaction. It is this assumption that can unravel the mystery in the energy behavior of the process $\pi^- p \to \eta n$ in its most recent measurements. We have assumed that a cut is also contributing substantially in this reaction.

If we assume that the $A_2 \otimes P$ cut contributes significantly only to the flip amplitude, then the two independent helicity amplitudes for this reaction may be written as

$$T_{++}(s,t)=\gamma^{A_2}_{++}(t)\xi_{A_2}(t)s^{\alpha_{A_2}(t)}\sqrt{\mu\mathrm{b}}\,\mathrm{GeV},$$

$$T_{+-}(s,t)=\sqrt{-t}\left(\gamma^{A_2}_{+-}(t)\xi_{A_2}(t)s^{\alpha_{A_2}(t)}+\gamma^{c}(t)\xi_{A_2}(t)\xi_P(t)\frac{s^{\alpha_c(t)}}{\ln s}\right)\sqrt{\mu\mathrm{b}}\,\mathrm{GeV},$$

where $\gamma^{A_2}_{++}(t)$, $\gamma^{A_2}_{+-}(t)$, and $\gamma^c(t)$ are unknown theoretically, $\xi(t)$ is the signature factor, $\alpha_{A_2}(t)$ stands for Regge trajectory A_2 and

$$\alpha_c(t)=\alpha_{A_2}(0)+\alpha_P(0)-1+\frac{\alpha'_{A_2}\alpha'_P t}{\alpha'_{A_2}+\alpha'_P}$$

gives the position of the branch point α_c. The differential cross section $d\sigma/dt$ and the polarization P are given by

$$\frac{d\sigma}{dt}=(|T_{++}(s,t)|^2+|T_{+-}|^2)/sp^2\ \mu\mathrm{b}\,(\mathrm{GeV}/c)^{-2},$$

$$P=2\,\mathrm{Im}(T_{++}T^*_{+-})/sp^2\frac{d\sigma}{dt}.$$

Here P is the c.m. momentum of the incident pion. The turnover near the forward direction suggests that the nonflip helicity amplitude plays a significant role near $t=0$.

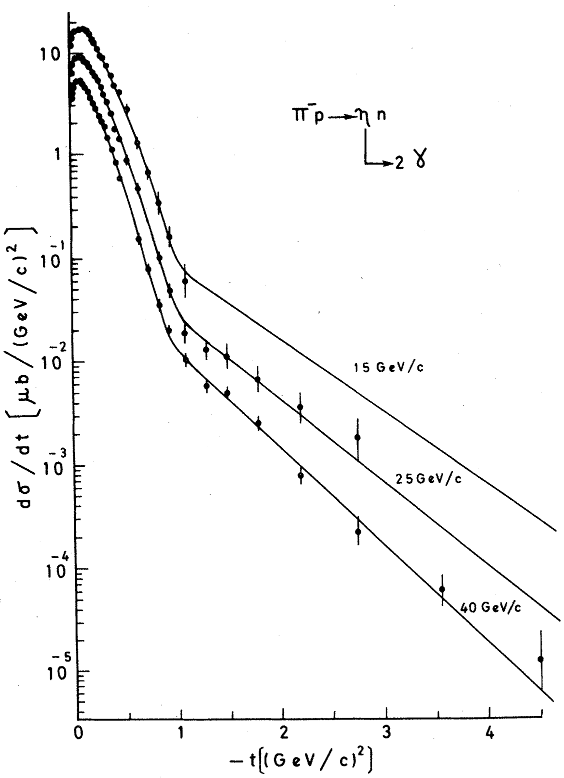

FIG. 1. Fits of the present model (curves) to the experimental differential-cross-section data of Apel *et al.* (Ref. 3) for $\pi^-p\to\eta n$ reaction at incident pion momenta $15\le P_{\mathrm{lab}}\le 40$ GeV/c and for the interval $0\le -t\le 4.5$ (GeV/c)^2.

