# Topological theory of electromagnetism

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A recently proposed topological theory of strong interactions is extended to electromagnetism. This extension is constructed so that, for reactions involving only leptons and photons, the topological expansion coincides with the usual perturbation expansion of QED. The lowest term in the topological expansion corresponds to pointlike elementary hadrons (as well as leptons); hadronic structure appears in nonleading topological terms. A natural explanation emerges for the quantization of electric charge and for the observed charges of leptons and hadrons. Fractional charges nowhere appear even though there is a notion of quark-photon coupling. No topological explanation has yet emerged for the zero photon mass or for the value of the fine-structure constant.

## I. INTRODUCTION

There has recently been proposed a theory of strong interactions based on an $S$-matrix topological expansion.[1] Components of the expansion are associated with two-dimensional surfaces and are ordered according to topological complexity or "entropy"; all components are "connected sums" of zero-entropy components. This paper describes an extension of topological particle theory to electromagnetism, achieved by associating a selected set of "minimum-entropy" topological components with "elementary" single-photon amplitudes. Minimal electromagnetic components cannot be obtained by connected sums from zero entropy; but any minimal amplitude is determined by a single universal parameter—the elementary unit of electric charge—and all higher electromagnetic components are to be obtained by connected sums from minimal components.

Our theory describes the electromagnetic interactions of leptons as well as hadrons and we conjecture that it reduces to perturbative (Feynman) quantum electrodynamics (QED) when hadrons are ignored. That is, the components of a Feynman-diagram lepton-photon expansion can be placed in correspondence with lepton-photon components of our topological expansion. Topological theory assigns to leptons a representation similar to that of the "quark" constituents of hadrons, but significant differences occur in our theory between hadron-photon and lepton-photon interactions.

Electric-charge-related highlights of topological theory have been described in a previous short paper,[2] an explanation there being given for the electric charges of both leptons and hadrons. This part of the story, being essential to the complete electromagnetic framework, will be repeated here but the bulk of this paper will concentrate on further issues, some of which we sketch now to prepare the reader.

A feature of topological theory is the appearance at the zero-entropy base of a relatively small number of "elementary" hadrons. (Most physical hadrons have no elementary counterpart and owe their existence to higher-order topologies. An example is the deuteron.) At the minimal electromagnetic level these elementary hadrons, like elementary leptons, are electromagnetically "structureless"—the coupling of each to a photon being characterized entirely by its electric charge together with its spin and energy-momentum. (In Lagrangian field theory the analog would be "point coupling".) The observed electromagnetic structure of physical hadrons, as for physical leptons, arises from higher-order topology. The reader accustomed to the idea that a baryon is built from three quarks and a meson from a quark-antiquark pair may be puzzled by the notion of "structureless" hadrons. It is necessary in this connection to realize that *topological quarks*, while carrying electric charge and spin, do *not* carry energy and momentum. If such a concept is to achieve meaning in topological theory it will be as an approximation associated with high entropy. As emphasized in Ref. 1 the low-entropy aspects of topological theory are complementary to perturbative quantum chromodynamics (QCD)—which is meaningful at high transverse momentum or high $q^2$ for intermediate photons. Low entropy, roughly speaking, means low $q^2$; only there is it useful to focus attention on leading terms of the topological expansion.

Topological quarks and elementary leptons are associated with certain triangles on a triangulated "quantum surface", but not all quantum triangles correspond to quarks or leptons. Baryon number is carried by "core triangles" and in our theory these core triangles must also carry electric charge (although no spin). Photons are coupled not only to quarks and to leptons but also to core triangles, and the rules for core-photon coupling constitute an essential aspect of the theory to be described in this paper. Also essential is the topological representation of photon spin and parity or, equivalently, of "vector currents." This representation will be achieved through topological orientations already introduced for strong interactions.

A distinction between electromagnetic and strong-interaction topology is the occurrence within the latter of zero-entropy *contractions* that blur meaning for the number of "intermediate" elementary hadrons. Because of contractions, single hadrons and multihadron resonances are often topologically indistinguishable. This characteristic of strong interactions leads to quark "confinement", i.e., to the impossibility of associating a single triangle with a hadron or, equivalently, of associating energy-momentum with an individual quark triangle. Conversely, the absence of contractions from electromagnetism not only allows association of certain single triangles with leptons but means that intermediate leptons and photons never disappear from the topology. Any lepton-photon Landau discontinuity graph generated by a connected sum may immediately be associated with a Feynman amplitude graph. At the same time, because our theory describes simultaneously leptons, photons, and hadrons, readers should not expect to be able to translate the entire content into familiar Feynman-diagram language. Energy-momentum is not carried by individual quarks, and intermediate hadrons are often invisible within the topology.

## II. LEPTON AND PHOTON AREAS ON THE QUANTUM SURFACE

As in Eq. (1) of Ref. 1, and maintaining the same notation, we postulate the topological expansion of a connected part $M_{fi}$ in a basis of "elementary particles":

$$M_{fi} = \sum_{\tau,\kappa} {}^{\tau}M_{fi}^{\kappa} . \tag{2.1}$$

The collection of elementary hadrons described in Ref. 1 is now to be augmented by elementary leptons and photons, but the topological index $\tau$ and the order index $\kappa$ retain their original meaning. All aspects of the topology described in Ref. 1 are maintained; no further features will be needed. It is necessary, however, to specify *a priori* the areas representing elementary leptons and photons on the quantum surface. Here we allow ourselves to be guided by experiment as well as by consistency requirements. We do not yet know how to deduce the existence of photons and leptons purely from consistency. Put differently, we are not proposing to explain the value of the elementary electric-charge unit, and setting this parameter equal to zero would effectively eliminate photons and leptons from the theory. We are also at this stage not explaining the zero photon mass.

Given the success of previous theories which assign a parallel status to quarks and to leptons, it is natural to postulate that a lepton, like a quark, corresponds to a single "peripheral" triangle on a quantum surface. For economy and for definiteness we propose that, standing alone, a lepton triangle is indistinguishable from a quark triangle. There is a clockwise or anticlockwise orientation, and the triangle is cut by the classical belt (the boundary of a classical surface $\Sigma_c$) which passes through one vertex and the opposite side. Each of the two edges not cut by the belt is oriented (Fig. 24 of Ref. 1).

The difference between quarks and leptons arises from the fact that lepton areas on the quantum surface, when combined into multiparticle areas, do not admit contractions. The contraction requirement for hadron areas demands at least two triangles to build a momentum-carrying particle (Sec. VI of Ref. 1), but a single triangle can represent a lepton. The positions of Landau-graph ends along the belt reflect the distinction. The belt always divides into particle pieces with one Landau arc ending within each piece. A *hadron* arc ends on a belt-intersected *edge* separating two triangles within the hadron area, while a *lepton* arc ends *inside* the triangle that represents the lepton. Thus the end of a hadron Landau arc is always "shared" by two adjacent quantum triangles, while a lepton-arc end "belongs" to a single triangle.

It is possible to expand the zero-entropy ordered Hilbert space (Secs. V and VI of Ref. 1) to admit single-lepton disk channels, which constitute trivial sectors. Each single-triangle channel disk can combine with its antidisk to cover a sphere representing a lepton propagator, but at zero entropy no lepton interactions are possible. Furthermore, no connected sums based on zero entropy can generate lepton interactions: A connected sum of 2 $\Sigma_Q$'s each covered by two triangles cannot produce a $\Sigma_Q$ covered by more than two triangles. In order to have lepton interactions we must first have photons.

An experimental characteristic of a photon is its lack of the internal quantum numbers (baryon number, lepton number, and flavor) that are carried by bounded areas of the quantum surface. We therefore represent the elementary photon as a complete sphere, disconnected from the remainder of the quantum surface and without boundary. We maintain the general patchwise orientation of $\Sigma_Q$ by dividing the photon sphere into two triangles of opposite orientation but because there is no photon perimeter there is no edge flavor. We assume that each of the two triangles building a photon sphere is cut by the belt in the manner characteristic of peripheral triangles rather than core triangles, with the photon Landau arc ending on the belt-intersected edge (not on the trivial vertex), as for a meson. Since any zero-entropy $\Sigma_Q$ is a single sphere, injection of photons into the topology raises the entropy.

Again allowing ourselves to be guided by more or less successful previous theories, we proceed next to postulate a set of minimum-entropy surface pairs $\Sigma^{0,\text{EM}} = (\Sigma_Q^{0,\text{EM}}, \Sigma_C^{0,\text{EM}})$ to represent the interaction of single photons with elementary hadrons and leptons. We here state the rules for $\Sigma_Q^{0,\text{EM}}$.

(a) $\Sigma_Q^{0,\text{EM}}$ consists of two disconnected spheres, one photon sphere and one zero-entropy hadron or lepton sphere. (Photon and lepton spheres each contain two triangles; a hadron sphere contains at least four triangles.)

(b) The two belt segments on the photon sphere, each a diameter of one triangle, have *opposite* ortho-para orientation (associated with the classical-patch orientation).

(c) On the hadron or lepton spheres the corresponding belt orientation is uniform, as at zero entropy, except for *one* triangle within which the belt orientation is reversed. This exceptional triangle may be either a core triangle or a peripheral triangle.

It should be recalled that strong-interaction topologies always have a uniform ortho or para orientation throughout any hadron disk, regardless of how high the entropy. The orientation reversal of a *single* triangle in an elementary electromagnetic interaction represents a kind of "locality", although this is far from the space-time locality of field-theoretic interactions. Energy-momentum is still carried by the hadron as a whole (or at least by pairs of triangles); it is the spin-parity content that is "locally" affected by the photon.

## III. MINIMAL ELECTROMAGNETISM ON $\Sigma_C$ FOR MESONS AND LEPTONS

In the theory described in Ref. 1 any quantum surface $\Sigma_Q$ is both oriented and patchwise oriented, and the same is true for each sheet of the classical surface $\Sigma_C$. We would like to maintain these properties for electromagnetism because they relate to $P$, $C$, $T$ symmetry (Appendix D of Ref. 1) as well as to the definition of Landau connected sums (Appendix B of Ref. 1). The global orientation of $\Sigma_C$, called HR (Harari-Rosner) orientation, is coupled to the orientations of $\Sigma_Q$ through HR arcs—or "quark lines"—which connect mated pairs of peripheral triangles on $\Sigma_Q$, each quark line being given a conventional direction from an anticlockwise triangle toward a clockwise triangle. [We adopt the convention that the global orientation of $\Sigma_Q$ is "clockwise". To say that the orientation of a triangle is clockwise (anticlockwise) really means that it agrees (disagrees) with the global orientation.] As an introduction to the classical surface $\Sigma_C^{0,\text{EM}}$ associated with minimal electromagnetism, let us look at an analogous purely mesonic (strong-interaction) $\Sigma_C$, belonging to a $\Sigma$ where $\Sigma_Q$ consists of two spheres and the belt is correspondingly disconnected. (This, of course, is a nonzero-entropy $\Sigma$.) The classical surface with two boundary components is a single-sheet cylinder, as will be the case for any $\Sigma_C^{0,\text{EM}}$ where the photon couples to mesons or to leptons.

