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Grand unified theories and the lepton number of the Universe

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Present observations are consistent with a number of neutrinos in the Universe much greater than the number of photons. We explore the possibility of the production and/or survival of a large lepton number within the framework of grand unified models. We find that standard unified theories with gauge-invariant initial asymmetries may give lepton numbers much greater than the photon number (even with *zero* initial baryon and lepton numbers), and lead to the possibility that the present Universe is hot in baryons ($n_B \ll n_\gamma$) but cold in leptons ($n_\nu \gg n_\gamma$). Detection of a large lepton number may thus provide a probe of initial conditions.

I. INTRODUCTION

The recent development of theories that attempt to unify the strong, weak, and electromagnetic interactions[1] has led to the speculation that the baryon- and lepton-number-violating interactions that occur in such theories provide a natural explanation for the observed cosmological baryon asymmetry.[2-4] One of the strong points of this mechanism is thought to be the lack of dependence of the final baryon asymmetry on the initial conditions. If any initial asymmetries are destroyed before baryon-number-generating decays take place, then the final baryon number depends only on the dynamics of the theory being considered. In this paper we demonstrate that initial conditions may be important, i.e., that grand unified interactions need not eradicate the memory of initial conditions, and we suggest that the lepton number of the Universe may provide a measurement of these initial conditions. In particular, we find that in SU(5) models both the final values of the baryon number (B) and lepton number (L) depend sensitively on initial conditions and may be independent of each other. The conditions necessary for a large L and small B in SU(5) models naturally lead us to a consideration of SO(10) models. We find that large initial SO(10)-invariant asymmetries (but with zero initial B and L) naturally lead to a present Universe with $L \gg B$.

Overall charge neutrality of the Universe requires that the excess of baryons over antibaryons be balanced by a corresponding excess of electrons over positrons:

$$L_e = \frac{n_e - n_{\bar{e}}}{n_\gamma} \simeq B = \frac{n_b - n_{\bar{b}}}{n_\gamma} \simeq 10^{-9}.$$

Any large lepton number in the Universe must thus be due to an excess of neutrinos over antineutrinos (or vice versa).

The existence of a large neutrino degeneracy would have several interesting cosmological effects. In particular, it would largely determine the results of primordial nucleosynthesis.[5] The primordial ^{4}He abundance is particularly sensitive to the value of the neutrino chemical potential. The existence of a large neutrino degeneracy may also prevent the high-temperature restoration of spontaneously broken gauge symmetries and the associated phase transitions.[6] This would prevent the possibility of any exponential expansion[7] and dissolve bounds on Higgs masses found by limiting the entropy produced in phase transitions.[8] It would also solve the problem of excess heavy stable monopoles produced in the phase transitions of hot models.[9] A large neutrino chemical potential may also make present-day detection of the background neutrinos possible due to the increase in number and energy of the neutrinos over the case of zero chemical potential.

The best limit on the neutrino number of the Universe comes from the limit on the total energy density of the Universe. In the absence of a large cosmological constant, the total energy density ρ_0 may be expressed in terms of the Hubble constant H_0, the deceleration parameter q_0, and the Planck mass $m_P = 1.2\times10^{19}$ GeV (Ref. 10):

$$\rho_0 = 2q_0\left(\frac{3H_0^2 m_P^2}{8\pi}\right). \tag{1.1}$$

The observational limits,[11] $100 \geq H_0$(km sec^{-1} Mpc^{-1}) ≥ 50 and $|q_0| \leq 2$, require the present energy density of the Universe to be $\rho_0 \leq 8 \times 10^{-29}$ g cm^{-3}. This limit restricts the possible number of primordial neutrinos, and hence restricts the lepton number. In order to apply this limit it is necessary to calculate the present number density of primordial neutrinos.

Neutrinos were in thermal equilibrium until about one second after the big bang when the temperature of the Universe had dropped to about 1 MeV.[10] After this time the mean free path of the neutrinos was larger than the size of the Universe, so neutrinos were effectively decoupled from the rest of the Universe. After decoupling, the number density of neutrinos was changed only by the overall expansion, and the present neutrino number density may be expressed in terms of the polylogarithm function[3]

$$n_\nu(T)=\left(\frac{T}{T_D}\right)^3\left[\int_0^\infty \frac{d^3\vec{p}}{(2\pi)^3}\frac{g_D}{\exp[(E-\mu_D)/T_D]+1}\right] \quad (T<T_D)$$

$$=-\left(\frac{T}{T_D}\right)^3\left[\frac{g_D T_D^3}{\pi^2}\,\mathrm{Li}_3(-e^{\mu_D/T_D})\right] \quad (m_\nu<T_D), \tag{1.2}$$

where g_D, T_D, and μ_D are the number of spin states in equilibrium, the temperature, and the chemical potential at neutrino decoupling. In Eq. (1.2) the factor $(T/T_D)^3$ represents the dilution of the neutrino number density due to expansion. If $m_\nu<T_D$, there is probably only one spin state of the neutrino in statistical equilibrium at decoupling, and below we will set $g_D=1$. In the cold limit $(\mu_D/T_D)\gg 1$ and in the hot limit $(\mu_D/T_D)\ll 1$, Eq. (1.2) becomes

$$n_\nu(T)=\left(\frac{T}{T_D}\right)^3\frac{T_D^3}{\pi^2}\frac{3}{4}\zeta(3)\left(1+\frac{2}{3}\frac{\zeta(2)}{\zeta(3)}\frac{\mu_D}{T_D}+\cdots\right) \quad (\mu_D\ll T_D),$$

$$n_\nu(T)=\left(\frac{T}{T_D}\right)^3\frac{\mu_D^3}{6\pi^2}\left[1+6\zeta(2)\left(\frac{T_D}{\mu_D}\right)^2+\cdots\right] \quad (\mu_D\gg T_D) \tag{1.3}$$

