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# Cosmic strings

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Phase transitions in the early universe can give rise to a network of vacuum strings. The dynamics of strings is studied in flat space-time and in expanding universes. The cosmological evolution of strings is discussed, and in particular the black-hole production by collapsing closed loops. Estimations are made for strings of electroweak ($10^2$ GeV) and grand unification ($10^{15}$ GeV) mass scale. It is shown that the effect of electroweak strings is negligible, while grand unification strings can have important cosmological consequences. In particular, massive black holes and large oscillating loops can provide seeds for galaxy formation.

## I. INTRODUCTION

In gauge theories with spontaneous symmetry breaking, the masses of vector bosons and of fermions are absent in the initial Lagrangian and arise due to the appearance of a nonzero vacuum average of a Higgs scalar field $\phi$. The original symmetry of the Lagrangian can, however, be restored[1] at sufficiently high temperatures, $T > T_0$. This phenomenon, combined with the hot big-bang cosmological model, leads to the prediction of a vacuum domain structure of the universe.[2,3]

As the universe cools below the critical temperature $T_c$, the Higgs field $\phi$ acquires a nonzero expectation value $\langle\phi\rangle \neq 0$. The direction of $\langle\phi\rangle$ in the manifold of degenerate vacuum states $M$ can be different in different regions of space. One can introduce a correlation length $\xi(t)$ such that the values of $\langle\phi\rangle$ at points separated by much more than $\xi$ are uncorrelated. It is clear that $\xi$ cannot be greater than the horizon[4]: $\xi \lesssim t$. The initial scale of the vacuum domain structure is given approximately by $\xi_0 = \xi(t_0)$, where $t_0$ is the time at which the phase transition occurs.

As the universe expands, most of the initial chaotic variation of $\langle\phi\rangle$ dies away, since, for energetic reasons, a constant or slowly varying $\langle\phi\rangle$ is preferred. However, a residual vacuum structure can remain. It can take the form of vacuum domain walls, strings, or monopoles, depending[3] on the topology of the manifold $M$. Cosmological monpole production has been discussed by a number of authors[5] who have found that the estimated number of monopoles is too large to be compatible with observations. Possible ways out of the difficulty have also been discussed.[5] Another group of authors[2,3,6] considered the cosmological effects of the domain walls. It has been shown[6] that the walls are gravitationally unstable and collapse at certain time $\sim t_w$ after their creation. This implies that models which lead to domain walls have to be ruled out, unless one can devise a mechanism by which the walls disappear at $t < t_w$. Kibble[3] has also discussed the evolution of cosmic strings. With certain assumptions concerning the interaction of strings with each other and with surrounding matter, he found that there can be only a few strings in the visible universe now. He notes, however, that the strings may have had an important effect at earlier times. Unusual gravitational properties of the strings have been discussed in Ref. 6.

The purpose of the present paper is to study in more detail the evolution and the cosmological consequences of cosmic strings. In the following two sections we shall study the dynamics of strings in flat space-time and in expanding universes. Sections IV and V discuss the cosmological evolution of the strings and, in particular, the black-hole formation by collapsing closed loops. The conclusions are summarized in Sec. VI.

## II. DYNAMICS OF STRINGS

Since we are interested in macroscopic effects of the strings, it is reasonable to approximate them by infinitely thin curves. (The transverse dimensions of the strings are comparable to the Higgs Compton wavelength.) The linear mass density of the strings is of the order[3]

$$\mu \sim \alpha^{-1} m^2 , \tag{2.1}$$

where $\alpha$ and $m$ are a typical coupling constant and boson mass, respectively, and it is assumed that all relevant masses and coupling constants have the same order of magnitude ($m$ and $\alpha$ can be different on different levels of symmetry breaking). For the numerical estimations below we shall take $\alpha \sim 10^{-2}$. Then on the electroweak scale ($m \sim 10^2$ GeV) Eq. (2.1) gives $\mu \sim 10^{-4}$ g/cm. For grand unification strings ($m \sim 10^{15}$ GeV) we get $\mu \sim 10^{22}$ g/cm.