Consider a four-meson $(A, B, C, D)$ connected part with three mesons $(A, B, C)$ on one sphere and a fourth meson $(D)$ on the other. Each meson is built from two quantum peripheral triangles (quarks) so the total number of belt segments is

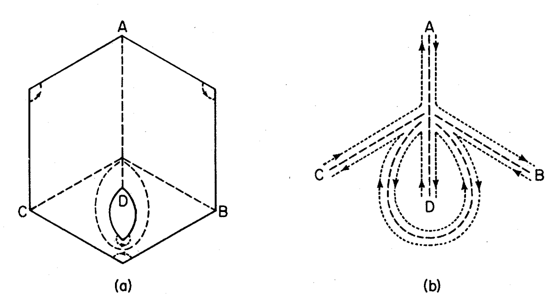


FIG. 1. (a) Two-boundary (cylindrical) classical surface for a four-meson interaction. Dashed lines are Landau arcs. Dotted lines are HR arcs. (b) The corresponding thickened Landau graph $[th(L)]$.

eight, six forming one boundary and two forming the other. The classical surface is shown in Fig. 1(a), together with the embedded single-vertex Landau graph and the HR arcs whose directions correspond to a clockwise HR (global) orientation of $\Sigma_C$ as the figure is drawn. (The figure may of course be "turned over" so as to make the global orientation appear anticlockwise.) As explained in Ref. 1, a closed Landau loop is always present on a strong-interaction cylinder. Figure 1(b) shows the corresponding thickened Landau graph, where a pair of oppositely directed quark lines runs along the boundaries of each strip that embeds a Landau arc. Because only mesons are involved, the topologies 1(a) and 1(b) are almost equivalent. When baryons or baryoniums occur there continues to be a well-defined HR orientation on each separate sheet of $\Sigma_C$ as well as on the thickened Landau graph, although the relation of $th(L)$ to $\Sigma_C$ is not as simple as in Fig. 1.

The quantum surface $\Sigma_Q^{0,\mathrm{EM}}$ of a reaction describing three mesons and a photon consists of two spheres, just as in the foregoing example. A $\Sigma_C^{0,\mathrm{EM}}$ that couples a photon to three mesons $(A, B, C)$ then should look similar to Fig. 1(a), with meson $D$ replaced by the photon, except that one does not expect the closed Landau loop in an elementary topology. In particular, if the three mesons $A$, $B$, $C$ were replaced by two leptons one would expect to be describing the basic amplitude of quantum electrodynamics, whose Feynman graph is a cubic tree (Fig. 4, below). Since quarks are supposedly similar to leptons, and Landau graphs are closely related to Feynman graphs, we anticipate *tree* Landau graphs to be embedded on $\Sigma_C^{0,\mathrm{EM}}$.

But if the Landau graph is a tree the HR arcs of Fig. 1(a) cannot lie along the boundary of $th(L)$. An arc with such a capacity must connect the two boundary components; an HR arc never couples two disconnected components of $\Sigma_Q$. It furthermore seems unnatural to attach HR arcs to a photon boundary. The photon's triangles are not peripheral and are not assigned spin and flavor—quantities attachable to quark lines. The photon triangles are nevertheless oriented, so we propose attaching to each a directed arc lying in $\Sigma_C$, the direction being *into* $\Sigma_C$ for the anticlockwise quantum triangle and *out of* $\Sigma_C$ for the clockwise triangle, just as for a hadronic HR arc. We shall refer to this new kind of object as an "active charge arc"; the connection with electric charge will be discussed below. In contrast to HR arcs, which always connect mated quantum triangles, an active charge arc connects two oppositely oriented quantum triangles that are not mates. However, active charge arcs always occur in pairs, and the other member of the pair will connect the mate of the first triangle to the mate of the second.

Figure 2(a) shows how all this works in an example parallel to Fig. 1(a). Notice that the Landau vertex to which the photon is attached is cubic (three incident arcs). Such a feature we postulate for any $\Sigma_C^{0,\mathrm{EM}}$; the reasons relate to gauge invariance and will be explained later, but *one* of the external particles must always be given a special role on $\Sigma_C^{0,\mathrm{EM}}$. It is particle $B$ that here is directly coupled to the photon, and within $B$'s quantum area it is the triangle labeled $t$ that is "directly activated" by the ortho-para reversal described above in Sec. II. The mate of $t$ is necessarily involved in the photon coupling, but indirectly.

Figure 2(b) shows the thickened Landau graph that accompanies 2(a). Notice how appropriately directed active charge arcs lie along *both* boundaries of the photon Landau arc and along *one* boundary of the Landau arcs belonging to mesons whose triangles have been "activated". Because of the convenience of attaching spin and flavor to quark lines, it is desirable for quantum-number bookkeeping to find a place on the thickened Landau graph for the "missing" $t\bar{t}$, active quark line. A natural convention is shown in Fig. 2(c), showing two "parallel" arcs (the *same* direction)—an active charge arc and an HR arc—attached to each member of the mated $t\bar{t}$ active triangle pair. The notion of photon coupling to one quark line is now manifest. [Figure 2(c) includes two "spectator" quark lines.]

Although the information conveyed by $th(L)$, when the active quark line is embedded, is the same as

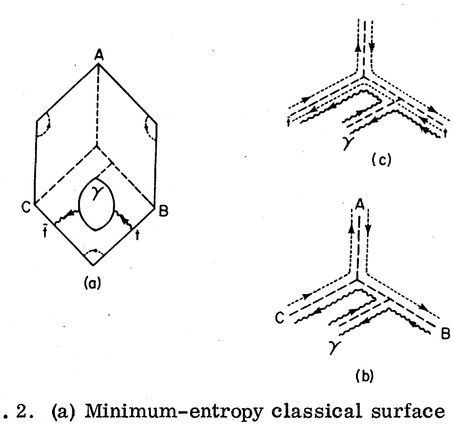


FIG. 2. (a) Minimum-entropy classical surface ($\Sigma_C^{0,\mathrm{EM}}$) for interaction of a photon with three mesons. The directly active quantum triangle belongs to meson $B$ and is labeled $t$. Wiggly lines are charge arcs. (b) The corresponding $th(L)$. (c) The $th(L)$ including the HR arc attached to the directly active quantum triangle.

that of $\Sigma_C$, the topology is different. The two-dimensional surface of Figs. 2(b) or 2(c) is "planar"—with a single boundary component, while that of Fig. 2(a) is "cylindrical"—with two boundary components. The closed nature of the photon quantum surface is not manifested by $th(L)$, although it may be inferred from the feature within Fig. 2(c) that the active quark line crosses the photon Landau arc. A crossing of Landau and HR arcs never occurs on $\Sigma_C$.

It is perhaps surprising but nevertheless true that the entire content of $\Sigma_C^{0,\mathrm{EM}}$ [e.g., Fig. 2(a)] is implied by $\Sigma_Q^{0,\mathrm{EM}}$. Identification of the active triangle on the hadron sphere completely determines the surface pair $\Sigma^{0,\mathrm{EM}}$. We shall explain in Sec. VII the spin-parity structure of an elementary electromagnetic interaction, as implied by the ortho-para orientation reversal of the active triangle.

The reader may wonder why we have not chosen to describe $\Sigma_C^{0,\mathrm{EM}}$ for two-hadron, one-photon amplitudes. Are these not simpler to discuss; why the extra hadron? The reason is the need to pick out *one* triangle on $\Sigma_Q$. Adding an extra hadron allows the two-vertex Landau graph to indicate the choice. Once ortho-para structure is included in $\Sigma_C^{0,\mathrm{EM}}$, the identity of the special triangle will be manifested though the classical surface, even in the two-hadron situation. An alternative device is to recognize that the Landau graph for $\Sigma^{0,\mathrm{EM}}$ always has *two* vertices, one of these being trivial when only two hadrons are involved. We may then, for example, represent photon coupling to one quark within $B$ of an $A, B$ meson quantum sphere by Fig. 3. The trivial vertex looks unnatural here, but once ortho-para structure is included, trivial Landau vertices will acquire clear significance.

Before extending the foregoing principles to baryons and baryoniums, it will be necessary to broaden the role of charge arcs—to include in the topology "passive" charge arcs that couple mated triangles on $\Sigma_Q$. It is possible, however, immediately to consider the coupling of photons to leptons. Here the counterpart of Fig. 3(a) is Fig. 4(a). The HR arc should now be described as a "lepton line" rather than a quark line. The leptonic sphere is covered by only two triangles and the lepton Landau arcs end in the triangle interiors. In the thickened Landau graph, Fig. 4(b), lepton arcs are bounded by one lepton line (HR arc) and one charge arc.

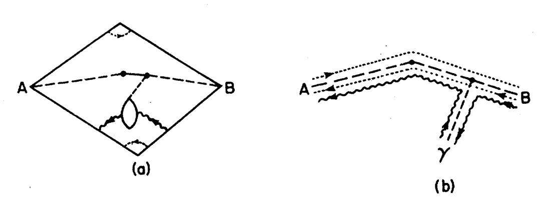


FIG. 3. (a) $\Sigma_C^{0,\mathrm{EM}}$ for a pair of mesons with two Landau vertices, one trivial. (b) The corresponding $th(L)$.

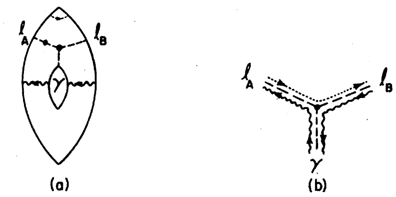


FIG. 4. (a) $\Sigma_C^{0,\mathrm{EM}}$ for a charged lepton pair. (b) The corresponding $th(L)$.