where ζ is the Riemann zeta function. Therefore in cold models the number density of neutrinos today is larger than would obtain for zero chemical potential by a factor of $\frac{2}{9}\zeta(3)(\mu_D/T_D)^3$. If a species of neutrinos has a mass greater than the present "blackbody" neutrino temperature $T_\nu=(1.4)^{-1}T_\gamma$, where T_γ is the present photon temperature $T_\gamma=2.9\,^\circ K=2.5\times10^{-4}$ eV and the factor of 1.4 accounts for the heating of the photons (but not the neutrinos) from electron-positron annihilation, then the effect of the chemical potential would be important in determining the mass density of the neutrinos. For instance the contribution to the mass density by a massive neutrino would increase by a factor of $\{1+\frac{2}{3}[\zeta(2)/\zeta(3)]\mu_D/T_D\}$ in the hot limit, and would increase by a factor of $\frac{2}{9}\zeta(3)(\mu_D/T_D)^3$ in the cold limit. If the neutrino is massless, its contribution to the energy density would be

$$\rho_\nu(T)=\left(\frac{T}{T_D}\right)^4\left[\int_0^\infty\frac{d^3\vec{p}}{(2\pi)^3}\frac{E}{\exp[(E-\mu_D)/T_D]+1}\right]$$

$$=-\left(\frac{T}{T_D}\right)^4\frac{3T_D^4}{2\pi^2}\,\mathrm{Li}_4(-e^{\mu_D/T_D}) \quad (T<T_D); \tag{1.4}$$

which has the hot and cold expansions

$$\rho_\nu(T)=\left(\frac{T}{T_D}\right)^4\frac{21}{8\pi^2}\zeta(4)T_D^4\left(1+\frac{\mu_D}{T_D}\frac{6\zeta(3)}{7\zeta(3)}+\cdots\right) \quad (\mu_D\ll T_D)$$

$$=\left(\frac{T}{T_D}\right)^4\frac{\mu_D^4}{8\pi^2}\left[1+12\zeta(2)\left(\frac{T_D}{\mu_D}\right)^2+\cdots\right] \quad (\mu_D\gg T_D). \tag{1.5}$$

If the only entropy increase subsequent to decoupling is the increase from electron-positron annihilation, then (μ/T) is constant, and in the cold limit the present neutrino energy density is

$$\rho_\nu(T)=\frac{\mu^4}{8\pi^2}\left[1+12\zeta(2)\left(\frac{T}{\mu}\right)^2+\cdots\right]. \tag{1.6}$$

The observational limit of Eq. (1.1) then implies

$$\mu\leqslant 1.3\times10^{-2}\text{ eV},\qquad \frac{\mu}{T}\leqslant 60. \tag{1.7}$$

Therefore, even though the present Universe is hot in baryons $(n_B/n_\gamma\simeq10^{-10\pm1})$, the only reliable limit on the neutrino number allows the Universe to be cold in leptons $(n_\nu/n_\gamma\leqslant 8\times10^4)$.

In the next section we consider the damping of initial asymmetries in a simple model. In Secs. III and IV we generalize these considerations and show how initial conditions may lead to a large lepton number in SU(5) and SO(10) unified theories. Section V contains our conclusions. Details of the evolution of a cold Universe are discussed in an appendix.

II. THE DAMPING OF INITIAL ASYMMETRIES

In this section we construct a simple model that illustrates the damping of initial asymmetries by particle interactions in the very early Universe. The results derived here describe the evolution of initial asymmetries in the absence of conserved quantum numbers. In subsequent sections when we treat actual unified models, the existence of conserved, or almost conserved, quantum numbers must be taken into account.

For the *production* of asymmetries, *CP* violation is crucial, but for the damping of an initial asymmetry the small corrections due to *CP* noninvariance are unimportant. This allows us to consider a model that is a simplification of one considered previously.[3] We will consider the damping of initial baryon number; the results may be easily generalized to the damping of asymmetries in any quantum number, e.g., L. In the model we consider, χ is a massive boson (vector or scalar) that violates baryon number, i.e., it decays to channels of different baryon number, and b and $\bar{b}$ are massless fermions with baryon number 1 and -1, respectively. Ignoring *CP* violation, the decay amplitudes for the χ and $\bar{\chi}$ are equal:

$$|M(\chi\to bb)|^2=|M(\chi\to\bar{b}\bar{b})|^2=\tfrac{1}{2}|M_0|^2\,,$$
$$|M(\bar{\chi}\to\bar{b}\bar{b})|^2=|M(\bar{\chi}\to bb)|^2=\tfrac{1}{2}|M_0|^2\,. \tag{2.1}$$

The Boltzmann equation for the evolution of the b number density is given by (the $\dot{R}/R$ term represents the dilution of the number density because of expansion).

$$\begin{aligned}\frac{dn_b}{dt}+3\frac{\dot{R}}{R}n_b=\Lambda^{\chi}_{12}\{&f_\chi(p_\chi)[1-f_b(p_1)][1-f_b(p_2)]|M(\chi\to bb)|^2+f_{\bar{\chi}}(p_{\bar{\chi}})[1-f_b(p_1)][1-f_b(p_2)]|M(\bar{\chi}\to bb)|^2\\&-f_b(p_1)f_b(p_2)[1+f_\chi(p_\chi)]|M(bb\to\chi)|^2-f_b(p_1)f_b(p_2)[1+f_{\bar{\chi}}(p_{\bar{\chi}})]|M(bb\to\bar{\chi})|^2\}\\&+\Lambda^{34}_{12}\{f_{\bar{b}}(p_1)f_{\bar{b}}(p_2)[1-f_b(p_1)][1-f_b(p_2)]|M'(\bar{b}\bar{b}\to bb)|^2\\&-f_b(p_1)f_b(p_2)[1-f_{\bar{b}}(p_3)][1-f_{\bar{b}}(p_4)]|M'(bb\to\bar{b}\bar{b})|^2\}\,,\end{aligned}\tag{2.2}$$

where $f_i(p)$ is the phase-space density of species i,

$$n_i=\int\frac{d^3\vec{p}}{(2\pi)^3}f_i(p)\,,\tag{2.3}$$

and $\Lambda^{ab\cdots}_{\alpha\beta\cdots}$ is the integral operator

$$\Lambda^{ab\cdots}_{\alpha\beta\cdots}=\int\frac{d^3\vec{p}_a}{2E_a(2\pi)^3}\int\frac{d^3\vec{p}_b}{2E_b(2\pi)^3}\cdots\int\frac{d^3\vec{p}_\alpha}{2E_\alpha(2\pi)^3}\int\frac{d^3\vec{p}_\beta}{2E_\beta(2\pi)^3}\cdots(2\pi)^4\delta^4(p_a+p_b+\cdots-p_\alpha-p_\beta-\cdots)\,.\tag{2.4}$$

