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Effect of a strong magnetic field and high temperature on broken gauge theories

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We explore the consequences on spontaneously broken gauge theories of a simultaneous presence of strong magnetic field and high temperature.

The idea that the vacuum may not display the symmetry of the Lagrangian has been exploited in several areas of physics. In the area of high-energy physics these ideas have been used to construct unified field theories.[1] In the area of solid-state physics examples of similar nature occur in magnetic ordering, superconductivity,[2] etc.. It is worthwhile to note here that the physical states share the symmetry of the vacuum state as opposed to the symmetry of the Lagrangian.

The symmetry of the vacuum depends on configurations of temperature[3] and external field.[4] In the area of high-energy physics it is well known that a phase transition may occur for sufficiently large values of the temperature or external magnetic field. However, a phase transition under the simultaneous impact of the external field and temperature has not been studied so far.

The motivation for studying these phase diagrams in field theory is twofold:

(i) The critical temperature in the simplest of gauge theories, the Weinberg-Salam theory is roughly $10^{15}-10^{16}$ degrees. Similarly, the critical field is estimated to be as high as 10^{20} G.[5] Such high magnitudes of temperature and field are not known to exist anywhere in the universe. Thus the realization of the symmetric vacuum remains problematic. It is necessary to point out that until the symmetric vacuum is realized experimentally, or observed, one of the fundamental assumptions of these theories remains unconfirmed.

The phase diagram of superconductivity may bring home the importance of the simultaneous impact of the external magnetic field and temperature.[6] It is obvious that to enact a phase transition it is not necessary to reach the maximum value of the critical magnetic field or the maximum value of critical temperature. Indeed, a point between these critical parameters could be substantially below their respective maximas and still effect a phase transition. This necessitates putting both external parameters (external field and temperature) together.

The simultaneous impact of the external magnetic field and temperature may reduce the estimates of both temperature and the magnetic field required to enact a phase transition. A high magnetic field is known to exist in nuclear matter. In stars hot nuclear matter abounds. Therefore, it may become possible to observe a symmetric vacuum in these objects.

(ii) The second motivation rests on an analogy between field theory and solid-state physics. Recently, it has been shown[7] that in theories that break *CP* symmetry softly through a vacuum, the *CP* asymmetry of the vacuum may persist at high temperatures (in fact, the *CP* asymmetry will probably not disappear at any temperature). The crucial feature of these theories is the simultaneous existence of more than one Higgs multiplet.

In solid-state physics such systems have recently been discovered.[8] Coexistence of magnetic ordering and superconductivity in the atmosphere of an external magnetic field and temperature has raised considerable interest of late. Analysis of this problem from field theory with more than one Higgs multiplet may give us some insight into ways of realizing superconductivity at high temperatures. With these goals in mind we set up a procedure for computing the effect on the ground state of a strong external magnetic field and temperature together.

Several points need to be emphasized in this context. First, our procedure will follow closely the earlier work (with the external magnetic field alone) by Salam and Strathdee.[4] Second, in their work they considered only small magnetic fields. In practice, however, in realistic gauge theories a small magnetic field is unlikely to yield the required critical field. Therefore, it is necessary to study the effect in the presence of an intense magnetic field. Finally, the effective potential in an intense magnetic field is beset with ambiguities. In particular, we obtain quantum corrections that have complex coefficients which are difficult to interpret. As will be mentioned later, this difficulty arises owing to nonpositive energy eigenvalues of the excitation spectrum of charged gauge bosons.[9] In the spirit of Salam and Strathdee[4] we avoid this difficulty by summing over the physical part of the energy spectrum that is positive definite. We only hope that this procedure of retaining only the physical states will prove reliable.

Further, we want to emphasize that we are unable to compute the effective potential to one loop exactly. Instead, we make a Euler-Maclaurin expansion of the effective potential.[10] Our earlier work convinces us that such an approximation is not far off the mark. In fact, by taking appropriate limits we have established that this approximation does make sense.