In comparing Fig. 4(b) to Fig. 3(b) an orientation difference between the lepton line and the active-quark line is observed. Maintaining the rule that an active charge arc is directed *from* an anticlockwise triangle *toward* a clockwise, it is necessary to say that the lepton line is oppositely directed. After developing in the following section a theory of electric charge, this difference will be interpretable by saying "charged leptons have an electric charge *opposite in sign* to that of charged quarks." The origin of the orientation difference is traceable to the fact that quarks are "confined" while leptons are not or, equivalently, to the fact that leptons carry energy-momentum while quarks do not. The Landau arc lies *between* the lepton line and the charge arc, requiring these to have opposite orientation, while an active quark line and the parallel charge arc lie on the *same* side of a hadron Landau arc.

## IV. TOPOLOGICAL REPRESENTATION OF ELECTRIC CHARGE

We have been led to introduce "active charge arcs" so as to fill a role for thickened Landau graphs, with respect to photons and particles coupled to photons, that for mesonic strong interactions is played by HR arcs. Now even for strong interactions (no photons) if a $th(L)$ is to be achieved for baryons and baryoniums, one needs in addition to HR arcs also arcs connecting mated *core* triangles, arcs whose direction runs from the anticlockwise triangle to the clockwise (Ref. 1, Sec. VIII). Such an arc is not a carrier of flavor and spin, like a quark or lepton line, because core triangles lack such attachments. Core-connecting arcs are thus similar in nature to active charge arcs except that mated triangles are connected. It is then natural to postulate that *every* quantum triangle has an attached "charge arc" lying in $\Sigma_C$. If this arc connects mates, it will be characterized as "passive", otherwise as "active". In strong interactions all charge arcs are passive. Electromagnetic interactions always involve some nonzero number of active charge arcs; those triangles with passive attached charge arcs may appropriately be described as "spectators" with respect to electromagnetism.

What about the directions of charge arcs? So far we have discussed only the case where the direction is from anticlockwise triangle to clockwise; if the charge arc is to lie along the boundary of a thickened Landau graph and to carry the HR orientation, then its direction is uniquely determined. But a *passive* charge arc attached to a *peripheral* hadronic triangle is not needed for the representation of HR orientation on $th(L)$. Here the quark line performs that task. We therefore allow both directions for charge arcs, *except when the charge arc is needed to represent HR orientation.* Charge arcs connecting *core* triangles always are constrained in direction, as are any *active* charge arcs that connect nonmated triangles. (The relaxation of this latter condition permits nonorientable $\Sigma_C$ and constitutes a promising basis for a theory of weak interactions.)

A new conserved quantity is implied by the introduction into the topology of directed charge arcs. Let $N_{ch}$ be defined as the number of clockwise triangles whose charge-arc directions are "out of" $\Sigma_C$ minus the anticlockwise number whose charge-arc directions are "in"; then $N_{ch}$ sums to zero over any $\Sigma_Q$. Because photons can only couple to triangles whose charge-arc direction is anticlockwise to clockwise, $N_{ch}$ counts "electrically charged" triangles minus antitriangles. If the charge-arc direction is reversed, the photon cannot couple. Peripheral triangles occur in two varieties, "charged" and "neutral", while core triangles are always charged.

The direction of a quark line is parallel to the charge arc if the quark is electrically charged, while the reverse is true for a lepton line. If we associate a directed quark or lepton line in the conventional (Feynman) way with a "particle" moving "forward" in time, then an electrically charged quark carries $N_{ch}=+1$ while an electrically charged lepton carries $N_{ch}=-1$. An electrically neutral quark carries $N_{ch}=0$, we cannot represent electrically neutral leptons so long as lepton lines are required to carry the (global) HR orientation; this is a physically acceptable situation since, from the standpoint of strong and electromagnetic interactions alone, neutrinos do not exist. Until topological theory is extended to weak interactions there is no need to represent neutrinos.

Although this paper is restricted to electromagnetism, known facts about weak interactions suggest two speculations about further extension of topological theory: (1) Even if global orientability of $\Sigma_C$ is not maintained, and charge arcs are allowed to connect two triangles of the same orientation, it will still be possible to maintain $N_{ch}$ as a conserved quantity if the triangle mating pattern persists. If charge arcs are paired—the two arcs that end on a mated triangle pair always having their other ends on a mated pair—then the charge-arc *pair* effectively "transports" a definite quantity of $N_{ch}$. If both charge arcs point in the same direction, a quantity $N_{ch}=+1$ flows in that direction. Two oppositely directed charge arcs carry zero $N_{ch}$. (For the theory described in this paper, the latter situation always obtains.) This speculation about future theory reinforces the natural postulate that electric charge is proportional to $N_{ch}$—one (+1) elementary unit of electric charge being carried by each clockwise triangle with an "out"-directed charge arc and −1 unit being carried by each anticlockwise triangle with an "in"-directed charge arc. The other two possibilities carry charge zero. One may then look forward to charge conservation even after global $\Sigma_C$ orientability is relaxed.

(2) The spin-parity structure of $\Sigma^{0,EM}$, described in Secs. VI and VII, is naturally expressed through left-handed and right-handed currents corresponding to ortho→ para and para→ ortho transitions, whose definition depends on HR arcs. It is then likely that if a weak-interaction topological theory emerges with only left-handed currents, the spin-parity behavior of leptons and quarks (rather than antileptons and quarks) will be similar. We thus expect that if electrically charged quarks are assigned positive charge, then the electron and the negatively charged muon and tau lepton will turn out to be topological leptons (rather than antileptons.)

It is shown in Ref. 1 that (outgoing) baryon number $B$ is the number of anticlockwise core triangles minus the clockwise number, so *beyond* the electric charge carried by quarks there is for hadrons an additional charge equal to $-B$. Thus the total charge of any hadron is

$$Q=N_{q_{ch}}-B\,, \tag{4.1}$$

if $N_{q_{ch}}$ is the number of charged quarks minus antiquarks. The total quark number $N_q$ is always equal to $3B$, while the number of neutral quarks is $N_{q_0}\equiv N_q-N_{q_{ch}}$, so

$$B=\tfrac{1}{3}(N_{q_0}+N_{q_{ch}}) \tag{4.2}$$

and

$$\begin{aligned}Q&=N_{q_{ch}}-\tfrac{1}{3}(N_{q_0}+N_{q_{ch}})\\&=\tfrac{2}{3}N_{q_{ch}}-\tfrac{1}{3}N_{q_0}\,.\end{aligned} \tag{4.3}$$

We see from formula (4.3) how the semblance of fractional quark charges is produced by the core

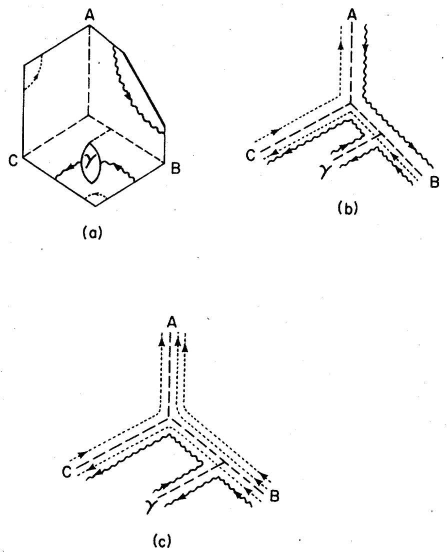


FIG. 5. (a) The main sheet of $\Sigma_C^{0,\mathrm{EM}}$ for interaction of a photon with two baryons and a meson. The directly active quantum triangle is a main-sheet quark belonging to baryon $B$. (b) The corresponding $th(L)$. (c) The $th(L)$ with the core charge arc replaced by a pair of HR arcs (diquark).

triangles.

Let us list rules that have thus far emerged for minimal electromagnetism.

(A) For each zero-entropy component of the topological expansion with $N$-charged triangles on the quantum surface, there are $N$ components to $\Sigma^{0,\mathrm{EM}}$—each coupling the photon to one of the charged triangles.

(B) Every triangle has an attached directed charge arc. A hadron triangle is charged if the charge direction agrees with HR orientation and neutral if these directions disagree. The reverse is true for lepton triangles.

(C) Peripheral triangles may be charged or neutral. Core triangles and photon triangles are always charged.

(D) Electric charge (outgoing) in elementary units is the sum of clockwise minus anticlockwise charged triangles. Charged quarks have charge +1 while charged leptons have charge −1. Hadrons have charge $N_{q_{\mathrm{ch}}} - B$. Electric charge is conserved.

## V. MINIMAL ELECTROMAGNETISM ON $\Sigma_C$ FOR BARYONS AND BARYONIUMS

There remains the task of describing $\Sigma_C^{0,\mathrm{EM}}$ when the photon couples to baryons (four triangles) or baryoniums (six triangles). Consider first the coupling to a "main-sheet" baryon quark. The classical surface is divided into sheets, connected

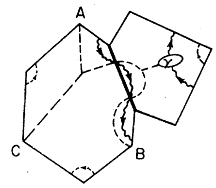


FIG. 6. The two sheets of $\Sigma_C^{0,\mathrm{EM}}$ containing the Landau graph when the photon couples to a baryon quark not on the main sheet (i.e., within the diquark).

through "junction lines", but at zero entropy the entire Landau graph resides on a single main sheet. One of the three quarks within a baryon is located on this main sheet, and in Fig. 5(a) we show the structure of the main sheet of $\Sigma_C^{0,\mathrm{EM}}$ when the photon couples to this quark in a photon, baryon-$A$, baryon-$B$, meson-$C$ amplitude. The pattern is similar to that of photon coupling to a meson quark except for the passive core-charge arc, which as discussed in Ref. 1 may be interpreted as a "diquark". Figure 5(b) shows the thickened Landau graph, which may be compared to Fig. 2(c). If desired, as explained in Ref. 1, it is possible to replace the passive core-charge arc on $th(L)$ by the two quark lines corresponding to the diquark. This device is shown in Fig. 5(c).

When the photon couples to one of the two quarks within the diquark (which do not lie on the main sheet of $\Sigma_C$) it is necessary to detour the baryon Landau arc via two "glitches," as shown in Fig. 6, onto the sheet where the electromagnetically active quark resides. In this case the corresponding $th(L)$ cannot easily represent all the information on $\Sigma_C$. We nevertheless remark that, on $th(L)$, photons interacting with any of the three quarks couple to the same side of the baryon as do mesons.

