Z_N topology and charge confinement in SU(N) Higgs models. II

Z. F. Ezawa and A. Iwazaki
Department of Physics, Tohoku University, Sendai 980, Japan
(Received 5 May 1981)

We analyze a mechanism of quark confinement by making use of a frozen Higgs model. By summing up all possible modes of topological excitations, we derive an effective Lagrangian of the model in the strong-coupling regime. These excitations are magnetic Z_N vortices and magnetic Z_N monopoles, which are labeled by the magnetic weight lattice and by the magnetic root lattice of SU(N), respectively. The effective Lagrangian is shown to contain electric Z_N vortices and electric Z_N charges as topological excitations, which are labeled by the electric weight lattice and by the electric root lattice of SU(N), respectively. From this fact we conclude that the SU(N) Higgs model gives the N-ality confinement of quarks in the strong-coupling regime.

I. INTRODUCTION

In a series of papers we are analyzing a mechanism of quark confinement in non-Abelian gauge theories.[1] Our ultimate purpose is to prove that the Yang-Mills vacuum is a magnetic superconductor in which electric flux is squeezed into physical vortices. Such a superconducting state is considered to be a condensed state of magnetic monopoles[2] or magnetic vortices.[3]

In the standard approach one evaluates the Wilson loop as a criterion of quark confinement; the well-known area law is expected to follow due to the linking of magnetic excitations with the Wilson loop.[2-5] However, we are more interested in deriving an effective Lagrangian of the Yang-Mills theory which would explicitly lead to electric vortices attached to quarks. It is very difficult to attain this object directly on the basis of the Yang-Mills Lagrangian. We recall that the SU(N) Higgs model would have precisely this type of vacuum in the strong-coupling regime.[3-5] Thus, we have decided to analyze it in detail. We hope that the Yang-Mills theory might be approached from the Higgs model in a certain limit and that the weak-coupling phase of the model would be removed in the same limit.

In our previous paper,[1] we analyzed the SU(N) Higgs model by freezing all the gauge degrees of freedom except for the minimal component that is necessary to introduce Z_N vortices and Z_N monopoles. It is given by the last component of the gauge potential, that is $A_\mu^{N^2-1}$ in the standard notation. However, it is impossible to argue on color confinement of quarks by making use of a single component of the gauge potential. It is also impossible to include the effects of all possible modes of Z_N excitations. We wish to make a more sensible approximation of the model.

Here, we note[6] that we may associate an $(N-1)$-dimensional charge vector $\vec{\epsilon}$ with a quark since the rank of SU(N) is $N-1$, and that we may label all states in all representations by vectors $\sum_{k=1}^{N-1} n_k \vec{\epsilon}_k$ with n_k being integers. The set of all such vectors composes the weight lattice of SU(N), which we call the electric weight lattice. For instance, charge components ϵ^H are the color isospin I_3 and the color hypercharge Y in SU(3). The states in the singlet representations, the triplet representations, etc., are labeled by vectors (I_3, Y) as $\{(0,0)\}$, $\{(\frac{1}{2},\frac{1}{3}), (-\frac{1}{2},\frac{1}{3}), (0,-\frac{2}{3})\}$, etc., in the weight lattice of SU(3), as we have illustrated in Fig. 1.

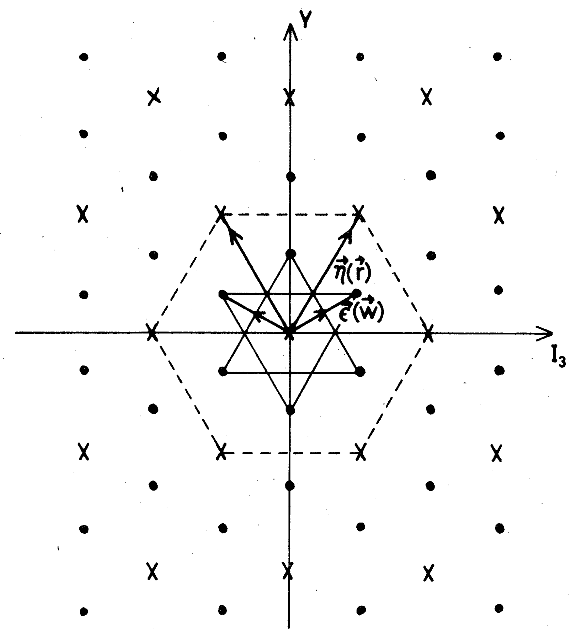

FIG. 1. The weight lattice of SU(3). The bases $\vec{\epsilon}_k$ of the electric weight lattice are given by the charge vectors of quarks, while the bases $\vec{\eta}_k$ of the electric root lattice are given by the charge vectors of Higgs fields in the adjoint representation. The bases $\vec{w}_k$ of the magnetic weight lattice are given by the minimum charge vectors of magnetic Z_N vortices, while the bases $\vec{r}_k$ of the magnetic root lattice are given by the minimum charge vectors of magnetic Z_N monopoles.

Now, let us assume that classical electric vortices are attached to quarks and confine them. These classical vortices are essentially Abelian objects because it is always possible to diagonalize color matrices along them. Namely, they are created in the maximal torus of the gauge group SU(N). Each electric vortex carries a definite amount of Abelian charge which is the charge $\vec{\epsilon}$ of the quark.

On the other hand, the SU(N) Higgs model in the adjoint representation contains magnetic Z_N vortices together with associated magnetic Z_N monopoles as topological excitations in the weak-coupling regime.[7,8] They are also introduced in the maximal torus of the gauge group SU(N). We may associate an $(N-1)$-dimensional magnetic charge vector $\vec{w}$ with each of these vortices. The set of all vectors $\vec{w}$ is easily shown to compose the weight lattice of SU(N), which we call the magnetic weight lattice.[9]

Thus, the electric vortices as well as the magnetic vortices are defined in the maximal torus of the gauge group SU(N) provided that we neglect non-Abelian quantum fluctuations. Therefore, even if we restrict the gauge group SU(N) to its maximal torus, we expect that we are still able to consider all possible modes of Z_N excitations and study color confinement in the SU(N) Higgs model. To achieve this simplification, we freeze all other gauge components by increasing the associated masses. The merit of making this approximation is that we may reduce the dynamical degrees of freedom to being Abelian without losing any essential group-theoretical properties of non-Abelian gauge theories.