The $|M(\chi\to bb)|^2$ terms in Eq. (2.2) represent the change in the b number density due to decay and inverse decay processes, while the $|M'(bb\to\bar{b}\bar{b})|^2$ terms represent the change in the b number density due to two-body scattering processes with the exchange of an intermediate χ. $|M'|^2$ is the square of the total amplitude for the scattering processes minus the part of the squared amplitude corresponding to the propagation of a real intermediate χ state, since the real intermediate χ state is already included in Eq. (2.2) as a sum of inverse decay and decay processes. The Boltzmann equation for $\dot{n}_{\bar{b}}$ may similarly be obtained and combined with the equation for $\dot{n}_b$ to form an equation for the evolution of the baryon number density:

$$\begin{aligned}\dot{n}_B\equiv\dot{n}_b-\dot{n}_{\bar{b}}=\Lambda^{\chi}_{12}|M_0|^2\{&f_\chi(p_\chi)[1-f_b(p_1)][1-f_b(p_2)]-f_\chi(p_\chi)[1-f_{\bar{b}}(p_1)][1-f_{\bar{b}}(p_2)]\\&-f_b(p_1)f_b(p_2)+f_{\bar{b}}(p_1)f_{\bar{b}}(p_2)\}\\&+2\Lambda^{34}_{12}|M'|^2\{f_{\bar{b}}(p_1)f_{\bar{b}}(p_2)[1-f_b(p_3)][1-f_b(p_4)]-f_b(p_1)f_b(p_2)[1-f_{\bar{b}}(p_3)][1-f_{\bar{b}}(p_4)]\}\\&-3\frac{\dot{R}}{R}n_B\,,\end{aligned}\tag{2.5}$$

where we have made the approximation that $f_\chi(p_\chi)=f_{\bar{\chi}}(p_{\bar{\chi}})\ll 1$ (i.e., no Bose condensate). The equation for $\dot{n}_B$ is in general quite complicated, but it has reasonably simple forms in the hot limit $(n_b/n_\gamma\simeq n_{\bar{b}}/n_\gamma\simeq 1)$, and in the cold limit $(n_b/n_\gamma\gg 1;\, n_{\bar{b}}/n_\gamma\ll 1)$.

In the hot limit $f_b(p)$, $f_{\bar{b}}(p)\ll 1$, $\mu/T\ll 1$, so the phase-space distributions may be approximated by the Maxwell-Boltzmann form

$$f_b=e^{-(E-\mu)/T}\simeq e^{-E/T}\left(1+\frac{\mu}{T}+\cdots\right)\,,$$
$$f_{\bar{b}}=e^{-(E+\mu)/T}\simeq e^{-E/T}\left(1-\frac{\mu}{T}+\cdots\right)\,.\tag{2.6}$$

The products of distribution functions appearing in the decay and inverse decay parts of Eq. (2.5) become

$$f_b(p_1)f_b(p_2)=2\frac{\mu}{T}e^{(E_1+E_2)/T}=2\frac{\mu}{T}e^{E_\chi/T}$$
$$=2\frac{\mu}{T}f_\chi^{eq}, \qquad (2.7)$$
$$f_{\bar b}(p_1)f_{\bar b}(p_2)=-2\frac{\mu}{T}e^{(E_1+E_2)/T}=-2\frac{\mu}{T}e^{E_\chi/T}$$
$$=-2\frac{\mu}{T}f_\chi^{eq},$$

where the energy conservation δ function in Λ_{12}^{χ} has been used to set $E_1+E_2=E_\chi$, and f_χ^{eq} is the equilibrium distribution function for χ. Therefore, in the hot limit, Eq. (2.5) becomes

$$\dot n_B+3\frac{\dot R}{R}n_B\simeq-4\frac{\mu}{T}\Lambda_{12}^{\chi}|M_0|^2 f_\chi^{eq}$$
$$-8\frac{\mu}{T}\Lambda_{12}^{34}|M'|^2e^{E_1/T}e^{E_2/T}$$
$$\simeq-4\frac{\mu}{T}\langle\Gamma_\chi\rangle n_\chi^{eq}-8\frac{\mu}{T}n_b{}^2\langle|v|\sigma'\rangle, \qquad (2.8)$$

where n_χ^{eq} is the equilibrium χ number density ($n_\chi^{eq}=\int[d^3\vec p/(2\pi)^3]f_\chi^{eq}$), $\langle\Gamma_\chi\rangle$ is the thermal averaged χ decay width

$$\langle\Gamma_\chi\rangle\simeq\frac{1}{2E}\int\frac{d^3\vec p_1}{(2\pi)^3E_1}\int\frac{d^3\vec p_2}{(2\pi)^3E_2}|M_0|^2(2\pi)^4\delta^4(p_\chi-p_1-p_2),$$

n_b is the baryon density ($n_b\simeq n_{\bar b}$), and $\langle\sigma'\rangle$ is the thermal averaged two-body cross section with real intermediate χ removed. Using Maxwell-Boltzmann statistics for photons and baryons gives

$$B\equiv\frac{n_B}{n_\gamma}\simeq2\frac{\mu}{T}. \qquad (2.9)$$

Assuming adiabatic expansion ($n_\gamma\sim R^{-3}$) the Boltzmann equation for $\dot B$ is

$$\dot B=n_\gamma{}^{-1}\left(\dot n_B+3\frac{\dot R}{R}n_B\right)$$
$$=-2B\langle\Gamma_\chi\rangle\left(\frac{n_\chi}{n_\gamma}\right)^{eq}-4Bn_b\langle|v|\sigma'\rangle. \qquad (2.10)$$

The first term in $\dot B$ represents the damping of B due to inverse decay ($bb\to\chi$), and the second term represents damping of B due to two-body scattering ($bb\to\bar b\bar b$).

