ρ, ω, and ϕ mixing mechanisms

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The ω-ϕ, ϕ-ρ, and ω-ρ mixing phenomena are reconsidered incorporating new experimental data. Reasonable agreement is obtained in terms of the conventional unitarity, one-photon, and tadpole contributions. No sizable gluonic contributions are required and their presence would tend to destroy the previously achieved agreement.

The dynamical origin of the violations of the Okubo-Zweig-Iizuka (OZI) rule has been much discussed and constitutes an interesting subject in meson physics.[1-3] Two main sources of violation are usually considered: (i) unitarity effects involving physical intermediate states of hadrons[1,4,5] and (ii) quark-antiquark annihilations into multigluon intermediate states.[1,2,6] However, the relative importance of these two sources and, in particular, a clear proof concerning the necessity of gluonic effects are still open questions. With minor modifications these two types of mechanisms are also expected to be relevant when dealing with the closely related subject of G-parity-violating transitions. Well-known examples are the $\omega \to \pi^+\pi^-$ and $\phi \to \pi^+\pi^-$ transitions and the related ω-ρ and ϕ-ρ mixing phenomena whose analyses in terms of these mechanisms (including now photonic states) and specific tadpole contributions have been performed by several authors.[7-11] The recent measurements of the $\phi \to \pi^+\pi^-$ decay amplitude[12] and the ω-ϕ interference in the reaction $e^+e^- \to \pi^+\pi^-\pi^0$ (Ref. 13) have considerably enriched our experimental knowledge of the G-parity-violating and/or OZI-rule-forbidden transitions ω-ρ, ω-ϕ, and ϕ-ρ. A combined study of these phenomena and an attempt to investigate the role played by the gluonic mechanisms constitute the main purpose of the present paper.

The formalism describing the mixing among three states is a trivial generalization of the well-known ω-ρ mass-mixing formalism.[7,8] We introduce the ideal basis formed by the three unmixed states

$$\rho_0 \equiv (u\bar{u} - d\bar{d})/\sqrt{2}\,,\quad \omega_0 \equiv (u\bar{u} + d\bar{d})/\sqrt{2}\,,\quad \phi_0 \equiv + s\bar{s}\,. \tag{1}$$

In terms of these, the physical, mixed states are given by the expressions

$$\rho \equiv \rho_0 - \epsilon_{\omega\rho}\,\omega_0 - \epsilon_{\phi\rho}\,\phi_0\,,\quad \omega \equiv \epsilon_{\omega\rho}\,\rho_0 + \omega_0 - \epsilon_{\omega\phi}\,\phi_0\,,\quad \phi \equiv \epsilon_{\phi\rho}\,\rho_0 + \epsilon_{\omega\phi}\,\omega_0 + \phi_0\,, \tag{2}$$

where only first-order terms in the (complex) mixing parameters $\epsilon_{vv'}$ have been retained. These are related to the off-diagonal mass-matrix elements $m^2_{vv'}$ through the expression

$$\epsilon_{vv'} = m^2_{vv'}/(m_v^2 - m_{v'}^2 + im_v\Gamma_v - im_{v'}\Gamma_{v'})\,, \tag{3}$$

where $v \neq v'$ stand for the different unmixed states (1). According to this, the coupling of the physical particles (2) to a given state receives two contributions: that coming from the corresponding unmixed state (direct coupling) and a second one involving the mixing parameters. As usual, direct couplings violating the OZI rule will be neglected, i.e., $g_{\phi_0\pi\pi} = g_{\phi_0\rho\pi} = 0$, and only first-order terms in the small, G-parity-violating parameters $\epsilon_{\omega\rho}$, $\epsilon_{\phi\rho}$, $g_{\omega_0\pi\pi}/g_{\rho\pi\pi}, \ldots$ will be considered.

