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Cloudy bag model of the nucleon

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A previously derived model in which a baryon is treated as a three-quark bag that is surrounded by a cloud of pions is used to compute the static properties of the nucleon. The only free parameter of the model is the bag radius which is fixed by a fit to pion-nucleon scattering in the (3,3)-resonance region to be about 0.8 fm. With the model so determined the computed values of the root-mean-square radii and magnetic moments of the neutron and proton, and g_A, are all in very good agreement with the experimental values. In addition, about one-third of the Δ-nucleon mass splitting is found to come from pionic effects, so that our extracted value of α_s is smaller than that of the MIT bag model.

I. INTRODUCTION

The MIT bag model[1-3] provides a reasonable description of hadronic properties. In its original form, however, the model did not possess chiral symmetry. Chodos and Thorn[4] and Brown and Rho[5] overcame this difficulty by introducing pions into the Lagrangian that defines the bag model. Thus the baryon can be thought of as containing three quarks inside a bag that is surrounded by a cloud of pions. An additional benefit of including pions, pointed out by Brown and Rho, is that two nucleons can interact by the exchange of pions.

If a baryon is to be regarded as partially consisting of pions, an important question can be raised: How large are the effects of pions on the properties of baryons? One might also ask: How large is the bag radius R? A rough equivalence between a small value of R and large pionic effects (or between a large value of R and small pionic effects) may be established by paraphrasing a classical-physics argument of Jaffe.[6] For positions just outside the bag surface the pion field $\vec{\phi}(\vec{r})$ may be written as

$$\phi_j(R\hat{r})=\frac{\vec{P}_j\cdot\hat{r}}{R^2}\ , \tag{1.1}$$

where $\vec{P}_j$ is a certain[6] vector-isovector quantity. The form (1.1) arises from neglecting the pion mass m_π in the pion wave equation ($m_\pi R$ is fairly small in any of the present models) and then using the P-wave solution of the resulting Laplace's equation. Hence the strength of the pion field, which determines the importance of pionic effects, is inversely proportional to the square of the bag radius.

In the MIT work $R\approx 1.0$ fm and pionic effects are expected to be small.[6] In the Brown and Rho work, the bag is little with $R\approx 0.35$ fm and pionic effects are very large. In the present work our radius $R\approx 0.80$ fm, and pion effects are modest, but not negligible.

In previous publications[7,8] we obtained a quantized theory of nucleons (N) and Δ's interacting with pions by incorporating chiral invariance in the MIT bag model. With a bag radius of about 0.72 fm we obtained a pion-nucleon (πN) scattering that gives a good fit to experimental data in the (3, 3)-resonance region.

In this work we extend our theory by calculating the renormalized πNN and $\pi N\Delta$ coupling constants. Use of these coupling constants gives the same πN scattering amplitude as in Ref. 8, but the bag radius is determined to be 0.82 fm. With this value of R we find that the charge radii and magnetic moments of the proton and neutron are very well described by our cloudy bag model (CBM). The computed value of the axial-vector coupling constant g_A is also found to be in agreement with the experimental value. Approaches similar to ours have been used by Cottingham *et al.*[9] and DeTar.[9]

The outline of the paper is as follows. In Sec. II the theory of Ref. 8 is reviewed. The calculation of the renormalized coupling constants and the resulting πN scattering cross section is presented in Sec. III.

The computation of the nucleonic charge radii and magnetic moments and comparison of the results with experiment provides a severe test of our model. This is done in Sec. IV where it is shown that the calculated electromagnetic properties of the nucleon are in very good agreement with the experimental ones.

The value of g_A provided by our model is discussed in Sec. V. Again we find that the predictions of our model agree with the experimental data.

The difference between the physical nucleon and Δ masses, ω_Δ, has two sources in our model. As in the MIT model a splitting is caused by one-gluon exchange between quarks.[2] However, there is also a contribution to ω_Δ because the pionic self-energies of the nucleon and Δ are different. In the MIT work the entire value of ω_Δ is assumed to come from gluon exchange alone. This can occur only with a very large value (0.55) of the quantum-chromodynamics (QCD) coupling constant α_s. In Sec. VI, we show that, within our model, about 30% of ω_Δ is due to pionic effects. If we ascribe the remainder of ω_Δ to gluon-exchange effects we obtain a value of α_s that is about 60% that of the MIT work.

A few concluding remarks are made in Sec. VIII. Some technical details are given in an appendix.

II. THE CLOUDY BAG MODEL

The cloudy bag model is defined by a Hamiltonian that is obtained, approximately, from a Lagrangian density with a partially conserved axial-vector current. This Lagrangian density $\mathcal{L}_{\text{CBM}}$ is given by

$$\mathcal{L}_{\text{CBM}}(x)=\left[\frac{i}{2}\sum_a \bar{q}_a(x)\overleftrightarrow{\partial\!\!\!/}q_a(x)-B\right]\theta_V-\tfrac{1}{2}\sum_a \bar{q}_a(x)e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f}q_a(x)\Delta_s+\tfrac{1}{2}[D_\mu\vec{\phi}(x)]\cdot[D_\mu\vec{\phi}(x)]-\frac{m_\pi^2}{2}\vec{\phi}^2(x)\,, \tag{2.1}$$

where

$$D_\mu\vec{\phi}=\partial_\mu\vec{\phi}-[1-j_0(\phi/f)]\hat{\phi}\times(\partial_\mu\vec{\phi}\times\hat{\phi})\,. \tag{2.2}$$

The term $q_a(x)$ is the Dirac wave function (color a) of the quarks in the bag, θ_V a function which is one inside the confinement volume and zero outside, Δ_s is a surface δ function, $\vec{\phi}$ is the isovector pseudoscalar pion field operator, and f is the pion decay constant, 93 MeV. The present $\mathcal{L}_{\text{CBM}}$ is not identical to the one of Ref. 8. An explanation for this is given in the Appendix.

The nonlinear pion-quark coupling is too difficult to handle in an exact manner so two approximations are made: (1) the terms nonlinear in $\vec{\phi}$ occurring in the exponential of (2.1) and in $D_\mu\vec{\phi}$ are neglected; and (2) the quark wave functions are taken as those of the MIT bag model. Thus for quarks in a $1s$ state we take

$$q_a(\vec{r})=\frac{N}{\sqrt{4\pi}}\begin{pmatrix} j_0\left(\frac{\omega r}{R}\right) \\ j_1\left(\frac{\omega r}{R}\right)i\vec{\sigma}\cdot\hat{r}\end{pmatrix}v_a\,, \tag{2.3}$$

where v_a is a spin and isospin wave function, we take $\omega=2.04$ (for the mode of lowest frequency), and

$$N^2=\frac{\omega^2}{R^3}\frac{1}{1-j_0^2(\omega)}\,. \tag{2.4}$$

There are several motivations and justifications for the two approximations. The linear pion-quark coupling leads to the familiar linear πNN and $\pi N\Delta$ couplings which suffice to explain a wide variety of phenomena. The quadratic term $\vec{\phi}^2$ leads to s-wave pion-nucleon scattering which is not under consideration here. Finally, the use of (2.3) gives a surface quark flux that vanishes identically. We are currently studying ways of improving on these approximations, but the current simple theory is of considerable interest—especially for nuclear-physics applications.

Employing our approximations and constructing the Hamiltonian in the usual manner we find

$$\hat{H}=\hat{H}_{\text{MIT}}+H_\pi+H_I\equiv H_0+H_I\,. \tag{2.5}$$

The first term $\hat{H}_{\text{MIT}}$ is the Hamiltonian describing baryons of the original MIT bag model, and is given by

$$\hat{H}_{\text{MIT}}=\sum_\alpha m_\alpha \alpha^\dagger\alpha\,. \tag{2.6}$$

The operator α ($\alpha^\dagger$) destroys (creates) the bare baryon. The Hamiltonian for a free quantized pion field is H_π which is given by

$$H_\pi=\sum_j\int d\vec{k}\,\omega_{\vec{k}}\,a^\dagger_{j\vec{k}}a_{j\vec{k}}\,, \tag{2.7a}$$

where $a_{j\vec{k}}$ destroys a free pion of quantum numbers $j,\vec{k}$. The free quantized pion field is then described by

$$\phi_j(\vec{x})=(2\pi)^{-3/2}\int\frac{d\vec{k}}{(2\omega_k)^{1/2}}(a_{j\vec{k}}e^{i\vec{k}\cdot\vec{x}}+a^\dagger_{j\vec{k}}e^{-i\vec{k}\cdot\vec{x}})\,. \tag{2.7b}$$