In the cold limit ($f_{\bar b}\simeq0; f_b\simeq1$) the fermion degeneracy blocks production of baryons, and Eq. (2.5) becomes

$$\dot n_B+3\frac{\dot R}{R}n_B=-\Lambda_{12}^{\chi}|M_0|^2[f_b(p_1)f_b(p_2)]$$
$$-2\Lambda_{12}^{34}|M'|^2[f_b(p_1)f_b(p_2)]. \qquad (2.11)$$

For large chemical potentials $\mu>m_\chi$, Eq. (2.11) simplifies to

$$\dot n_B+3\frac{\dot R}{R}n_B=-2n_b{}^2\langle|v|\sigma'\rangle. \qquad (2.12)$$

Now as $n_B\simeq n_b$ is damped by the creation of antibaryons, the number of "photons" (in this case a $b\bar b$ pair is equivalent to a photon) increases and $\dot R/R\neq-\dot T/T$, i.e., the expansion is no longer adiabatic in the sense that n_γ is not constant. However if we now define B as the baryon number in a comoving volume element, then Eq. (2.12) becomes

$$\dot B=-2Bn_b\langle|v|\sigma'\rangle. \qquad (2.13)$$

Only in the limit that $\dot B$ is small can the B in Eq. (2.13) be associated with the "baryon to *photon*" ratio.

The cross sections for $bb\to\bar b\bar b$ as a function of total center-of-momentum energy squared $s\equiv(p_1+p_2)^2\simeq4\langle E\rangle^2$ for $s>m_\chi$ are given by

$$|v|\sigma\simeq\frac{4\pi\alpha_H{}^2}{3s}\quad(\text{scalar }\chi) \qquad (2.14a)$$
$$\simeq\frac{4\pi\alpha^2}{m_\chi{}^2}\quad(\text{vector }\chi), \qquad (2.14b)$$

with $\alpha=g^2/4\pi$ where g is the gauge coupling constant at the scale $\sim m_\chi$, and $\alpha_H=h^2/4\pi$ where h is the Yukawa coupling of scalars to fermions at the same scale. The fact that the vector χ cross section is a constant for high energies is evidence of the fact that at large s, the *total* cross section for two-body scattering through t-channel exchange of a spin-j particle is $\sigma\sim(s/m^2)^j/s$. In the cold limit, the baryon number density is $n_b=\mu^3/6\pi^2$, the average energy is $\langle E\rangle=3\mu/4$ ($s=9\mu^2/4$), and the age of the Universe is (see Appendix A) $t\simeq3m_P/2\mu^2$. In the cold limit the final value of B is thus

$$B=B_0\exp\left(-\frac{16}{81}\frac{\alpha_H{}^2}{\pi}(6m_Pt)^{1/2}\right)$$
$$=B_0\exp\left[-\frac{16}{81}\frac{\alpha_H{}^2}{\pi}\frac{3m_P}{\mu}\right] \qquad (2.15)$$

for scalar χ, while for vector χ

$$B=B_0\exp\left[-2\sqrt6\,\frac{\alpha^2}{\pi}\left(\frac{m_P}{m_\chi}\right)^2\right]. \qquad (2.16)$$

In grand unified gauge theories the vector gauge coupling is typically $\alpha\sim1/40$, while the Yukawa coupling is given in terms of the W-boson mass and a typical fermion mass for the heaviest family of fermions m_f by $\alpha_H=h^2/4\pi$ with $h=gm_f/\sqrt2 m_W$. Since quark masses typically decrease by a factor of three between present energies and the unification scale, we expect that $(m_f/m_W)^2\leq10^{-1}$ even for fermion masses that saturate the bound $m_f\leq100$ GeV from stability of the effective potential.[12] The baryon-number-violating reactions will freeze out when $\langle E \rangle \leq m_\chi$, so the destruction of the initial baryon number due to *scalar* exchange would be

$$\frac{B}{B_0} = \exp\left(-\frac{64}{81}\frac{\alpha_H{}^2}{\pi}\frac{m_P}{m_\chi}\right) \leq \exp\left(-10^{-6}\frac{m_P}{m_\chi}\right) \text{ (scalar } \chi) \qquad (2.17)$$

with $(m_f/m_W)^2 = 0.1$. Therefore if $m_\chi \geq 10^{13}$ GeV, essentially no damping will occur. However for damping due to exchange of a *vector* χ, the baryon number is reduced by

$$\frac{B}{B_0} = \exp\left[-10^{-3}\left(\frac{m_P}{m_\chi}\right)^2\right] \text{ (vector } \chi). \qquad (2.18)$$

For a vector mass of $m_\chi = 5\times 10^{14}$ GeV $= 0.5$ ΠeV, $B/B_0 \simeq \exp(-10^6)$.

We conclude that scalar exchange is ineffective in damping the initial baryon asymmetry if the scalar mass is greater than about $10^{-6} m_p \simeq 10^{-2}$ ΠeV. Vector interactions, however, are very efficient in damping initial asymmetries, and unless the vector is sufficiently heavy ($m_\chi \geq 10^{-2} m_p \simeq 10^2$ ΠeV) initial asymmetries will be destroyed by vector interactions.

In the next section we will see that the existence of quantum numbers conserved by vector interactions may change the above conclusions as shown by Treiman and Wilczek in Ref. 2. Equation (2.17) may be used to estimate the damping of these quantum numbers by scalar interactions.

III. LEPTON NUMBER IN SU(5)

The results of the previous section indicate that the damping of initial asymmetries is governed by the structure of the vector-gauge-boson couplings. In this section we consider the damping of initial asymmetries in unified gauge theories based on the group SU(5).[1,13]

SU(5) is the simplest group (and the only group of rank 4) which contains the group necessary to describe low-energy phenomenology, $SU(3)_C \otimes SU(2)_L \otimes U(1)$. A family of fermions consisting of fifteen left-handed fermion fields is placed into the reducible representation $\underline{\bar{5}}_f \oplus \underline{10}_f$. Such a family has the generic particle content

$$\underline{\bar{5}}_f = (\vec{D}_L^C, \nu_L, E_L), \qquad \underline{10}_f = (\vec{U}_L, \vec{U}_L^C, \vec{D}_L, E_L^C), \qquad (3.1)$$

where U, D, ν, and E represent the charge $\frac{2}{3}$ quark, the charge $-\frac{1}{3}$ quark, the neutrino, and the charged lepton in the family. The vector sign represents transformation as a $SU(3)_C$ triplet. The subscript L indicates projection of the left-handed component and the superscript C indicates the charge conjugate state. The CP conjugates of the particles in Eq. (3.1) transform as $\underline{5}_f \oplus \underline{\overline{10}}_f$:

$$\underline{5}_f = (\vec{D}_R, \nu_R^C, E_R^C), \qquad \underline{\overline{10}}_f = (\vec{U}_R^C, \vec{U}_R, \vec{D}_R^C, E_R). \qquad (3.2)$$

The vector bosons transform as the adjoint 24-dimensional representation and have gauge couplings to the fermions given by

$$\mathcal{L}_g = \frac{g}{\sqrt{2}}[\underline{\bar{5}}_f \times \underline{5}_f + \underline{\overline{10}}_f \times \underline{10}_f] \times 24_V, \qquad (3.3)$$

where g is the gauge coupling constant.