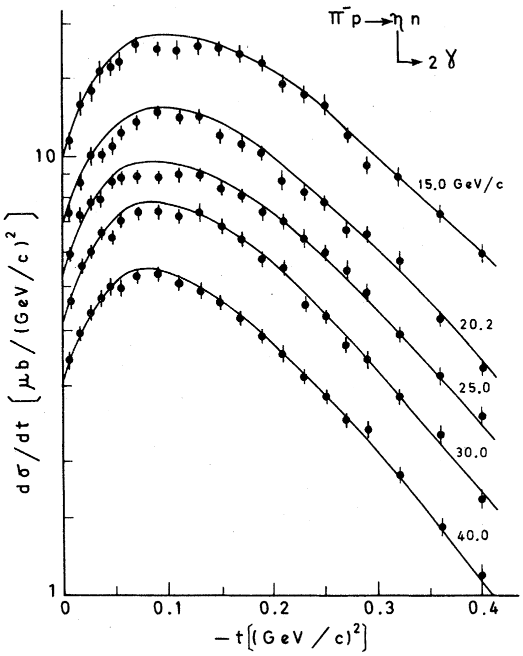

FIG. 2. The same as in Fig. 1 but for small $-t$.

Following Saleem *et al.*,[4] we assign phenomenological values to the γ's. We then find that a good fit with experiment is obtained by the following choices:

$$\gamma^{A_2}_{++}(t)=11.33e^{1.17t}\sqrt{\mu\mathrm{b}}\ \mathrm{GeV},$$
$$\gamma^{A_2}_{+-}(t)=65.6e^{4.57t-0.1t^2}\,c\sqrt{\mu\mathrm{b}},$$
$$\gamma^c(t)=11.48e^{0.53t}\,c\sqrt{\mu\mathrm{b}},$$

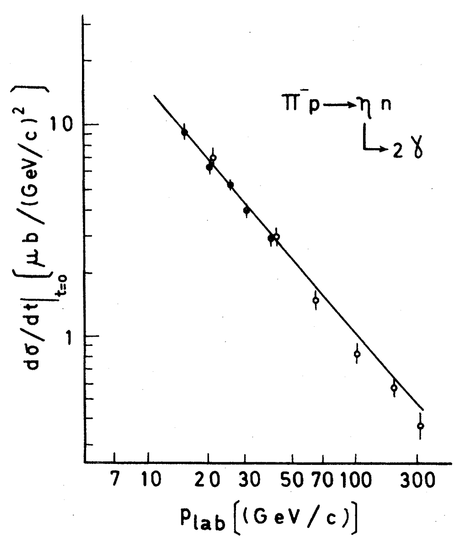

FIG. 3. The forward differential cross section for $\pi^-p\to\eta n$ plotted vs $-t$. The solid curve represents the results obtained by using the model described in the test. The experimental points have been taken from Apel *et al.* (Ref. 3).

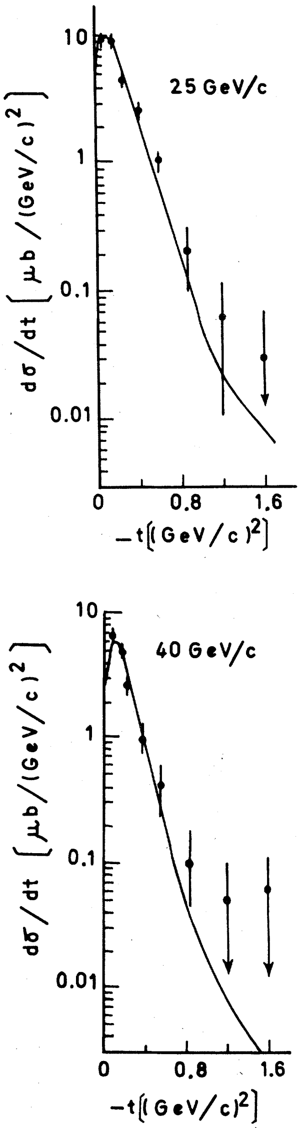

FIG. 4. Fits of the present model to the experimental differential-cross-section data of Bolotov *et al.* (Ref. 1) for $\pi^- p \to \eta n$ at incident pion momenta $P_{lab} = 15$ and 40 GeV/c, and for the interval $0 \leq -t \leq 1.6$ (GeV/c)^2.

where t is in (GeV/c)^2. The branching ratio and the $1/\sin(\pi\alpha/2)$ factors have been absorbed in γ's. The equation for A_2 and the Pomeron P have, respectively, been taken as

$$\alpha_{A_2}(t) = 0.4 + 0.7t,$$

$$\alpha_P(t) = 1 + 0.2t,$$

where t is in (GeV/c)^2. The curves in Figs. 1–3 show the results of the fit of the present model to the experimental data of Ref. 3. Figure 1 gives $d\sigma/dt$ plotted vs $-t$. Figure 2 shows the differential cross section in the small-momentum-transfer region plotted against $-t$. Figure 3 exhibits the forward differential cross section plotted against $-t$. In all the figures, we find satisfactory agreement.