Using the above definitions of $\hat{H}_{\text{MIT}}$ and H_π we may obtain the matrix elements of H_I in the set of basis states of H_0. We find

$$H_I=\sum_{\alpha,\beta,j}\int\frac{d\vec{k}}{(2\pi)^{3/2}}(v_k^{\alpha\beta}\alpha^\dagger\beta a_{j\vec{k}}+\text{H.c.})\,, \tag{2.8}$$

with

$$v_k^{NN}=\left(\frac{4\pi}{2\omega_k}\right)^{1/2}i\frac{f_{\pi NN}}{m_\pi}u(k)\tau_k\vec{\sigma}\cdot\vec{k} \tag{2.9a}$$

and

$$v_k^{\Delta N}=\left(\frac{4\pi}{2\omega_k}\right)^{1/2} i\frac{f_{\pi\Delta N}}{m_\pi}u(k)T_k\vec{S}\cdot\vec{k}\,, \tag{2.9b}$$

$$v_k^{\Delta\Delta}=\left(\frac{4\pi}{2\omega_k}\right)^{1/2} i\frac{f_{\pi\Delta\Delta}}{m_\pi}u(k)\mathcal{T}_k\vec{\Sigma}\cdot\vec{k}\,. \tag{2.9c}$$

The form factor $u(k)$ is given by

$$u(k)=\frac{3j_1(kR)}{kR}\,. \tag{2.10}$$

The operator $\vec{S}(\vec{T})$ changes spin (isospin-)-$\frac{1}{2}$ states to spin (isospin-)-$\frac{3}{2}$ states, and is defined by the reduced matrix element

$$\langle\tfrac{3}{2}\|S\|\tfrac{1}{2}\rangle=\langle\tfrac{3}{2}\|T\|\tfrac{1}{2}\rangle=2\,. \tag{2.11}$$

[The convention of Messiah[10] (p. 1076) is used here.] The constants $f_{\pi NN}$ and $f_{\pi N\Delta}$ are unrenormalized coupling constants and have the relation

$$\frac{f_{\Delta N\pi}}{f_{NN\pi}}=\left(\frac{72}{25}\right)^{1/2} \tag{2.12a}$$

in the quark model. Finally $\mathcal{T}_k$ and $\vec{\Sigma}$ are the spin and isospin operators for a spin-$\frac{3}{2}$—isospin-$\frac{3}{2}$ object and

$$\frac{f_{\Delta\Delta\pi}}{f_{NN\pi}}=\frac{4}{5}\,. \tag{2.12b}$$

In principle $f_{NN\pi}$ may be obtained from $q_a(x)$ and H_I, with

$$\sqrt{4\pi}\,\frac{f_{NN\pi}}{m_\pi}=\frac{5}{18f}\,\frac{\omega}{\omega-1}\,. \tag{2.13}$$

However, we treat $f_{\pi NN}$ as a free parameter, to be determined in a fit to πN scattering in the (3,3)-resonance region. In Sec. III we find that the phenomenologically determined value of $f_{NN\pi}$ agrees with that of (2.13) to within 20%.

The cloudy bag model is thus defined by the relations (2.5)—(2.12) and the specification of the parameters R and $f_{\pi NN}$.

An additional number needed for computations is the difference between the physical masses of the Δ and nucleon, ω_Δ. In practice a fit to πN scattering data is used to determine the value of ω_Δ. However, the difference between the Δ and nucleon bag masses, $m_\Delta^{(0)}-m_N^{(0)}$; and the difference between the pion contributions to the Δ and nucleon self-energies (which we calculate) make up ω_Δ. Thus, in our theory there is a relationship between ω_Δ and $m_\Delta^{(0)}-m_N^{(0)}$, and either one may be regarded as a free parameter. However, $m_\Delta^{(0)}-m_N^{(0)}$ can be estimated in one-gluon-exchange models, so that ω_Δ can be obtained from a calculation. We find (Sec. VI) that the theoretical expectation for the value of ω_Δ corresponds closely to the phenomenologically determined value, so that ω_Δ is not to be regarded as a (totally) free parameter.

In the theory presented above, as in Ref. 4, the pion field $\phi_\pi(\vec{r})$ does not vanish for $r<R$. That is, the pion penetrates the confinement region. However, when the pion is inside the bag it does not interact with quarks so that the asymptotic freedom property of QCD is maintained. A version of the CBM in which the pion does *not* enter the bag is being developed and will appear elsewhere.

The theory presented above employs a static bag: the quarks are contained within, and pions interact with the surface of a fixed sphere. Thus the eigenfunctions of energy are not eigenfunctions of the total-momentum operator. Recently Donoghue and Johnson[11] have presented a method for improving such eigenstates by projecting these on to states of good momentum. In Secs. IV, V, and VI we use the static model to compute properties of the nucleon, then discuss how the "recoil corrections" of Ref. 11 change the results. The importance of these corrections has been emphasized by DeTar.[9] We are also investigating alternate ways of making such corrections.

III. RENORMALIZED COUPLING CONSTANTS AND PARAMETER SPECIFICATION

In previous work we discussed the renormalization procedures which simplify the use of our Hamiltonian. However, the renormalized πNN and $\pi N\Delta$ coupling constants, $f_{\pi NN}^{(R)}$, $f_{\pi N\Delta}^{(R)}$ were taken to be independent of energy with

$$f_{\pi NN}^{(R)\,2}=0.06 \tag{3.1a}$$

and

$$f_{\pi N\Delta}^{(R)}=(\tfrac{72}{25})^{1/2}f_{\pi NN}^{(R)}\,. \tag{3.1b}$$

In this work we take $f_{\pi NN}$ as a free parameter, use (2.12) to obtain $f_{\pi N\Delta}$, and employ the theory of Théberge *et al.*[8] to obtain the renormalized coupling constants. These depend on an energy parameter ϵ. Finally the parameter $f_{\pi NN}$ (for a given R and ω_Δ) is limited by the condition that the renormalized πNN coupling constant (for pions of zero four-momentum) has approximately the correct value

$$f_{\pi NN}^{(R)\,2}(\epsilon=0)\approx 0.08\,. \tag{3.2}$$

Then the parameters R and ω_Δ are specified by the πN scattering data. The fitted value of $f_{\pi NN}$ is then compared with the theoretical value of (2.13).

According to Ref. 8 (but in a slightly different notation) the relationship between $f_{\pi NN}^{(R)}(\epsilon)$ and $f_{\pi NN}$ is

$$f_{\pi NN}^{(R)}(\epsilon)=\frac{V_N(\epsilon)f_{\pi NN}}{Z_N}\,, \tag{3.3}$$

where Z_N^{-1} is the probability (less than one) that the physical nucleon is a bare three-quark state. To lowest order $(f_{\pi NN}{}^2)$, the factor Z_N may be obtained by using (2.9) in Eq. (3.17) of Ref. 8. (The quantity Z_N is Z^{-1} of that equation.) We find

$$Z_N = 1 + \frac{3 f_{\pi NN}{}^2}{\pi m_\pi{}^2} \int_0^\infty dq \frac{q^4 u^2(q)}{\omega_q{}^3} + \frac{96}{25\pi} \frac{f_{\pi NN}{}^2}{m_\pi{}^2} \int_0^\infty dq \frac{q^4 u^2(q)}{\omega_q(\omega_q + \omega_\Delta)^2}. \tag{3.4a}$$

The second and third terms of (3.4) arise from $N\pi$ and $\Delta\pi$ components of the wave function of the physical nucleon, respectively. The quantity Z_N can also be given in terms of the nucleon self-energy $\Sigma_N(E)$ (Fig. 1),

$$Z_N = 1 - \frac{\partial \Sigma_N(E)}{\partial E}\bigg|_{E=m_N}. \tag{3.4b}$$

For future reference we also display $\Sigma_N(E)$:

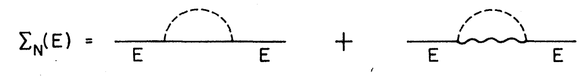

FIG. 1. Nucleon self-energy. In all the figures the pion, nucleon, and Δ are represented by dashed, solid, and wiggly lines, respectively.

$$\Sigma_N(E) = \frac{3 f_{\pi NN}{}^2}{\pi m_\pi{}^2} \int_0^\infty \frac{q^4 dq\, u^2(q)}{\omega_q(E^+ - \omega_q - m_N)} + \frac{96}{25} \frac{f_{\pi NN}{}^2}{\pi m_\pi{}^2} \int_0^\infty \frac{q^4 dq\, u^2(q)}{\omega_q(E^+ - \omega_q - m_\Delta)}. \tag{3.4c}$$