We derive an effective Lagrangian of the frozen Higgs model in the strong-coupling regime by summing up all possible excitations of magnetic Z_N vortices as well as magnetic Z_N monopoles. A technique for this calculation has been developed in our previous paper.[1] As is expected, the effective Lagrangian describes a magnetic superconductor in which electric flux is squeezed into physical vortices. Furthermore, the effective Lagrangian contains excitations of not only electric vortices but also electric charges. We demonstrate that they are labeled by the electric weight lattice and the electric root lattice of SU(N), respectively; the electric weight lattice is precisely the one which we have associated with the quark charge.

We obtain the following conclusions: (i) A physical electric vortex is generated and accompanies each system of quarks, which is on the electric weight lattice. (ii) The electric vortex is unstable when it is on the root lattice. In contrast, an electric vortex accompanying a single quark is concluded to be stable. This implies that a single quark cannot exist as a physical state and must be confined. More generally, we show that only the states in nonzero N-ality representations are confined in the Higgs model.

However, this type of confinement scheme might be peculiar to the Higgs model in the adjoint representation. The reason for N-ality confinement is that the magnetic superconducting vacuum allows electric charge excitations labeled by the electric root lattice. We may identify these charges with Higgs charges liberated into an incoherent plasma state due to magnetic vortex condensation.[1] Indeed, within the present model, if we consider only the effect of monopole condensation, we are able to check that such charge excitations do not emerge. Therefore, it may well be that only a singlet combination of quarks will escape from confinement in the Yang-Mills theory where Higgs fields are absent. However, we shall not attempt to elaborate this statement any further since we still lack a consistent renormalization scheme by way of which the Yang-Mills theory might be approached.

In Sec. II, we remark about the correspondence between the electric Dirac string and the path-dependent phase factor. We also associate an $(N-1)$-dimensional charge vector $\vec{\epsilon}$ with a quark and introduce the electric weight lattice. In Sec. III, we review on the classification of magnetic vortices as well as magnetic monopoles in terms of the magnetic weight lattice and the magnetic root lattice. In Sec. IV, we derive an effective Lagrangian of a frozen Higgs model in the strong-coupling regime and show that it leads to electric vortices to confine quarks. In Sec. V, we discuss the problem of deciding the magnetic gauge group. Finally, in Sec. VI, we summarize our conclusions.

II. ELECTRIC DIRAC STRINGS AND ELECTRIC WEIGHT LATTICE

We consider the SU(N) gauge theory

$$L=-\frac{1}{2g^2}\,\mathrm{Tr}(F_{\mu\nu}{}^2+4g^2A_\mu J_\mu)\,,\qquad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu-i[A_\mu,A_\nu] \tag{2.1}$$

in the presence of an external source J_μ. Here, J_μ is supposed to be given by

$$J^a_\mu=\tfrac{1}{2}\bar{\psi}\lambda^a\gamma_\mu\psi \tag{2.2}$$

in terms of quark fields which transform according to the fundamental representation of the gauge group SU(N).

Our common belief is that a single quark is always accompanied by a vortex which is generated by dressing a "string" $W(\infty, x)$ via radiative corrections. Here

$$W(y,x)=P\exp\left(i\int_x^y dz_\mu A_\mu\right) \tag{2.3}$$

is the well-known path-dependent phase factor.

It is essential to note that operator (2.3) transports the color charge of a quark at point x to another point y. Namely, the object

$$\phi(y)=W(y,x)\psi(x) \tag{2.4}$$

transforms according to the fundamental representation of SU(N) at point y. Therefore, we may classify a set of external quarks into color multiplets by transporting all their charges to one point y. In an instance of the SU(3) gauge theory, a $\bar{\phi}\phi$ system is decomposed into irreducible representations by the scheme $\underline{3}^*\times\underline{3}=\underline{1}+\underline{8}$, while a $\phi\phi\phi$ system is represented by $\underline{3}\times\underline{3}\times\underline{3}=\underline{1}+\underline{8}+\underline{8}+\underline{10}$. Here, combinations $\sum\bar{\phi}^i\phi^i$ and $\sum\epsilon_{ijk}\phi^i\phi^j\phi^k$ provide the singlet representations.

As we have argued in the Introduction, we shall restrict the gauge group SU(N) to its maximal torus in this paper. Accordingly, we consider the gauge potential

$$A_\mu=\sum_{H=1}^{N-1}A_\mu^H\lambda^H/2\,, \tag{2.5}$$

which takes values in the maximal Abelian subalgebra. Without loss of generality, we way set

$$\lambda^H=\left(\frac{2}{H(H+1)}\right)^{1/2}\mathrm{diag}(1,\ldots,1,-H,0,\ldots,0)\,, \tag{2.6}$$

where 1 appears H times, which are the diagonal Gell-Mann matrices in SU(N).

In our formalism,[1] operator expressions for the quark current (2.2) and the phase factor (2.3) are not convenient. We replace the quark current by a classical current

$$J_\mu(x)=\sum_{H=1}^{N-1}J_\mu^H(x)\lambda^H/2\,,$$
$$J_\mu^H(x)=\sum_p\epsilon_p^H\int d\tau\,\delta^{(4)}(x-z^p)\dot{z}_\mu^p\,, \tag{2.7}$$

where ϵ_p^H are the electric charges of the pth quark:

$$\epsilon^H=q^\dagger\frac{\lambda^H}{2}q\,. \tag{2.8}$$

Here, q is an N-dimensional vector which reads

$$q_k^\dagger=(0,\ldots,0,1,0,\ldots,0)\,, \tag{2.9}$$

where $k-1$ zeros precede the 1, when the quark has a definite color k. Thus, a quark with color k has an $(N-1)$-dimensional charge vector $\vec{\epsilon}_k$ whose components are given by $\epsilon_k^H=q_k^\dagger(\lambda^H/2)q_k$:

$$\vec{\epsilon}_1=(e_1,e_2,\ldots,e_{N-1})\,,$$
$$\vec{\epsilon}_k=(0,\ldots,0,(1-k)e_{k-1},e_k,\ldots,e_{N-1}) \quad (2\le k\le N-1)\,, \tag{2.10a}$$
$$\vec{\epsilon}_N=(0,\ldots,0,(1-N)e_{N-1})\,,$$

with

$$e_k=1/[2k(k+1)]^{1/2}\quad(1\le k\le N-1)\,. \tag{2.10b}$$

Note that

$$\vec{\epsilon}_j\cdot\vec{\epsilon}_k=\begin{cases}(N-1)/2N & \text{for } j=k\,,\\ -1/2N & \text{for } j\ne k\end{cases} \tag{2.11a}$$

and

$$\sum_{j=1}^N\vec{\epsilon}_j=0\,. \tag{2.11b}$$

It is well known[6] that we may construct the weight lattice of SU(N) on the basis of the vectors $\vec{\epsilon}_k$.