It may at first sight seem strange that an "elementary" photon interaction requires glitches, but when we consider the photon spin and parity it will be found that $\Sigma_C^{0,\mathrm{EM}}$ always is divided into two ortho-para patches—another feature that does not look elementary. Electromagnetism unavoidably requires some topological complexity. For

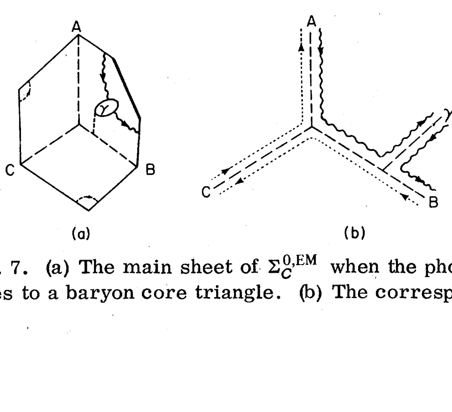


FIG. 7. (a) The main sheet of $\Sigma_C^{0,\mathrm{EM}}$ when the photon couples to a baryon core triangle. (b) The corresponding $th(L)$.

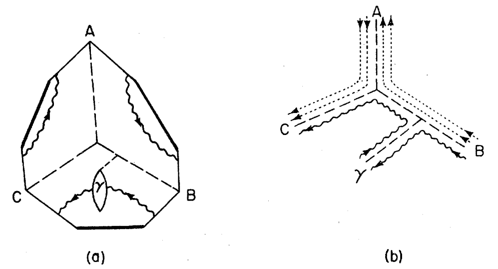


FIG. 8. (a) The main sheet of $\Sigma_C^{0,\mathrm{EM}}$ when the photon couples to a baryonium core triangle. (b) The corresponding $th(L)$. Here the diquark belonging to the active core triangle is represented by the core charge arc.

strong interactions, glitching and ortho-para patching are among the simplest and quantitatively most important forms of topological complexification, and their appearance at the lowest level of electromagnetism is not an indication of inconsistency. Perhaps it is a hint that in some future theory electromagnetism will not be postulated on an *a priori* basis.

No glitches are required for $\Sigma^{0,\mathrm{EM}}$ when the photon couples to a core triangle, as shown by Fig. 7(a). The thickened Landau graph of Fig. 7(b) reveals the core-coupled photon attached to the *opposite* side of the baryon with respect to mesons. This is the side of the baryon to which baryoniums may be attached in a $th(L)$.

The rules for minimal photon coupling to (six-triangle) baryoniums are analogous to the foregoing rules for baryons. If coupling is to either core triangle, no glitches are needed, as shown by the three-baryonium, one photon $\Sigma^{0,\mathrm{EM}}$ main sheet exhibited in Fig. 8(a). The corresponding $th(L)$ is shown in Fig. 8(b). In contrast, photon coupling to any of the two quarks or two antiquarks requires a pair of baryonium glitches, as shown in Fig. 9. It is noteworthy that baryonium glitches never occur in any strong-interaction topologies.

## VI. ORTHO-PARA PATCHING OF $\Sigma_C^{0,\mathrm{EM}}$

Topological representation of spin and parity in strong interactions involves "ortho-para" patching of $\Sigma_C$. Each $\Sigma_C$ is divided into patches of alternating orientation. A patch whose orientation agrees (disagrees) with HR orientation is described as "ortho" ("para"). Adjacent patches are separated by "transition arcs" whose orientation is induced by the patch orientations; belt segments have a corresponding induced ortho-para orientation. A zero-entropy $\Sigma_C$ consists of single-patch sheets which are either all ortho or all para. [It has subsequently been found that, even at zero entropy, each quark can be ortho or para independently. See G. F. Chew, J. Finkelstein, and M. Levinson, Phys. Rev. Lett. 47, 767 (1981).]

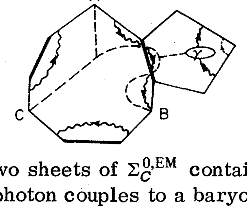


FIG. 9. The two sheets of $\Sigma_C^{0,\mathrm{EM}}$ containing the Landau graph when the photon couples to a baryonium quark.

Among the leading strong-interaction corrections to zero entropy are connected sums that glue together ortho and para classical surfaces—with introduction of a transition arc that cuts certain HR arcs. The Stapp spin rules[3]—expressed through HR arcs as described in Appendix B—associate with ortho→ para or para→ ortho transition a definite quark or lepton spin dependence that has the structure of a right-handed or left-handed vector "current". It is then natural to guess that, in any $\Sigma_C^{0,\mathrm{EM}}$ where a photon couples to a peripheral triangle, the associated HR arc is cut by a transition arc "tied" to the photon. At the same time photon "factorization"—the independence of photon identity from the system to which it couples—demands a general rule for tieing transition arcs to photons. We now propose such a rule, which in Appendix B and Sec. VII is verified to yield the standard (Dirac) vector coupling to leptons.

Any $\Sigma_C^{0,\mathrm{EM}}$ may be seen as an ortho or para zero-entropy $\Sigma_C^0$ into one sheet of which a photon boundary is inserted. The photon boundary consists of two belt segments each coupled by an active charge arc to a triangle on the lepton or hadron quantum surface; *one* of these triangles, which we shall call the "directly active" triangle, is to have its ortho-para orientation reversed. We thus postulate that attached to the corresponding photon boundary segment there is a patch—whose orientation opposes that of the rest of the sheet—which also attaches to the directly active triangle. The four-sided patch perimeter consists of two belt segments, one belonging to the photon and the other to the directly active triangle, plus two transition arcs connecting the ends of the photon segment to the ends of the hadron or lepton segment; the charge arc connecting these segments lies within the patch.

A feature of strong-interaction topology is that each patch contains at least one Landau vertex. We perpetuate this rule by drawing the transition arcs so as to enclose the photon vertex together with the photon Landau arc. We may then refer to a minimum-entropy photon as "ortho" or "para" depending on the patch containing its Landau arc, just as we do for zero-entropy particles. Figure

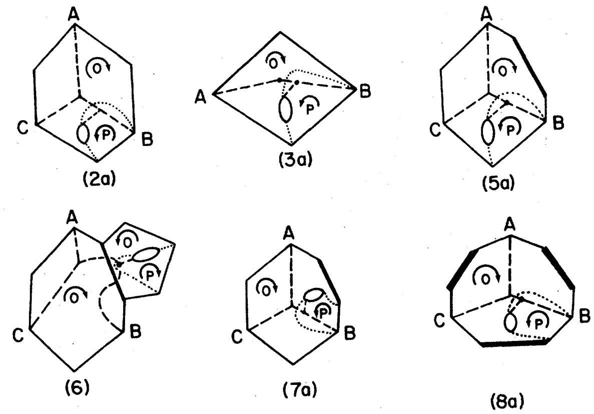


FIG. 10. Transition arcs (dotted lines) attached to $\Sigma_C^{0,\mathrm{EM}}$ presented in the indicated earlier figures for photon coupling to hadrons. Charge and HR arcs are here omitted.

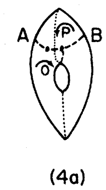


FIG. 11. Transition arcs when a photon is attached to a lepton propagator.

10 shows the patch structure for each of the hadron examples of $\Sigma_C^{0,\mathrm{EM}}$ previously discussed. We have here, for clarity, omitted HR arcs and charge arcs, but it may be verified that when the directly active triangle is peripheral, the attached HR arc is cut by a transition arc (see Appendix B). Notice in the patched version of Fig. 3(a) that the trivial Landau vertex is the only vertex inside its patch. Patch structure endows trivial vertices with significance.

The rules for patching $\Sigma_C^{0,\mathrm{EM}}$ when the photon couples to a lepton pair are the same as the foregoing. Again a trivial vertex is needed, as shown in Fig. 11. We shall see that for quantum electrodynamics (QCD)—the sum of lepton-photon topologies with neglect of hadrons—the trivial vertex is inert and might be ignored; also for many computations the net effect of ortho-para patching is simply to generate vector coupling. These features nevertheless are needed for consistent enumeration of different topological components in a complete theory. By keeping them we maintain the uniform principle that each component of $\Sigma^{0,\mathrm{EM}}$ couples a photon to one particular triangle on the lepton or hadron quantum surfaces. Also uniform is the statement that every $\Sigma^{0,\mathrm{EM}}$ has *two* Landau vertices. The only topologies in the topological expansion with a single Landau vertex are zero-entropy $\Sigma^0$.

## VII. CLASSICAL ELECTRIC CHARGE

Associated with each topology $\Sigma^{0,\mathrm{EM}}$ there is an amplitude—more precisely an $M$ function—that depends analytically on particle energies and momenta. All higher electromagnetic topologies are to be constructed by successive Landau connected sums starting from the set of $\Sigma^{0,\mathrm{EM}}$ and $\Sigma^0$; each sum corresponds to a discontinuity formula, which allows calculation of the amplitude belonging to the higher topology. But we need a prescription for the amplitudes belonging to $\Sigma^{0,\mathrm{EM}}$. The prescription to be proposed below specifies each elementary amplitude in terms of a dimensionless parameter $e$, which will be seen to determine the "classical" electric charge of each particle—i.e., the force exerted on the particle by a classical electric field of unit strength. Each $\Sigma^{0,\mathrm{EM}}$ amplitude will be directly proportional to $e$, so if this parameter were set equal to zero all electromagnetic effects would disappear. Experiments yield a small value for $e$, a fact harmonious with assignment to each $\Sigma^{0,\mathrm{EM}}$ of nonzero entropy (two quantum surface components, two classical patches, two Landau vertices), but we do not yet understand how to calculate $e$. In this respect out present theory of electromagnetism is incomplete.