Under these assumptions we consider the possibility of symmetry restoration. Our results indicate that the possibility of symmetry restoration becomes likely in the simultaneous presence of an external magnetic field and temperature. However, owing to difficulties associated with the choice of the renormalization scale our observations would have to be qualified.

In Sec. I we outline the method of calculation of the effective potential under the external magnetic field and temperature. In Sec. II we carry out the calculations for contributions of scalar particles. In Sec. III we discuss the effect of gauge bosons. In Sec. IV we compare our results with the ones previously known and touch upon the difficulties that still need to be overcome in order to carry out realistic calculations.

I. EFFECTIVE POTENTIAL

To be definite let us consider a scalar field multiplet ϕ that belongs to an irreducible representation of a group containing electromagnetism. Let us also assume that the potential part of the Lagrangian of this field is of the Higgs type so that we have spontaneous symmetry breaking. We of course assume that the electromagnetic symmetry stays intact.

The Lagrangian for such a system is given by

$$\mathcal{L} = -\tfrac{1}{4}F^{\alpha}_{\mu\nu}F^{\alpha}_{\mu\nu} - \tfrac{1}{2}(D_\mu\phi)^\dagger(D_\mu\phi) - V(\phi)\,, \tag{1}$$

where $V(\phi)$ is the classical potential. The above Lagrangian describes the field system at absolute-zero temperature. We will be interested in the behavior of the field system away from absolute zero, i.e., for finite values of temperature. The expectation is that for some critical temperature, the symmetry (which is spontaneously broken at 0° K) will be partially or completely restored.

Aside from that, we will also introduce an external magnetic field into the above Lagrangian. The procedure for doing this has been amplified in Ref. 4 and we will not repeat those steps here. The introduction of an external magnetic field is also meant to restore symmetry.

The study of phase transition in field theory is done by studying the effective potential. This question has been broadly studied in Ref. 3. The procedure replaces the classical potential by studying the effect of one-particle-irreducible graphs with all the external momenta set at zero. Thus, we get the loop expansion

$$V^{\text{eff}} = V_0 + V_1 + V_2 + \cdots\,, \tag{2}$$

where V_i gives the correction to V_0, the tree potential, due to one-particle-irreducible diagrams with i loops with "amputated" external legs. Thus, for the scalar field theory above we get (following Ref. 3)

$$V_1 = -\tfrac{1}{2}i\int (\ln iD^{-1})\frac{d^4k}{(2\pi)^4}\,, \tag{3}$$

where D is the scalar propagator.

We find it convenient at this stage to divide the particles in our system into two groups: electrically neutral and electrically charged. The reason for this grouping is that the external magnetic field affects only the charged particles while temperature affects all the particles. The relevant quantities for the charged particles will from now on be written with a subscript c, and that for the neutrals will be given with the subscript n.

Define $M_n^2(\phi)$ as the second derivative with respect to the neutral fields evaluated at the classical minima while $M_c^2(\phi)$ is the corresponding second derivative with respect to charged fields evaluated at $\phi^{\pm}=0$.

So far as the neutral fields are concerned, since they are unaffected by the external magnetic field, the results obtained by Jackiw and Dolan[3] are taken without any modifications. For charged scalars the procedure is to be modified as follows. We will first carry out the dk_0 integration by the procedure outlined by Jackiw and Dolan. Subsequently, we shall sum over all the zero-point excitations (keeping only the positive-energy part of the spectrum for gauge bosons). To carry out the dk_0 integration we note that it can be replaced by a sum (as explained by Jackiw and Dolan). Thus,

$$\int \frac{dk_0}{2\pi} \to \frac{1}{-i\beta}\sum_a \tag{4}$$

and

$$D_{c,n} = \frac{-i}{4\pi^2a^2/\beta^2 + E_{c,n}^{\,2}(m)}\,, \tag{5}$$

where $E_{c,n}^{\,2}(m)$ is the excitation energy spectrum to be defined shortly. Note that the subscripts refer to either charged or neutral particles. We shall, of course, concentrate on the charged particles since the results for the uncharged particles have been obtained by Jackiw and Dolan. Thus, we get

$$V_1 = -\tfrac{1}{2} i \int \frac{d^3k}{(2\pi)^3} \sum_a \frac{1}{-i\beta} \ln\left[\frac{4\pi^2 a^2}{\beta^2} - E_c{}^2(m)\right] . \quad (6)$$