The breaking of SU(5) to $SU(3)_C \otimes SU(2)_L \otimes U(1)$ is usually effected by means of a $\underline{24}_H$ of Higgs scalars which is postulated to obtain a vacuum expectation value of order 10^{15} GeV $\equiv 1$ ΠeV. Scalar fields which couple to fermions must transform according to representations that appear in the Lorentz scalar products of fermion fields:

$$\underline{\bar{5}}_f \otimes \underline{\bar{5}}_f = \underline{\overline{10}} \oplus \underline{\overline{15}}, \quad \underline{\bar{5}}_f \otimes \underline{10}_f = \underline{5} \oplus \underline{45}, \quad \underline{10}_f \otimes \underline{10}_f = \underline{\bar{5}} \oplus \underline{\overline{45}} \oplus \underline{50}. \qquad (3.4)$$

The only representations in (3.4) which have a neutral component and may thus have nonzero vacuum expectation values are the $\underline{5}$, $\underline{15}$, and $\underline{45}$. A $\underline{15}_H$ Higgs field is usually excluded since it would give a Majorana mass to the left-handed neutrino that cannot be made naturally small.

In this section we consider only a single $\underline{5}_H$ of Higgs. The results derived here are essentially independent of the Higgs sector as long as there are no scalars with anomalously large Yukawa couplings. The Yukawa coupling of a $\underline{5}_H$ to fermions is given schematically by

$$\mathcal{L}_Y = [\underline{10}_i (h_U)^{ij} \underline{10}_j] \times \underline{5}_H + [\underline{\bar{5}}_i (h_D)^{ij} \underline{10}_j] \times \underline{\bar{5}}_H, \qquad (3.5)$$

where i and j are family indices and h_U and h_D are the Yukawa coupling matrices. In the single family model we consider, $h_U = (g/\sqrt{2}) m_U/m_W$, $h_D = (g/\sqrt{2}) m_D/m_W$, where m_W is the mass of the $SU(2)_L$ gauge boson, $m_W \simeq 100$ GeV, and m_U (m_D) is the mass of the charge $\frac{2}{3}$ ($-\frac{1}{3}$) quark in the family.

If we consider only vector couplings, then it is clear that the couplings in Eq. (3.3) are invariant under two global phase transformations: $\underline{5}_f \to e^{i\alpha}\underline{5}_f$ and $\underline{10}_f \to e^{i\beta}\underline{10}_f$. The corresponding conserved quantum numbers are given by $\chi_5 = +1$ (-1) for each field in the $\underline{5}_f$ ($\underline{\bar{5}}_f$), and $\chi_{10} = +1$ (-1) for each field in the $\underline{10}_f$ ($\underline{\overline{10}}_f$). Scalar interactions violate χ_5 and χ_{10}, but from Eq. (3.5) we see that it is possible to take a linear combination of χ_5 and χ_{10} that is still a conserved quantum number, $Z=3$ (-3) for $\underline{5}_f$ ($\overline{\underline{5}}_f$), $Z=+1$ (-1) for $\underline{10}_f$ ($\overline{\underline{10}}_f$), and $Z=-2$ ($+2$) for the $\underline{5}_H$ ($\overline{\underline{5}}_H$). When $SU(3)_C \otimes SU(2)_L \otimes U(1)$ breaks to $SU(3)_C \otimes U(1)_{EM}$, Z is spontaneously broken, but a combination of Z and the hypercharge remains unbroken. This combination is just the baryon number minus the lepton number $B-L$. Although the full SU(5) theory does not separately conserve χ_5 and χ_{10}, the results of Sec. II indicate that to a good approximation the scalar interactions may be neglected in the damping of initial asymmetries. In this approximation both χ_5 and χ_{10} will be conserved. Since the linear combination of χ_5, χ_{10} and hypercharge given by $B-L$ is conserved by the Higgs sector as well, it will be convenient to work with the quantum numbers χ_5 and $B-L$ rather than χ_5 and χ_{10}.

We now consider the damping of initial asymmetries. f_- will denote the asymmetry between a left-handed fermion field f and its CP-conjugate antiparticle, normalized to the photon number density

$$\begin{aligned} U_- &\equiv (n_{U_L} - n_{U_R^C})/n_\gamma , \\ D_-^C &\equiv (n_{D_L^C} - n_{D_R})/n_\gamma , \\ \nu_- &\equiv (n_{\nu_L} - n_{\nu_R^C})/n_\gamma \cdots . \end{aligned} \tag{3.6}$$

$SU(3)_C \otimes SU(2)_L \otimes U(1)$ invariance requires the equality of asymmetries between different colors of quarks and between weak isodoublets and also requires that the total hypercharge be zero.[3] The independent fermion asymmetries may thus be parametrized by the quantities

$$\begin{aligned} B &= U_- + D_- - U_-^C - D_-^C , \\ B-L &= U_- + D_- - U_-^C - D_-^C + E_-^C - E_- - \nu_- , \\ \chi_5 &= -3D_-^C - E_- - \nu_- , \\ \nu_- &= \nu_- , \end{aligned} \tag{3.7}$$

where the asymmetries in the quark fields are summed over the three possible colors.