What is the precise definition of the classical electric charge of a zero-entropy elementary particle $B$—corresponding to some $N$-triangle quantum disk? The electric charge $Q_B$ of any particle $B$ determines the value taken by a $(B_{\mathrm{out}}, \gamma, B_{\mathrm{in}})$ amplitude in the limit as photon energy-momentum approaches zero. With the convention that a momentum four-vector $p_i$ describes an outgoing particle when the energy is positive and an ingoing particle when the energy is negative, so that in the latter case the physical energy-momentum is $-p_i$, the amplitude for $(B_{\mathrm{out}}(p_2), \gamma(q), B_{\mathrm{in}}(p_1))$, with $p_1+p_2+q=0$, in the $q\to 0$ limit is equal to

$$Q_B(p_2-p_1)\cdot A\,, \tag{7.1}$$

where $A$ is the (unit) photon polarization four-vector, satisfying $q\cdot A=0$. Now at the minimal topological level this amplitude is a sum over $N$ distinct $\Sigma^{0,\mathrm{EM}}$ amplitudes, each corresponding to a different "directly active" triangle on the $(B_{\mathrm{out}}, B_{\mathrm{in}})$ quantum sphere. Guided by formula (7.1) we shall require that if the directly active triangle has quantum electric charge $N_{\mathrm{ch}}^i(0,\pm 1)$ then the $\Sigma^{0,\mathrm{EM}}$ amplitude for $(B(p_2),\gamma(q)B(q_1))$ has the $q\to 0$ value

$$eN_{\mathrm{ch}}^i p_i\cdot A\,, \tag{7.2}$$

where $p_i$ is the (conventional) momentum of the *particle* to which triangle $i$ belongs. It follows from (7.1) and (7.2) that

$$Q_B = e\sum_{i=1}^{N/2} N_{ch}^i\,, \tag{7.3}$$

where the sum runs over the triangles in the outgoing particle disk. The corresponding sum over ingoing triangles is equal to $-Q_B$. For an arbitrary process, with or without photons, the general conservation of $N_{ch}$ evidently implies the general conservation of classical electric charge.

In Appendix B the Stapp rules[3] for topological spin dependence are extended to electromagnetism, with results for $\Sigma^{0,EM}(B(p_2),\gamma(q)B(p_1))$ that we now state in a form as close as possible to Feynman rules with four-component Dirac spinors. Each of the $N$ elementary amplitudes is a product of $N/2$ factors, one for each mated triangle pair on the leptonic or hadronic quantum surfaces. One factor belongs to the active pair, all the others to spectator pairs. (Each active pair appears in two different elementary amplitudes.) Spectator factors are the same as at zero entropy.

Suppose that we label a mated triangle pair $(i', i)$ with the HR-arc direction running from $i$ to $i'$ for peripheral triangles (anticlockwise to clockwise for quarks, the reverse for leptons). A spectator core-triangle pair simply gives a factor 1 while a spectator peripheral-triangle pair gives a factor

$$\bar{u}(i')(1\pm\gamma_5)u(i)\,, \tag{7.4}$$

with $\bar{u}(i)u(i)=1$, when $i$ is incoming and $i'$ outgoing. The $1+\gamma_5$ applies to ortho and $1-\gamma_5$ to para. Appendix B may be consulted for details about the notation $u(i)$. Because $\bar{u}\gamma_5 u \to 0$ as $q\to 0$ all spectator factors thus approach 1 in the $q\to 0$ limit.

What about an *active* core-triangle pair? Guided by gauge invariance and analyticity requirements, we postulate that the factor here is simply equal to (7.2) even for $q$ different from zero. The *sum* of the *two* amplitudes belonging to the two members of a mated core pair then has a factor

$$eN_{ch}^{i'}(p_{i'}-p_i)\cdot A\,, \tag{7.5}$$

where $N_{ch}^{i'}$ belongs to the core triangle attached to the particle with (conventional) momentum $p_i'$. Because $q=-p_{i'}-p_i$ while $p_{i'}{}^2=p_i{}^2=(m_B^0)^2$, where $m_B^0$ is the zero-entropy hadron mass, it follows that (7.5) satisfies the gauge condition of vanishing when $A$ is replaced by $q$.

Finally we come to an active peripheral-triangle mated pair. If the HR direction runs *toward* the directly active triangle *from* its mate (see Appendix B) then the factor is

$$e m_B^0 N_{ch}^{i'}\bar{u}(i')(1\mp\gamma_5)\frac{\gamma\cdot A}{2}(1\pm\gamma_5)u(i)\,, \tag{7.6}$$

when $i$ is incoming and $i'$ outgoing. If the directly active triangle is the one *from* which the HR arc is directed, then the factor is

$$-e m_B^0 N_{ch}^{i}\bar{u}(i')(1\pm\gamma_5)\frac{\gamma\cdot A}{2}(1\mp\gamma_5)u(i)\,. \tag{7.7}$$

Each of these factors is equal to (7.2) in the $q\to 0$ limit, and if the two factors are added we find

$$2m_B^0 e N_{ch}^{i'}\bar{u}(i')\gamma\cdot A\,u(i)\,, \tag{7.8}$$

the familiar Dirac form that correctly vanishes if $A$ is replaced by $q$. In all electromagnetic computations the factors (7.5) or (7.8) may be immediately inserted as the contribution from an active triangle pair.

## VIII. QUANTUM ELECTRODYNAMICS (PHOTONS AND LEPTONS)

A natural question concerns the connection between photon-lepton quantum electrodynamics (QED) and the nonhadronic components of the topological expansion. Although proof is lacking, we shall now argue that precise correspondence probably exists between Feynman amplitudes—each associated with a distinct Feynman graph—and topological lepton-photon amplitudes.

A persuasive case has long ago been made[4-8] that the Feynman QED expansion in the Feynman gauge can be generated from $S$-matrix unitarity-analyticity considerations provided one assumes that each $S$-matrix connected part is expandable in powers of $e$:

$$M_{fi} = \sum_{n=N_f+N_i-2}^{\infty} e^n M_{fi}^{(n)}\,, \tag{8.1}$$

where $N_i$ and $N_f$ are, respectively, the total numbers of particles in initial and final channels. The discontinuities of $M_{fi}^{(n)}$ involve products $M_{jk}^{(n')}M_{lm}^{(n'')}$ where $n'+n''=n$, with $n'$ and $n''\neq 0$ so that $n'<n$ and $n''<n$. Successive terms in the expansion can then be calculated through dispersion relations from lower-order terms, the whole structure flowing from the two-lepton, one-photon lowest-order amplitude proportional to $e$. It has furthermore been shown in a collection of examples, and with no counterexamples, that $M_{fi}^{(n)}$ as calculated from $S$-matrix unitarity-analyticity is identical with $M_{fi}^{(n)}$ as calculated from Feynman graphs; renormalization is straightforward through dispersion relations. The obstacle to a general equivalence proof is inadequate understanding of the structure of the complex-momentum Riemann surface when the number of variables becomes large.

Now the leading lepton-photon components in the topological expansion have been seen in Sec. VII to be identical with the basic Feynman vertex.

This fact justifies our using the same symbol $e$ in formulas (8.1) and (7.8). Since higher-order topological components are to be generated by $S$-matrix unitarity-analyticity, it is then immediately plausible that the topological sequence can be placed in correspondence with the expansion (8.1). This expectation becomes stronger when one realizes that thickened Landau photon-lepton graphs, minus trivial vertices or other ortho-para structure, stand in one-to-one correspondence with Feynman graphs. The only contractions allowable in electron-photon topologies generated by connected sum from $\Sigma^{0,EM}$ are replacement along a lepton propagator of two adjacent trivial vertices of the same orientation (ortho or para) by a single trivial vertex of that orientation. So if a lepton propagator is understood as summing over all numbers of ortho-para trivial vertices along a lepton Landau arc (equivalent to omitting trivial vertices from the topology), there is precise correspondence between Feynman graphs and lepton-photon topologies within our topological expansion.

An interesting feature of this correspondence relates to the distinction between ordinary Feynman graphs and *ordered* Feynman graphs—where the three arcs incident on each vertex stand in a definite cyclic sequence. Thickened Landau graphs have ordered vertices, while an ordinary Feynman graph gives no significance to vertex order. Figure 4(b) shows, however, that only one lepton-photon vertex order is allowed in a thickened Landau graph. When drawn on a plane, photons are always emitted from the "same side" of a lepton. With such a rule any ordinary Feynman graph translates into a unique ordered Feynman graph.

Another interesting question connected to trivial Landau vertices relates to parity doubling. At zero entropy each lepton and also the photon appears in two forms, ortho and para, just as do hadrons[1]; the linear combinations $O+P$ and $O-P$ have opposite intrinsic parity. As shown by Stapp,[3] superposition of components in the topological expansion then eliminates $O-P$ states, both externally and internally. It nevertheless seems impossible to formulate topological theory directly in terms of $O+P$, even though Lagrangian theory (QED) succeeds in so doing. On the other hand, topological theory explains why only one intrinsic parity, and not both, finally is physically manifested, a feature arbitrarily inserted into QED. It of course looks promising for a future theory of weak interactions to have already at zero entropy an ortho-para topological base.

Returning to the expansion (8.1), which is really in powers of $\alpha=e^2$ for any given amplitude $M_{fi}$, we remark that each power of $\alpha$ belongs to an uncontractible photon-vertex pair. No photon vertex can ever be removed by contraction, so the number of photon vertices is effectively an entropy index. This association of the fine-structure constant with entropy growth suggests that the small value of $\alpha$ ultimately will be related to amplitude suppression through phase interference since for strong interactions the smallness of high entropy components is so understood.[9] Our present theory, however, accepts an arbitrary value for $\alpha$, just as does QED.

Assuming that individual Feynman-graph amplitudes are in fact equal to topological amplitudes (summed over trivial Landau vertices and over ortho-para), one cannot avoid the idea that Feynman off-mass-shell rules might *generally* be invoked for evaluating higher components of the topological expansion. Each zero-entropy amplitude corresponds to a single Landau vertex and cannot itself be perturbatively evaluated, but perhaps the special singularity structure attached to zero entropy permits a unique off-mass-shell significance for ordered vertices with any number of incident arcs. In the following section we discuss how such a possibility relates to $\Sigma^{0,EM}$ when hadrons are involved.

## IX. MORE GENERAL HADRON TOPOLOGIES

We have prescribed explicit gauge-invariant amplitudes for $\Sigma^{0,EM}$ couplings of a photon to two elementary hadrons or to two leptons. Section VIII has discussed how more general topologies not involving hadrons, either externally or internally, correspond to uncontracted (apart from adjacent trivial vertices) connected sums built from $\Sigma^0$ and $\Sigma^{0,EM}$, but if any hadrons occur, attention must be paid to the possibility of (nontrivial) contractions.