For a later convenience we remark about the Higgs charges when Higgs fields are introduced in the adjoint representation. Then, there are $N-1$ independent charge vectors $\vec{\eta}_k$ associated with the Higgs charge. For instance, they are given by

$$\vec{\eta}_k=\vec{\epsilon}_k-\vec{\epsilon}_N\quad(k=1,\ldots,N-1)\,. \tag{2.12}$$

Then, the root lattice of SU(N) is constructed on the bases of the vectors $\vec{\eta}_k$. Obviously, the root lattice is a sublattice of the weight lattice in SU(N) and labels the elements in the multiplets composed of the Higgs particles.

Now, we introduce electric Dirac strings attached to the quarks by

$$J_{\mu\nu}(x)=\sum_{H=1}^{N-1}J_{\mu\nu}^H(x)\lambda^H/2\,,$$
$$J_{\mu\nu}^H(x)=\sum_p\epsilon_p^H\int d^2\tau\,\delta^{(4)}(x-z^p)[z_\mu^p,z_\nu^p]\,, \tag{2.13}$$
$$[z_\mu,z_\nu]\equiv\frac{\partial(z_\mu,z_\nu)}{\partial(\tau_1,\tau_2)}\,.$$

Note that they satisfy

$$\partial_\nu J_{\mu\nu}^H(x)=J_\mu^H(x)\,. \tag{2.14}$$

It is important to notice that phase factor (2.3) with gauge potential (2.5) satisfies the commutation relation

$$[F_{0k}^H(x),q^\dagger W(y_1,y_2)q]=q^\dagger W(y_1,y_2)q\epsilon^H\int d^2\tau\,\delta^{(4)}(x-y)[y_0,y_k]\,, \tag{2.15}$$

with electric field F_{0k}^H. Therefore, $q^\dagger W(y_1,y_2)q$ creates an electric Dirac string connecting two points y_1 and y_2 along the integration path in phase factor (2.3). We emphasize that these strings are unphysical.

We now arrange all strings to meet each other at one point y. Then, cutting them at this point, we obtain

$$\partial_\nu J^H_{\mu\nu}(x) = J^H_\mu(x) - \delta_{\mu 0}\sum_p \epsilon^H_p \delta^{(3)}(x-y)\,, \qquad (2.16)$$

instead of (2.14). The total charge $\sum_p \epsilon^H_p$ is thus induced at a point y. It is obvious that the charge vector constructed from Abelian charges $\sum_p \epsilon^H_p$ is labeled by the electric weight lattice. We may associate a single electric Dirac string to this charge by solving (2.14). Hence, we conclude that all possible electric Dirac strings in SU(N) gauge theories are also labeled by the electric weight lattice.

In Sec. III, we define magnetic Dirac strings in SU(N) gauge theories. In Sec. IV, we shall discuss the relation between magnetic and electric Dirac strings.

III. MAGNETIC DIRAC STRINGS AND MAGNETIC WEIGHT LATTICE

In deriving an effective Lagrangian of the SU(N) Higgs model in the strong-coupling regime, we need to sum up all possible modes of topological excitations. They are defined with respect to the center Z_N of the gauge group SU(N).[7] Here, we review the classification of all Z_N excitations in the Higgs model.

We consider the Lagrangian density

$$L = -\frac{1}{2g^2}\,\mathrm{Tr}(F_{\mu\nu}{}^2 - 4g^2 A_\mu J_\mu) + L_m(A_\mu, \Phi_1, \ldots, \Phi_k)\,, \qquad (3.1)$$

where, in order to break the gauge symmetry SU(N) completely except for the center Z_N, we have introduced a sufficient number of Higgs fields in the adjoint representation. First, we note that all topological excitations are parametrized by magnetic Dirac strings and magnetic monopoles which singular gauge transformations generate.[7] Indeed, performing a singular gauge transformation

$$A_\mu \to S A_\mu S^{-1} - i(\partial_\mu S)S^{-1}\,, \quad \Phi_i \to S\Phi_i S^{-1}\,, \quad J_\mu \to S J_\mu S^{-1} \qquad (3.2)$$

to the Lagrangian (3.1), we obtain

$$L_\rho = \frac{-1}{2g^2}\,\mathrm{Tr}[(F_{\mu\nu} - \rho_{\mu\nu})^2 - 4g^2(A_\mu - iS^{-1}\partial_\mu S)J_\mu] + L_m(A_\mu, \Phi_1, \ldots, \Phi_k)\,, \qquad (3.3)$$

$$\rho_{\mu\nu} = iS^{-1}[\partial_\mu, \partial_\nu]S\,.$$

Then, $\rho_{\mu\nu}$ provides us with a boundary condition for classical field equations to create topological excitations.[7] In the present formalism,[1] by integrating over field variables for given $\rho_{\mu\nu}$ in the generating functional, we obtain an interacting system of magnetic strings and monopoles described by $\rho_{\mu\nu}$. Namely, $\rho_{\mu\nu}$ may be regarded as bare excitations which are to be dressed by way of quantum corrections.

It is always possible to diagonalize the matrix $\rho_{\mu\nu}$. Thus, we may set

$$\rho_{\mu\nu} = \sum_{H=1}^{N-1} \rho^H_{\mu\nu}\lambda^H/2\,, \qquad (3.4)$$

where λ^H are defined by (2.6), where

$$\rho^{H*}_{\mu\nu}(x) = 4\pi\sum_q w^H_q \int d^2\tau\, \delta^{(4)}(x-y^q)[y^q_\mu, y^q_\nu]\,, \qquad (3.5a)$$

$$k^H_\mu(x) = 4\pi\sum_p r^H_p \int d\tau\, \delta^{(4)}(x-y^p)\dot{y}^p_\mu \qquad (3.5b)$$

together with

$$\partial_\nu \rho^{H*}_{\nu\mu}(x) = k^H_\mu(x) \qquad (3.5c)$$

and $\rho^{H*}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\rho^H_{\alpha\beta}$. Here, w^H and r^H are Abelian charges of each of the strings and monopoles, respectively. These charges are quantized according to generalized Dirac quantization conditions. In fact, a singular gauge transformation S should satisfy[8]

$$P\exp\left(\oint dx_\mu(\partial_\mu S)S^{-1}\right) \in Z_N \qquad (3.6a)$$

to create strings, and

$$P\exp\left(\oint dx_\mu(\partial_\mu S)S^{-1}\right) = 1 \qquad (3.6b)$$

to create monopoles, which are equivalent to

$$\exp\left(i2\pi\sum_{H=1}^{N-1} w^H\lambda^H\right) \in Z_N\,, \qquad (3.7a)$$

$$\exp\left(i2\pi\sum_{H=1}^{N-1} r^H\lambda^H\right) = 1\,. \qquad (3.7b)$$

We note that these quantization conditions have been studied in the classification problem of non-Abelian monopoles.[9]