The suppression due to the wave-function-renormalization constant Z_N is mitigated by the inclusion of the vertex correction $V_N(\epsilon)$. This quantity is displayed, to order $f_{\pi NN}{}^2$, in Fig. 2. A straightforward set of manipulations gives the result

$$V^N(\epsilon) = 1 + \frac{f_{\pi NN}{}^2}{3\pi m_\pi{}^2} \int_0^\infty \frac{q^4 dq\, u^2(q)}{\omega_q{}^2(\omega_q - \epsilon)} + \frac{32}{15} \frac{f_{\pi NN}{}^2}{\pi m_\pi{}^2} \int_0^\infty \frac{q^4 dq\, u^2(q)}{\omega_q(\omega_q + \omega_\Delta)(\omega_q + \omega_\Delta - \epsilon)} + \frac{128}{75} \frac{f_{\pi NN}{}^2}{\pi m_\pi{}^2} \int_0^\infty \frac{q^4 dq\, u^2(q)}{\omega_q(\omega_q - \epsilon)(\omega_q + \omega_\Delta)} + \frac{128}{75} \frac{f_{\pi NN}{}^2}{\pi m_\pi{}^2} \int_0^\infty \frac{q^4 dq\, u^2(q)}{\omega_q{}^2(\omega_q + \omega_\Delta - \epsilon)}. \tag{3.5}$$

The expressions (3.3)—(3.5) specify the renormalized πNN coupling constant. Prior to obtaining the renormalized $\pi N\Delta$ coupling constant it is worthwhile to discuss the parameter ϵ used in obtaining the crossed Born graph of Fig. 3(a). By considering the energy denominators of Fig. 3(b), one may show that for the calculation of Fig. 2(a), $V(\epsilon)$ must be evaluated at the point

$$\epsilon = -E, \tag{3.6}$$

where E is the energy of the incident and outgoing pion.

The Δ is not an eigenstate of the Hamiltonian. However, in quark models the Δ and N are members of an SU(6) multiplet. In order to maintain the SU(6) symmetry and treat the Δ and N in the same manner, we must apply the renormalization techniques of Ref. 1 to the Δ as well as the nucleon. Thus we write in analogy with (3.3)

$$f_{\pi N\Delta}^R(\epsilon) = \frac{V_\Delta(\epsilon)}{\sqrt{Z_\Delta}\sqrt{Z_N}} f_{\pi N\Delta}. \tag{3.7}$$

The Δ wave-function-renormalization constant Z_Δ is given by

$$Z_\Delta = 1 - \frac{\partial \operatorname{Re} \Sigma_\Delta(E)}{\partial E}\bigg|_{E=m_\Delta}, \tag{3.8}$$

where $\Sigma_\Delta(E)$ is the pion contribution to the Δ self-energy, Fig. 3. The evaluation of $\Sigma_\Delta(E)$ gives

$$\operatorname{Re} \Sigma_\Delta(E) = \frac{24}{25} \frac{f_{\pi NN}{}^2}{m_\pi{}^2 \pi} \mathrm{P} \int_0^\infty \frac{q^4 dq\, u^2(q)}{\omega_q(E - \omega_q - m_N)} + 3 \frac{f_{\pi NN}{}^2}{m_\pi{}^2 \pi} \mathrm{P} \int_0^\infty \frac{q^4 dq\, u^2(q)}{\omega_q(E - m_\Delta - \omega_q)}. \tag{3.9}$$

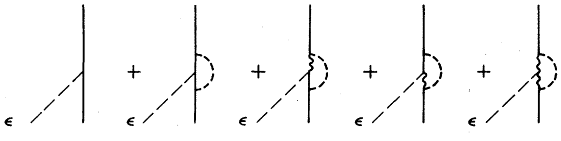

FIG. 2. The $\pi N \to N$ vertex function. ϵ is the energy of the incident pion.

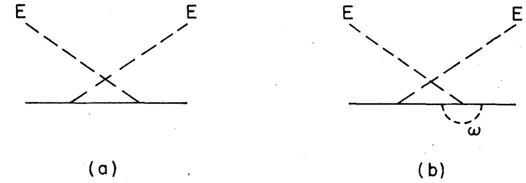

FIG. 3. Pion-nucleon crossed Born term. (a) Lowest order. (b) With a vertex correction. The energy of the virtual pion is ω.

The vertex correction $V_\Delta(\epsilon)$ is shown in Fig. 4. Some care must be used in evaluating the terms of Fig. 4. For example, the term of Fig. 4(e) is already included in the pion-nucleon t matrix of Ref. 1 [see Fig. 8 therein and Fig. 4(f) here]. It should not be recomputed in calculating the pion-nucleon scattering amplitude. Similarly the term of Fig. 4(b) would have been included if the crossed Δ term, Fig. 5, had been included in the driving term of the integral equation of the pion-nucleon t matrix. Indeed the term of Fig. 5 may be regarded as a correction to the Chew term of Fig. 3(a). However, the ratio of the term of Fig. 5 to that of Fig. 3(a) is ~ 0.03. We therefore neglect the term of Fig. 4(b) in treating πN scattering. We do, however, include the terms of Figs. 4(b) and 4(e) in computations of the renormalized $N\Delta\pi$ coupling constant to be used in calculating the electromagnetic properties of the nucleon (Sec. IV) and the Δ-N mass splitting (Sec. VI).

The evaluation of the terms of Figs. 4(a), 4(c), and 4(d) gives

$$V_\Delta(\epsilon) = 1 + \frac{5}{3}\frac{f_{NN\pi}^{\;2}}{\pi m_\pi^{\;2}} \int_0^\infty \frac{q^4 dq\, u^2(q)}{\omega_q^{\;2}(\omega_q + \omega_\Delta - \epsilon)} + \frac{4}{3}\frac{f_{NN\pi}^{\;2}}{\pi m_\pi^{\;2}} \int_0^\infty \frac{q^4 dq\, u^2(q)}{\omega_q(\omega_q + \omega_\Delta)(\omega_q + \omega_\Delta - \epsilon)}\,. \tag{3.10}$$

A consideration of the term of Fig. 5(b) leads to the result that the value of ϵ to be used in (3.10) appropriate for computing πN scattering is the pion energy E. We further note that for energies of our current interest the denominators in the second and third terms of (3.10) do not vanish.

The equations (3.7)–(3.10) specify the renormalized $\pi N \Delta$ coupling constant. The next step is to determine the parameters R and ω_Δ by computing the scattering phase shifts as a function of energy

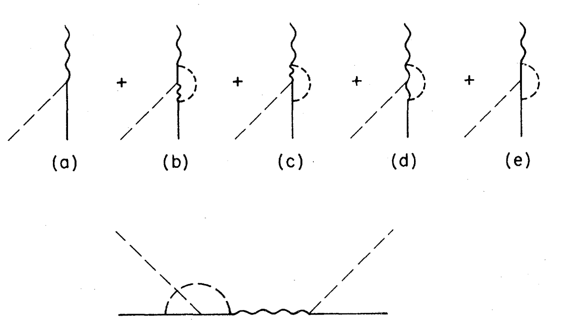

FIG. 4. (a)–(e) The $\pi N \to \Delta$ vertex function. (f) A term included previously in the pion-nucleon T matrix.

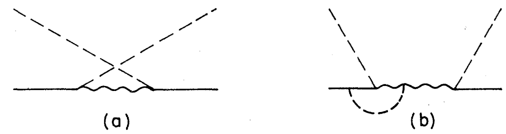

FIG. 5. (a) Small correction to pion-nucleon Born term. (b) Vertex correction in π-nucleon scattering.

and obtaining the best possible fit. This is done by using Eq. (4.41) of Ref. 8, but replacing the energy-independent coupling constants of (3.1) by the ones of (3.3) and (3.7). The best fit shown in Fig. 6 is obtained with the parameters

$$R = 0.82 \text{ fm}\,,\quad \omega_\Delta = 280 \text{ MeV}\,,\quad f_{NN\pi}^{\;2} = 0.078\,. \tag{3.11}$$

The above value of $f_{NN\pi}^{\;2}$ leads to

$$f_{NN\pi}^{(R)\,2}(\epsilon = 0) = 0.064\,. \tag{3.12}$$

For comparison with the usual value of 0.08 one should multiply the results of (3.12) by $u^2(|\vec{q}| = i m_\pi)$. This leads to an enhancement of the value of (3.11) by about 7%. The result (3.12) is then in reasonable agreement with the usual value of 0.080. The energy dependence of the renormalized coupling constants is displayed in Table I.