We shall assume that the strings can be either infinite or closed; finite strings connecting two monpoles will not be discussed. The space-time trajectory of a string can be parametrized as

$$x^{\mu}=x^{\mu}(\sigma,\tau)\,, \tag{2.2}$$

where $\sigma$ is a spacelike and $\tau$ is a timelike parameter. Then the action functional of a string is given by[7,8]

$$S=-\mu\int d\tau\int d\sigma\left[\left(\frac{\partial x}{\partial\sigma}\cdot\frac{\partial x}{\partial\tau}\right)^2-\left(\frac{\partial x}{\partial\sigma}\right)^2\left(\frac{\partial x}{\partial\tau}\right)^2\right]^{1/2}, \tag{2.3}$$

where $a\cdot b=a^0b^0-\vec{a}\cdot\vec{b}$. This action is invariant with respect to a reparametrization

$$\sigma\to\tilde{\sigma}(\sigma,\tau),\quad \tau\to\tilde{\tau}(\sigma,\tau)\,. \tag{2.4}$$

If we take $\sigma$ to be the length along the string $l$ and $\tau$ to be the time $t$ then Eq. (2.3) takes the form[7]

$$S=-\mu\int dt\int dl(1-v_{\perp}^2)^{1/2}\,, \tag{2.5}$$

where

$$\vec{v}_{\perp}=\frac{\partial\vec{x}}{\partial t}-\frac{\partial\vec{x}}{\partial l}\left(\frac{\partial\vec{x}}{\partial t}\cdot\frac{\partial\vec{x}}{\partial l}\right) \tag{2.6}$$

is the transverse velocity. Equation (2.5) is easily understood if we note that only the transverse motion of the string is physically observable (see, e.g., Ref. 6).

The equations of motion corresponding to the action (2.3) are

$$\frac{\partial}{\partial\tau}\frac{\partial L}{\partial\dot{x}^{\mu}}+\frac{\partial}{\partial\sigma}\frac{\partial L}{\partial x'^{\mu}}=0\,, \tag{2.7}$$

where $L$ is the Lagrangian,

$$S=\int d\tau\int d\sigma L\,, \tag{2.8}$$

and dots and primes stand for derivatives with respect to $\tau$ and $\sigma$, respectively.

An important special case of the string dynamics is the propagation of waves on a straight string. Let us assume for simplicity that the string (including the wave) lies in the $(x,y)$ plane $(z=0)$. We want to find an equation for $y=y(x,t)$. Suppose that $x(\sigma,t)$ is a one-to-one function of $\sigma$ (we take $\tau=t$). Then we can choose $x(\sigma,t)$ to be a new parameter $\tilde{\sigma}$ [see Eq. (2.4)]:

$$\tilde{\sigma}(\sigma,t)=x(\sigma,t),\quad \tilde{\tau}=t\,. \tag{2.9}$$

The action then becomes

$$S=-\mu\int dt\int dx(1+y'^2-\dot{y}^2)^{1/2} \tag{2.10}$$

and the equation of motion is

$$\frac{\partial}{\partial t}\left[\dot{y}(1+y'^2-\dot{y}^2)^{-1/2}\right]=\frac{\partial}{\partial x}\left[y'(1+y'^2-\dot{y}^2)^{-1/2}\right], \tag{2.11}$$

where the prime now stands for $x$ derivative.

The conservation laws corresponding to the action (2.10) can be easily derived. For example, the energy is given by

$$E=\mu\int dx(1+y'^2)(1+y'^2-\dot{y}^2)^{-1/2}\,, \tag{2.12}$$

This can be also written in the form

$$E=\mu\int dl(1-v_{\perp}^2)^{-1/2}\,, \tag{2.13}$$

where $v_{\perp}=\dot{y}(\partial x/\partial l)$.

It is easily verified that

$$y=f(x-t)$$

and (2.14)

$$y=f(x+t)\,,$$

with an arbitrary function $f(x)$, are solutions of Eq. (2.11). These solutions describe waves propagating with the velocity of light without changing their shapes. In contrast to electromagnetism, the general solution cannot be written as a superposition of solutions (2.14), because Eq. (2.11) is nonlinear. In the case of small perturbations $(y'^2,\dot{y}^2\ll 1)$ Eq. (2.11) reduces to the wave equation

$$\ddot{y}-y''=0 \tag{2.15}$$

and the general solution is $y=f(x-t)+g(x+t)$. (Here I assume that the unperturbed string is parallel to the $x$ axis.)

For a circular loop

$$y=\pm[R^2(t)-x^2]^{1/2}\,, \tag{2.16}$$

the action is given by

$$\begin{aligned} S&=-2\mu\int dt\,R(1-\dot{R}^2)^{1/2}\int_{-R}^{R}dx(R^2-x^2)^{-1/2}\\ &=-2\pi\mu\int dt\,R(1-\dot{R}^2)^{1/2} \end{aligned} \tag{2.17}$$

and the equation of motion is[9]

$$\ddot{R}(1-\dot{R}^2)^{-1}+R^{-1}=0\,. \tag{2.18}$$

The energy corresponding to the action (2.17) is

$$E=2\pi\mu R(1-\dot{R}^2)^{-1/2}\,. \tag{2.19}$$

If at $t=0$ the radius of the loop is $R_0$ and $\dot{R}(0)=0$, then $R(t)=R_0\cos(t/R_0)$. The loop collapses to a point at $t=\pi R_0/2$.