Because λ^H are diagonal matrices explicitly given by (2.6), it is quite easy to solve these equations. Obviously, (3.7a) is equivalent to

$$4\pi\sum_{H=1}^{N-1} w^H\epsilon^H_k = \frac{2\pi}{N}n + 2\pi n_k \quad (k=1,\ldots,N)\,, \qquad (3.8)$$

n and n_k being arbitrary integers, where ϵ^H_k are the Hth components of vectors (2.10a). We associate an $(N-1)$-dimensional vector $\vec{w}$ to each of solutions of (3.8). Then, by making use of relation (2.11) it is trivial to show that the general solutions of (3.7a) are given by

$$\vec{w}=\sum_{j=1}^{N-1} n_j \vec{\epsilon}_j \tag{3.9a}$$

with n_j being integers. Thus, the set of all vectors $\vec{w}$ compose the weight lattice of SU(N), which we call the magnetic weight lattice.[9] On the other hand, the general solutions of (3.7b) are given by

$$\vec{r}=\sum_{j=1}^{N-1} n_j \vec{\eta}_j\,, \tag{3.9b}$$

n_j being integers, and hence they constitute a subset of the magnetic weight lattice, which is called the magnetic root lattice. Therefore, up to regular gauge transformations, all Z_N vortices (Z_N monopoles) are labeled by the weight (root) lattice of SU(N).[9] In Fig. 1, we have illustrated these lattices in case of SU(3). All points therein correspond to magnetic vortex excitations while the crossed points correspond to magnetic monopole excitations.

It is clear that we are able to describe all these excitations within the maximal torus of SU(N). For the sake of simplicity, we freeze the other gauge components by increasing their masses so that we may approximate the gauge potential by (2.5). Moreover, let us increase Higgs masses as well and approximate the Higgs Lagrangian (3.1) by

$$L=-\frac{1}{4g^2}\sum_{H=1}^{N-1}(F^H_{\mu\nu}F^H_{\mu\nu}-2m_V{}^2A^H_\mu A^H_\mu-4g^2A^H_\mu J^H_\mu)\,. \tag{3.10}$$

Then, Lagrangian (3.3) amounts to

$$L_\rho=-\frac{1}{4g^2}\sum_{H=1}^{N-1}[(F^H_{\mu\nu}-\rho^H_{\mu\nu})^2-2m_V{}^2A^H_\mu A^H_\mu - 2g^2(F^H_{\mu\nu}-\rho^H_{\mu\nu})J^H_{\mu\nu}]\,, \tag{3.11}$$

where $J^H_{\mu\nu}$ denotes electric Dirac strings (2.13) attached to quarks. It is well known[10] that Lagrangian (3.11) describes magnetic vortices parametrized by $\rho^H_{\mu\nu}$. In this linearized version, we need an ultraviolet cutoff which is provided by the Higgs mass m_S.[10]

In the next section we analyze the effect of these magnetic excitations to the generating functional of the Higgs model.

IV. MAGNETIC SUPERCONDUCTOR AND ELECTRIC VORTICES

The essential step towards quark confinement is to prove that phase-factor (2.3) indeed creates physical electric vortices. In this section, we analyze the SU(N) Higgs model instead of the Yang-Mills theory. We derive an effective Lagrangian in the strong-coupling regime and show that the Lagrangian describes a magnetic superconductor.

For this purpose, we evaluate the generating functional

$$Z=\int [dA_\mu][d\Phi_i]\exp\left(-\int L\right) \tag{4.1}$$

together with the Higgs Lagrangian (3.1) in Euclidean metric, where field variables are to be integrated over all possible topological excitations. As we have remarked in the preceding section, they are parametrized by magnetic string singularities (3.5). Thus, we rewrite (4.1) as

$$Z=\int [d(\text{string})]\int [dA_\mu][d\Phi_i]\exp\left(-\int L_\rho\right) \tag{4.2}$$

together with (3.3). Here, $\int [d(\text{string})]$ stands for an integration over all possible configurations of magnetic string singularities (3.5). By integrating over A_μ and Φ_i for given $\rho_{\mu\nu}$ in generating functional (4.2), we obtain an interacting system of magnetic vortices and monopoles.

We evaluate generating functional (4.2) in the frozen limit, where we may use Lagrangian (3.11). Then, the system is dynamically Abelian and the self-energies of the Z_N excitations are calculated easily.[10] Corresponding to the magnetic vortex excitation (3.5a) and the magnetic monopole excitation (3.5b), they read

$$\mathfrak{M}_1(x)=\alpha_1\frac{m_V{}^2}{g^2}\sum_q \vec{w}_q{}^2\int d^2\tau\,\delta^{(4)}(x-y^q) \times|\det[y^q_\mu,y^q_\nu]|^{1/2}\,, \tag{4.3a}$$

and

$$\mathfrak{M}_2(x)=\alpha_2\frac{m_S}{g^2}\sum_p \vec{r}_p{}^2\int d\tau\,\delta^{(4)}(x-y^p)(\dot{y}^p_\mu\dot{y}^p_\mu)^{1/2}\,, \tag{4.3b}$$

respectively, where α_k are dimensionless positive quantities whose precise values are not important and m_S is a typical mass of the frozen Higgs fields. Vectors $\vec{w}$ and $\vec{r}$ denote points on the magnetic weight lattice and the magnetic root lattice, respectively.