II. CONTRIBUTION FROM THE SCALARS

We now evaluate V_1 for scalar particles. The a sum is performed following Jackiw and Dolan. The addition of the magnetic field only changes the excitation spectrum. Thus, we get

$$V_1 \equiv V_1^0 + V_1^T = \int \frac{d^3k}{(2\pi)^3} \sum_m \frac{E_m}{2} + \frac{1}{\beta} \int \frac{d^3k}{(2\pi)^3} \sum_m \ln(1 - e^{-\beta E_m}) , \quad (7)$$

where V_1^0 is the part independent of temperature and V_1^T depends on temperature. Thus the first integral on the right-hand side is V_1^0 while the second corresponds to V_1^T. In the presence of a magnetic field in the direction labeled by subscript H, the density of states is no longer $d^3k/(2\pi)^3$ but is given by $(eH/2\pi)\, dk_H/2\pi$. The excitation spectrum is given by

$$E_m{}^2 = K_H{}^2 + M_c{}^2 + (m + \tfrac{1}{2})2eH . \quad (8)$$

Thus, V_1 becomes

$$V_1 \equiv V_1^0 + V_1^T = \frac{eH}{2\pi} \int_{-\infty}^{\infty} \frac{dk_H}{2\pi} \sum_{m=0}^{\infty} \frac{E_m}{2} + \frac{eH}{2\pi\beta} \int_{-\infty}^{\infty} \frac{dk_H}{2\pi} \sum_{m=0}^{\infty} \ln(1 - e^{-\beta E_m}) . \quad (9)$$

A. Computation of the temperature-independent part V_1^0

Consider V_1^0. We have

$$V_1^0 = \frac{eH}{2\pi} \int_{-\infty}^{\infty} \frac{dk_H}{2\pi} \sum_{m=0}^{\infty} \frac{[k_H{}^2 + M_c{}^2 + (m + \frac{1}{2})2eH]^{1/2}}{2} \quad (10)$$

$$= \frac{eH}{2\pi} \frac{1}{2} \int_{-\infty}^{\infty} \frac{dk_H}{2\pi} \sum_{m=0}^{\infty} \frac{1}{\Gamma(\nu - \frac{1}{2})} \int_0^{\infty} dt\, t^{\nu - 3/2} e^{-tE_m^2} . \quad (11)$$

We shall, in what follows, take the $\nu = 0$ limit of this integral. Carrying out the m summation, we get

$$V_1^0 = \frac{eH}{4\pi^2} \frac{1}{\Gamma(\nu - \frac{1}{2})} \int_0^{\infty} \frac{dt\, t^{\nu-3/2} e^{-t(M_c{}^2 + eH)}}{1 - e^{-2eHt}} \int e^{-tk_H^2} dk_H . \quad (12)$$

The rest of the integrals are easily evaluated and we get

$$V_1^0 = -\frac{1}{16\pi^2} \frac{(2eH)^{2-\nu}}{\nu - 1} \frac{1}{2\nu} \zeta\left(\nu - 1, \frac{M_c{}^2}{2eH} + \tfrac{1}{2}\right) , \quad (13)$$

where ζ is the Riemann zeta function.[11] Since the limit of ν going to zero is singular, we multiply by ν and take the derivative with respect to ν and then go to the $\nu = 0$ limit. This gives

$$V_1^0 = \left[-\frac{(2eH)^2}{(4\pi)^2} \frac{\ln 2eH}{2} + \frac{(2eH)^2}{(4\pi)^2} \frac{1}{2} \right] \zeta\left(-1, \frac{M_c{}^2}{2eH} + \tfrac{1}{2}\right) + \frac{(2eH)^2}{(4\pi)^2} \frac{1}{2} \zeta'\left(-1, \frac{M_c{}^2}{2eH} + \tfrac{1}{2}\right) , \quad (14)$$

where ζ' stands for the derivative with respect to ν.