In terms of these parameters the available experimental information can be briefly summarized as follows:

(i) ω-ϕ *mixing*. A detailed analysis of this case has already been performed in Ref. 2. It makes use of the experimental data concerning the 3π, $\pi\gamma$, and e^+e^- decay modes of the ω and ϕ mesons and their relative production through the reactions $\pi^-p \to \omega n$ and $\pi^-p \to \phi n$. All this information is found[2] to be compatible with $\epsilon_{\omega\phi} \simeq +0.05$. The phase of this complex parameter $\epsilon_{\omega\phi}$ has been fully confirmed by the recent data on the reaction $e^+e^- \to \pi^+\pi^-\pi^0$ in the whole ω-ϕ region and a more accurate value for $|\epsilon_{\omega\phi}|$ can be deduced from the ratio[14] $\Gamma(\phi \to 3\pi)/\Gamma(\omega \to 3\pi) = 0.066 \pm 0.005$ and the ρ-pole dominance of these decays.[5] One has $\epsilon_{\omega\phi} \simeq g_{\phi\rho\pi}/g_{\omega\rho\pi}$ and

$$|\mathrm{Im}\,\epsilon_{\omega\phi}| \ll \mathrm{Re}\,\epsilon_{\omega\phi} = +0.046 \pm 0.005\,, \tag{4a}$$

$$|\mathrm{Im}\,m^2_{\omega\phi}| \ll \mathrm{Re}\,m^2_{\omega\phi} = (20 \pm 2) \times 10^{-3}\ \mathrm{GeV}^2\,, \tag{4b}$$

where the standard values[14] for the ω and ϕ masses and widths have been introduced.

(ii) ϕ-ρ *mixing*. The relevant experimental information comes from the recent measurement[12] of the $e^+e^- \to \phi \to \pi^+\pi^-$ amplitude, which implies

$$\mathrm{Re}\,\frac{f_\rho g_{\phi\pi\pi}}{f_\phi g_{\rho\pi\pi}} = (0.36 \pm 0.12) \times 10^{-3}\,,$$

$$\mathrm{Im}\frac{f_\rho g_{\phi\pi\pi}}{f_\phi g_{\rho\pi\pi}}=(0.59\pm0.20)\times10^{-3}.$$

On the other hand, from Eqs. (2) and (4a) and the smallness of the ratio $g_{\omega_0\pi\pi}/g_{\rho\pi\pi}$ one gets

$$g_{\phi\pi\pi}/g_{\rho\pi\pi}=\epsilon_{\phi\rho}+\epsilon_{\omega\phi}g_{\omega_0\pi\pi}/g_{\rho\pi\pi}\simeq\epsilon_{\phi\rho}.$$

Comparing these last equations and introducing the experimental ratio[14] $f_\phi/f_\rho=-2.65\pm0.20$ [whose sign has been deduced from the corresponding SU(3) result $f_{\phi_0}/f_{\rho_0}=-3/\sqrt{2}$], one obtains

$$\mathrm{Re}m^2_{\phi\rho}=-(0.58\pm0.18)\times10^{-3}\ \mathrm{GeV}^2,\qquad(5)$$
$$\mathrm{Im}m^2_{\phi\rho}=(0.53\pm0.18)\times10^{-3}\ \mathrm{GeV}^2,$$

where use has been made of the inequality $\Gamma_\phi\ll\Gamma_\rho$.

(iii) *ω-ρ mixing*. The experimental data concerning this case are more abundant and come mainly from photoproduction of $\pi^+\pi^-$ pairs and $e^+e^-\to\pi^+\pi^-$ annihilation around the ω peak.[11,14] The phase of the ratio between physical couplings $g_{\omega\pi\pi}/g_{\rho\pi\pi}$ is accurately fixed by the e^+e^- annihilation data and its modulus is most conveniently deduced from the precise value[14] $\Gamma(\omega\to\pi^+\pi^-)/\Gamma(\rho\to\pi\pi)=(8.9\pm1.3)\times10^{-4}$. One obtains

$$\left|\mathrm{Re}\frac{g_{\omega\pi\pi}}{g_{\rho\pi\pi}}\right|\ll\mathrm{Im}\frac{g_{\omega\pi\pi}}{g_{\rho\pi\pi}}=0.030\pm0.002.$$

The mixing formalism now implies

$$g_{\omega\pi\pi}/g_{\rho\pi\pi}=\epsilon_{\omega\rho}+g_{\omega_0\pi\pi}/g_{\rho\pi\pi}$$

and contains the unknown parameter $g_{\omega_0\pi\pi}/g_{\rho\pi\pi}$ which in this case cannot be neglected. In spite of this, some valuable information can be deduced[7] from the last two equations in the limit of $m_\rho\simeq m_\omega$ and $\Gamma_\omega\ll\Gamma_\rho\simeq\Gamma_{\rho\to\pi\pi}$, namely,

$$|\mathrm{Im}m^{2\prime}_{\omega\rho}|\ll\mathrm{Re}m^2_{\omega\rho}=-(3.5\pm0.2)\times10^{-3}\ \mathrm{GeV}^2,\qquad(6)$$

where $m^{2\prime}_{\omega\rho}$ contains all the contributions to the ω_0-ρ_0 transition except that originated by the direct coupling $\omega_0\to\pi^+\pi^-\to\rho_0$.