At temperature above the unification mass the full SU(5) invariance will enforce equality in the asymmetries among members of irreducible representations of SU(5). We thus consider SU(5) invariant but CP-noninvariant initial conditions with an asymmetry η_5 in each member of the $\underline{5}_f$ and an asymmetry η_{10} in each member of the $\underline{10}_f$. In fact this is a quite general result. The fast vector interactions will convert *any* asymmetries into gauge-invariant initial conditions with equal asymmetries in all fermion fields within an irreducible representation of the gauge group as long as there are no net asymmetries corresponding to gauged quantum numbers. The initial values of the quantum numbers in (3.7) are then

$$\begin{aligned} B^0 &= \eta_5 + \eta_{10} , \\ (B-L)^0 &= 3\eta_5 + 2\eta_{10} , \\ \chi_5^0 &= 5\eta_5 , \\ \nu_-^0 &= -\eta_5 . \end{aligned} \tag{3.8}$$

The initial lepton number is given by $L^0 = -2\eta_5 - \eta_{10}$.

Vector interactions will exactly conserve these initial values since they conserve χ_5 and χ_{10} separately and will maintain the equality of asymmetries among members of the $\underline{5}_f$ and $\underline{10}_f$. This may also be seen by explicitly considering the terms in the Boltzmann equation for the damping of asymmetries through, for example, vector inverse decay. These terms give[3]

$$\begin{aligned} \dot{\chi}_5 &= 0 , \\ \dot{B} - \dot{L} &= 0 , \\ \dot{B} &\propto [2B - \nu_- - (B-L)] , \\ \dot{\nu}_- &\propto [5\nu_- + \chi_5] . \end{aligned} \tag{3.9}$$

With the initial conditions (3.8) these equations then give $\dot{\chi}_5 = (\dot{B} - \dot{L}) = \dot{B} = \dot{\nu}_- = 0$.

Scalar interactions violate χ_5 and χ_{10} but conserve $B-L$. They will thus eventually relax all the initial asymmetries except $B-L$. The usual weak doublet of Higgs scalars may violate ν_- and χ_5 but not B or L. They will thus relax all asymmetries except B and L to zero since their effects are important until $T \lesssim 100$ GeV. The results of Sec. II indicate, however, that the *massive* B- and L-violating scalars are ineffective in reducing the initial values of B and L. In particular, if the initial conditions consist of an equal asymmetry in each left-handed fermion field so that $\eta_5 = -\eta_{10}$, we have

$$\begin{aligned} B^0 &= 0 , \\ L^0 &= \eta_{10} . \end{aligned} \tag{3.10}$$

Scalar interactions will give final values of B and L which differ from these initial values by $\leq O(\exp(-10^{-6} m_P/m_s))$ (cf. Eq. 2.18) which for the conditions (3.10) could yield a large lepton number and a small baryon number compatible with present observations. Since the initial B and L are independent it is also possible that both a large L and a large B survived which were later diluted by entropy production to yield the presently observed value of n_B/n_γ.

IV. LEPTON NUMBER IN SO(10)

In grand unified theories[14] based on the gauge group SO(10) all the fermions in a single family are assigned to the complex spinor representation $\underline{16}_f$:

$$\underline{16}_f^T = (\vec{U}_L, \vec{U}_L^C, \vec{D}_L, \vec{D}_L^C, E_L, E_L^C, \nu_L, N_L^C). \quad (4.1)$$

The $\overline{\underline{16}}_f$ contains the CP-conjugate states:

$$\overline{\underline{16}}_f^T = (\vec{U}_R^C, \vec{U}_R, \vec{D}_R^C, \vec{D}_R, E_R^C, E_R, \nu_R^C, N_R). \quad (4.2)$$

Since there are only 15 known fermion fields per family (assuming the existence of the top quark) it is necessary to postulate the existence of a particle, the N_L^C, that is neutral under $SU(3)_C \otimes SU(2)_L \otimes U(1)$. The existence of this particle has interesting consequences for the lepton number of the Universe as well as for low-energy neutrino phenomenology.

The gauge vector bosons in SO(10) transform as the 45-dimensional adjoint representation. The gauge coupling to fermions has the form

$$\mathcal{L}_g = \frac{g}{\sqrt{2}} \overline{\underline{16}}_f \times \underline{16}_f \times \underline{45}_V . \quad (4.3)$$

The vector interactions in SO(10) conserve a quantum number, $\chi_{16} = +1$ (-1) for each field in the $\underline{16}_f$ $(\overline{\underline{16}}_f)$, analogous to the χ_5 and χ_{10} conservation in SU(5).

The Higgs fields which can couple to fermions appear in the decomposition of $\underline{16} \otimes \underline{16}$:

$$\underline{16} \otimes \underline{16} = (\underline{10} + \underline{126})_S + (\underline{120})_A . \quad (4.4)$$

If only the $\underline{10}_H$ contributes to the fermion masses, then to lowest order at unification energies

$$m_U = m_D = m_E = m_\nu . \quad (4.5)$$

This very simple mass relation is unfortunately very wrong. The most obvious contradiction is the prediction that the neutrino in a family has the same mass as the charged particles. However, if the N_L^C acquires a very large Majorana mass M_N presumably through a nonzero vacuum expectation value for the $\underline{126}_H$ (Ref. 15) or through radiative corrections,[16] then the neutral lepton mass matrix in the ν, N basis will have the form

$$\begin{pmatrix} 0 & m_q \\ m_q & M_N \end{pmatrix}, \quad (4.6)$$

where m_q is the mass of the charge $\frac{2}{3}$ quark in the family. The approximate eigenvalues of this matrix are m_q^2/M_N and M_N. The observed low-energy neutrinos will thus have masses $O(m_U^2/M_N)$ which can be made compatible with present observations if M_N is sufficiently large. In order to adjust the other mass relations in Eq. (4.5) various machinations in the Higgs sector are often performed; these complications will not concern us.

We now consider the damping of asymmetries in SO(10) models with an initial asymmetry η_{16} in each member of the $\underline{16}_f$:

$$U_- = \vec{U}_-^C = \vec{D}_- = \vec{D}_-^C = E_- = E_-^C = \nu_- = N_-^C = \eta_{16}^0 . \quad (4.7)$$

The global $B-L$ symmetry present in SU(5) models is gauged in SO(10) models with the N_L^C assigned $B-L=1$. This initial condition thus corresponds to zero net B and L. It should be noted that in the limit of exact SO(10) invariance the presence of an unbroken charge conjugation operator C requires any asymmetries in quantum numbers that are odd number C (e.g., B, L, $Q, \ldots$) to be zero.