The measurements of Bolotov *et al.*[1] for $d\sigma/dt$ can also be fitted by the Regge pole + cut model using exactly the same parametrization as has been used for the data obtained by Apel *et al.*[3] The agreement between our model and the experimental data is good. This is exhibited graphically for $P_{lab} = 25$ and 40 GeV/c in Figs. 4(a) and 4(b), where $d\sigma/dt$ has been plotted vs $-t$. We thus conclude that the Serpukhov data of Refs. 1 and 3 are consistent within errors.

It is interesting to note that the forward differential cross sections for $\pi^- p \to \eta n$ as extrapolated from the measurements of Bolotov *et al.*[1] are not always consistent with the corresponding extrapolated values of Apel *et al.*[3] This is due to the fact that owing to large errors in the $d\sigma/dt$ measurements by Bolotov *et al.*,[1] their extrapolated values for $d\sigma/dt$ at $t = 0$ are not very reliable.

It is also found that extrapolated values of $d\sigma/dt$ at $t = 0$ as obtained from Fermilab data[2] and by Apel *et al.*[3] are consistent with each other within errors (Fig. 3). Moreover, for $t \neq 0$, the differential cross sections at $P_{lab} \simeq 20$ GeV/c as given in Refs. 2 and 3 are compatible. However, for $P_{lab} \simeq 40$ GeV/c, the results for $t \neq 0$ as obtained in the two laboratories start differing from each other as we move away from $t = 0$. We find that our model with the parameters used for the Serpukhov data[1,3] cannot fit the Fermilab data[2] at $P_{lab} \geq 40$ GeV/c. This difference becomes quite appreciable for high values of laboratory momentum of the incident pions. In fact, an examination of the data from the two laboratories shows that they are not always consistent with each other. A renormalization of the data from one of the two laboratories would not help because the energy dependence of the data from the two laboratories is not the same. It is therefore very essential that a thorough probe for the causes of this discrepancy be made.

It would be interesting at this stage to determine the effective trajectory from $d\sigma/dt$ measurements at Serpukhov.[3] Within the framework of the Regge theory, the energy dependence of the differential cross sections for any reaction can be described in a single-pole approximation as

$$\frac{d\sigma}{dt} = B(t) s^{2\alpha(t) - 2}.$$

Here $\alpha(t)$ is the effective trajectory. The values found for the $\alpha(t)$ trajectory for the reaction $\pi^- p \to \eta n$ are given in Fig. 5. For $-t \leq 0.7$ (GeV/c)^2 the points we obtain are located close to the straight line passing in the region $t > 0$ through the A_2 mesons. This trajectory does in fact also provide a reasonable description of the cross sections at lower momenta. However, the effective trajectory $\alpha(t)$ calculated according to the data of Ref. 2 in the region of higher energies is systematically lower compared with the trajectory deduced using only $P_{lab} \leq 40$ GeV/c data. For $0 \leq -t \leq 0.7$ (GeV/c)^2 the trajectory obtained from the data of Ref. 3 decreases linearly with an increase in $-t$: $\alpha(t) \approx 0.4 + 0.7t$. But at larger $-t$ the trajectory has a very complex shape: Apel *et al.*[3] have stated that over the range $0.7 \leq -t \leq 1.6$ (GeV/c)^2 it is practically constant; subsequently it falls again.

We have recalculated the last two points of the effective trajectory $\alpha(t)$ from the experimental values of $d\sigma/dt$ in Ref. 3 in the range $2.2 \leq -t \leq 2.75$ (GeV/c)^2 and have found that probably some error has crept into the calculations of Apel *et al.*[3] The points corresponding to the effective trajectory $\alpha(t)$ as obtained from the data of Ref. 3 have been shown in Fig. 5. The effective trajectory as calculated from our model has been represented by a solid curve. We find that the effective trajectory as calculated by using our model is consistent with the experimental data of Ref. 3.

The intercept of the effective trajectory for the Fermilab data as obtained by Dahl *et al.*[2] is found to be 0.371 ± 0.008 while intercept for the effective trajectory for the Serpukhov data is 0.4. It is for this reason that the extrapolated forward differential-cross-section data from the two laboratories are consistent within errors. However, the slopes of the effective trajectories for the Serpukhov and Fermilab data being quite different, the results disagree as we move away from $t = 0$.