Having summarized the relevant experimental information in Eqs. (4b), (5), and (6), the remainder of this paper will be devoted to discussing the different theoretical contributions to the real and imaginary parts of $m^2_{vv'}$. The latter are expected to be given by the contributions of the different open channels to the unitarity sum and can be reliably evaluated[7,9-11] from well-established data.[14] One easily finds

$$\mathrm{Im}m^2_{\omega\phi}=(2.5\pm0.3)\times10^{-3}\ \mathrm{GeV}^2,$$
$$\mathrm{Im}m^2_{\phi\rho}=(0.37\pm0.15)\times10^{-3}\ \mathrm{GeV}^2,\qquad(7)$$
$$\mathrm{Im}\,m^{2\prime}_{\omega\rho}\simeq-0.24\times10^{-3}\ \mathrm{GeV}^2,$$

where the dominant K^+K^-, $K^0\bar{K}^0$, and $\eta\gamma$ contributions with their appropriate sign have been included in the $\omega^0\phi^0$ and $\phi^0\rho^0$ transitions, while only the $\pi^0\gamma$ state contributes sensibly to $\mathrm{Im}m^{2\prime}_{\omega\rho}$. As expected, the experimental information on $\mathrm{Im}m^2_{vv'}$ quoted in Eqs. (4b), (5), and (6) is compatible with the theoretical estimates (7) or, more generally, with unitarity.

The situation is much more involved when dealing with the real parts of $m^2_{vv'}$. However, also in this case two types of contributions to $\mathrm{Re}m^2_{\phi\rho}$ and $\mathrm{Re}m^2_{\omega\rho}$ can be safely evaluated. The first corresponds to the one-photon transitions[15] $\phi_0\to\gamma\to\rho_0$ and $\omega_0\to\gamma\to\rho_0$ which, using vector-meson-dominance ideas and well-known data,[14] lead to

$$\mathrm{Re}m^2_{\phi\rho}(\gamma)=e^2m_\rho^2/f_\phi f_\rho=-(0.82\pm0.07)\times10^{-3}\ \mathrm{GeV}^2,\qquad(8a)$$

$$\mathrm{Re}\,m^2_{\omega\rho}(\gamma)=e^2m^2_\rho/f_\omega f_\rho=(0.73\pm0.08)\times10^{-3}\ \mathrm{GeV}^2.\qquad(8b)$$

The second type of contribution is due to unitarity and includes the effects of both open and closed intermediate channels. In spite of this, it has been shown in Refs. 10 and 11 that these unitarity contributions to $\mathrm{Re}m^2_{\phi\rho}$ and $\mathrm{Re}m^2_{\omega\rho}$ can safely be neglected when compared with the experimental values quoted in Eqs. (5) and (6). Unfortunately, the same type of unitarity contribution to $\mathrm{Re}m^2_{\omega\phi}$ is not negligible and requires the introduction of unknown form factors in its evaluation. A detailed analysis including the dominant channels $K\bar{K}$ and $\bar{K}K^*+K\bar{K}^*$ has already been performed[4] and leads to

$$\mathrm{Re}m^2_{\omega\phi}\simeq19\times10^{-3}\ \mathrm{GeV}^2.\qquad(9)$$

Notice, however, that in the context of dual models the effects of the different channels [involving F and D SU(3) couplings] tend to cancel and a much smaller value is obtained for $\mathrm{Re}m^2_{\omega\phi}$.[2] A final contribution is conventionally[7-9] attributed to the tadpole mechanism which recently has been reconsidered by Langacker[11] and reinterpreted in terms of current-quark mass differences. It contributes only to $\mathrm{Re}m^2_{\omega\rho}$ and gives

$$\mathrm{Re}\,m^2_{\omega\rho}(\mathrm{tad})\simeq2r(m_\rho^2-m_{K^*}^2)=-4.2\times10^{-3}\ \mathrm{GeV}^2,\qquad(10)$$

where one has introduced $r_-\equiv(m_d-m_u)/2m_s=0.011$, as follows from a combined analysis[11] of the tadpole effects in kaon and baryon mass differences, $\eta\to3\pi$, decays, and ω-ρ mixing.