## III. STRINGS IN EXPANDING UNIVERSE

In this section we shall discuss how the dynamics of strings is affected by the expansion of the universe. For simplicity, we shall consider a flat-space cosmological model

$$ds^2 = a^2(\eta)(d\eta^2 - dx^2 - dy^2 - dz^2), \quad (3.1)$$

where $a(\eta)d\eta = dt$. The action integral for a string in a curved space-time is given by Eq. (2.3) with $a \cdot b = g_{\mu\nu} a^\mu b^\nu$. As in the previous section, we shall assume that the string lies in the plane $z = 0$ and that the $x$ coordinate can be chosen as a parameter on the string. Then with $\sigma = x$, $\tau = \eta$ the Lagrangian of the string becomes

$$L = -\mu a^2(\eta)(1 + y'^2 - \dot{y}^2)^{1/2}, \quad (3.2)$$

where $\dot{y} = \partial y/\partial\eta$. The equation of motion takes the form

$$\left(\frac{\partial}{\partial\eta} + 2\frac{\dot{a}}{a}\right)[\dot{y}(1 + y'^2 - \dot{y}^2)^{-1/2}] = \frac{\partial}{\partial x}[y'(1 + y'^2 - \dot{y}^2)^{-1/2}]. \quad (3.3)$$

For small perturbations ($y'^2, \dot{y}^2 \ll 1$) this equation reduces to

$$\ddot{y} + 2\frac{\dot{a}}{a}\dot{y} - y'' = 0. \quad (3.4)$$

In a radiation-dominated universe ($p = \epsilon/3$), $a(\eta) \sim \eta$ and Eq. (3.4) becomes

$$\ddot{y} + 2\eta^{-1}\dot{y} - y'' = 0. \quad (3.5)$$

The solutions of Eq. (3.5) are easily found:

$$y(x, \eta) = \eta^{-1} f(x \pm \eta), \quad (3.6)$$

where $f(x)$ is an arbitrary function. Any solution of (3.5) can be represented as a superposition of standing waves

$$y = \eta^{-1}\sin(k\eta)\cos(kx), \quad (3.7)$$

$$y = \eta^{-1}\cos(k\eta)\cos(kx), \quad (3.8)$$

and similar solutions with $\sin(kx)$.

It is natural to assume that at the time of formation ($\eta = \eta_0$) the strings are at rest with respect to the surrounding matter:

$$\dot{y}(\eta_0) = 0. \quad (3.9)$$

For sufficiently long waves ($k\eta_0 \ll 1$) this initial condition can be replaced by

$$\dot{y}(\eta) \to 0 \quad (\eta \to 0). \quad (3.10)$$

The condition $k\eta_0 \ll 1$ means that at $t = t_0$ the wavelength $\lambda_0 = 2\pi a(\eta_0)k^{-1}$ is much greater than the horizon $t_0 = \frac{1}{2}a(\eta_0)\eta_0$. The requirement (3.10) selects waves of the form (3.7) and prohibits waves of the form (3.8).

To give a physical picture of the evolution of a string with a standing wave of the form (3.7), we shall consider two limiting cases when the wavelength $\lambda(t)$ is much greater and much smaller than the horizon $t$. If $\lambda \gg t$ ($k\eta \ll 1$), then $\dot{y} \approx 0$. This means that the string is conformally stretched by the expansion of the universe. When $\lambda$ becomes much smaller than the horizon ($\lambda \ll t, k\eta \gg 1$), then the wavelength keeps increasing proportionally to the scale factor $a(\eta)$, but the amplitude remains constant [$a(\eta)\eta^{-1} = \text{const}$].

The energy of the perturbation related to one wavelength can be written as [compare with Eq. (2.12)]

$$E = \frac{1}{2}\mu\alpha\int_0^{2\pi k^{-1}} dx(y'^2 + \dot{y}^2). \quad (3.11)$$

The energy grows proportionally to the scale factor when $\lambda \gg t$ and it decreases as $a^{-1}$ when $\lambda \ll t$. It can be shown that this result holds in the general case of a power-law expansion: $a(\eta) \sim \eta^\alpha$.

The Lagrangian of a circular loop in the metric (3.1) is given by

$$L = -2\pi\mu a^2(\eta) r(1 - \dot{r}^2)^{1/2}, \quad (3.12)$$

where $R = a(\eta)r$ is the radius of the loop. The equation of motion is

$$\ddot{r}(1 - \dot{r}^2)^{-1} + 2\frac{\dot{a}}{a}\dot{r} + r^{-1} = 0. \quad (3.13)$$

Analysis similar to that for small perturbations shows that the loop is conformally stretched until its radius becomes smaller than the horizon. Then it collapses with a relativistic speed.