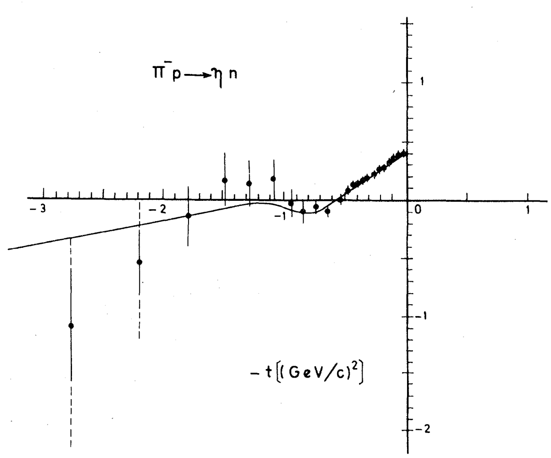

FIG. 5. The effective trajectory $\alpha(t)$ plotted vs $-t$. The experimental values of $\alpha(t)$ have been taken from the data of Apel *et al.* (Ref. 3).

A more interesting situation has arisen by the most recent measurements of $d\sigma/dt$ for $\pi^- p \to \eta' n$ at Serpukhov.[5] These measurements have been made in the momentum range $15 \leq P_{lab} \leq 40$ GeV/c and have been extended to large values of $-t$. For this reaction, the previous investigations had been made at a statistical level of only 50 $\eta' \to 2\gamma$ decays. In Ref. 5, however, a total of 6000 decays were recorded. This made it possible to widen the range of the determination of differential cross sections to $-t \sim 1.8$ (GeV/c)^2 and also to study the region of small momentum transfers. We can obtain the differential cross sections for $\pi^- p \to \eta' n$ from those for $\pi^- p \to \eta n$ as follows.

The η and η' mesons can be expressed as combinations of pure states belonging to the unitary singlet η_0 and octet η_8 of pseudoscalar mesons:

$$|\eta\rangle = D_1(-\sin\beta_1|\eta_0\rangle + \cos\beta_1|\eta_8\rangle),$$

$$|\eta'\rangle = D_2(\cos\beta_2|\eta_0\rangle + \sin\beta_2|\eta_8\rangle).$$

At high energies, where the difference in the phase spaces of reactions $\pi^- p \to \eta' n$ and $\pi^- p \to \eta n$ is insignificant, the value of $R(\eta'/\eta)_{t=0}$, the ratio of

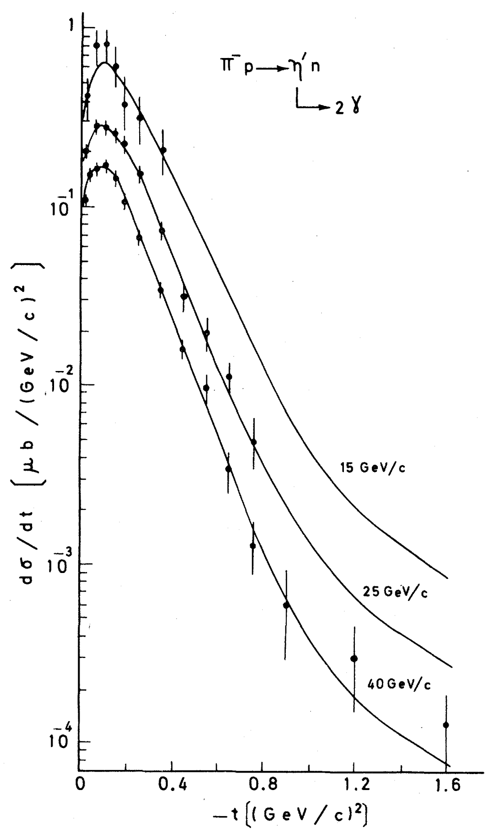

FIG. 6. Fits of the present model to the experimental differential-cross-section data of Apel *et al.* (Ref. 5) for $\pi^- p \to \eta' n$ at incident pion momenta $15 \leq P_{lab} \leq 40$ GeV/c and in the interval $0 \leq -t \leq 1.6$ (GeV/c)^2.