The conclusion of the preceding paragraph is that a reasonable description of the experimental values of $\mathrm{Re}\,m^2_{vv'}$ has been achieved as can be seen by comparing Eqs. (4b) and (9), Eqs. (5) and (8a) and, finally, Eq. (6) with the sum of values quoted in Eqs. (7b) and (10). Unfortunately, it is a rather complicated description which requires the cooperation of well-known effects, such as (i) the one-photon and (ii) the negligible unitarity contribution to $\mathrm{Re}\,m^2_{\phi\rho}$ and $\mathrm{Re}\,m^2_{\omega\phi}$ with other less reliable effects, such as (iii) a non-negligible unitarity contribution to $\mathrm{Re}\,m^2_{\omega\phi}$ and (iv) a large tadpole term contributing only to $\mathrm{Re}\,m^2_{\omega\rho}$. Therefore, it seems natural to ask whether a new mechanism could be used in cooperation with the just mentioned and well-known effects (i) and (ii) to reobtain a more simple and satisfactory description.

Gluonic corrections inspired by quantum chromodynamics are usually accepted as a candidate for such a mechanism whose main characteristic is that it contributes not only to $\mathrm{Re}\,m^2_{\omega\phi}$ and $\mathrm{Re}\,m^2_{\omega\rho}$ but also to $\mathrm{Re}\,m^2_{\phi\rho}$. The contributions of intermediate states containing n gluons and one photon are expected to satisfy ($n \geq 2$).

$$\frac{\mathrm{Re}\,m^2_{\phi\rho}(ng+\gamma)}{\mathrm{Re}\,m^2_{\omega\rho}(ng+\gamma)} = \frac{\sqrt{2}\,Q_s}{Q_u+Q_d}\,\frac{\alpha_s^{\,n}(m_\phi^{\,2})}{\alpha_s^{\,n}(m_\omega^{\,2})}$$

$$= -\sqrt{2}\,\frac{\alpha_s^{\,n}(m_\phi^{\,2})}{\alpha_s^{\,n}(m_\omega^{\,2})}, \qquad (11)$$

where $Q_{u,d,s}$ are the charges of the quarks and $\alpha_s(m^2)$ is the running coupling constant of QCD. Similarly, one expects

$$\mathrm{Re}\,m^2_{\phi\rho}(ng+\gamma) = -k\,\mathrm{Re}\,m^2_{\omega\phi}(ng+g)\,, \qquad (12)$$

where k is a positive parameter of order $\alpha/\alpha_s(m_\phi^{\,2}) \sim 10^{-2}$. Therefore, if the tadpole mechanism (iv) is substituted by large and negative contributions of the type $\mathrm{Re}\,m^2_{\omega\rho}(ng+\gamma)$, Eq. (11) implies that a large and positive contribution should be expected for $\mathrm{Re}\,m^2_{\phi\rho}$, which would destroy the previously achieved agreement. Similarly, one can try to explain the large and positive value of $\mathrm{Re}\,m^2_{\omega\phi}$ as a gluonic effect but then Eq. (12) predicts a small (but significant) negative contribution to $\mathrm{Re}\,m^2_{\phi\rho}$ which, again, tends to increase the so-far moderate discrepancy between Eqs. (5) and (8a).

In conclusion, the available information on the imaginary parts of the nondiagonal mass-matrix elements $m^2_{vv'}$ among the neutral vector mesons ρ_0, ω_0, and ϕ_0 can be reasonably interpreted in terms of simple unitarity corrections. The corresponding real parts can be described only in a rather complicated context including unitarity corrections, one-photon contributions, and the conventional tadpole mechanism. In this case, the presence of sizable gluonic contributions is excluded by the experimental data. Moreover, if one tried to substitute the unreliable effects of the previous context by the effects of gluonic mechanisms the so-far acceptable description of the data (particularly, those concerning the $\phi \to \pi^+\pi^-$ transition) tends to be lost.

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