The phenomenological value of $f_{\pi N \bar{N}}$ is [by (3.11)] 0.28, whereas the theoretical value [from

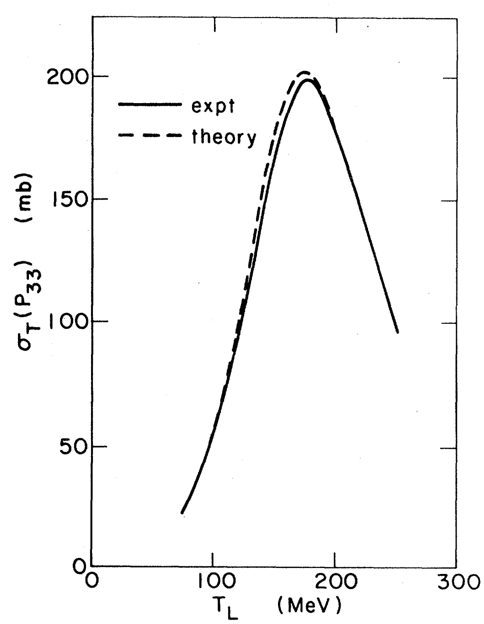

FIG. 6. Best fit (dashed curve) to the experimental P_{33} total cross section (solid).

(2.13)] is 0.23, so that the phenomenological value agrees with the theoretical one to within 20%. Such a deviation is to be expected from the inherent inaccuracies of the Goldberger-Trieman relation, and the static approximations we use. Thus the phenomenological value of $f_{\pi NN}$ is reasonably well predicted by the CBM.

This specification of R, ω_Δ, and $f_{\pi NN}$ completely defines the theory. Thus the computation of additional observables is free of arbitrary parameters.

IV. ELECTROMAGNETIC PROPERTIES OF THE NUCLEON

The comparison of the computed electromagnetic properties of the nucleon with the experimental values should provide a severe test of our model. In particular, the root-mean-square (rms) charge radius of the neutron is expected to be extremely sensitive to the pionic components of the neutron.

The organization of this section is as follows. First the pionic charge $\hat{\rho}_\pi(x)$ and current $\vec{j}_\pi(x)$ density operators are defined. Then the pionic contribution to the charge density is obtained by evaluating $\hat{\rho}_\pi(x)$ in the physical nucleon state. The bag charge density $\rho_B(x)$ is computed, and the total charge density is the sum of $\rho_B(x)$ and the pion charge density. The rms radii of the proton and neutron are then computed. The expectation value of $\vec{j}_\pi(x)$ in the physical nucleon state is evaluated to obtain the pionic contribution to the magnetic moment of the nucleon. The quark contribution to the nucleon magnetic moment is calculated and added to the pionic contribution to get the nucleon magnetic moments. The parameters of (3.11) are used in these computations.

TABLE I. Energy-dependent renormalized coupling constants. The quantity $C(E)$ is defined by $C(E) = \frac{25}{72} f_{\pi N\Delta}^{(R)\,2}/f_{\pi NN}^{(R)\,2}(E)$. The terms of Figs. 4(b) and 4(e) are not included in the computation of $C(E)$.

E (MeV)	$f_{\pi NN}^{R\;2}(-\epsilon)$	$C(E)$
0	0.064	0.85
260	0.054	1.13
285	0.054	1.16
312	0.053	1.19
337	0.053	1.21
368	0.053	1.25
391	0.052	1.27

To establish our notational conventions we present a few definitions. The matrix element of the electromagnetic current operator $\hat{j}_\mu(0)$ is given by

$$\langle p'|\hat{j}_\mu(0)|p\rangle = \frac{1}{(4EE')^{1/2}}\,\bar{u}(p')\{[F_1^S(q^2)+F_1^V(q^2)\tau_3]\gamma_\mu + i\sigma_{\mu\nu}q^\nu[F_2^S(q^2)+F_2^V(q^2)\tau_3]\}u(p)\,, \tag{4.1}$$

where $|p\rangle$ represents a physical nucleon state of four-momentum p and $q=p'-p$. For future reference the $q^2=0$ values of the form factors are listed below:

$$\begin{aligned} &F_1^S(0)=e/2\,,\\ &F_1^V(0)=e/2\,,\\ &F_2^S(0)=-0.06\,e/2m_N\,,\\ &F_2^V(0)=+1.85\,e/2m_N\,. \end{aligned} \tag{4.2}$$

Because we use the static solution of the free bag equations it is appropriate to compute $\langle p'|\hat{j}^\mu|p\rangle$ in the static limit ($m_N\to\infty$). We define $\hat{J}^\mu(q^2=-\vec{q}^{\,2})$ by the relation

$$\langle\chi_{\lambda'}|\hat{J}^\mu(-\vec{q}^{\,2})|\chi_\lambda\rangle = \lim_{m_N\to\infty}\langle p'|\hat{j}_\mu(0)|p\rangle\,, \tag{4.3}$$

where $|\chi_\lambda\rangle$, $|\chi'_\lambda\rangle$ are the spin-isospin wave functions of the nucleon. By taking the $m_N=\infty$ limit of the matrix elements on the right-hand side of (4.1) we find

$$\hat{J}^0(-\vec{q}^{\,2}) = F_1^S(-\vec{q}^{\,2})+F_1^V(-\vec{q}^{\,2})\tau_3 - [F_2^S(0)+F_2^V(0)\tau_3]\frac{\vec{q}^{\,2}}{2m_N} \tag{4.4a}$$

$$\equiv G_{ES}(-\vec{q}^{\,2})+G_{EV}(-\vec{q}^{\,2})\tau_3 \tag{4.4b}$$

and

$$\vec{J}(-\vec{q}^{\,2}) = i(\vec{\sigma}\times\vec{q})[F_2^S(-\vec{q}^{\,2})+F_2^V(-\vec{q}^{\,2})\tau_3]\,. \tag{4.5}$$

In taking the static limit of (4.1) we have kept a term that arises from the $\sigma_{0\nu}q^\nu$ operator and gives rise to the F_2 term of (4.4a). Because of the large value of the anomalous magnetic moment, this is effectively of order zero in m_N, and makes an important contribution to the effective rms charge radius of the neutron.

It is our task, in this paper, to compute the theoretical values of the nucleonic electromagnetic properties at small $\vec{q}^2$. Thus we employ the CBM to compute the theoretical value of the current $\hat{J}^{\mu}_{\rm CBM}(q^2)$. In making the computations it is useful to define the Fourier transform $\hat{j}^{\mu}(r)$ of $\hat{J}^{\mu}_{\rm CBM}(q^2)$ as

$$\hat{j}^{\mu}(\vec{r}) = \int \frac{d^3q}{(2\pi)^3}\, \hat{J}^{\mu}_{\rm CBM}(q^2) e^{-i\vec{q}\cdot\vec{r}} \tag{4.6}$$

with

$$\hat{j}^{\mu}(\vec{r}) = (\hat{\rho}(\vec{r}), \vec{j}(\vec{r}))\,. \tag{4.7}$$

By using the Fourier inverse of (4.6) and comparing the result with (4.5) we obtain

$$G^{\rm CBM}_{ES}(-\vec{q}^{\,2}) + G^{\rm CBM}_{EV}(-\vec{q}^{\,2})\tau_3 = \left\langle p' \left| \int d^3r\, e^{i\vec{q}\cdot\vec{r}}\, \hat{\rho}(\vec{r}) \right| p \right\rangle \tag{4.8}$$

and

$$[F_2^S(-\vec{q}^{\,2}) + F_2^V(-\vec{q}^{\,2})\tau_3]_{\rm CBM} = -i\,\frac{(\vec{\sigma}\times\vec{q})}{2\vec{q}^{\,2}} \cdot \left\langle p' \left| \int d^3r\, e^{i\vec{q}\cdot\vec{r}}\, \vec{j}(\vec{r}) \right| p \right\rangle . \tag{4.9}$$

To obtain the theoretical values of the various electromagnetic form factors, $G^{\rm CBM}_{ES}(-\vec{q}^{\,2})$, etc., we compute $\hat{\rho}(\vec{r})$ and $\vec{j}(\vec{r})$.