Now, $\zeta(-1, M^2/2eH + \frac{1}{2})$ is simple to compute. The second term contains the derivative of the zeta function ζ'. We assume that $eH \gg M^2$ and expand ζ' around $\zeta'(-1, \frac{1}{2})$. The procedure for doing this is sketched in Appendix A.

The result is

$$V_1^0 = -\frac{1}{8\pi^2} (\ln 2eH - 1)(\tfrac{1}{2} M_c{}^4) - \frac{2}{5} \frac{(2eH)}{(4\pi)^2} M_c{}^2 - \tfrac{1}{4}(2eH) M_c{}^4 . \quad (15)$$

We remind the reader that to this we must add the temperature-independent quantum contributions of the neutral scalars as given in Jackiw and Dolan's work.

B. Computation of the temperature-dependent part V_1^T

The temperature-dependent contribution is given in Eq. (9) as

$$V_1^T = \frac{eH}{2\pi\beta} \int_{-\infty}^{\infty} \frac{dk_H}{2\pi} \sum_{m=0}^{\infty} \ln(1 - e^{-\beta E_m}) , \quad (16)$$

where E_m is the energy spectrum defined in Eq. (8). The m summation is replaced by an integral (for gauge bosons this procedure is to be modified as will be discussed). Thus, we have an Euler-Maclaurin expansion of V_1^T. We obtain

$$V_1^T = \frac{eH}{2\pi\beta} \int_{-\infty}^{\infty} \frac{dk_H}{2\pi} \int_0^{\infty} \ln(1 - e^{-\beta E_m}) dm \quad (17)$$

$$= \frac{eH}{2\pi^2\beta} \int_0^{\infty} dk_H \int_0^{\infty} \ln(1 - e^{-\beta E_m}) dm . \quad (18)$$

We make the following change of variables:

$$\beta k_H = x$$

and

$$2\beta^2 eHm = y^2 . \quad (19)$$

Therefore, we get

$$V_1^T=\frac{1}{2\pi^2\beta^4}\int_0^\infty y\,dy\int_0^\infty dx\,\ln\{1-\exp[-(x^2+y^2+\beta^2M_c^2+\beta^2eH)^{1/2}]\}\,. \tag{20}$$

Expanding around small β^2M^2 we get

$$V_1^T\approx\frac{\beta^2M_c^2}{4\pi^2\beta^4}\int_0^\infty y\,dy\int_0^\infty dx\frac{1}{(x^2+y^2+\beta^2eH)^{1/2}}\,\frac{1}{\exp[(x^2+y^2+\beta^2eH)^{1/2}]-1}\,, \tag{21}$$

where the term independent of M_c^2 has been disregarded. We expand the integral around $\beta^2eH\approx1$. We are therefore expanding around $T^2\approx eH$. Thus, we are in the high-temperature, high-magnetic-field domain. The result is

$$V_1^T=-\frac{M_c^2eH}{8\pi^2}(0.68)+\frac{M_c^2}{4\pi^2\beta^2}(0.95)\,. \tag{22}$$

We would like to state the obvious fact that the integrals encountered in the expansion of Eq. (21) are done in polar coordinates. Several of these integrals have been obtained numerically and these integrals have been summarized in Appendix B.

To this equation, once again, we must add the contributions due to neutral scalar particles. For that, we take the results obtained by Jackiw and Dolan.