The damping of asymmetries will in general depend on the pattern of symmetry breaking. We first consider the case $SO(10) \to SU(5) \to SU(3)_C \times SU(2)_L \times U(1)$. The $\underline{16}_f$ has the SU(5) decomposition

$$\underline{16} = \underline{10} + \overline{\underline{5}} + \underline{1} \quad (4.8)$$

with N_C^L corresponding to the SU(5) singlet. In terms of SU(5), the initial conditions (4.7) thus correspond to

$$\eta_{10} = -\eta_5 = \eta_1 \quad (4.9)$$

with $\eta_1 = N_-^C$. If only vector interactions are included then the initial conditions (4.7) will be maintained and hence no B or L will be produced. In order to convert the asymmetries in (4.7) into a net B or L, Higgs bosons and the breaking of SO(10) and C invariance must be taken into account. As discussed in Sec. II, $2 \to 2$ reactions mediated by scalar bosons may be ineffective in damping initial asymmetries and will give only small corrections to the relations in Eq. (4.7). However, the presence of a Majorana mass for the N_L^C requires that N_-^C be zero at temperatures below the N mass.

The dominant process that modifies the relations (4.7) and thus generates a net B and L is the damping of the asymmetry in N_L^C through the Majorana mass term and the subsequent rearrangement of asymmetries by vector interactions. The asymmetry in N_L^C is relaxed by the Majorana mass term which induces the transition $N_L^C \to N_R$ at a rate given approximately by

$$\Gamma(N_L^C \to N_R) \simeq M_N(M_N/E), \quad (4.10)$$

where the factor (M_N/E) accounts for the effect of time dilation. The equation governing the damping of the N_L^C asymmetry in a cold Universe is thus given by

$$\dot{\eta}_1 = -\eta_1 \frac{M_N^2}{E} \simeq -\eta_1 \frac{4}{3} \frac{M_N^2}{\mu} \tag{4.11}$$

with the solution

$$\eta_1 \simeq \eta_{16}^0 \exp\left(-\frac{2}{9} \frac{M_P M_N^2}{\mu^3}\right), \tag{4.12}$$

where we have used the equations in the Appendix with $\xi = 48$ representing the existence of three families of fermions and where η_{16}^0 is the initial value of η_1.

As η_1 is driven to zero the vector interactions will redistribute the asymmetry in the other fields. As an example, we consider the interactions of the B-violating vector boson X' which occurs in the SU(5) representation 10 under the decomposition of the SO(10) gauge bosons:

$$\underline{45}_V = \underline{24} + \overline{\underline{10}} + \underline{10} + \underline{1}. \tag{4.13}$$

The X' induces transitions between the SU(5) representations of fermions of the form $\underline{10}_f + \underline{10}_f \leftrightarrow \underline{1}_f + \overline{\underline{5}}_f$. When $\eta_{10} = -\eta_5 = \eta_1$, the forward and reverse interactions occur at the same rate. As η_1 is destroyed at a rate given by (4.12), the reaction $\underline{10}_f + \underline{10}_f \to \underline{1}_f + \overline{\underline{5}}_f$ proceeds at a faster rate than the reverse reaction $\overline{\underline{5}}_f + \underline{1}_f \to \underline{10}_f + \underline{10}_f$. Since the baryon number is given by $B = \eta_5 + \eta_{10}$, these reactions will tend to produce a nonzero baryon number by destroying the balance between η_5 and η_{10}. We can estimate the baryon number produced by assuming the X' interactions are fast enough to establish chemical equilibrium so that

$$2\eta_{10} = -\eta_5 + \eta_1 \tag{4.14}$$

or, using $B = \eta_5 + \eta_{10}$,

$$B = \eta_1 - \eta_{10}. \tag{4.15}$$

Assuming that the X' interactions freeze out at $\mu = M_{X'}$ and using (4.12) then gives for B

$$B = \eta_{16}^0 \left[\exp\left(-\frac{2}{9} \frac{m_P m_N^2}{m_{X'}^3}\right) - 1\right]$$
$$\simeq \eta_{16}^0 \left(\frac{2}{9} \frac{m_N^2 m_P}{m_{X'}^3}\right), \tag{4.16}$$

where we have assumed $m_N \ll m_{X'}$. As shown in Sec. III, the conservation of χ_5 and χ_{10} in SU(5) keeps this baryon number from subsequently being damped by the vector bosons of SU(5).

The lepton number, given by

$$L = -2\eta_5 - \eta_{10} - \eta_1, \tag{4.17}$$

is of order B at the temperature at which the X' bosons freeze out. However, since η_1 is eventually driven to zero by the Majorana mass term, the final value of L is given approximately by

$$L \simeq -2\eta_5 - \eta_{10} = \eta_{16}^0. \tag{4.18}$$

L may be reduced somewhat from this value by the interactions of the usual weak doublet of Higgs bosons which may couple to the N. At temperatures $T < m_N$ these bosons are effectively L conserving. Equation (2.16) suggests that these interactions should reduce L by an approximate factor of

$$\exp\left[-5\times10^{-7}\left(\frac{1\ \text{IIeV}}{M_N}\right)\right] \simeq 1 \quad \text{for } M_N \geq 10^9\ \text{GeV}, \tag{4.19}$$

where we have used $\alpha = 1/40$ and $m_\tau/M_W \simeq 0.02$ at the unification scale.

Comparison of Eqs. (4.19) and (4.16) reveals a large lepton number and a small baryon number

$$\left|\frac{B}{L}\right| \simeq \frac{2M_N^2 M_P}{9M_{X'}^3}. \tag{4.20}$$

If $M_{X'} \simeq 10^{17}$ GeV, $M_N \simeq 10^{12}$ GeV, then this gives $|B/L| \simeq 1.3\times10^{-9}$.