## IV. EVOLUTION OF COSMIC STRINGS

### A. Straight and almost straight strings

We shall start with a rather unrealistic case when the strings form as chaotically oriented straight lines. It has been shown in Ref. 6 that the energy-momentum tensor of a string parallel to the $z$ axis is given by

$$T^\nu_\mu = \mu\,\mathrm{diag}(1, 0, 0, 1). \quad (4.1)$$

Assuming that the distribution of strings is uniform and isotropic and averaging over all directions we get

$$T^\nu_\mu = \epsilon\,\mathrm{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}), \quad (4.2)$$

where $\epsilon$ is the energy density of the strings. Equation (4.2) shows that the strings behave like a medium with $p = -\epsilon/3$. This equation of state is easily understood if we note that the energy of strings in a comoving volume $V$ grows as $V^{1/3}$. Then $dE = (E/3V)dV = -p\,dV$, $E = \epsilon V$, and $p = -\epsilon/3$.

Let $\epsilon_0$ be the initial energy density of strings at the phase transition ($t = t_0$) and $\rho_0$ the total matter density at the same time. We shall assume that the matter equation of state is $p = \epsilon/3$. If $\epsilon_0 \ll \rho_0$, then the universe expands as $a(t) \sim t^{1/2}$ and $\epsilon(t)$

$=\epsilon_0 t_0/t$, $\rho(t)=\rho_0(t_0/t)^2$. At $t_1 \sim t_0\rho_0/\epsilon_0$ the universe becomes string dominated. The following expansion is governed by the equation[10]

$$\frac{1}{2}\left(\frac{da}{dt}\right)^2 = \frac{4\pi}{3} G\epsilon a^2 = \text{const} \tag{4.3}$$

and thus $a(t) \sim t$ $(t > t_1)$.

This scenario remains unchanged if the strings are not exactly straight but have small perturbations. The energy of the perturbations decreases as $a^{-1} \sim t^{-1/2}$ when their wavelength becomes smaller than the horizon. Thus the strings tend to straighten out and are practically straight on scales smaller than $t$. At $t > t_1$ the strings are conformally stretched, since then $\lambda \sim t$.

### B. Brownian strings

Although simple, the case of straight strings is unrealistic. If the correlation length at $t=t_0$ is $\xi_0$, we cannot expect the directions of a string to be correlated over distances much greater than $\xi_0$. We thus expect the strings to form as something like Brownian trajectories with a persistence length of order $\xi_0$. The energy density of strings at formation is

$$\epsilon_0 \sim \mu \xi_0^{-2} . \tag{4.4}$$

As the universe expands, the strings tend to straighten out. However, Brownian strings can never become straight: the rate of growth of the persistence length is limited by causality. The persistence length at time $t$ cannot be greater than $t$, since that would require superluminal velocities. Thus, we can write

$$\epsilon(t) \gtrsim \mu t^{-2} . \tag{4.5}$$

This inequality means simply that there should be at least one string within the horizon.

The evolution of cosmic strings depends on their interaction with the surrounding matter. In the present paper this interaction is neglected, i.e., we assume that the strings move without friction. This assumption is probably justified for sufficiently late time when the density of matter is sufficiently low. We note also that if the strings interact only with massive particles of mass greater than $m$, then friction can be neglected at $T < m$.

Another important factor is the interaction of strings with each other. When two strings cross they may be apt to change partners. Here we assume that this does not happen and the strings go through each other without interaction. Finally, we disregard the possibility that some of the strings can be in the form of closed loops.

The effects of friction and of changing partners are briefly discussed in Sec. IV C. Closed loops are considered in the next section.

The system of Brownian strings at formation can be thought of as a superposition of standing waves with wavelengths $\xi_0 \lesssim \lambda < \infty$ and with spectral density $\nu_0(\lambda)$:

$$\epsilon_0 = \int_{\xi_0}^{\infty} \nu_0(\lambda) d\lambda , \tag{4.6}$$

$$\nu_0(\lambda) \sim \mu\lambda^{-3} . \tag{4.7}$$

The power-law form of the spectrum (4.7) is due to the fact that Brownian curves are self-similar and do not have any intrinsic scale. $\xi_0$ serves only as a lower cutoff of the wavelengths. (If oscillations on scales smaller than $l$ are smoothed out then we get a system of Brownian strings with persistence length $l$ and with $\epsilon \sim \mu l^{-2}$.)