We observe that the charged scalar particles work against restoration of symmetry. The uncharged scalar particles contribute, according to Jackiw and Dolan, a temperature-dependent term as follows:

$$V_1^T=\frac{M_n^2}{24\beta^2}\,, \tag{23}$$

where M_n is as defined earlier. Thus, these uncharged scalars tend to restore the broken symmetry. The situation changes somewhat for the gauge bosons all of which work towards restoration of symmetry.

III. CONTRIBUTIONS OF GAUGE BOSONS

In computing the gauge-boson contribution, we shall follow closely the work of Salam and Strathdee. Let us once again group the gauge bosons into two groups—the charged and the uncharged. The contributions of the uncharged gauge bosons have been computed, for example, by Jackiw and Dolan, and these are unaffected by the introduction of an external magnetic field. Once again we shall use the subscripts c and n to distinguish between the charged and neutral particles.

The dk_0 integration can be done as before (see Sec. I). However, the excitation spectrum now is different:

$$E_c^2(m)=K_H^2+M_{c1}^2+(2m+1-2qS_H)eH\,, \tag{24}$$

where S_H denotes the component of spin in the direction of the external magnetic field.

While for small eH (which is the case in Salam and Strathdee's work), the above expression for E is positive, the same is not true for an intense magnetic field. In fact, it is easy to observe that the spectrum of eigenvalues contains nonpositive-energy states. In what follows we shall, while summing over the excitation spectrum, carry the summation only over the positive-energy states. Once again, we get

$$V_1\equiv V_1^0+V_1^T=\int\frac{d^3k}{(2\pi)^3}\sum_{m,S_H}\frac{E_m}{2}+\frac{1}{\beta}\int\frac{d^3k}{(2\pi)^3}\sum_{m,S_H}\ln(1-e^{-\beta E_m})\,. \tag{25}$$

A. Computation of the temperature-independent part V_1^0

The temperature-independent part is given as

$$V_1^0=\int\frac{d^3k}{(2\pi)^3}\sum_{m,S_H}\frac{E_m}{2} \tag{26}$$

$$=\frac{eH}{2\pi}\int_{-\infty}^{\infty}\frac{dk_H}{2\pi}\sum_{m,S_H}\frac{E_m}{2}\,. \tag{27}$$

The procedure for evaluation of the above is as outlined in Sec. II. The difference lies in the case in which $S_H=1$. Since the $m=0$ state has nonpositive energy the sum is carried out from $m=1$ to infinity, instead of from 0 to infinity. Thus, for example, for $S_H=1$ we get the contribution to V_1^0 as follows

$$V_1^0(S_H=1)=\frac{1}{8\pi^2}(\ln2eH-1)(\tfrac{1}{2}M_{c1}^2)-\tfrac{2}{5}(2eH)\frac{M_{c1}^2}{(4\pi)^2}-\tfrac{1}{4}(2eH)M_{c1}^4\,. \tag{28}$$

Following this approximation we readily evaluate the temperature-independent contribution to effective potential due to charged gauge bosons and we obtain

$$V_1^0=\frac{eHM_{c1}^2}{(4\pi)^2}(\ln2eH+\tfrac{2}{5})\,. \tag{29}$$

To this the contribution of the neutral gauge bosons have to be added, as in Jackiw and Dolan's work for example.

B. Computation of the temperature-dependent part

The temperature-dependent part is computed as in the case of scalar particles, except that we now have to contend with nonpositive-energy eigenvalues.