After extracting self-energies of these excitations, we may rewrite (4.2) as[1]

$$Z=\sum_{\{\vec{w},\vec{r}\}}\prod_{H=1}^{N-1}\int [dF^H_{\mu\nu}][dA^H_\mu]\delta(F^H_{\mu\nu}-\partial_\mu A^H_\nu+\partial_\nu A^H_\mu+\rho^H_{\mu\nu})\exp\left[-\int\left(\frac{1}{4g^2}F^H_{\mu\nu}F^H_{\mu\nu}+\frac{m_V{}^2}{2g^2}A^H_\mu A^H_\mu + \mathfrak{M}^H_1+\mathfrak{M}^H_2+\frac{i}{2}J^H_{\mu\nu}F^H_{\mu\nu}\right)\right], \tag{4.4}$$

together with (3.5) and

$$\mathfrak{M}_1^H(x)=\alpha_1\frac{m_V{}^2}{g^2}\sum_q (w_q^H)^2\int d^2\tau\,\delta^{(4)}(x-y^q)\left|\det[y_\mu^q,y_\nu^q]\right|^{1/2}, \tag{4.5a}$$

$$\mathfrak{M}_2^H(x)=\alpha_2\frac{m_S}{g^2}\sum_p (r_p^H)^2\int d\tau\,\delta^{(4)}(x-y^p)(\dot y_\mu^p\dot y_\mu^p)^{1/2}, \tag{4.5b}$$

where $\sum_{\{\vec w,\vec r\}}$ stands for a summation over magnetic vortices ($\vec{\mathrm{w}}$) and magnetic monopoles ($\vec{\mathrm{r}}$) labeled by the magnetic weight lattice and the magnetic root lattice. We now express the Bianchi identity (3.5c) which represents the magnetic flux conservation by

$$\delta(\partial_\mu\rho_{\mu\nu}^{H*}-k_\nu^H)=\int[dB_\nu^H]\exp\Big(i\int[B_\nu^H(\partial_\mu\rho_{\mu\nu}^{H*}-k_\nu^H)]\Big). \tag{4.6}$$

Then, (4.4) reads

$$Z=\prod_{H=1}^{N-1}\int[dA_\mu^H][dB_\mu^H][dC_{\mu\nu}^H]I(B,C)\exp\Big[-\int\Big(\frac{g^2}{4}(C_{\mu\nu}^H+J_{\mu\nu}^H)^2+\frac{m_V{}^2}{2g^2}A_\mu^HA_\mu^H+\frac{i}{2}C_{\mu\nu}^HA_{\mu\nu}^H\Big)\Big] \tag{4.7}$$

with

$$I(B,C)=\sum_{\vec w}\exp\Big[-\sum_{H=1}^{N-1}\int\Big(\mathfrak{M}_1^H+\frac{i}{2}\rho_{\mu\nu}^H(G_{\mu\nu}^{H*}+C_{\mu\nu}^H)\Big)\Big]\sum_{\vec r}\exp\Big(-\sum_{H=1}^{N-1}\int(\mathfrak{M}_2^H+ik_\mu^HB_\mu^H)\Big), \tag{4.8}$$

and

$$G_{\mu\nu}^H=\partial_\mu B_\nu^H-\partial_\nu B_\mu^H.$$

Here, the summation $\sum_{\vec w}$ ($\sum_{\vec r}$) runs over all points on the magnetic weight lattice (magnetic root lattice) of SU(N).

In our previous paper[1] we have analyzed the summation of magnetic Z_N excitations where charges only with the single component ($H=N-1$) are taken into account. The generalization to the present case is simple but needs a technical improvement. We shall describe this technique in the Appendix. Here we only cite the result. By making use of Poisson resummation formulas, we may change the summation over magnetic Z_N excitations into the summation over electric Z_N excitations. Thus, (4.8) leads to

$$I(B,C)=\sum_{\vec\epsilon}\sum_{\vec\eta}\exp\Big[-\sum_{H=1}^{N-1}\int\Big(\frac{g^2}{16\alpha_1m_V{}^2b^2}(G_{\mu\nu}^H+4\pi C_{\mu\nu}^{H*}+\sigma_{\mu\nu}^{H*})^2+\frac{g^2}{4\alpha_2m_Sb^3}B_\mu^HB_\mu^H\Big)\Big], \tag{4.9}$$

with b being a lattice spacing introduced in order to fix the integration measure of magnetic Z_N excitations, and

$$\sigma_{\mu\nu}^H(x)=4\pi\sum_q\epsilon_q^H\int d^2\tau\,\delta^{(4)}(x-z^q)[z_\mu^q,z_\nu^q], \tag{4.10a}$$

$$j_\mu^H(x)=4\pi\sum_p\eta_p^H\int d\tau\,\delta^{(4)}(x-z^p)\dot z_\mu^p \tag{4.10b}$$

together with

$$\partial_\nu\sigma_{\nu\mu}^H=j_\mu^H. \tag{4.10c}$$

Here, $\sigma_{\mu\nu}^H$ and j_μ^H represent electric Dirac strings and electric charges carrying Abelian flux ϵ^H and charge η^H, respectively. The summation $\sum_{\vec\epsilon}$ ($\sum_{\vec\eta}$) runs over all points on the electric weight lattice (electric root lattice) of SU(N).

Combining (4.7) and (4.9), we derive the effective Lagrangian

$$L_\sigma^{\text{eff}}=-\frac{1}{4(1+\gamma)g^2}\sum_H(F_{\mu\nu}^HF_{\mu\nu}^H-2m_A{}^2A_\mu^HA_\mu^H+2g^2F_{\mu\nu}^HJ_{\mu\nu}^H)$$
$$-\frac{\gamma g^2}{64\pi^2(1+\gamma)}\sum_H[(G_{\mu\nu}^H-\sigma_{\mu\nu}^{H*}+4\pi J_{\mu\nu}^{H*})^2-2m_B{}^2B_\mu^HB_\mu^H]-\frac{\gamma}{8\pi(1+\gamma)}\sum_H F_{\mu\nu}^{H*}(G_{\mu\nu}^H-\sigma_{\mu\nu}^{H*}), \tag{4.11}$$

with $\gamma = 4\pi^2/\alpha_1 b^2 m_V{}^2$, $m_A{}^2 = (1+\gamma)m_V{}^2$, and $m_B{}^2 = 8\pi^2(1+\gamma)/\alpha_2\gamma b^3 m_S$ in the unitary gauge. This Lagrangian describes a magnetic superconductor in field variables B^H_μ, just as Lagrangian (3.11) describes an electric superconductor in field variables A^H_μ. It is obvious[10] that the system contains electric vortex solutions in B^H_μ whose boundary conditions are given by electric Dirac strings (4.10a). These vortices may have open ends when they terminate on charge excitations described by charge singularities (4.10b). The mass of an open vortex is composed of two terms[10,11]; the mass density $\sim g^2 m_B{}^2 \ln(\Lambda/m_B)$ of the vortex, and the mass $\sim g^2\Lambda$ of the end-point charge, with Λ being an ultraviolet cutoff. In case of magnetic vortices, the cutoff is given by the size of the vortex core in which the electric superconductor is broken and it is provided by the Higgs mass m_S. Similarly in case of electric vortices, the cutoff Λ is given by the size of the vortex core in which the magnetic superconductor is broken and it must be provided by the monopole mass. Hence, we expect that $\Lambda \simeq m_S$.