We must first obtain the pion contribution to $\hat{j}$. The pion current operator $\hat{j}^{\mu}_{\pi}(\vec{r})$ is the conserved current of that part of $\mathcal{L}_{\rm CBM}(x)$ (2.1) corresponding to the free pion field. We have

$$\hat{j}^{\mu}_{\pi}(x) = -ie[\phi(x)\partial^{\mu}\phi^*(x) - \phi^*(x)\partial^{\mu}\phi(x)]\,, \tag{4.10a}$$

where

$$\phi(x) = \frac{1}{\sqrt{2}}[\phi_1(x) - i\phi_2(x)]\,. \tag{4.10b}$$

By using the expression for the free pion field (2.7a) in (4.10) we can obtain expressions for the pion charge density $\hat{\rho}_{\pi}(x)$ and current density $\vec{j}_{\pi}(x)$ in terms of pion creation and destruction operators. After suitable manipulations one derives

$$\hat{\rho}_{\pi}(\vec{x}) = -\frac{ie}{2} \sum_{i,j=1,3} \frac{\epsilon_{ij3}}{(2\pi)^3} \int d^3k\, d^3k' \left(\frac{\omega_{k'}}{\omega_k}\right)^{1/2} e^{i(\vec{k}-\vec{k}')\cdot\vec{x}} (-a_{i,-\vec{k}'} + a^{\dagger}_{i,\vec{k}'})(a_{j,\vec{k}} + a^{\dagger}_{j,-\vec{k}}) \tag{4.11}$$

and

$$\vec{j}_{\pi}(\vec{x}) = -\frac{ie}{2} \sum_{i,j=1,3} \frac{\epsilon_{ij3}}{(2\pi)^3} \int \frac{d^3k\, d^3k'}{(\omega_k\omega_{k'})^{1/2}}\, \vec{k}\,(a^{\dagger}_{i\vec{k}'} + a_{i,-\vec{k}'})(a_{j,\vec{k}} + a^{\dagger}_{j,-\vec{k}})\, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}. \tag{4.12}$$

To compute the expectation value of $\hat{\rho}_{\pi}(x)$ we use the expression

$$|\tilde{N}\rangle = Z_N^{-1/2}\left[1 + \Lambda\frac{1}{m_N - \tilde{H}_0}\,\tilde{H}_I + \Lambda\left(\frac{1}{m_N - \tilde{H}_0}\,\Lambda\tilde{H}_I\right)^2\right]|N\rangle \tag{4.13}$$

for the physical state $|\tilde{N}\rangle$, valid to second order in $f_{\pi NN}$. (The ket $|\tilde{N}\rangle$ describes $|p\rangle$ in the limit that m_N is infinite.) In (4.13) $|N\rangle$ is the bare nucleon state and $\Lambda = 1 - |N\rangle\langle N|$. The operators $\tilde{H}_0$ and $\tilde{H}_I$ are given by

$$\tilde{H}_I = H_I - \Sigma_N\,, \qquad \tilde{H}_0 = H_0 - \Sigma_N\,. \tag{4.14}$$

The pion-charge-density operator can be defined by

$$\rho_{\pi}(x)\tau_3 = \langle\tilde{N}|\hat{\rho}_{\pi}(\vec{x})|\tilde{N}\rangle \tag{4.15}$$

because pions contribute only to the isovector form factors. The application of (4.13) to (4.15) leads to four terms, but only two are different from zero. These are depicted in Fig. 7. The contributions of the terms of Figs. 7(a)–7(b) to $\rho_{\pi}(x)$ are defined according to the relevant intermediate state as $\rho_{\pi,N}(\vec{x})$ and $\rho_{\pi,\Delta}(\vec{x})$ so that

$$\rho_{\pi}(\vec{x}) = \rho_{\pi,N}(\vec{x}) + \rho_{\pi,\Delta}(\vec{x})\,. \tag{4.16}$$

After a lengthy evaluation we find

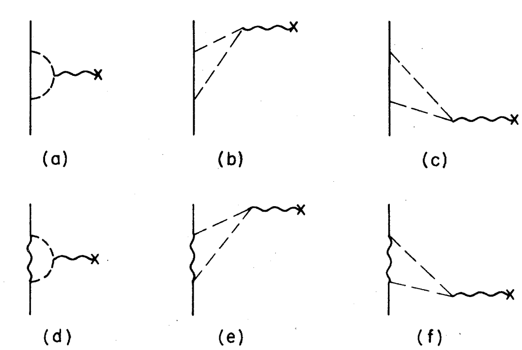

FIG. 7. Photon-pion interactions. The wiggly line with a × at the end is used to describe the photon.

$$\rho_{\pi,N}(\vec{x}) = \frac{4e}{(2\pi)^5 m_\pi^2} f^{R\,2}_{\pi NN} \int\int \frac{d^3k\, d^3k'\, u(k)u(k')}{\omega_k\omega_{k'}(\omega_k+\omega_{k'})}\, \vec{k}\cdot\vec{k}'\, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}} \tag{4.17a}$$

and

$$\rho_{\pi,\Delta}(\vec{x}) = -\frac{e}{(2\pi)^5}\,\frac{8}{9}\,\frac{f^{R\,2}_{\pi\Delta N}}{m_\pi^2}\,\frac{d^3k\, d^3k'}{(\omega_\Delta+\omega_k)}\,\frac{u(k)u(k')}{(\omega_\Delta+\omega_{k'})(\omega_k+\omega_{k'})}\,\vec{k}\cdot\vec{k}'\, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}\,. \tag{4.17b}$$

Only terms of second order in $f^R_{\pi NN}$ have been kept in obtaining (4.17). The renormalized coupling constants of (4.17) are evaluated at $\epsilon = 0$ [see (3.3) and (3.7)]. Using the parameters obtained in Sec. III we compute $\rho_{\pi,N}(\vec{x})$ and $\rho_{\pi,\Delta}(\vec{x})$ and present the results in Fig. 8.

The computation of the nucleon charge density is completed by the specification of the contribution due to the photon-quark interactions. These are shown schematically in Fig. 9.

The electromagnetic current of the quarks, $j^\mu_Q(\vec{x})$, is given by the expression

$$j^\mu_Q(\vec{x}) = \sum_a eQ_a \bar{q}_a(\vec{x})\gamma^\mu q_a(x)\,. \tag{4.18}$$

For the moment we are interested in the zeroth component of $j^\mu_Q(\vec{x})$, which is the quark charge density $\rho_Q(\vec{x})$. We have

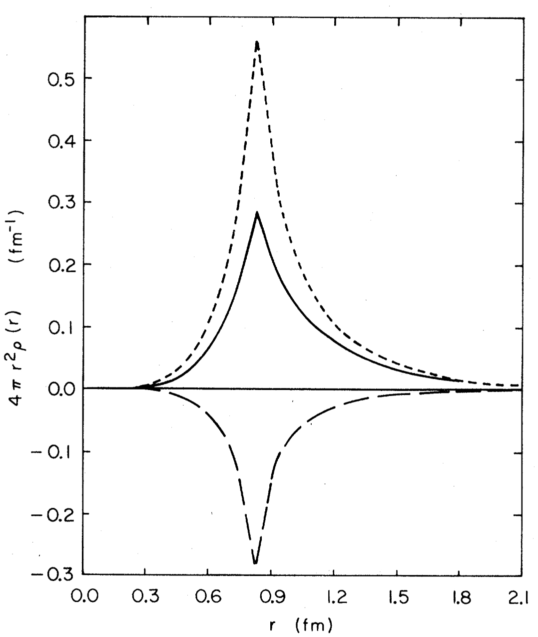

FIG. 8. Pionic contributions to the nucleon charge density. Short dashed curve, $4\pi r^2\rho_{\pi,N}$. Long dashed curve, $4\pi r^2\rho_{\pi,\Delta}$. Solid curve, $4\pi r^2\rho_\pi(r)$.

$$\rho_Q(\vec{x}) = \sum_a eQ_a q_a^\dagger(x) q_a(x)\,. \tag{4.19}$$

The matrix element of ρ_Q between the Δ and nucleon bag states vanishes, so the terms of Figs. 9(d) and 9(e) do not contribute to the quark charge density. The expression for the bag contribution may be further simplified by the observation that all quarks, whether in the nucleon or Δ, have the wave function of (2.3). The use of that expression in (4.19) gives the result

$$\rho_Q(x) = eC\left[j_0^2\left(\frac{\omega x}{R}\right) + j_1^2\left(\frac{\omega x}{R}\right)\right]\theta(R-x)\,, \tag{4.20}$$

where C is obtained from the condition that

$$Q^{p,n} = \int d^3x\, \rho^{p,n}(\vec{x})\,, \tag{4.21a}$$

with

$$\rho^{p,n}(\vec{x}) = \rho_Q(\vec{x}) \pm \rho_\pi(\vec{x})\,. \tag{4.21b}$$

The upper (lower) sign of the right-hand side of (4.21b) refers to the proton (neutron). The charge density of the proton, as well as $\rho_Q(r)$, is shown in Fig. 10. The charge density of the neutron is shown in Fig. 11. It must be noted that no corrections for center-of-mass motion have been applied in the results of Figs. 8, 10, and 11.

From the densities of Figs. 10 and 11 we may compute the rms charge radii of the proton and neutron. We find $\langle r_c^2\rangle_p^{1/2} = 0.69$ fm and $|\langle r_c^2\rangle_n^{1/2}| = 0.34$ fm in very good agreement with the experimental values[3,12] of 0.83 and 0.34 fm. The expectation value $\langle r_c^2\rangle_n$ has a negative sign, also in agreement with experiment.

It is worthwhile to estimate the effects of

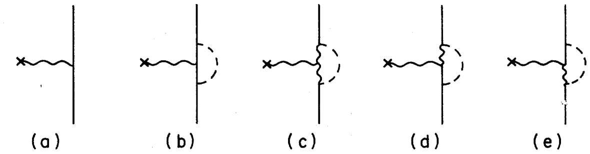

FIG. 9. Photon-quark interactions.