This is done once again by summing only through the positive part of the energy spectrum. We have

$$V_1^T=\frac{1}{\beta}\int\frac{d^3k}{(2\pi)^3}\sum_{m,S_H}\ln(1-e^{-\beta E_m})\,. \tag{30}$$

Changing the density of the states we get

$$V_1^T=\frac{eH}{2\pi\beta}\int_{-\infty}^{\infty}\frac{dk_H}{2\pi}\sum_{m,S_H}\ln(1-e^{-\beta E_m})\,. \tag{31}$$

For $S_H=0$ and -1 we encounter no difficulties and follow the procedure laid out for the scalars. For $S_H=1$ we carry the m summation from 1 to infinity. This avoids going through nonpositive-energy eigenstates.[12] Equation (21) is now altered as follows:

$$V_1^T=\frac{M_{c1}{}^2}{4\pi^2\beta^2}\int_{(2\beta^2eH)^{1/2}}^{\infty}y\,dy\int_0^{\infty}\frac{dx}{(x^2+y^2+\beta^2eH)^{1/2}}\,\frac{1}{\exp[(x^2+y^2+\beta^2eH)^{1/2}]-1}\,. \tag{32}$$

Once again, we convert this integral to polar coordinates and then expand around $\beta^2eH\approx 1$. (Some relevant integrals have been evaluated and recorded in Appendix B.) The result is

$$V_1^T=\frac{(0.42)}{4\pi^2\beta^2}M_{c1}{}^2-\frac{M_{c1}{}^2eH}{4\pi^2}(0.09)\,. \tag{33}$$

To this the effect of temperature on the neutral gauge bosons has to be added. The results for the neutral scalars can be obtained from the work of Jackiw and Dolan.

IV. DISCUSSION

The case of fermions is straightforward. The energy spectrum is positive and the procedure of summation is identical to the case of scalars. Since no new difficulties are encountered we have not dealt with this case.

In our earlier work we dealt with the case of weak magnetic field and high temperature. To lowest order we showed that the limit of small temperature produces the results of Salam and Strathdee and the limit of no field reproduces the results of Jackiw and Dolan. This need not be taken to mean that the effect of putting temperature and field together is just the naive summation of the results of previous papers. First, the external field impacts only on the charged particles in the system. Second, the naive summation of the previous works will yield wrong results as shown in this paper. We, however, agree with the general trends that the previous authors have established. Our results are valid for strong magnetic field unlike the previous work.

However, there are several hurdles to get over before we can embark on realistic calculation. The most important among these is the choice of renormalization scale. In the work of Salam and Strathdee the renormalization scale has been chosen to be given by the classical value of the fields. While this choice may be appropriate for their case we find it to be particularly indefensible in the case under consideration. This is because we have two external parameters, and choosing the scale arbitrarily as the vacuum expectation value of the field is going to go counter to the aim of our work. This is evident from Eq. (29) where the choice of scale would determine crucially the question of surviving symmetry. It therefore seems that a careful renormalization-group analysis is required. We note in passing that the external magnetic field contributes a divergent piece to the mass and wave-function renormalization of the relevant fields.

We have adopted a somewhat unorthodox procedure of keeping only the physical degrees of freedom in our calculations. While this process can equally well be adopted for calculations in arbitrary gauges, the results may become numerically ambiguous though the trend may be identical.[3]

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APPENDIX A

We list some of the well-known results on Riemann zeta functions for the convenience of the reader. First, $\zeta(-1,M^2/2eH+\frac{1}{2})$ is evaluated by using the following relationship:

$$\zeta(-m,\alpha)=-\frac{B_{m+1}(\alpha)}{m+1} \tag{A1}$$

where B's are the Bernoulli's polynomial defined as

$$B_n(x)=\sum_{l=0}^{n}\binom{n}{l}B_l x^{n-l}\,, \tag{A2}$$

where B_l's are the Bernoulli numbers. To expand $\zeta'(-1, M^2/2eH+\frac{1}{2})$ around $M^2 \ll eH$ we use the following method. The first term, which is independent of M^2 is disregarded. The second term in the Taylor expansion is

$$\left(\frac{M^2}{2eH}+\tfrac{1}{2}\right)\frac{d}{da}\frac{d}{d\nu}\zeta(\nu,a)\bigg|_{\substack{\nu=-1\\ a=1/2}}=I\,. \tag{A3}$$