At $t > t_0$ we can write

$$\epsilon(t) = \int \nu(\lambda) d\lambda . \tag{4.8}$$

We shall find $\nu(\lambda)$ assuming that the behavior of the waves is not very different from that of small perturbations on straight strings, i.e., we shall assume that the waves are conformally stretched if $\lambda > t$ and that the energy of the wave decreases as $a(t)^{-1}$ if $\lambda < t$. In both cases the wavelength changes as

$$\lambda = \frac{a}{a_0} \lambda_0 , \tag{4.9}$$

where $a_0 = a(t_0)$, $\lambda_0 = \lambda(t_0)$.

Let us concentrate on waves with initial wavelength $\sim\lambda_0$ in the interval $d\lambda_0$. The wavelength becomes of order $t$ at $t=t_1$:

$$t_1 = \frac{a_1}{a_0} \lambda_0 . \tag{4.10}$$

For $t_0 < t < t_1$ the energy of waves grows as $a$ and the energy density decreases as $a^{-2}$. Thus,

$$\begin{aligned} &\nu(\lambda) d\lambda = (a_0/a)^2 \nu_0(\lambda_0) d\lambda_0 , \\ &\nu(\lambda) = (a_0/a)^3 \nu_0(\lambda_0) \quad (t_0 < t < t_1) . \end{aligned} \tag{4.11}$$

For $t > t_1$ the energy density decreases as $a^{-4}$ and

$$\nu(\lambda) = (a_1/a)^5 \nu_1(\lambda_1) \quad (t > t_1) , \tag{4.12}$$

where the index "1" denotes quantities at time $t_1$. Combining Eqs. (4.9), (4.11), and (4.12) we obtain the spectral density at time $t$:

$$\begin{aligned} &\nu(\lambda) \sim \mu\lambda^{-3} \quad (\lambda > t) , \\ &\nu(\lambda) \sim (a_1/a)^2 \mu\lambda^{-3} \quad (\lambda < t) . \end{aligned} \tag{4.13}$$

If the expansion law is

$$a(t) \sim t^{\alpha} , \tag{4.14}$$

then

$$\frac{a_\perp}{a} = \left(\frac{t}{\lambda}\right)^{\alpha/(\alpha-1)} . \tag{4.15}$$

and Eqs. (4.8) and (4.13) give

$$\epsilon(t) = \int_\xi^t \nu(\lambda)d\lambda + \int_t^\infty \nu(\lambda)d\lambda$$

$$\sim \mu t^{-2} \int_{\xi/t}^1 dx\, x^{-(5\alpha-3)/(\alpha-1)} + \mu t^{-2} . \tag{4.16}$$

Here

$$\xi = \xi_0(t/t_0)^\alpha \tag{4.17}$$

is the minimum wavelength at time $t$.

For $\alpha > \frac{1}{2}$ the power of $x$ in the integrand of (4.16) is greater than $-1$ and

$$\epsilon(t) \sim \mu t^{-2} \quad (\alpha > \tfrac{1}{2}) . \tag{4.18}$$

For $\alpha = \frac{1}{2}$,

$$\epsilon(t) \sim \mu t^{-2} \ln(t/\xi) \quad (\alpha = \tfrac{1}{2}) . \tag{4.19}$$

For $\alpha < \frac{1}{2}$,

$$\epsilon(t) \sim \mu t^{-2}(t/\xi)^{2(1-2\alpha)/(1-\alpha)} \quad (\alpha < \tfrac{1}{2}) . \tag{4.20}$$

The expansion law (4.14) corresponds to the equation of state $p = \gamma\rho$ with $\gamma = 2/3\alpha - 1$. Equation (4.18) then means that for $\gamma < \frac{1}{3}$ the straightening of the strings is complete: the contribution of waves with $\lambda \ll t$ to the energy density is negligible, the persistence length at time $t$ is of order $t$ and there are only a few strings within the horizon. In particular, this is the case in a matter-dominated universe ($p = 0$). The ratio of string and matter energy densities is [see Eq. (2.1)]

$$\epsilon/\rho \sim G\mu \sim \alpha^{-1}Gm^2 \ll 1 . \tag{4.21}$$

(In this equation $\alpha \sim 10^{-2}$ is the coupling constant.)

In a radiation-dominated universe ($\gamma = \frac{1}{3}$, $\alpha = \frac{1}{2}$) the straightening of strings is almost complete. The contribution of short waves to the energy density is greater than that of waves with $\lambda > t$, but only by a logarithmic factor. In this case

$$\epsilon/\rho \sim G\mu \ln(t/t_0) . \tag{4.22}$$

In the standard cosmological model the radiation era ends at $t \sim t_{\rm rec} \sim 5 \times 10^{12}$ sec. The grand unification phase transition occurs[11] at $t_0 \sim 10^{-39}$ sec and the electroweak phase transition at $t_0 \sim 10^{-13}$ sec. Therefore, the logarithmic factor in Eq. (4.22) does not exceed $10^2$ and $\epsilon/\rho \ll 1$. At $t > t_{\rm rec}$ the universe is matter dominated and $\epsilon/\rho$ is given by (4.21). Thus, in the standard model the energy density of strings is always much smaller than that of matter.