A simple example is the Fig. 12 connected sum, expressed here through thickened Landau graphs, of a zero-entropy (one vertex) three-meson $\Sigma^0$ with a three-meson, one-photon (two-vertex) $\Sigma^{0,EM}$. The orientation of the transition arc corresponds

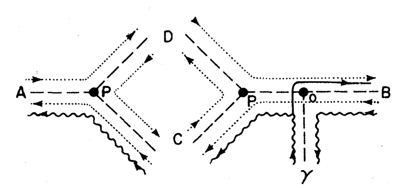


FIG. 12. A connected sum leading to the $\Sigma^{0,EM}$ of Fig. 3. The solid line is a transition arc, wiggly lines are charge arcs, dotted lines are HR arcs, and dashed lines are Landau arcs. The symbols $O$ and $P$ designate ortho and para patches, inside each of which there is a Landau vertex.

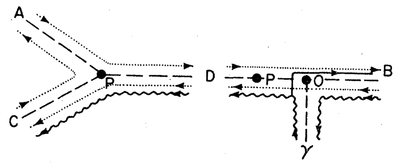


FIG. 13. A connected sum leading to the $\Sigma^{0,\mathrm{EM}}$ of Fig. 2 after contraction of the adjacent para vertices to a single vertex.

to an ortho-photon coupling to mesons that are all para, as indicated by the $O, P$ labels on the vertices. The contraction rules of Ref. 1 specify that the closed loop generated by this connected sum should be erased, leading to the two-vertex $\Sigma^{0,\mathrm{EM}}$ of Fig. 3. Thus the trivial meson vertex of Fig. 3 is to be understood as representing an indefinite number and assortment of intermediate hadrons whose topology is contractible in the manner exemplified by Fig. 12. Any purely hadronic vertex has such a character. In contrast a vertex to which a photon attaches is always "simple"—never combinable by contraction with other vertices and always interpretable as interaction of an "elementary" photon with a pair of "elementary" particles.

Maintenance of gauge invariance will require mixing different topological complexities, once one goes beyond two-particle, one-photon $\Sigma^{0,\mathrm{EM}}$ topologies. We have seen how gauge invariance for two-particle, one-photon (either ortho-photon or para-photon) $\Sigma^{0,\mathrm{EM}}$ amplitudes requires combining both photon couplings (either both ortho or both para) to two mated triangles on $\Sigma^0_Q$. Now suppose we build the $\Sigma^{0,\mathrm{EM}}$ three-meson, one-photon topology of Fig. 2 from the connected sum of a zero-entropy three-meson $\Sigma^0$ with a two-meson, one-photon $\Sigma^{0,\mathrm{EM}}$ as in Fig. 13, where the photon is ortho. (For simplicity imagine that the passive quarks here are electrically neutral.) The associated amplitude is not gauge invariant, and gauge invariance fails to be achieved merely by adding the $\Sigma^{0,\mathrm{EM}}$ amplitude where the ortho photon attaches to particle $C$ rather than to particle $B$. These two amplitudes contain different meson poles, one in the variable $S_{AB}=(p_A+p_B)^2$ and one in the variable $S_{AC}=(p_A+p_C)^2$, and the residue of each pole must contain a gauge-invariant photon coupling in order to achieve overall gauge invariance.[4] But the residue of the $S_{AC}$ pole factors according to Fig. 13 so the photon coupling here—to particle $B$ but *not* to particle $D$—fails to be gauge invariant. To achieve gauge invariance a pole whose residue is of the form of Fig. 14(a), where the ortho photon couples to $D$, must be added. Such a pole will indeed occur within the topological expansion, but not from a $\Sigma^{0,\mathrm{EM}}$ component, because the connected sum depicted in Fig. 14(a) does not contract to two Landau vertices. Instead one finds the three-vertex topology of Fig. 14(b). To achieve a gauge-invariant coupling of an ortho photon to three or more hadrons, one must include such components of the topological expansion—which are more complex than $\Sigma^{0,\mathrm{EM}}$.

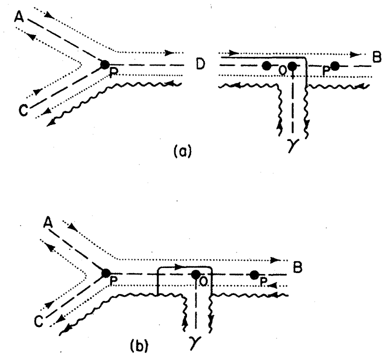


FIG. 14. (a) A connected sum that admits no contractions. (b) The three-patch structure of this connected sum.

It has long been recognized that individual complexity levels of the topological expansion may violate certain symmetries of the full expansion without destroying the expansion's usefulness. (An example is $G$-parity invariance, which is not satisfied by the discontinuities of planar amplitudes.[9]) One might worry with respect to gauge invariance, however, that through its absence for individual topologies the expansion components could depend on the choice of gauge. Our $S$-matrix theory, however, effectively has selected a special gauge by its association with topology of definite spin and momentum dependence. If it turns out that Feynman-graph (off-mass-shell) computation methods are possible, we conjecture from the results of Refs. 5–8 that the special gauge is the so-called Feynman gauge; the standard Feynman rules and the spin rules of Appendix B are closely connected.

We assume, in accord with the corresponding assumption in Ref. 1, that standard $S$-matrix unitarity-causality requirements not only specify the higher-order electromagnetic topologies but give explicit formulas (dispersion relations) for computation of the associated amplitudes. The development of a practical approach for evaluating higher-order components we shall not attempt in this paper. A concrete example of a plausible formula, however, may be worthwhile to illustrate what may be expected. An amplitude of the type of Fig. 14(b) or Fig. 2 plausibly may be expressed through a single pole in the variable $S_{AC}$ at the point $S_{AC}=(m^0_B)^2$. The spin structure would be topologically determined by the rules of Appendix B and factored from the dependence on $S_{AC}$ and $S_{AB}$, leaving the pole residue in this simple case a constant. By factorization the constant is a product of the electric charge of the active quark and the zero-entropy three-meson coupling. It remains to be seen whether such a pole formula correctly yields the amplitude in question and, if it does, whether this kind of formula can be generalized.

## X. SUMMARY AND CONCLUSIONS

This paper has extended to the electromagnetic interactions the topological theory of strong interactions proposed in Ref. 1. The lowest term in the topological expansion for electromagnetic amplitudes, which we call the "minimum-entropy" term, consists of a sum of pieces, where each piece can be described as a zero-entropy amplitude into which a photon has been inserted to interact with a particular quantum triangle (quark, lepton, or core) of a particular external particle.

The simplest of the minimum-entropy terms are the cubic vertex functions, i.e., the amplitude for the interaction of a single propagating particle with a photon; the rest of the minimum-entropy terms, and then the rest of the expansion, can be obtained from these simplest terms. Explicit formulas for these three-vertex terms are given above in Sec. VII. It can be seen that terms where the photon interacts with a quark or with a lepton have the structure of an interaction with a pointlike spin-$\frac{1}{2}$ object, while terms where the photon interacts with the core have the structure of an interaction with a pointlike spin-0 object. However, although this theory may be visualized in terms of hadronic "constituents", there is at the minimum-entropy level no notion of energy or momentum being carried by the constituents, and so it turns out that hadrons themselves behave as pointlike objects, so long as higher-than-minimum-entropy terms are not included.

Once the minimum-entropy terms are given, the higher-order correction terms will be generated by connected sums, just as in the purely strong-interaction theory of Ref. 1. Restricting ourselves to leptons and photons, where the *only* interactions are the electromagnetic ones (we are of course ignoring weak and gravitational interactions at this point), we conjecture that the topological expansion merely reproduces the standard perturbation expansion of QED. Hadrons of course do have strong interactions, and these will enter also in the higher terms of the topological expansion. Thus our theory could be characterized as follows: start with pointlike couplings for both leptons and hadrons, and then include, in the same expansion, both perturbative (QED) corrections and nonperturbative strong interactions.

Every triangle on the quantum surface is the end point of a charge arc; the photon can only couple when the direction of the charge arc agrees with, rather then opposes, the global (HR) orientation of the classical surface. This leads to the appearance in our theory of neutral, as well as unit-charged, quarks. However, the charges of all hadrons turn out to be the same as in conventional models (with fractionally charged quarks). The fact that the baryon "core" also carries charge makes integral charge for quarks compatible with the known charges of hadrons.

To compute electromagnetic properties of hadrons beyond that of their charge involves going beyond the minimum-entropy terms, and this is not done in this paper. For the most part, this problem is for the future; however, in a separate paper we shall report on a calculation of baryon magnetic moments. This calculation includes non-minimum-entropy corrections, and seems to agree with experimental results somewhat better than does a naive (fractionally charged) quark-model calculation.

We speculate that the charge arcs may play an important role in a future weak-interaction topological theory. In the present theory, the charge arcs have two functions: first, they ensure charge conservation, and second, they maintain classical-surface orientability and so help ensure separate $P$, $C$, and $T$ invariance. Obviously in a theory of weak interactions the first function must be retained and the second not. In Appendix A we discuss a possible generalization of our charge arcs into elements of nontrivial fiber bundles; perhaps this generalization will ultimately be relevant to a topological theory of weak interactions.

## APPENDIX A: REAL LINE BUNDLES

In the main body of this paper, the following statements have been made:

(i) Since each triangle of the quantum surface forms part of the boundary of the classical surface, then to each triangle we may associate a direction, which is either *into* or *out of* the classical surface.

(ii) We seek to associate this direction with electric charge, but then, since mated quantum triangles must carry corresponding charges, a triangle whose "*charge direction*" is in (out) must be mated with a triangle whose charge direction is out (in).

The aim of this appendix is to show how such requirements occur naturally in the theory of fiber bundles, in particular of real-line bundles. [The mathematical reader will notice that these will actually be "line bundles with local (fiber) orientation."] The fiber bundles that we discuss here generalize the notion of charge direction in two ways: first, we allow bundles which are "twisted" so that the notion of "in or out" cannot be globally defined; and second, we consider two ways of gluing together such objects (which are analogous to connecting electrical resistors either in series or in parallel), only one of which corresponds to the gluing of the charge directions implied by connected sums of surface pairs.