Interchanging the order of differentiation, we get

$$I=\frac{M^2}{2eH}\frac{d}{d\nu}\frac{d}{da}\zeta(\nu,a)\bigg|_{\substack{\nu=-1\\ a=1/2}}\,, \tag{A4}$$

where once again we have dropped the term that has no field dependence. Thus

$$I=\frac{M^2}{2eH}\frac{d}{d\nu}[-\nu\zeta(\nu+1,a)]\,, \tag{A5}$$

where we have used the formula

$$\frac{d}{da}\zeta(\nu,a)=-\nu\zeta(\nu+1,a)\,. \tag{A6}$$

Carrying out the differentiation with respect to ν, we get

$$I=\frac{M^2}{2eH}\left[-\zeta(\nu+1,a)-\nu\frac{d}{d\nu}\zeta(\nu+1,a)\right] \tag{A7}$$

$$=\frac{M^2}{2eH}[-(\tfrac{1}{2}-a)+\ln\Gamma(a)-\tfrac{1}{2}\ln 2\pi]\,, \tag{A8}$$

where we have used

$$\zeta(0,a)=\tfrac{1}{2}-a \tag{A9}$$

and

$$\frac{d}{dz}\zeta(z,a)\bigg|_{z=0}=\ln\Gamma(a)-\tfrac{1}{2}\ln 2\pi\,. \tag{A10}$$

It is now a straightforward exercise to obtain the next term in the Taylor expansion.

APPENDIX B

We list several integrals here that are useful in the evaluation of the Taylor expansion of the temperature-dependent part of the effective potential around $\beta^2 eH \approx 1$:

$$\int_0^\infty \frac{\gamma^2 d\gamma}{(\gamma^2+1)}\frac{1}{\exp[(\gamma^2+1)^{1/2}]-1}=0.689\,, \tag{B1}$$

$$\int_0^\infty \frac{\gamma^2 d\gamma}{(\gamma^2+1)^{3/2}}\frac{1}{\exp(\gamma^2+1)^{1/2}-1}=0.185\,, \tag{B2}$$

$$\int_0^\infty \frac{\gamma^2 d\gamma}{(\gamma^2+1)}\frac{\exp(\gamma^2+1)^{1/2}}{[\exp(\gamma^2+1)^{1/2}-1]^2}=0.401\,, \tag{B3}$$

$$\int_{\sqrt{2}}^\infty \frac{\gamma^2 d\gamma}{(\gamma^2-1)^{1/2}}\frac{1}{\exp(\gamma^2-1)^{1/2}-1}=0.996\,, \tag{B4}$$

$$\int_{\sqrt{2}}^\infty \frac{\gamma^2 d\gamma}{(\gamma^2-1)^{3/2}}\frac{1}{\exp(\gamma^2-1)^{1/2}-1}=0.354\,, \tag{B5}$$

$$\int_{\sqrt{2}}^\infty \frac{\gamma^2 d\gamma}{(\gamma^2-1)}\frac{\exp(\gamma^2-1)^{1/2}}{[\exp(\gamma^2-1)^{1/2}-1]^2}=0.704\,, \tag{B6}$$

$$\int_0^\infty \frac{\gamma^2 d\gamma}{(\gamma^2+3)^{1/2}}\frac{1}{\exp(\gamma^2+3)^{1/2}-1}=0.369\,, \tag{B7}$$

$$\int_0^\infty \frac{\gamma^2 d\gamma}{(\gamma^2+3)^{3/2}}\frac{1}{\exp(\gamma^2+3)^{1/2}-1}=0.47\,, \tag{B8}$$

$$\int_0^\infty \frac{\gamma^2 d\gamma}{(\gamma^2+3)}\frac{\exp(\gamma^2+3)^{1/2}}{\exp(\gamma^2+3)^{1/2}-1}=0.136\,. \tag{B9}$$

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