If the equation of state, during a certain period, is harder than $p = \rho/3$, then the straightening of strings is incomplete, the dominant contribution to the energy of strings is given by short wavelengths, and the mass density of the strings can become comparable to that of matter.

### C. Effects of friction and of changing partners

We have discussed the evolution of strings neglecting their interaction with each other and with the surrounding matter. The straightening of the strings in this case is due only to red-shifting of short-wavelength perturbations.

Friction due to interaction with particles gives an additional damping mechanism. Depending on its strength, friction can speed up or slow down the straightening process. If the friction is so strong that the strings cannot move with relativistic speeds, then it slows down the straightening. In this case Eq. (4.5) is replaced by $\epsilon(t) \gtrsim \mu(vt)^{-2}$, where $v(t)$ is the characteristic velocity of the strings at time $t$.

If the friction force is small enough, so that the strings can be relativistic, then it can only speed up the straightening. However, comparing Eqs. (4.5), (4.18), and (4.19), we see that, in the standard cosmological model, the maximum the friction can do is to remove the logarithmic factor in Eq. (4.19). We thus see that in this case the results of subsection B remain practically unchanged in the presence of friction. A quantitative analysis of the role of friction will be given elsewhere.

Another straightening mechanism has been discussed by Kibble.[3] As the strings cross, they may be apt to change partners. This leads to the formation of closed loops which then collapse. As a result, the overall length of the strings decreases. As in the case of small friction, this mechanism can only speed up the straightening, but the maximum it can do is to remove the logarithmic factor in Eq. (4.19). The loop formation, however, can be very important for production of black holes. This is discussed in Sec. V.

## V. CLOSED LOOPS AND BLACK-HOLE FORMATION

A certain number of closed loops are formed during the phase transition at $t \sim t_0$. Additional loops are produced as a result of the intersection of the strings if changing partners is allowed. Let us consider a loop with characteristic scale $\lambda$ (the length of the loop $l \sim \lambda^2/\xi$ is greater than $\lambda$; here $\xi$ is the peristence length). If $\lambda > t$, then the loop expands with the universe ($\lambda \sim a$) while small scale irregularities are smoothed out. When $\lambda$ becomes of order $t$, the loop collapses during time $\sim t$. The mass of the loop at this time is of order $\mu t$. The important question is: What does the loop collapse to? There are several possibilities: (i) the energy of the loop is completely dissipated, as a result of friction or of gravitational radiation; (ii) the loop decays into relativistic particles; and (iii) the loop collapses to a black hole.

In the present paper we do not attempt an estimation of the density or scale distribution of the loops formed during the phase transition. We shall only estimate the black-hole production due to loops formed by intersecting strings and discuss its cosmological consequences.

Let us first consider the simplest case of a circular loop assuming that friction can be neglected. It seems reasonable to assume that decay into particles becomes possible when the radius of the loop becomes comparable to the width of the string, so that different parts of the loop can interact with each other. This has to happen before the loop falls inside its Schwarzschild sphere, so that

$$GM \sim G\mu t < m^{-1}$$

or (5.1)

$$t < t_2 \sim \alpha G^{-1} m^{-3} .$$

Here I used the fact that the width of the strings is comparable to the Higgs wavelength $m^{-1}$. For grand unification strings $t_2 \sim 10^{-33}$ sec and for electroweak strings $t_2 \sim 10^6$ sec. Circular loops collapsing at $t > t_2$ collapse to black holes.

In the general case of an irregular loop with a complicated initial velocity distribution, the loop will violently oscillate losing its energy to friction and gravitational radiation. Important questions here are as follows. (i) What is the lifetime of an oscillating loop? (ii) What is the probability of black-hole formation? (iii) How much energy does the loop loose before a black hole is formed? These problems will not be tackled in the present paper. I hope to return to them in later publications.

In this section we shall estimate the black-hole production assuming that friction can be neglected and that the behavior of irregular loops is not much different from that of circular loops. (The latter assumption means that the probability of black-hole formation is of order 1, the collapse time is of the order of initial loop size, and the mass of the resulting black hole is not much different from the initial energy of the loop.) Obviously, such an estimation will give an upper limit for the black-hole production.