We have associated with each triangle on the quantum surface one of two charge directions; suppose we look at the boundaries between triangles, and label a boundary (+) if the two triangles it separates have the same direction, and (−) if they have opposite directions. Then instead of studying the charge directions themselves, we might instead study the possible patterns of (+) and (−) on the boundaries. This leads us to our first generalization: we can now consider other patterns of (+) and (−) on the boundaries which do not correspond to any consistent assignment of (in) and (out) on the entire quantum surface; these will turn out to represent what we call "nontrivial" line bundles. As an analogy, consider instead of the quantum surface a circle, with various segments on the circle assigned a direction (in) or (out): then if we associate (+) or (−) with the points separating the segments as before, we will surely have an even number of (−) points. A pattern with an odd number of (−) points will represent a nontrivial bundle.

We now recall some standard facts about real-line bundles; the reader is referred to the book by Steenrod[10] for more information. A real-line bundle is a pair of spaces $(E, \Sigma)$ and a surjective map $\pi$: $E \to \Sigma$, satisfying certain requirements; $E$ is called the total space, $\Sigma$ the base space, and $\pi$ the projection. For the case we consider $\Sigma$ will be the quantum surface and $E$ a three-dimensional manifold.

The projection $\pi$ has the property that for each triangle $\sigma \subset \Sigma$, $\pi^{-1}(\sigma)$ is the "trivial" Cartesian product $\sigma \times R$ ($R$ will denote the real line). In more precise terms, for each $\pi^{-1}(\sigma)$ there is a local "coordinate system"

$$\pi^{-1}(\sigma) \xrightarrow[\psi_\sigma]{} \sigma \times R ,$$

which is a homeomorphism carrying our $\pi$ into the natural projection $\sigma \times R \to \sigma$.

We call $\pi^{-1}(x)$ the *fiber* at $x$ $(x \in \Sigma)$. Of course, the way in which the various $\sigma \times R$'s are glued together can be *twisted*. This twist is described as follows: let us say that two adjacent triangles $\sigma_i$, $\sigma_j$ have the common edge $A$. For each point $x \in A$, the transformation $\psi_{\sigma_j} 0 \psi_{\sigma_i}^{-1}$ induces a homeomorphism of the real line into itself, which we denote by $g_{ji}(x)$. This $g_{ji}(x)$ will be thought of as an element of the group $O(1)$, depending continuously on $x$. Hence $g_{ji}$ depends only on $A$. It will be denoted by $g(A)$ and it is equal to +1 or −1. Here +1 stands for $t \to t$ and −1 for $t \to -t$. Thus *if* there is a direction (in) or (out), then $g(A) = +1$ (or −1) if $A$ separates triangles of the same (or opposite) directions. However, we can also study more general patterns of $g(A)$. We will impose the following "co-cycle condition" on the $g(A)$'s: if $P$ is a vertex of $\Sigma$ and if $A_1, \ldots, A_k$ are the various edges starting from $P$, then the product

$$g(A_1) \cdot g(A_2) \cdots g(A_k) = +1 .$$

Several features of this definition should be noted:

(1) One can completely reconstruct the bundle $E \xrightarrow{\pi} \Sigma$ out of the collection of "coordinate transformations" $g(A)$ (defined for all edges $A$ and satisfying the co-cycle condition), and any such abstract collection defines a bundle. One simply uses the $g(A)$'s as a recipe for gluing things together.

(2) Let us call an abstract collection of $g(A)$'s a "directory". We will consider "gauge transformations" of the following type: take a triangle $\sigma$, with edges $A_1, A_2, A_3$, and make the transformation

$$(g(A_1), g(A_2), g(A_3)) \to (-g(A_1), -g(A_2), -g(A_3))$$

(note that this does *not* violate the co-cycle condition). Then two directories define "equivalent" bundles (which means that we can identify the two $E$'s in a way compatible with the projection $\pi$) if and only if they are related by gauge transformation.

(3) We can identify $\Sigma$ with a part of $E$. Namely attach to each $x \in S$ the middle (or zero) point of the corresponding fiber.

Now a new image emerges: Our surface $\Sigma$ is embedded in a larger three-dimensional object $E$, made out of lines crossing $\Sigma$. The coordinate chart orients their lines *locally*, independently for each triangle $\sigma$ (however, there may not be a consistent *global* orientation).

The function $g(A)$ tells us whether the local

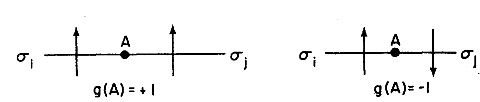


FIG. 15. The cases where the orientations (represented by arrows) of the triangles $\sigma_i$ and $\sigma_j$ agree $[g(A) = +1]$ or disagree $[g(A) = -1]$.

transverse orientations of $\sigma_i$ and of $\sigma_j$ agree or disagree (Fig. 15). An elementary gauge transformation just means reversing our local transverse orientation.

By definition the bundle $E \xrightarrow{\pi} \Sigma$ is trivial (or orientable) if by gauge transformations all $g(A)$'s can be made +1, or equivalently, if we can define a coherent *global* transverse orientation. A word of caution here: this is *not* the same thing as the orientability of the three-dimensional object $E$. It means just that one can unambiguously define a notion of in or out throughout our bundle, "transversally to $\Sigma$", just in the same way as the fact that $\Sigma$ is orientable, means that one can unambiguously define the notion of clockwise or counterclockwise triangle *on $\Sigma$ itself*.

*Remarks.* So what is classically called a fiber bundle, is what we would call an "equivalence class" of bundles, up to gauge transformation. The "structure group" of our bundles is $O(1) \cong Z/2$ (sometimes also known as the Ising group). But other bundles, with continuous structure groups appear in various gauge theories. For example in QED we use U(1) (representing the phase of the wave function of the electron) or in non-Abelian gauge theories SU(2) or SU(3) (rotations in the space of various internal parameters, such as isospin or color). From this standpoint it is natural that our discrete theory, which is very close to zero entropy, should only use the very simple and discrete "flip-flop" group ($Z/2$, like "spin up and spin down").

Locally, each triangle $\sigma$ has four possibilities, since it can be oriented either clockwise or counterclockwise, and have a (local) transverse direction (in) or (out). There are also four global possibilities; $\Sigma$ can be orientable or not, and the bundle can be orientable or not.

The only case we are actually investigating in detail in this paper is the case where both $\Sigma$ and the bundle are orientable. (We speculate that the other possibilities may be important in a theory of weak interactions.) So in this case there are global notions of clockwise-anticlockwise, and in-out, and each triangle $\sigma$ can receive two independant orientations (see Fig. 16).

If $\Sigma$ is orientable but the bundle is not, then there is only one global orientation we can talk about: clockwise-anticlockwise.

If $\Sigma$ is nonorientable but the bundle is orientable, then again there is only one global orientation one can talk about: in-out.

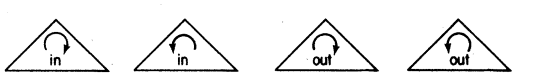

FIG. 16. The four possible orientations of quantum triangles. Here arrows represent quantum-surface orientations, and "in" or "out" represent bundle orientations.

In both of the last two cases the three-dimensional $E$ is nonorientable. But assume now that both $\Sigma$ and the bundle are *non*orientable, in such a way that those closed circuits of $\Sigma$ along which the orientation of $\Sigma$ gets reversed coincide with the closed circuits of $\Sigma$ along which the bundle orientation gets reversed. Then $E$ is *orientable*. One can think of this orientation as being defined by

$$\begin{aligned}(\text{clockwise}) \times (\text{in}) &= (\text{anticlockwise}) \times (\text{out}) \\ &= -(\text{clockwise}) \times (\text{out}) \\ &= -(\text{anticlockwise}) \times (\text{in}).\end{aligned}$$

We now turn to the question of gluing together of line bundles. Suppose we have two bundles $\pi_1$: $E_1 \to \Sigma_1$ and $\pi_2$: $E_2 \to \Sigma_2$, and assume that we make the connected sum $\Sigma_1 \# \Sigma_2$ by identifying the triangles $\sigma_1$ (with edges $A_1, A_2, A_3$) and $\sigma_2$ (with corresponding edges $B_1, B_2, B_3$) and erasing the interiors.

The point we want to explain is that there are two distinct procedures for defining a connected sum of bundles which we will call "parallel"-connected sum and "series"-connected sum. In heuristic terms the difference is depicted in Figs. 17(a) and 17(b). The question is how we "glue" together the fibers $\pi_1^{-1}(x)$, $\pi_2^{-1}(y)$ for corresponding points $x \in \sigma_1$, $y \in \sigma_2$. Let us say that the coordinate transformations of the bundles are denoted by $g_j (j=1, 2)$.

For *parallel-connected sum* the coordinate transformations $g'$ are

(1) $g'(A) = g_1(A)$ for an edge $A$ of $\Sigma_1$ different from $A_1, A_2, A_3$.

(2) $g'(B) = g_2(B)$ for an edge $B$ of $\Sigma_2$ different from $B_1, B_2, B_3$.

(3) $g'(A_i) = g_1(A_i) g_2(B_i)$ for the edges $A_i = B_i$ $(i=1,2,3)$. For a *series-connected sum*, the coordinate transformations $g''$ satisfy conditions (1) and (2) above, but instead of (3) one has

(4) $g''(A_i) = -g_1(A_i) g_2(B_i)$.

If the quantum surface $\Sigma$ is endowed with a parallel or a series line bundle, the two contraction rules (corresponding to forward scattering) are depicted in Fig. 18. For a "parallel" contraction, there is a conserved quantity which is depicted

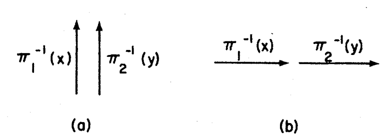

FIG. 17. The two kinds of connected sums: (a) represents the parallel, and (b) the series, sums.


FIG. 18. The contraction rules: (a) contracts to nothing in the parallel case, while (b) contracts to nothing in the series case.

in Fig. 19. In contrast, for a "series" contraction, the following quantity is conserved: number of "in"–number of "out".

The connected sums discussed in the body of this paper correspond to the series case. The conservation law for series bundles is very much like "conservation of flavors". It is natural, in our context, to think of such a law as coming from the fact that two mated triangles are joined by a charge fiber *line*, containing the two charge fibers, the in and out directions providing us with a unique orientation of this charge fiber line. This is exactly what is done in the main body of our paper.

## APPENDIX B: PHOTON SPIN AND PARITY: VECTOR CURRENTS

Photons may be coupled either to spin-$\frac{1}{2}$ peripheral triangles (quarks or leptons) or to spin-0 core triangles. Coupling to spin-$\frac{1}{2}$ is usually expressed in QED through four-component Dirac spinors, so we begin by restating with $4\times 4$ Dirac matrices the Stapp rule for zero-entropy spin dependence.[3] A natural prescription will then emerge for the dependence of $\Sigma^{0,\mathrm{EM}}$ connected parts on photon polarization.