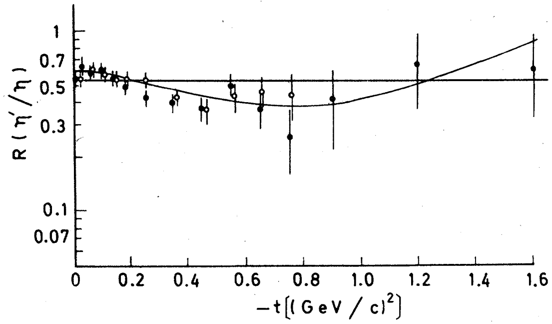

FIG. 7. Fits of the present model to the ratios of the differential cross sections of reactions $\pi^- p \to \eta' n$ and $\pi^- p \to \eta n$ at various t and for $P_{lab} = 25$ GeV/c (open circles) and $P_{lab} = 40$ GeV/c (solid circles).

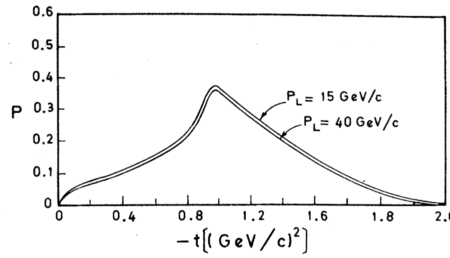

FIG. 8. The polarization for the reaction $\pi^- p \to \eta n$ at $P_{lab} = 15$ and 40 GeV/c as predicted by our model.

the differential cross sections of these reactions, is linked with the mixing angles by the relation

$$R(\eta'/\eta)_{t=0} = \frac{D_2 \sin(\beta_2 + \beta_0)}{D_1 \cos(\beta_1 + \beta_0)},$$

where $\tan\beta_0 = \sqrt{2}$.[6-8] If there is a full overlap between the singlet and orbital wave functions, i.e., $D_2 = D_1$, then the above formula reduces to

$$R(\eta'/\eta)_{t=0} = \frac{\sin(\beta_2 + \beta_0)}{\cos(\beta_1 + \beta_0)}.$$

A recent phenomenological analysis of the pseudoscalar nonet, based on chromodynamics,[9] shows that the mixing angles β_1 and β_2 are equal to $-17°$ and $21°$, respectively. This implies $R(\eta'/\eta)_{t=0} = 0.5$. Assuming that this ratio does not depend upon t and noting that $B(\eta \to 2\gamma) = 0.38 \pm 0.01$ and $B(\eta' \to 2\gamma) = 0.020 \pm 0.003$, we can calculate $d\sigma/dt$ and $R(\eta'/\eta)$ for the reaction $\pi^- p \to \eta' n$ from the data of process $\pi^- p \to \eta n$. The results are shown in Figs. 6 and 7, respectively. The agreement is found to be quite good.

We may take this opportunity to remark that assuming that only the ρ and A_2 trajectories were exchanged in $\pi^- p \to \pi^0 n$ and $\pi^- p \to \eta n$, respectively, it is believed that the ρ and A_2 trajectories which would fit the Fermilab data alone are

$$\alpha_\rho(t) = 0.48 + 0.80t,$$

$$\alpha_{A_2}(t) = 0.37 + 0.80t.$$

The spacing of 0.11 in the intercepts of two trajectories clearly indicate a substantial break in the exchange degeneracy of the two trajectories. In fact, the same value of the intercept for ρ can fit the experimental data from Fermilab for the difference of total cross sections for $\pi^- p$ and $\pi^+ p$. Thus if we confine ourselves to the Fermilab data, then the concept of exchange degeneracy which has often been taken as an article of faith must be considered a poor approximation to reality. On the other hand, the present analysis has shown that in $\pi^- p \to \eta n$, in addition to A_2, there is a vital contribution from the $A_2 \otimes P$ cut. The equation of the A_2 trajectory then comes out to be

$$\alpha_{A_2}(t) = 0.4 + 0.7t.$$

An analysis of most recent measurements of $\pi^- p \to \pi^0 n$ by Apel *et al.*[10] has not yet been made. If another trajectory contributes substantially at all values of $-t$ to this process, then the equation of the trajectory may be quite different from the one usually assigned to it. The exchange degeneracy of ρ and A_2 therefore still remains an open question.

According to the present model, the polarization in general is nonzero. We have calculated the polarization at $P_{lab} = 15$ and 40 GeV/c. The results are exhibited in Figs. 7 and 8. The theoretical curves show that the polarization is energy dependent and has zero in the vicinity of $-t = 2$ (GeV/c)^2. The experimental measurements of polarization for $\pi^- p \to \eta n$ at those energies have not yet been made. An accurate measurement of P at these momenta will further check the validity of our model.

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