At time $t$ the typical curvature radius of the strings is $\sim t$, the minimum distance from a point on one string to another string is also $t$, the motion of the strings on scales smaller than $t$ is relativistic, and it is clear from dimensionality that the number density of closed loops formed during the interval $\Delta t \sim t$ cannot be much different from $t^{-3}$, so that

$$\frac{dn}{dt} \sim t^{-4} . \tag{5.2}$$

The right-hand side of Eq. (5.2) can be multiplied by a small numerical factor, but it can do no more than take off a few orders of magnitude. If $t > t_2$ [see Eq. (5.1)], then the loops collapse to black holes of mass

$$M \sim \mu t . \tag{5.3}$$

(Here it is essential that we neglect friction: if friction is strong, the loop can loose most of its energy as it collapses.)

Black holes of mass $M$ evaporate[12] at

$$t \sim t_{\rm BH} \sim \beta \nu^{-1} G^2 M^3 , \tag{5.4}$$

where $\nu$ is the number of particle species contributing to the black-hole radiation (for estimations below we take $\nu \sim 10^2$) and[13] $\beta \sim 2 \times 10^3$ is a numerical coefficient. In Eq. (5.4) I assume that $t_{\rm BH}$ is much greater than the time of formation of black holes ($t \sim M/\mu$). It is easily checked that this condition is satisfied for black holes formed at $t > t_2$.

The number density of black holes with masses from $M$ to $M + dM$ at time $t$ can be written as

$$dn(t) = \sigma_t(M) dM = \left(\frac{a'}{a}\right)^3 \frac{dt'}{t'^4} , \tag{5.5}$$

where $t' = M/\mu$ is the time of formation of the black holes, $a \equiv a(t)$ and $a' = a(t')$. The total mass density of black holes equals

$$\rho_{\rm BH}(t) = \int \sigma_t(M) M \, dM = \mu \int_{\tilde{t}}^{t} \left(\frac{a'}{a}\right)^3 \frac{dt'}{t'^3} , \tag{5.6}$$

where

$$\tilde{t} = \max\{t_*, t_2\} \tag{5.7}$$

and

$$t_*(t) \sim \beta^{-1/3} \nu^{1/3} G^{-2/3} \mu^{-1} t^{1/3} \tag{5.8}$$

is the time of formation of black holes evaporating at time $t$.

In a radiation-dominated universe $a \sim t^{1/2}$ and

$$\rho_{\rm BH}(t) \sim \mu t^{-3/2} \tilde{t}^{-1/2} , \tag{5.9}$$

$$\rho_{\rm BH}/\rho \sim G\mu (t/\tilde{t})^{1/2} . \tag{5.10}$$

Now, from Eqs. (5.1) and (5.8) we get

$$t_*/t_2 \sim \nu^{1/3} \beta^{-1/3} G^{1/3} m t^{1/3} . \tag{5.11}$$

For grand unification strings $t_*$ becomes greater than $t_2$ at $t \sim 10^{-30}$ sec and the mass density of black holes becomes comparable to that of radiation at $t \sim t_3 \sim 10^{-16}$ sec. The maximum contribution to $\rho_{BH}$ is given by the black holes of smallest mass, $M_{min} \sim \mu t_*$. These smallest black holes evaporate and contribute to the radiation density $\rho_\gamma$, and we expect $\rho_{BH}$ and $\rho_\gamma$ to remain roughly of the same order. If we assume that at $t > t_3$ the universe becomes black hole dominated so that the radiation pressure can be neglected and $a(t > t_3) = a(t_3)(t/t_3)^{2/3}$, then Eq. (5.6) gives

$$\rho_{BH}(t > t_3) \sim \mu\left(\frac{t_3}{t}\right)^2 \int_{t_*}^{t_3}\left(\frac{t'}{t_2}\right)^{3/2}\frac{dt'}{t'^3} + \mu\int_{t_3}^{t}\left(\frac{t'}{t}\right)^2\frac{dt'}{t'^3}$$

$$\sim \frac{1}{Gt^2}\left(\frac{t_3}{t}\right)^{1/6} + \frac{\mu}{t^2}\ln\frac{t}{t_3} < \frac{1}{Gt^2}\,. \qquad (5.12)$$

This contradicts the assumption that $\rho_{BH} > \rho_\gamma$ at $t > t_3$. Thus, at $t > t_3$ the universe is dominated by mini-black holes and their high-energy radiation.

We shall not attempt to calculate the expansion law at $t > t_3$ and to discuss the further evolution in any detail. At this point it can already be shown that the model just described contradicts observations. The radiation of evaporating black holes can be thermalized only if[14] it is emitted before the thermalization time, $t_{therm} \sim 1$ sec. It is therefore clear that in our model the radiation spectrum can never become Planckian. We conclude that, with the assumptions made, the existence of grand unification strings is incompatible with standard cosmology. (We stress again that the actual density of black holes may be much smaller than estimated here if the probability of black-hole formation by irregular loops is very small.)