We now discuss color confinement of quarks. As we have proved, the effective Lagrangian (4.11) contains electric vortices labeled by the electric weight lattice of SU(N). Furthermore, the string singularities (4.10a) have precisely the same expression as the electric Dirac strings (2.13) attached to quarks. Therefore, we conclude that the SU(N) Higgs model contains electric vortices parametrized by (2.13) as topological excitations in the strong-coupling regime; equivalently, phase-factor (2.3) creates physical electric vortices.

Next, let us arrange these strings (2.13) to join with each other at a point y and cut them as in (2.16). Then, the total charge $\sum_p \epsilon^H_p$ is induced, which is labeled by the electric weight lattice of SU(N). On the other hand, the system (4.11) contains excitations of electric charges (4.10b) which are labeled by the electric root lattice of SU(N). If and only if the induced charge is on the electric root lattice, we may screen it. Hence, electric vortices are terminated and the system has a finite energy. Therefore, we conclude that the SU(N) Higgs model provides us with the N-ality confinement of quarks.

V. ELECTRIC AND MAGNETIC GAUGE GROUPS

As we have remarked in Sec. III, all magnetic excitations in gauge theories with the electric gauge group SU(N) are labeled by the magnetic weight lattice of SU(N). Based on this fact, the existence of the underlying magnetic gauge group SU(N) has been conjectured.[9,12] We are yet unable to give a proof of this conjecture since in this paper we are analyzing solely the maximal torus of SU(N). Nonetheless, we may present an affirmative step towards this goal. Namely, we shall show that electric Dirac strings $\sigma^H_{\mu\nu}$ in (4.11) may be removed by performing a singular gauge transformation in the underlying magnetic gauge group SU(N), just as magnetic Dirac strings $\rho^H_{\mu\nu}$ in (3.11) have been generated by performing a singular gauge transformation in the electric gauge group SU(N).

First we rewrite effective Lagrangian (4.11) as

$$L^{\text{eff}}_\sigma = -\frac{1}{2(1+\gamma)g^2}\,\text{Tr}(F_{\mu\nu}{}^2 - 2m_A{}^2 A_\mu{}^2 + 2g^2 F_{\mu\nu}J_{\mu\nu}) - \frac{\gamma g^2}{32\pi^2(1+\gamma)}\,\text{Tr}[(G_{\mu\nu} - \sigma^*_{\mu\nu} + 4\pi J^*_{\mu\nu})^2 - 2m_B{}^2 B_\mu{}^2] - \frac{\gamma}{4\pi(1+\gamma)}\,\text{Tr}F^*_{\mu\nu}(G_{\mu\nu} - \sigma^*_{\mu\nu}) \tag{5.1}$$

in the matrix notation, where we have set

$$A_\mu = \sum_{H=1}^{N-1} A^H_\mu \lambda^H/2\,,\quad B_\mu = \sum_{H=1}^{N-1} B^H_\mu \lambda^H/2\,,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]\,, \tag{5.2a}$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu - i[B_\mu, B_\nu]\,,$$

$$J_{\mu\nu} = \sum_{H=1}^{N-1} J^H_{\mu\nu}\lambda^H/2\,,$$

and

$$\sigma^*_{\mu\nu} = \sum_{H=1}^{N-1} \sigma^{H*}_{\mu\nu}\lambda^H/2 \tag{5.2b}$$

with λ^H being defined by (2.6). We remark that all matrices are taken in the maximal torus of SU(N). This Lagrangian reads

$$L^{\text{eff}}_\sigma = -\frac{1}{2(1+\gamma)g^2}\,\text{Tr}[F_{\mu\nu}{}^2 - 2m_A{}^2(A_\mu + i(\partial_\mu\Omega_A)\Omega_A{}^{-1})^2 + 2g^2 F_{\mu\nu}J_{\mu\nu}] - \frac{\gamma g^2}{32\pi^2(1+\gamma)}\,\text{Tr}[(G_{\mu\nu} - \sigma^*_{\mu\nu} + 4\pi J^*_{\mu\nu})^2 - 2m_B{}^2(B_\mu + i(\partial_\mu\Omega_B)\Omega_B{}^{-1})^2] - \frac{\gamma}{4\pi(1+\gamma)}\,\text{Tr}F^*_{\mu\nu}(G_{\mu\nu} - \sigma^*_{\mu\nu})\,, \tag{5.3}$$

in the covariant gauge.[5,13] Here, Ω_A is essentially the Higgs phase factor[13] which carries the electric gauge symmetry. In the present frozen case, it reads

$$\Omega_A = \exp\left(i\sum_{H=1}^{N-1} \chi^H_A \lambda^H/2\right) \tag{5.4}$$

with χ^H_A being Goldstone fields. Recall that the multivalueness in χ_A^H has created magnetic Z_N vortices in the weak-coupling regime.[13] Now, χ_A^H are single-valued functions in the strong-coupling regime and Ω_A is a regular function in (5.3). On the other hand, Ω_B carries the magnetic gauge degrees of freedom in effective Lagrangian (5.3). It reads

$$\Omega_B = \exp\left(i\sum_{H=1}^{N-1} \chi_B^H \lambda^H/2\right) \tag{5.5}$$

in the present frozen case.[14] We may trace back the origin of the "Goldstone" field χ_B^H to the Bianchi identity (4.6), which does not define the magnetic potential B_μ^H uniquely. Indeed, (4.6) is invariant by the replacement

$$B_\mu^H \to B_\mu^H + \partial_\mu \chi_B^H , \tag{5.6}$$

because the monopole current (3.5b) is conserved, $\partial_\mu k_\mu = 0$. It is possible to identify phase-factor (5.5) with quantum monopole fields whose radial components are frozen[14]; these monopoles are condensed and create a magnetic superconductor in the strong-coupling regime.