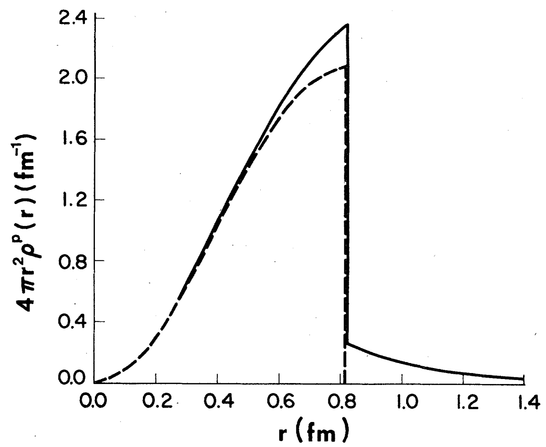

FIG. 10. Proton charge density, solid curve. $\rho_Q(r)$, dashed curve.

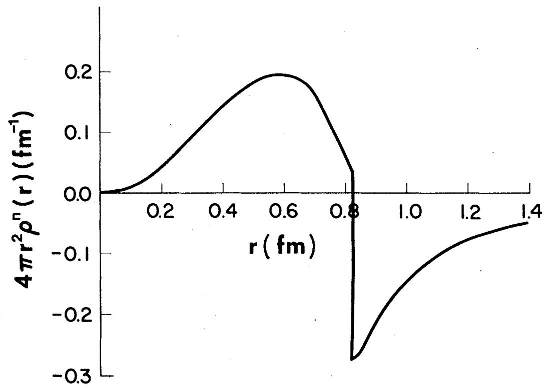

FIG. 11. Neutron charge density.

center-of-mass motion on our results. If we apply the Donoghue-Johnson[11] correction procedure to the calculation of $\langle r_c^2\rangle_{n,p}^{1/2}$ we find small changes: the value of $\langle r_c^2\rangle_p^{1/2}$ is changed from 0.69 to 0.73 fm, and the value of $|\langle r_c^2\rangle_n^{1/2}|$ is changed from 0.34 to 0.36 fm.

The agreement with the low-q^2 behavior of the neutron electric form factor $G_{En}(q^2)$ is significant. Just as in all the old static source theories the process $n \to p\pi^-$ gave rise to a negative tail for the intrinsic neutron charge distribution, so does our model. These early models had, however, one essential problem, namely that the core was not understood, and its properties were incalculable. In our model the core is a simple three-quark bag. Second the interpretation of $G_{En}(q^2)$ was always clouded by the presence of the Darwin-Foldy term, whereby a Dirac particle with an anomalous magnetic moment appears, because of the *Zitterbewegung* to have an intrinsic charge distribution.

In the quark model the photon interacts not with a Dirac nucleon, but with three confined quarks (and the pion in the CBM) and there is no Darwin-Foldy term. Thus the interpretation of $G_{En}(q^2)$ in terms of an intrinsic charge distribution is unambiguous in this model, and the agreement with $\langle r_c^2\rangle_n$ is very significant. Further, if we take seriously the phenomenological fits of $\rho_{\text{ch}}^n(r)$ to the admittedly very poor data for $G_{En}(q^2)$, we see that they tend to give the zero in $\rho_{\text{ch}}^n(r)$ (where it switches from positive to negative) at radii between 0.7 and 0.9 fm. In the cloudy bag model the pion field is a maximum at the surface of the bag and this switch in sign should occur very close to R, the bag radius, Fig. 11. The qualitative agreement between the experimental value quoted above and the value of 0.82 fm extracted from πN scattering is at the very least a remarkable coincidence. Better data for $G_{En}(q^2)$ are essential.

In the CBM pions penetrate the confinement region, and one might wonder about the fraction λ of $\langle r_c^2\rangle_p$ that results from the contribution pions with $r < R$. To do this define the quantity λ as

$$\lambda = \frac{\int_0^R r^4 \rho_\pi(r)dr}{\int_0^\infty r^4 \rho_\pi(r)dr} . \tag{4.22}$$

We find that λ is about 15%, so pions inside the bag contribute only a negligible amount to $\langle r_c^2\rangle_p$.

Next we evaluate the magnetic moments of the nucleon. To do this we need the expectation value of $\vec{j}_\pi(\vec{x})$. We define a vector $\vec{J}_\pi(\vec{x})$ so that

$$\tau_3 \vec{J}_\pi(\vec{x}) = \langle \tilde{N} | \vec{j}_\pi(\vec{x}) | \tilde{N} \rangle . \tag{4.23}$$

The terms contributing to $\vec{J}_\pi(\vec{x})$ are shown in Fig. 7. We define $\vec{j}_{\pi,N}(\vec{x})$ and $\vec{j}_{\pi,\Delta}(\vec{x})$ with the two terms coming from nucleon and Δ intermediate states:

$$\vec{J}_\pi(\vec{x}) = \vec{j}_{\pi,N}(\vec{x}) + \vec{j}_{\pi,\Delta}(\vec{x}) . \tag{4.24}$$

By defining the pion contributions to $F_2^{S,V}(q^2)$ in analogy with (4.9) as

$$F_{2,N}^{V,\pi}(-\vec{q}^2) = -\frac{i(\vec{\sigma}\times\vec{q})}{2\vec{q}^2} \int d^3x \, \vec{j}_{\pi,N}(\vec{x}) \tag{4.25a}$$

and

$$F_{2,\Delta}^{V,\pi}(-\vec{q}^2) = -\frac{i(\vec{\sigma}\times\vec{q})}{2\vec{q}^2} \int d^3x \, \vec{j}_{\pi,\Delta}(\vec{x}) , \tag{4.25b}$$

and performing a lengthy manipulation we find

$$F_{2,N}^{V,\pi}(-\vec{q}^2)=\frac{e}{2\pi^2}\frac{f_{\pi NN}^{R\,2}}{m_\pi^2}\int d^3k\,\frac{\sin^2\theta\,k^2u(k)u(k')}{\omega_k^2\omega_{k'}^2} \tag{4.26}$$

and

$$F_{2,\Delta}^{V,\pi}(-\vec{q}^2)=+\frac{e}{9}\frac{f_{\pi N\Delta}^{R\,2}}{m_\pi^2}\frac{1}{2\pi^2}\int d^3k\,\frac{k^2\sin^2\theta\,u(k)u(k')}{\omega_k\omega_k'}\,\frac{(\omega_\Delta+\omega_k+\omega_{k'})}{(\omega_\Delta+\omega_k)(\omega_\Delta+\omega_{k'})(\omega_k+\omega_{k'})}\,. \tag{4.27}$$

In (4.26) and (4.27) θ is the angle between $\vec{k}$ and $\vec{q}$, and $\vec{k}'=\vec{k}+\vec{q}$. (There is no contribution to the isoscalar form factors.) Once again the renormalized coupling constant are evaluated at $\epsilon=0$. The contributions to the magnetic moment are obtained by taking the $|\vec{q}|=0$ limit of (4.26) and (4.27).

Next we must compute the contribution of the magnetic moment due to the photon-quark interactions. The magnetic-moment operator $\hat{\mu}$ is given by

$$\hat{\mu}=\mu_0\sum_a U_a^\dagger\sigma_3 Q\,U_a\,, \tag{4.28}$$

where U_a is a quark spin-isospin wave function, Q is the quark charge matrix

$$Q=\begin{pmatrix}\frac{2}{3} & 0\\ 0 & -\frac{1}{3}\end{pmatrix}. \tag{4.29}$$

and

$$\mu_0=\frac{R}{12}\frac{1}{\omega^2-\sin^2\omega}(4\omega-3\sin 2\omega+2\omega\cos 2\omega)\,. \tag{4.30}$$

Unlike the charge operator, $\hat{\mu}$ can cause transitions between the nucleon and Δ bag states. We obtain the quark contribution to the magnetic moment by a lengthy evaluation of the terms of Fig. 9. The photon-quark contribution to the magnetic moment is defined as $F_2^{p,n}(0,\text{bag})$. The result is

$$\begin{pmatrix}F_2^p(0,\text{bag})\\ F_2^n(0,\text{bag})\end{pmatrix}=\mu_0\left[\begin{pmatrix}1\\ -\frac{2}{3}\end{pmatrix}\tilde{Z}_N^{-1}+\frac{1}{27}\begin{pmatrix}1\\ -4\end{pmatrix}P_{N\pi}+\frac{5}{27}\begin{pmatrix}4\\ -1\end{pmatrix}P_{\Delta\pi}+\frac{4}{9}\begin{pmatrix}1\\ -1\end{pmatrix}P_{N\Delta\pi}\,.\right] \tag{4.31}$$