A similar analysis shows that the density of black holes produced by electroweak strings is always negligible. At the end of the radiation era ($t \sim 5 \times 10^{12}$ sec)$\rho_{BH}/\rho \lesssim 10^{-26}$ and at present ($t \sim 5 \times 10^{17}$ sec)$\rho_{BH}/\rho \lesssim 10^{-27}$.

## VI. CONCLUSIONS AND DISCUSSION

We have discussed the evolution of cosmic strings in the hot big-bang cosmological model. In most of the paper we have assumed that friction due to interaction of strings with particles can be neglected.

Shortly after the phase transition the strings are expected to have the form of Brownian trajectories with the persistence length of order $\xi_0$. ($\xi_0$ is the correlation length at $t \sim t_0$.) As the universe expands, the strings straighten out and the persistence length at time $t > t_0$ is of order $t$. The straightening is a result of red-shifting of short-wavelength perturbations (Sec. IV). Friction and loop formation can also be important,[3] but they cannot substantially increase the rate of straightening: persistence length greater than $t$ is forbidden by causality. (If friction is very strong, it can slow down the straightening. See Sec. IV C.)

As the strings cross, they may be apt to change partners. This leads to formation of closed loops which then collapse. Additional loops are produced during the phase transition at $t \sim t_0$. Sufficiently large circular loops collapse to black holes. Loops of irregular shape undergo violent oscillations losing their energy to friction and gravitational radiation.

To estimate the potential importance of black-hole formation, we have found an upper limit for the black-hole density assuming that changing partners is possible and that the behavior of irregular loops is not much different from that of circular loops (so that the probability of black-hole formation is of order 1). The estimations have been made for strings of electroweak ($m \sim 10^2$ GeV) and grand unification ($m \sim 10^{15}$ GeV) mass scale. For electroweak strings, the black-hole density is negligible; it never exceeds $10^{-26}$ of the total mass density of the universe. For grand unification strings, the (maximum) number of black holes produced is unacceptably large. In particular, the spectrum of cosmic radiation becomes essentially nonequilibrium as a result of high-energy radiation of mini-black holes. We stress that the actual density of black holes may be much smaller if the probability of black-hole formation by irregular loops is very small.

In our estimations we assume that friction can be neglected. If friction is strong, then collapsing loops can lose most of their energy and the black-hole production can be substantially reduced. A detailed analysis of the role of friction will be given elsewhere. Since the force of friction decreases with temperature, one can expect that it becomes small at sufficiently late times, so that the collapse of large loops is not affected. Massive black holes and large oscillating loops can serve as seeds for galaxy formation.

In some models the phase transition can be of the first order (see, e.g., Linde,[1] and Guth and Tye,[5]). In this case, a considerable supercooling is possible and the strings can form at a temperature much lower than that suggested by the mass scale of the theory. Then the horizon and the correlation length at the phase transition increase correspondingly and the black-hole mass spectrum starts at a higher value of $M$.

We note also that the string formation is possible only if the manifold $M$ of the degenerate vacuums of the theory is not simply connected. A discussion of the conditions for the existence of vacuum strings can be found in Refs. 3, 15, and 16. If subsequent research shows that grand unification strings are incompatible with the big-bang cosmology, then we shall have another criterion for selecting the models. On the other hand, if overproduction of black holes can be avoided and no other contradictions arise, then grand unified models with strings may be preferred since they can give a natural explanation for the galaxy formation.

### *Note added in proof*

After this paper was submitted, I noticed a recent paper by Zel'dovich[17] in which a possible role of grand unification strings in the galaxy formation is discussed.

Dr. F. Klinkhammer has pointed out to me that my estimation of the radius of the loop at which particle creation becomes important may be incorrect. Particle creation by moving strings requires a special analysis and a conclusive answer to the question cannot be given at this point. A. E. Everett [Phys. Rev. D 24, 858 (1981)] has given a plausible argument suggesting that particle creation becomes important when the proper acceleration of the loop becomes comparable to the symmetry-breaking mass $m$. In this case Eq. (5.1) should be replaced by $t_2 \sim (G\mu)^{-2} m^{-1}$, $t_2 > t_*$ for $t < 10^{-12}$ sec, and the universe becomes black-hole dominated at $t \sim 10^{-15}$ sec. All the conclusions of the paper remain unchanged.

An analysis of the density perturbations produced by the strings has been published. [A. Vilenkin, Phys. Rev. Lett. 46, 1169 (1981). Because of typographic errors, this paper is incomprehensible without the erratum, Phys. Rev. Lett. 46, 1496(E) (1981).] It is shown that the probability of black-hole formation by closed loops is very small and that the density fluctuations produced by the strings of grand unification mass scale are large enough to explain the galaxy formation.

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[9]This equation can also be obtained by substituting Eq. (2.16) in Eq. (2.11).

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