Bounds on $e^+e^- \to l^*\bar{l}$ and $lp \to l^*X$ (l^* = excited lepton), and prospects for visible l^* tracks in cosmic-ray emulsion events

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The small limits of discrepancy between experimental and theoretical values of $(g-2)_{e,\mu}$ are shown to eliminate any chance of seeing *single* excited e^*'s or μ^*'s in colliding-beam or inelastic electro- or muoproduction experiments. On the other hand, visible l^* tracks in very-high-energy ($E > 10^{19}$ eV) cosmic-ray events are possible for $m_l^* \sim 1$ TeV. The results presuppose a new dynamics, disjoint from QED, underlying the l^* excitations.

The close agreement between the experimental values of the anomalous magnetic moments (a_e, a_μ) of the electron[1] and muon,[2] and the best theoretical estimates[3] using "known" dynamics places severe constraints[4,5] on the mass scales associated with any subconstituents of the electron and muon.[6] As shown in Refs. 4 and 5, a variety of considerations lead to the result

$$|\delta a_l^{\text{non-QED}}| \sim O(m_l/M_{\text{const}}), \tag{1}$$

where $\delta a_l^{\text{non-QED}}$ ($l = e, \mu$) is the contribution to $a_l \equiv \frac{1}{2}(g_l - 2)$ from a dynamical structure associated with a constituent of mass M_{const}, in the limit $\alpha_{\text{EM}} \to 0$. The bounds placed on $\delta a_l^{\text{non-QED}}$ by the combined theoretical and experimental uncertainty in a_e (about 5×10^{-10}) or a_μ (about 2×10^{-8}) requires (in both cases) that

$$M_{\text{const}} \geq 10^6 \text{ GeV}. \tag{2}$$

In this note we will implement a version of this analysis, using the Drell-Hearn-Gerasimov (DHG) sum rule,[7] to extract severe bounds on the single l^* production processes $e^+e^- \to l^*\bar{l} + l\bar{l}^*$ and $lp \to l^*X$, where l^* is an excited state of l associated with compositeness. The result, in essence, is that such processes will never be observed, regardless of the energy of the interaction, as long as $m_{l^*} \leq 10^3$ TeV. We will also comment on the feasibility of obtaining visible emulsion tracks due to l^*'s produced in high-energy ($>10^{19}$ eV) cosmic-ray events.

Consider the matrix element for the photo- or electroexcitation of a lepton resonance of mass m_{l^*}. For the present, we consider only parity-conserving dipole excitations of a spin-$\frac{1}{2}$ resonance, and choose[8]:

$$\langle l^* | eJ_\mu | l\rangle = \frac{ie}{2M}\bar{U}(p+q,\Lambda)\sigma_{\mu\nu}q^\nu\begin{pmatrix}1\\ \gamma_5\end{pmatrix}u(p,\lambda), \tag{3}$$

where the mass M is a measure of the strength of the excitation. (In a composite model, M^{-1} may be very small, but it is formally of zero order in α.) From Eq. (3), the contribution of l^* to $\delta a_l^{\text{non-QED}}$, as obtained through the DHG sum rule, is easily calculated

$$(\delta a_l^{\text{non-QED}})^2 = \frac{m_l^2}{2\pi^2\alpha}\int_0^\infty \frac{d\nu}{\nu}[\sigma_{3/2}(\nu) - \sigma_{1/2}(\nu)]_{l^*} = -(m_l/M)^2 \tag{4}$$

and hence[9]

$$|\delta a_l^{\text{non-QED}}| \simeq (m_l/M). \tag{5}$$

This is in agreement with the results of Refs. 4 and 5, if M is taken to be the mass of the constituent. However, we do not demand this identification of M. The lower limit on M is then fixed by the uncertainties in $a_{e,\mu}$, just as was the limit on M_{const}

$$M \geq 10^6 \text{ GeV}. \tag{2$'$}$$

However, we now have an additional piece of information since, *independent of the interpretation of M in Eq. (2′)*, the matrix element (3) is now bounded. Since this matrix element controls the processes $e^+e^- \to l^*\bar{l}$ ($+l\bar{l}^*$) and $lp \to l^*X$, and the radiative decay rate $l^* \to l\gamma$, we can apply our bound (2′) directly to these processes.

(1) Consider the one-photon annihilation process

$$e^+e^- \to l^*\bar{l}\ (+l\bar{l}^*).$$
$$\quad\ \searrow l\gamma$$

It is a straightforward algebraic exercise to evaluate the contribution to R from the $l\bar{l}\gamma$ final state, using the matrix element (4) (properly crossed). It is

$$\delta R(l\bar{l}\gamma) = \sigma^{\text{non-QED}}_{e^+e^-\to l\bar{l}\gamma}/\sigma_{e^+e^-\to\mu^+\mu^-} = \frac{1}{4}\left(\frac{m_{l^*}}{M}\right)^2\left(\frac{s}{m_{l^*}^2} - \frac{3m_{l^*}^2}{s} + \frac{2m_{l^*}^4}{s^2}\right) \times F(s)\left(\frac{\Gamma_{l^*\to l\gamma}}{\Gamma_{l^*}^{\text{tot}}}\right), \tag{6}$$

where $F(s)$ is a form factor ($|F(s)| \leq 1$). It is clear, in comparing Eqs. (6) and (2′), that

$$\delta R(l\bar{l}\gamma) \leq \tfrac{1}{4}(m_{l^*}/10^6\ \mathrm{GeV})^2\,, \qquad (7)$$

assuming that F decreases fast enough with s to tame the kinematic factor. Hence this channel will not be seen at any s if m_l^* is of the order of 10^3 TeV or less [since $R^{\mathrm{QED}}(l\bar{l}\gamma) \simeq 0.2$].[10]