The quantity $\tilde{Z}_N$ is defined by replacing the unrenormalized coupling constants of (3.4a) by renormalized ones evaluated at $\epsilon=0$, and has the value 1.55. $P_{N\pi}$ and $P_{\Delta\pi}$ are the probabilities that the physical nucleon have $N\pi$ and $\Delta\pi$ components. These are given by the expressions

$$P_{N\pi}=\frac{3}{\pi}\tilde{Z}_N^{-1}\frac{f_{\pi NN}^{(R)\,2}}{m_\pi^2}\int_0^\infty\frac{k^4dk\,u^2(k)}{\omega_k^3} \tag{4.32}$$

and

$$P_{\Delta\pi}=\frac{4}{3\pi}\tilde{Z}_N^{-1}\frac{f_{\pi N\Delta}^{(R)\,2}}{m_\pi^2}\int_0^\infty\frac{k^4dk\,u^2(k)}{\omega_k(\omega_k+\omega_\Delta)^2}\,. \tag{4.33}$$

For our parameters we find $P_{N\pi}=0.20$ and $P_{\Delta\pi}=0.15$. The term $P_{N\Delta\pi}$ comes from the graphs of Figs. 7(d) and 7(e) and has the expression

$$P_{N\Delta\pi}=\frac{8\sqrt{2}}{3\pi}\frac{f_{\pi NN}^R f_{\pi N\Delta}^R}{m_\pi^2}\int_0^\infty\frac{k^4dk\,u^2(k)}{\omega_k^2(\omega_\Delta+\omega_k)}\,. \tag{4.34}$$

The magnetic moment of the nucleon is given by the expression

$$\mu^{p,n}=\pm[F_{2,N}^{V,\pi}(0)+F_{2,\Delta}^{V,\pi}(0)]+F_2^{p,n}(0,\text{bag})\,, \tag{4.35}$$

where the plus (minus) sign refers to the proton (neutron). The evaluation of (4.31) leads to the values

$$\mu_p=2.2e/2m_N\,,\qquad \mu_n=-1.7e/2m_N\,, \tag{4.36}$$

which are in fairly good agreement with the experimental values of $2.79e/2m_N$ and $-1.91e/2m_N$.

It is worthwhile to examine the effects of correcting for the motion of the center of mass. According to Ref. 11, the static values of μ_p and μ_n are increased by 18%, so that the corrected values of μ_p and μ_n are given by

$$\mu_p=2.60\,e/2m_N\,,\qquad \mu_n=-2.01e/2m_N\,. \tag{4.37}$$

Thus the inclusion of recoil effects substantially improves the agreement with experiment.

A summary of the calculated electromagnetic properties of the nucleon along with a comparison to experiment and the results of the MIT bag model is given in Table II. We stress that these results are obtained in a parameter-free calculation.

Our proton rms charge radius is about the same as that of the MIT bag, even though our bag radius (0.82 fm) is smaller than theirs (1.0 fm). This is due to the presence of positively charged pions outside the bag.

There is no mechanism in the original MIT bag model that gives a nonzero value of the neutron

TABLE II. Static electromagnetic properties of the nucleon. Magnetic moments are given in units of Bohr magnetons.

Quantity	This work	Experiment	MIT bag model[2]
$\langle r_c^2\rangle_p^{1/2}$	0.73 fm	0.83 fm	0.73 fm
$\lvert\langle r_c^2\rangle_n\rvert^{1/2}$	0.36 fm	0.35 fm	0.00 fm
μ_p	2.60	2.79	1.9
μ_n	−2.01	−1.91	−1.2

rms charge radius. In our model the π^- components give a value of $\langle r^2\rangle_n^{1/2}$ in good agreement with experiment. There have been attempts to explain the value of $\langle r^2\rangle_n^{1/2}$ in terms of charge segregation caused by the gluon-exchange interaction.[12] The size of such effects is proportional to α_s and the bag radius R. In our theory both R and α_s (Sec. VI) are smaller than in the MIT model. Hence for our model it is reasonable to expect that such contributions to $\langle r_c^2\rangle_n$ would be fairly small.

The inclusion of the pion cloud and recoil corrections leads to a substantial enhancement of the MIT-bag-model magnetic moment. Indeed the experimental values of μ_n and μ_p are very well reproduced.

It seems that the pion contributions make modest but significant corrections to quantities derived from the MIT bag model. Furthermore, from the relatively small values $P_{N\pi}=0.20$ and $P_{\Delta\pi}=0.15$ we conclude that the nucleon wave function is reliably calculated in lowest order. (A more detailed study of the convergence of our perturbation treatment is in progress, and preliminary results show that the lowest-order treatments we employ are reliable.) Thus the cloudy bag model provides a very reasonable description of the structure of the nucleon and Δ.

V. AXIAL-VECTOR COUPLING CONSTANT g_A

Quark-model calculations of the quantity g_A, which is the ratio of effective strengths of axial-vector and vector currents in nucleon β decay, have been of considerable interest. Evaluations of g_A using the nonrelativistic quark model[13] give $g_A=1.67$ which is significantly larger than the experimental value $g_A^{\rm exp}=1.24$. Use of the MIT bag model[2] leads to a value of $g_A=1.09$. The reduction can be understood by a consideration of the quark contribution to the axial-vector $\vec{A}_Q^i$:

$$\vec{A}_Q^i(x)=\sum_a \bar{q}_a(x)\vec{\gamma}\gamma_5\tfrac{1}{2}\tau^i q_a(x)\,, \qquad (5.1)$$

where

$$\vec{\gamma}\gamma_5=\begin{pmatrix}\vec{\sigma} & 0\\ 0 & -\vec{\sigma}\end{pmatrix}. \qquad (5.2)$$

From the quark wave function (2.3) and with $x=2.04$, we note that the lower component of $q_a(x)$ is comparable in magnitude with the upper component. Hence the minus sign in the lower diagonal term of (5.2) leads to a reduction in the calculated value of g_A.

To obtain g_A in our model, observe that our model has a partially conserved axial-vector current $\vec{A}_\mu$ such that

$$\vec{A}_\mu=\sum_a \bar{q}_a(x)\gamma_\mu\gamma_5\tfrac{1}{2}\vec{\tau}q_a(x)-f\partial_\mu\vec{\phi} \qquad (5.3)$$

and

$$\partial^\mu\vec{A}_\mu=m_\pi^2 f\vec{\phi}\,. \qquad (5.4)$$

[The relation (5.4) is valid even if one uses the approximations discussed in Sec. II.] It is well known that using (5.4) and standard techniques of taking various matrix elements between physical states one may derive the Goldberger-Treiman relation. In our notation we have

$$g_A=\frac{2f}{m_\pi}\sqrt{4\pi}f_{\pi NN}^{(R)}=\frac{2f}{m_\pi}\sqrt{4\pi}\,f_{\pi NN}\frac{V_N(\epsilon=0)}{Z_N}\,, \qquad (5.5)$$

where the quantity in parentheses is the renormalized pion-nucleon coupling constant. The result (5.5) can also be obtained from the direct evaluation of the matrix element of $e^{i\vec{q}\cdot\vec{x}}A_\mu(x)$ for small $\vec{q}$. Using our value of $f_{\pi NN}$ ($f_{\pi NN}^2=0.078$) (which is consistent, within the error[14] in the Goldberger-Treiman[15] relationship, with our theory) and Eqs. (3.4) and (3.5) we find

$$g_A=1.19 \qquad (5.6)$$

in excellent agreement with the experimental value of 1.24.

If the Donoghue-Johnson procedure for center-of-mass corrections is applied to our result for g_A we find

$$g_A=1.33\,, \qquad (5.7)$$

which is also in good agreement with the experimental value of 1.24.

Before concluding this section we compare our results with the calculation of Jaffe.[6] In a classical lowest-order calculation he finds that pionic effects increase the value of g_A (1.09, as obtained in a bag calculation) by a factor of $\frac{3}{2}$. Our value for g_A is smaller then Jaffe's because pions enter the bag and also because of renormalization

TABLE III. Values of g_A.

Source	g_A
Experiment	1.24
Nonrelativistic quark model	1.67
MIT bag model	1.09
Jaffe	1.63
This work	1.33

effects.

The values of g_A discussed in this section are presented in Table III.

VI. GLUON-EXCHANGE CONTRIBUTION TO ω_Δ

The difference between the physical masses of the Δ and nucleon has been determined from our fit to scattering data in the (3,3)-resonance region to be 280 MeV. In the MIT bag work of DeGrand *et al.*,[3] this splitting is entirely ascribed to the difference between the one-gluon-exchange contribution, Fig. 12 to the Δ and nucleon masses. With a bag radius of 1.0 fm DeGrand *et al.* find that the strong coupling constant $\alpha_s = 0.55$, a value that is somewhat too large compared with more recent determinations[16] and the expectation that perturbative treatments of QCD are valid at short distances. The one-gluon-exchange energy is expected to be a perturbative correction to the bag energy and the large value of α may not be consistent with this expectation. In this work there is an additional contribution to ω_Δ, namely the difference between the pion self-energies of the Δ ($\tilde{\Sigma}_\Delta$) and nucleon ($\tilde{\Sigma}_N$). Hence the extracted value of α_s is reduced.