We proceed to use the fact that electric Dirac strings $\sigma_{\mu\nu}$ are labeled by the electric weight lattice of SU(N), just as magnetic Dirac strings $\rho_{\mu\nu}$ are labeled by the magnetic weight lattice of SU(N). Mathematically, $\sigma_{\mu\nu}^*$ and $\rho_{\mu\nu}$ are the same objects, as is obvious from (3.5), (4.10), and (3.9). Therefore, corresponding to (3.3), it is possible to choose a singular gauge function S such that

$$\sigma_{\mu\nu}^* = iS^{-1}[\partial_\mu, \partial_\nu]S . \tag{5.7}$$

Then, (5.3) gives rise to

$$L^{\text{eff}} = -\frac{1}{2(1+\gamma)g^2}\,\mathrm{Tr}[\tilde{F}_{\mu\nu}{}^2 - 2m_A{}^2(\tilde{A}_\mu + i(\partial_\mu\tilde{\Omega}_A)\tilde{\Omega}_A{}^{-1})^2 + 2g^2\tilde{F}_{\mu\nu}\tilde{J}_{\mu\nu}]$$
$$-\frac{\gamma g^2}{32\pi^2(1+\gamma)}\,\mathrm{Tr}[(\tilde{G}_{\mu\nu} + 4\pi\tilde{J}_{\mu\nu}^*)^2 - 2m_B{}^2(\tilde{B}_\mu + i(\partial_\mu\tilde{\Omega}_B)\tilde{\Omega}_B{}^{-1})^2] - \frac{\gamma}{4\pi(1+\gamma)}\,\mathrm{Tr}\tilde{F}_{\mu\nu}^*\tilde{G}_{\mu\nu} , \tag{5.8}$$

where we have set

$$\tilde{A}_\mu = SA_\mu S^{-1} + iS\partial_\mu S^{-1} ,$$
$$\tilde{F}_{\mu\nu} = SF_{\mu\nu}S^{-1} , \quad \tilde{\Omega}_A = S\Omega_A , \tag{5.9a}$$
$$\tilde{B}_\mu = SB_\mu S^{-1} + iS\partial_\mu S^{-1} ,$$
$$\tilde{G}_{\mu\nu} = S(G_{\mu\nu} - \sigma_{\mu\nu}^*)S^{-1} , \quad \tilde{\Omega}_B = S\Omega_B , \tag{5.9b}$$
$$\tilde{J}_{\mu\nu} = SJ_{\mu\nu}S^{-1} . \tag{5.9c}$$

We are able to interpret (5.9) as a singular gauge transformation performed in the magnetic gauge group SU(N).

A comment is in order. Literally, we should have written

$$\tilde{F}_{\mu\nu} = S(F_{\mu\nu} - \sigma_{\mu\nu}^*)S^{-1}$$

in (5.9a), and hence $(\tilde{F}_{\mu\nu} + \sigma_{\mu\nu}^*)^2$ instead of $\tilde{F}_{\mu\nu}{}^2$ in (5.8). However, we have dropped it since this type of singularity is entirely unphysical in (5.8). This is so because the Higgs phase-factor $\tilde{\Omega}_A$ has acquired a singular phase S simultaneously, which cancels string singularities $\sigma_{\mu\nu}^*$ in field equations. Namely, $\sigma_{\mu\nu}^*$ do not provide a boundary condition for A_μ to create physical magnetic vortices, as should be the case. For brevity, we may say that singular gauge transformations in the magnetic (electric) gauge group create electric (magnetic) vortex strings in the strong-(weak-) coupling regime of the Higgs model.

Finally we wish to make a conjecture on "unfrozen" SU(N) Higgs models where the complete gauge group SU(N) is respected. A concise example of such a Higgs model is given by

$$L = -\tfrac{1}{2}\mathrm{Tr}[F_{\mu\nu}{}^2 - 2m_V{}^2(A_\mu + i(\partial_\mu\Omega)\Omega^{-1})^2] , \tag{5.10}$$

in the large mass limit ($m_S \gg m_V$) of Higgs fields. Here, Ω is the Higgs phase factor and denotes the unphysical Goldstone mode carrying the complete electric gauge symmetry in the model.[13] Lagrangian (5.10) itself is known as the massive Yang-Mills Lagrangian.[5,15] Comparing the resemblance between (5.8) and (5.10), we conjecture that (5.8) might be the effective Lagrangian of (5.10) in the strong-coupling regime. Then, phase-factor (5.5) would also be replaced by the general SU(N) matrix which carries the complete magnetic gauge symmetry. However, in order to prove this conjecture, we need to improve a technical device of calculating non-Abelian quantum fluctuations around classical Abelian vortices.

VI. CONCLUSIONS

In this paper we have analyzed an SU(N) Higgs model in the strong-coupling regime by freezing out all the gauge degrees of freedom except for the minimal components that are necessary to argue color confinement of quarks. These components are given by the maximal torus of the gauge group SU(N). Then, we have associated an ($N-1$)-dimensional charge vector $\vec{\epsilon}$ with the quark. All possible charge vectors $\vec{\epsilon}$ in the model compose the electric weight lattice of SU(N).

In the weak-coupling regime the SU(N) Higgs model contains magnetic vortices together with end-point monopoles as topological excitations. These excitations have $(N-1)$-dimensional magnetic charge vectors $\vec{w}$. The set of all possible charge vectors $\vec{w}$ in the model composes the magnetic weight lattice of SU(N).

By summing up all magnetic excitations, we have derived effective Lagrangian (4.11) from frozen Higgs-model (3.11). It is notable that the effective Lagrangian is local but it contains two kinds of fields A_μ and B_μ. We could have performed the integration over A_μ in the generating functional, but the resulting effective Lagrangian would be unmanageably nonlocal.

The effective Lagrangian describes a magnetic superconductor where topological excitations are electric vortices labeled by the electric weight lattice and electric charges labeled by the electric root lattice. We may identify these electric charges with Higgs charges liberated into an incoherent plasma state due to the magnetic vortex condensation.[1] Because of electric charge screening by these charge excitations, we have concluded that the SU(N) Higgs model gives the N-ality confinement of quarks in the strong-coupling regime.

We would like to emphasize the importance of the monopole condensation in the problem of quark confinement.[1] Indeed, if only the condensation of closed magnetic vortices are to be considered, it is easy to show that we do not get the mass term for the magnetic potential B_μ^H in the effective Lagrangian. Namely, in the absence of monopole condensation, the effective Lagrangian is unable to produce physical electric vortices to confine quarks.

Finally, we remark on the magnetic gauge group of effective Lagrangian (4.11). We have shown that it is given by the maximal torus of the gauge group SU(N). This is so because we have restricted the electric gauge group to the maximal torus of the electric gauge group SU(N). Accordingly, if the complete gauge degrees of freedom could be taken into account, the magnetic gauge group would be SU(N) itself. However, in order to justify this conjecture, we need a technical improvement to include non-Abelian quantum corrections to classical vortices and monopoles which are essentially Abelian. We shall discuss this problem elsewhere.