To illustrate the Stapp zero-entropy rule we use as an example the same three-meson $(A,B,C)$ amplitude employed in Appendix D of Ref. 1. For ortho zero-entropy topology, as in Fig. 20, the Stapp $S$-matrix spin dependence translates into the form (B1):

$$[\bar{U}^{\alpha}(A)(1+\gamma_5)\bar{U}^{\alpha}(B)][\bar{U}^{\beta}(B)(1+\gamma_5)U^{\beta}(C)] \times[\bar{U}^{\gamma}(C)(1+\gamma_5)U^{\gamma}(A)]\,, \quad \text{(B1)}$$

where $U$ is a conventional $u$ or $v$ spinor depending on whether the attached particle energy is positive or negative (outgoing or ingoing). The para spin dependence is similar with $(1+\gamma_5)$ replaced by $(1-\gamma_5)$. The notation here is tied as closely as possible to Feynman rules, but topological quarks do not carry momentum, so what do we mean, for example, by the symbol $u^{\alpha}(B)$? The index $\alpha$

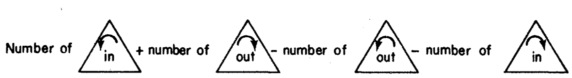


FIG. 19. A conserved quantity for the parallel contraction.

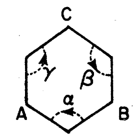


FIG. 20. A three-meson zero-entropy $\Sigma^0_C$.

labels a mated quark pair, as shown in Fig. 20, and the Dirac spinor $u^{\alpha}(B)$ describes the spin of that member of this quark pair that resides within the disk of meson $B$. For example, in the Pauli-Dirac representation, the spinor describing a quark spin state with $s_z=+\frac{1}{2}\hbar$ in the rest system of particle $B$ would be

$$u^{\alpha}(B)=\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\,,\quad p_B=(m_B,0,0,0)\,. \quad \text{(B2)}$$

We here assume $B$ to be ingoing, so from Fig. 20 $\alpha(B)$ is an ingoing quark. In some other frame of reference where $p_B=(E_B,\vec{p}_B)$, Lorentz transformation of the spinor gives

$$u^{\alpha}(B)=\left(\frac{1+v_B^0}{2}\right)^{1/2}\begin{bmatrix}1\\0\\ \dfrac{\sigma\cdot\vec{v}_B}{1+v_B^0}\begin{pmatrix}1\\0\end{pmatrix}\end{bmatrix}\,, \quad \text{(B3)}$$

where

$$v_B=(v_B^0,\vec{v}_B)\equiv p_B/m_B \quad \text{(B4)}$$

and $\vec{\sigma}$ denotes the Pauli $2\times 2$ spin matrices. The prescription (B3) makes it look as if the quark carries the momentum $p_B$, but that is an inappropriate interpretation. It is only the *velocity* of the frame of reference that matters. Once this distinction is grasped, the usual field theory convention may safely be employed in using formulas such as (B1). [A disadvantage of formulas such as (B1) is their awkwardness with respect to crossing. Formula (B3) is only appropriate when $B$ is an ingoing particle so that $\alpha(B)$ is an ingoing quark. If $B$ is outgoing then $\alpha(B)$ becomes an outgoing antiquark and the form of (B3) is different. It is well known, however, that the nice $M$-function crossing properties (see Appendix D of Ref. 1) hold for the $4\times 4$ matrices that are sandwiched between $\bar{U}$ and $U$ spinors.]

The transitivity (self-reproducing character) of (B1) and of the corresponding para spin dependence in zero-entropy unitarity products follows from the form of the projection operator

$$\frac{1+\gamma\cdot v_B}{2} \quad \text{(B5)}$$

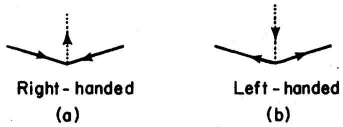


FIG. 21. Patch-boundary orientations on $\Sigma_C$ that correspond to right-handed and left-handed "currents" when the dotted transition arc is cut by an HR arc.

that results from summing over all intermediate spin values when a plug is made of particle $B$. (It is necessary here to remember an essential feature of the zero-entropy meson spectrum: the quartet spin pattern. A spin sum runs over the three states of $S=1$ plus the one $S=0$ state. A similar observation applies to baryons and baryoniums.) If the plug is ortho to ortho, then for the Dirac matrices acting on quark $\alpha$ we have

$$(1+\gamma_5)\left(\frac{1+\gamma\cdot v_B}{2}\right)(1+\gamma_5)=1+\gamma_5\,, \tag{B6}$$

and, if para to para,

$$(1-\gamma_5)\frac{1+\gamma\cdot v_B}{2}(1-\gamma_5)=1-\gamma_5\,, \tag{B7}$$

both results independent of the intermediate velocity $v_B$ and of the original form. In contrast, if the plug is ortho to para we have

$$(1-\gamma_5)\frac{1+\gamma\cdot v_B}{2}(1+\gamma_5)=\gamma\cdot v_B(1+\gamma_5)\,, \tag{B8}$$

a result that remembers the intermediate velocity.

The Dirac matrix (B8) is that associated with a right-handed current

$$\gamma_\mu(1+\gamma_5)\,, \tag{B9}$$

coupled to a vector that is parallel to $v_B$, while the para to ortho transition corresponds to a left-handed current. We remark here that the topological distinction between left-handed and right-handed transitions can be economically expressed through belt (parity) orientations as shown in Fig. 21. No matter what direction is assigned to the HR arc, if one follows the HR direction the transition is ortho→ para in case (a) and para→ ortho in case (b).

We are now ready to state the dependence of a $\Sigma^{0,\mathrm{EM}}$ connected part on photon polarization and quark spin. Consider the example of two mesons $(A, B)$ coupled to a photon. There are two pairs of mated quarks which we label $(\alpha,\beta)$. Suppose that before photon insertion the topology is ortho and that the photon couples to the quark-pair $\alpha$, as shown in Fig. 22, where we have omitted the Landau graph. The spin dependence of the quark-pair $\beta$ remains that of Stapp, but quark $\alpha$ makes an ortho→ para or a para→ ortho transition, depending on whether the photon parity patch touches particle $A$ or particle $B$. Suppose, as indicated in Fig. 22, that $B$ is touched, changing its active quark from ortho to para. (If $B$ is ingoing, its quark in Fig. 22 is para and its antiquark is ortho.) Then the photon causes quark $\alpha$ to change from para to ortho, a left-handed transition, so the following spin dependence is natural:

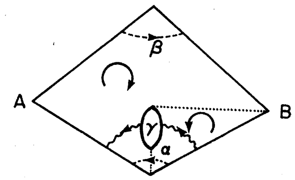


FIG. 22. A left-handed $\Sigma_C^{0,\mathrm{EM}}$ coupling a photon to a meson pair.

$$[\bar U^\beta(B)(1+\gamma_5)U^\beta(A)]\left[\bar U^\alpha(A)(1+\gamma_5)\frac{\gamma_\mu}{2}(1-\gamma_5)U^\alpha(B)\right], \tag{B10}$$

where the index $\mu$ characterizes the photon polarization. The Dirac matrix in the second bracket is equal to the standard left-handed form $\gamma_\mu(1-\gamma_5)$, but the form (B10) more clearly exhibits the transitivity property. For example, if a particle-$A$ plug were made between the surface of Fig. 22 and an ortho zero-entropy surface then the topological character would remain unchanged (a para photon patch within an ortho zero-entropy surface, the photon patch touching an ingoing quark). Formula (B6) together with (B10) shows that the left-handed character of the spin dependence is correctly preserved.

Were the photon patch attached to the outgoing quark of particle $A$ rather than to the ingoing quark of particle $B$, the electromagnetic current would be right handed. In the topological expansion there always occurs the sum of two such alternatives—the sum corresponding to a pure vector electromagnetic current coupled to a quark pair, because the terms proportional to $\gamma_\mu\gamma_5$ cancel each other.

In addition to this right- and left-handed superposition of photon couplings, needed for gauge invariance, there is a further topological doubling associated with parity-patch orientation reversal for the entire $\Sigma_C$. As discussed in Ref. 1, this latter doubling is responsible for parity invariance and eliminates particles of the "wrong" intrinsic parity.

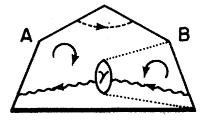


FIG. 23. $\Sigma_C^{0,\mathrm{EM}}$ for photon coupling to the core triangle of an ortho baryon.

What is the dependence on photon polarization when the photon couples to a (zero-spin) core triangle? In Fig. 23 we show the main sheet of $\Sigma_C$ for a two-baryon $\Sigma^{0,\mathrm{EM}}$ amplitude where the active charge arc now belongs to a pair of core triangles. Assume baryon $B$ ingoing and baryon $A$ outgoing. Here there is no ortho-para quark transition—the photon is not coupled to quark spin. When baryon $B$ is active, as in Fig. 23, transitivity is achieved if the dependence on photon polarization is of the form $(p_B)_\mu$. Adding the contribution from the photon-patch attachment to particle $A$, remembering that the core-triangle charge is reversed in sign, we have

$$(p_B - p_A)_\mu \tag{B11}$$

the expected form for the coupling of a photon to zero spin. At the same time the three-baryon quark spins, (one on the main sheet of $\Sigma_C$ and two on adjoining sheets) are propagating independently of the photon according to the ortho form $(1+\gamma_5)$, as in formula (B1).

Although we have here given the two-hadron $\Sigma^{0,\mathrm{EM}}$ spin-polarization coupling rules through special examples—quarks within mesons and core triangles within baryons—the extension is straightforward to quarks within baryons and baryoniums and to core triangles within baryoniums. The further extension to $\Sigma^{0,\mathrm{EM}}$ with more than two hadrons may be accomplished through a connected sum of some $\Sigma^0$ (zero entropy) with a two-hadron $\Sigma^{0,\mathrm{EM}}$.

We have not bothered to mention here the minimal photon spin-parity coupling to two leptons because it is identical with that to quarks. It is impossible within any $\Sigma^{0,\mathrm{EM}}$ to couple a photon to more than two leptons.

## ACKNOWLEDGMENTS

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