If one assumes that the entire contribution to ω_Δ comes from pionic and one-gluon-exchange effects one may write

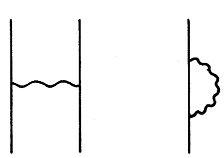

FIG. 12. Gluon contributions to the nucleon self-energy. Here the wiggly line describes a gluon, and the solid lines the quarks.

$$\omega_\Delta = \mathrm{Re}\,\tilde{\Sigma}_\Delta - \tilde{\Sigma}_N + \omega_\Delta^{\mathrm{QCD}}, \qquad (6.1)$$

where $\omega_\Delta^{\mathrm{QCD}}$ is the mass splitting caused by gluon exchange. The quantities $\tilde{\Sigma}_\Delta$ and $\tilde{\Sigma}_N$ are defined by replacing the unrenormalized coupling constants of (3.4a) and (3.9) by renormalized ones evaluated at the energies ω_Δ for $\tilde{\Sigma}_\Delta$ and zero for $\tilde{\Sigma}_n$. Using the parameters obtained in Sec. III, and Eqs. (3.4c) and (3.9) we find $\omega_\Delta^{\mathrm{QCD}} = 200$ MeV. The parameter $\omega_\Delta^{\mathrm{QCD}}$ is well determined; slight shifts in R and $f_{\pi NN}$, which change the calculated phase shifts by modest amounts, do not change $\omega_\Delta^{\mathrm{QCD}}$. The corresponding value of $\omega_\Delta^{\mathrm{QCD}}$ used by DeGrand *et al.* is 300 MeV. Using the fact that $\omega_\Delta^{\mathrm{QCD}}$ is proportional to α_s and inversely proportional to R we find

$$\alpha_s = \frac{200}{300} \times \left(\frac{0.82\ \mathrm{fm}}{1.0\ \mathrm{fm}}\right) \times 0.55 \qquad (6.2)$$

or

$$\alpha_s = 0.30\,. \qquad (6.3)$$

The result (6.3) is in better agreement with the idea that QCD effects in the bag can be treated in a perturbative manner.

The smaller value of α_s has important implications for bag-model calculations of the nucleon-nucleon (*NN*) force. In DeTar's work[17] the color-electrostatic interaction between quarks leads to a minimum in the value of the deformation energy of about -200 MeV which occurs at an internucleon separation of about 1 fm. This is a feature not found in phenomenological *NN* forces. However, this attraction is very sensitive to the value of α_s, and the use of $\alpha_s = 0.30$ would very significantly reduce the magnitude of the calculated attraction.[17]

VII. CONCLUSIONS

In the cloudy bag model the baryon is described as three quarks in a bag surrounded by a pion cloud. There is only one uncalculated parameter in our treatment of the field equations. This is the bag radius R, which is determined from π-nucleon scattering to be about 0.82 fm. With this radius the calculated electromagnetic properties (at zero momentum transfer) are in very good agreement with the experimental ones. The computed value of the axial-vector coupling constant g_A is also in good agreement with experiment. Thus, the cloudy bag model provides a very good description of the static properties of the nucleon and Δ.[18]

Still to be answered is the question of whether the cloudy bag model provides a good description of the energy spectrum of the baryons. In this model the energy splitting due to pion contributions to baryonic mass splitting essentially replaces the splittings (of the MIT bag model) caused by gluon exchanges. Very good agreement with the data has been achieved in the MIT bag model,[2] and it will be interesting to see if similar results can be obtained in the cloudy bag model.

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APPENDIX

The CBM equations presented in Ref. 8 (CBM 1) were sufficient to guarantee PCAC (partial conservation of axial-vector current), but even in the limit $m_\pi \to 0$ were not exactly chirally symmetric. As discussed in many places,[6,19] exact chiral symmetry usually implies considerable nonlinearity in both the transformation on the pion field, and in the Lagrangian density. For completeness we shall give the full, nonlinear Lagrangian density and field equations in this appendix. We observe, however, that from the point of view adopted in CBM 1, and also in this paper, where only terms of order ϕ are retained in actual calculation, the formalism presented here changes nothing.

The lack of exact chiral symmetry in the Lagrangian density (2.8) of CBM 1 under the chiral transformation

$$q(x) \to q(x) + \frac{i}{2}\vec{\tau}\cdot\vec{\epsilon}\,\gamma_5 q(x)\,, \tag{A1}$$

$$\vec{\phi} \to \vec{\phi} - f\vec{\epsilon} \tag{A2}$$

comes from the bag surface term. In fact, under the transformation (A1) and (A2) we find

$$\mathcal{L}_{\rm CBM}(x) \to \mathcal{L}_{\rm CBM}(x) + \frac{1}{2f}\sum_a \bar{q}_a(x) i\gamma_5 j_1(\phi/f) \times \vec{\tau}\cdot\vec{\phi}(x) \times [\vec{\epsilon}\times\hat{\phi}(x)] q_a(x)\,\Delta_s, \tag{A3}$$

where

$$\phi = |\vec{\phi}|\,; \quad \hat{\phi} = \vec{\phi}/\phi\,, \tag{A4}$$

and Δ_s is a surface δ function. This problem can be cured by making a more complicated transformation on $\vec{\phi}(x)$, namely

$$\vec{\phi} \to \vec{\phi} - f\vec{\epsilon} + f[1-(\phi/f)\cot(\phi/f)]\hat{\phi}\times(\vec{\epsilon}\times\hat{\phi})\,. \tag{A5}$$

Having cured the problem with the surface term, we have now introduced difficulties with the pion kinetic energy term. In order to cure that we must finally introduce the covariant derivative

$$D_\mu\vec{\phi} = \partial_\mu\vec{\phi} - [1-j_0(\phi/f)]\hat{\phi}\times(\partial_\mu\vec{\phi}\times\hat{\phi})\,. \tag{A6}$$

It is a difficult, but instructional algebraic exercise to prove that the new Lagrangian density

$$\mathcal{L}'_{\rm CBM}(x) = \left[\frac{i}{2}\sum_a \bar{q}_a(x)\overleftrightarrow{\not{\partial}} q_a(x) - B\right]\theta_V - \tfrac{1}{2}\sum_a \bar{q}_a(x) e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a(x)\,\Delta_s + \tfrac{1}{2}(D_\mu\vec{\phi})^2 \tag{A7}$$

is invariant under the chiral transformations (A1) and (A5). This invariance is of course associated with a conserved axial-vector current ($\vec{A}^\mu$) which one can calculate in the canonical way

$$\vec{A}^\mu = \tfrac{1}{2}\sum_a \bar{q}_a\gamma^\mu\gamma_5\vec{\tau} q_a\theta_V - f\hat{\phi}(\partial^\mu\phi) - f\frac{\sin(2\phi/f)}{2\phi/f}\hat{\phi}\times(\partial^\mu\vec{\phi}\times\hat{\phi})\,. \tag{A8}$$

Finally we write down the field equations which follow when one demands that the action associated with the Lagrangian density (A7) be invariant under arbitrary variations in the quark and pion fields and under variations along the normal to the bag surface:

$$i\not{\partial} q_a(x) = 0\,, \quad x\in V\,, \tag{A9}$$

$$i\gamma\cdot n q_a(x) = e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a(x)\,, \quad x\in S\,, \tag{A10}$$

$$B = -\tfrac{1}{2} n\cdot\partial \sum_a [\bar{q}_a(x) e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a(x)]\,, \quad x\in S\,, \tag{A11}$$

$$\partial^2\vec{\phi}(x) - \partial_\mu\left[\left(1-\frac{\sin(2\phi/f)}{2\phi/f}\right)\hat{\phi}\times(\partial_\mu\vec{\phi}\times\hat{\phi})\right] = -\frac{i}{2f}\sum_a \bar{q}_a\gamma_5\left[\cos(\phi/f)\hat{\phi}\times(\vec{\tau}\times\hat{\phi}) + \frac{i\gamma_5}{f}\vec{\phi}\cos(\phi/f) + \frac{\vec{\tau}\cdot\vec{\phi}}{f}\hat{\phi}\frac{\cos(\phi/f)}{\tan(\phi/f)}\right] q_a\Delta_s, \quad Vx\,. \tag{A12}$$

These equations are very closely related to those of Jaffe, but we do not exclude the pion field from the bag at this stage.

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