APPENDIX

The well-known Poisson resummation formula

$$\sum_n \exp(-an^2+iBn)=\left(\frac{\pi}{a}\right)^{1/2}\sum_m \exp[-(B-2\pi m)^2/4a] \tag{A1}$$

relates the one-dimensional "magnetic" lattice point labeled by n with the one-dimensional "electric" lattice point labeled by m. In our previous paper,[1] where we have considered a single component of the gauge potential, we have used (A1) in order to change the summation over magnetic Z_N excitations into the summation over electric Z_N excitations.

In this appendix, we derive the following Poisson resummation formulas:

$$\sum_{\vec{w}} \exp(-a\vec{w}^2+i\vec{B}\cdot\vec{w}) =\left(\frac{\pi}{a}\right)^{(N-1)/2}\sum_{\vec{\eta}} \exp[-(\vec{B}-4\pi\vec{\eta})^2/4a] \tag{A2}$$

and

$$\sum_{\vec{r}} \exp[-a\vec{r}^{\,2}+i\vec{B}\cdot\vec{r}] =\left(\frac{\pi}{a}\right)^{(N-1)/2}\sum_{\vec{\epsilon}} \exp[-(\vec{B}-4\pi\vec{\epsilon})^2/4a]\,, \tag{A3}$$

where $\vec{w}$ ($\vec{r}$) runs over all points on the magnetic weight (root) lattice of SU(N), while $\vec{\eta}$ ($\vec{\epsilon}$) runs over all points on the electric root (weight) lattice of SU(N). Then, it is quite easy to derive (4.9) from (4.8) in the present case by following the method presented in our previous paper.[1]

Let us prove (A2). In general, a point $\vec{w}$ on the magnetic weight lattice is represented as

$$\vec{w}=\{(n_1-n_2)e_1,\ldots,(n_1+\cdots+n_k-kn_{k+1})e_k,\cdots,(n_1+\cdots+n_{N-1})e_{N-1}\}, \tag{A4}$$

n_k being integers, where we have used (2.10a) and (3.9a). Then, we may express

$$\begin{aligned}\sum_{\vec{w}} \exp(-a\vec{w}^2+i\vec{B}\cdot\vec{w}) =\left(\frac{1}{4a\pi}\right)^{(N-1)/2}\prod_{k=1}^{N-1}\int dx_k \exp\left(-\frac{1}{4a}\sum_{k=1}^{N-1}x_k^2\right)\\ \times\sum_{\{n_k\}}\exp\bigg(i\sum_{k=1}^{N-2}(x_k+B_k)(n_1+\cdots+n_k-kn_{k+1})e_k\\ +i(x_{N-1}+B_{N-1})(n_1+\cdots+n_{N-1})e_{N-1}\bigg)\,.\end{aligned} \tag{A5}$$

By making use of a Poisson resummation formula

$$\sum_n \exp(ixn) = 2\pi \sum_m \delta(x + 2\pi m) , \tag{A6}$$

we rewrite in (A5)

$$\sum_{\{n_k\}} \exp(\cdots) = \prod_{j=0}^{N-1} \sum_{m_{j+1}} 2\pi\delta\left(-j(x_j + B_j)e_j + \sum_{k=j+1}^{N-1} (x_k + B_k)e_k + 2\pi m_{j+1}\right) . \tag{A7}$$

These δ functions restrict the values of x_k in (A5) as

$$x_k = -B_k + 4\pi e_k(m_1 + \cdots + m_k - k m_{k+1}) \tag{A8a}$$

for $1 \leq k \leq N-2$, and

$$x_{N-1} = -B_{N-1} + 4\pi N e_{N-1}(m_1 + \cdots + m_{N-1}) , \tag{A8b}$$

where (2.10b) was used. Recalling that the bases $\vec{\eta}_k$ of the root lattice are defined by (2.12), we may summarize (A8) as

$$\vec{x} = -\vec{B} + 4\pi \sum_{k=1}^{N-1} m_k \vec{\eta}_k . \tag{A9}$$

Therefore, (A5) is reduced to the right-hand side of (A2). The proof of (A3) is similar.

[1] Z. F. Ezawa and A. Iwazaki, Phys. Rev. D 23, 3036 (1981).
[2] S. Mandelstam, Phys. Rep. 23C, 245 (1976).
[3] G. 't Hooft, Nucl. Phys. B138, 1 (1978).
[4] M. Stone and P. R. Thomas, Phys. Rev. Lett. 41, 351 (1978); D. Foerster, Phys. Lett. 77B, 211 (1978); 78B, 473 (1978); M. E. Peskin, Ann. Phys. (N. Y.) 113, 122 (1978); M. B. Einhorn and R. Savit, Phys. Rev. D 19, 1198 (1979).
[5] J. M. Cornwall, Nucl. Phys. B157, 392 (1979). Here, Higgs fields are introduced in the fundamental representation to implement an appropriate symmetry breakdown. However, in order to break SU(N) completely except for the center Z_N, it is necessary to introduce Higgs fields in the adjoint representation. It is possible to do so by using N^2-1 Higgs fields which are taken to be orthogonal to each other.
[6] See, e.g., B. G. Wybourne, *Classical Groups for Physicists* (Wiley, New York, 1974).
[7] Z. F. Ezawa and H. C. Tze, Nucl. Phys. B100, 1 (1975); Phys. Rev. D 14, 1006 (1976).
[8] Z. F. Ezawa and H. C. Tze, J. Math. Phys. 17, 2228 (1976); Phys. Rev. D 15, 1647 (1977).
[9] F. Englert and P. Windey, Phys. Rev. D 14, 2728 (1976); P. Goddard, J. Nuyts, and D. Olive, Nucl. Phys. B125, 1 (1977).
[10] Y. Nambu, Phys. Rev. D 10, 4262 (1974).
[11] R. Brout, F. Englert, and W. Fischler, Phys. Rev. Lett. 36, 649 (1976).
[12] M. B. Halpern, Phys. Rev. D 19, 517 (1979); S. Mandelstam, *ibid.* 19, 2391 (1979).
[13] Z. F. Ezawa, Phys. Rev. D 15, 2091 (1978).
[14] Z. F. Ezawa, Phys. Lett. 86B, 313 (1979).
[15] A. A. Slavnov, Phys. Lett. 98B, 57 (1981).