The ratio $|B/L|$ could be easily smaller than the estimate given in Eq. (4.20). So far we have considered the breaking scheme $SO(10) \to SU(5) \to SU(3)_C \times SU(2)_L \times U(1)$. Other breaking schemes such as $SO(10) \to SU(4)\times SU(2)_L \times U(1)_R \to SU(3)_C \times SU(2)_L \times U(1)$ are also possible. Since the gauge bosons of $SU(4)\times SU(2)_L \times U(1)_R$ do not violate baryon number, this intermediate symmetry may persist to lower temperatures than the SU(5) symmetry. Since the $U(1)_R$ symmetry forbids a Majorana mass for the N, the N becomes massive only after the $SU(4)\times SU(2)_L \times U(1)_R$ intermediate symmetry is broken. At this temperature all B- and L-violating vector bosons will have frozen out and as a result the final L will be of order η_{16}^0 while the final B will be produced only through $2\to2$ Higgs exchange processes and will thus be much smaller than the estimate (4.16).

V. CONCLUSIONS

We find that the interactions of grand unified theories combined with CP noninvariant initial fermion asymmetries can naturally lead to the present lepton number of the Universe being much larger than the baryon number. We thus disagree with others who have claimed that $L \sim B$ as a consequence of grand unification.[17] In SU(5) models the requirement that $L \gg B$ requires a cancellation between different contributions to the initial baryon number. This cancellation has a natural explanation in SO(10) models where all fermions in a family are placed in a single irreducible representation. While we have considered explicitly only SU(5) and SO(10) unified models, our results can be easily generalized to other theories. The reason that SU(5) and SO(10) theories allow $L \gg B$ can be related to the fact that they also predict a discrepancy between quark and neutrino masses. In SU(5) $m_\nu = 0$ as a result of the global $B-L$ symmetry which in turn is related to the reducibility of the fermion representation. It is this reducibility that allows B and L to be independent. In SO(10), $m_\nu \ll m_q$ is a result of a large $SU(3)_C \times SU(2)_L \times U(1)$-invariant Majorana mass term for the right-handed component of the neutrino. This mass term rapidly destroys any net lepton number residing in the right-handed neutrino field thus leaving any initial asymmetry in the left-handed neutrino unbalanced. We expect that any theory that predicts $m_\nu \ll m_q$ in a natural way will also allow $L \gg B$. In light of this fact, we feel that further investigation of the evolution of a Universe with a large neutrino degeneracy is warranted.

Finally, we remark that in SU(5) theories and in SO(10) theories with $M_N \sim M_{X'}$, B and L may both be large. A large B present after B-violating interactions have frozen out would invalidate the constraints on cosmological models due to limiting the dilution of B due to entropy production.[8,18]

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APPENDIX A: THE DYNAMICS OF A COLD UNIVERSE

In this appendix we derive the equations describing the evolution of a cold Universe. By cold, we mean a Universe with at least one fermion species with a chemical potential μ larger than the thermodynamic temperature T.

The number density of a fermion species is given by

$$n = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\exp[(E-\mu)/T]+1} . \tag{A1}$$

Since we are interested in the evolution of the Universe for $T \gg m_f$ we may ignore the mass of the fermion in Eq. (A1), and the number density becomes

$$n = -\frac{T^3}{\pi^2} \mathrm{Li}_3(-e^{\mu/T}) , \tag{A2}$$

where Li_n is the polylogarithm function

$$\mathrm{Li}_n(x) = \int_0^x \frac{\mathrm{Li}_{n-1}(t)}{t} dt = \sum_{k=1}^{\infty} \frac{x^k}{k^n} . \tag{A3}$$

For $\mu/T \gg 1$ the number density becomes

$$n = \frac{\mu^3}{6\pi^2}\left[1 + 6\zeta(2)\left(\frac{T}{\mu}\right)^2 + \cdots\right] , \tag{A4}$$

where $\zeta(n)$ is the Riemann zeta function.

The energy density of a massless degenerate fermion species is

$$\begin{aligned} \rho &= \int \frac{d^3p}{(2\pi)^3} \frac{E}{\exp[(E-\mu)/T]+1} \\ &= -\frac{3}{2}\frac{T^4}{\pi^2} \mathrm{Li}_4(-e^{\mu/T}) \\ &= \frac{\mu^4}{8\pi^2}\left[1 + 12\zeta(2)\left(\frac{T}{\mu}\right)^2 + \cdots\right] . \end{aligned} \tag{A5}$$

The average energy of a degenerate species of massless fermions is

$$\langle E \rangle = \rho/n = \tfrac{3}{4}\mu . \tag{A6}$$

In a homogeneous and isotropic Universe the dynamic equation that describes the expansion at early times is (assuming zero cosmological constant)

$$\begin{aligned} \left(\frac{\dot R}{R}\right)^2 &= \frac{8\pi G}{3}\rho \\ &= \frac{8\pi}{3}\frac{\rho}{m_P^2} , \end{aligned} \tag{A7}$$

where R is the scale factor in the Robertson-Walker metric, and may be considered as the radius of the Universe. If μ is associated with an exactly conserved quantum number, then conservation of that quantum number implies $\mu^3 R^3 = \text{constant}$, and

$$\begin{aligned} -\frac{\dot R}{R} &= \frac{\dot\mu}{\mu} \\ &= \frac{\dot T}{T} , \end{aligned} \tag{A8}$$

where the last equality comes from conservation of the number of photons. If there were a significant change in the quantum number associated with μ, then (A8) is not correct since $\dot\mu$ would change due to interactions, as well as expansion. In that case the relevant equations are an equation describing how μ changes from interactions and the energy-conservation equation

$$\frac{d}{dR}(\rho R^3) = -3pR^2 , \tag{A9}$$

where ρ and p will receive contributions from the degenerate fermions and the photons. In the limit that $\dot{\mu}$ from interactions is less than $\dot{\mu}$ from expansion,

$$\frac{\dot{R}}{R} = \frac{-\dot{\mu}}{\mu} = \left(\frac{8\pi\rho}{3m_P{}^2}\right)^{1/2} = \left(\frac{\xi}{3\pi}\right)^{1/2} \frac{\mu^2}{m_P}. \tag{A10}$$

With the initial condition $\mu(t=0)=\infty$, Eq. (A10) may be integrated to yield

$$t = \frac{m_P}{2\mu^2}\left(\frac{3\pi}{\xi}\right)^{1/2}. \tag{A11}$$

If Eqs. (A4), (A5), (A6), and (A11) are compared to the analogous results for a hot big bang, the order of magnitude of ρ, n, $\langle E\rangle$, and t may be found by the substitution $\mu \leftrightarrow T$.

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