(2) An analogous exercise shows that

$$\frac{d\sigma}{d^3p'/E'}(lp \to l^*X) \sim \left(\frac{m_{l^*}}{10^6\ \mathrm{GeV}}\right)^2 \frac{d\sigma}{d^3p'/E'}(lp \to lX) \quad (8)$$

and the same conclusions hold as in (1) above. These results also carry over to the case of spin-$\frac{3}{2}$ resonances. Hence, it is fair to conclude that lower-mass l^*'s (in the TeV range) are best sought in experiments designed for associated production (e.g.,

$$e^+e^- \to l^*\bar{l}^* \quad \text{or} \quad pp \to l^*\bar{l}^* + X)\,.$$

(3) Finally, there is an optimistic aspect following from the bound (2′) established on M: From the matrix element (4) we may calculate the proper lifetime for the radiative decay of the l^* to be $\tau_{l^*} = 4M^2/\alpha m_{l^*}{}^3$. A bit of arithmetic then shows that if the radiative mode dominates the decay channels of the l^*, then an l^* carrying energy E in the laboratory has a mean free path of

$$\lambda_{\mathrm{lab}}(\text{in mm}) \geq \frac{E(\text{in eV}) \times 10^{-19}}{[m^*(\text{in TeV})]^4}\,. \qquad (9)$$

Hence if $m_{l^*} \sim 1$ TeV, cosmic-ray events in the energy range $>10^{19}$eV could give rise to visible l^* paths in emulsions.

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[1]R. S. Van Dyck, Jr., Bull. Am. Phys. Soc. 24, 758 (1979).

[2]J. Bailey, *et al.*, Nucl. Phys. B150, 1 (1979).

[3]For the electron, see the recent review by T. Kinoshita, Cornell Report No. CLNS-70/437, 1979 (unpublished); for the muon, see the review by F. J. M. Farley and E. Picasso, Annu. Rev. Nucl. Sci. 29, 243 (1979).

[4]S. J. Brodsky and S. D. Drell, Phys. Rev. D 22, 2236 (1980).

[5]G. L. Shaw, D. Silverman, and R. Slansky, Phys. Lett. 94B, 57 (1980).

[6]Various aspects of constituent quark and/or lepton models have been discussed by J. G. Pati and A. Salam, Phys. Rev. D 10, 275 (1974); O. W. Greenberg and C. A. Nelson, *ibid.* 10, 2567 (1974); G. R. Kalbfleisch and B. C. Fowler, Nuovo Cimento 194, 173 (1974); J. D. Bjorken (unpublished); J. C. Pati, A. Salam, and J. Strathdee, Phys. Lett. 58B, 265 (1975); O. W. Greenberg, Phys. Rev. Lett. 35, 1120 (1976); E. Nowak, J. Sucher, and C. H. Woo, Phys. Rev. D 16, 2874 (1977); G. L. Shaw, Phys. Lett. 81B, 343 (1979); H. Harari, Phys. Lett. 86B, 83 (1979); M. A. Shupe, *ibid.* 86B, 87 (1979); J. G. Taylor, *ibid.* 88B, 291 (1979); W. Krolikowski, *ibid.* 85B, 335 (1979); R. Casalbuoni and R. Gatto, *ibid.* 93B, 47 (1980); J. Ellis, M. K. Gaillard, L. Maiani, and B. Zumino, Report No. LAPP TH-15/CERN TH-2481, 1980 (unpublished); J. Ellis, M. K. Gaillard, and B. Zumino, Phys. Lett. 94B, 343 (1980); V. Visnjić-Triantafillou, *ibid.* 95B, 47 (1980); and Fermilab Report No. 80/34-THY, 1980 (unpublished); E. Derman, Phys. Lett. 95B, 369 (1980); Phys. Rev. D 23, 1623 (1981); H. Terazawa, *ibid.* 22, 184 (1980); Prog. Theor. Phys. 64, 1763 (1980); I. Montvay, Phys. Lett. 95B, 227 (1980); G. L. Shaw and R. Slansky, Phys. Rev. D 22, 1750 (1980); S. L. Adler, *ibid.* 21, 2903 (1980); H. Harari and N. Seiberg, Phys. Lett. 98B, 269 (1981); R. Casalbuoni, G. Domokos, and S. Kovesi-Domokos, SLAC Report No. 2580 (unpublished); O. W. Greenberg and J. Sucher, Phys. Lett. 99B, 339 (1981); T. L. Curtright and P. G. O. Freund, Report No. EFI 79/24 (unpublished); H. S. Shantchandra, LPTHE 80/01 (unpublished); L. Gonzalez Mestres, Phys. Rev. D 23, 2055 (1981); A. A. Anselm, Pis'ma Zh. Eksp. Teor. Fiz. 31, 150 (1980) [JETP Lett. 31, 138 (1980)]; J. C. Pati, Trieste report, 1980 (unpublished); S. Dimopoulos, S. Raby, and L. Susskind, Nucl. Phys. B173, 208 (1980); K. Matumoto and K. Kahazu, Toyama Report No. DRUR-1 (unpublished); Prog. Theor. Phys. 65, 390 (1981); R. Barbieri, L. Maiani, and R. Petronzio, Phys. Lett. 96B, 63 (1980); H. Schnitzer, Brandeis report, 1980 (unpublished). The above reference list was compiled by the last-named author.

[7]S. D. Drell and A. C. Hearn, Phys. Rev. Lett. 16, 908 (1966); S. B. Gerasimov, Yad. Fiz. 2, 598 (1965) [Sov. J. Nucl. Phys. 2, 430 (1966)].

[8]The most general gauge-invariant transition matrix element in an orthogonal basis is

$$\langle l^*(p)|eJ_\mu|l(p)\rangle = e\,\bar{U}(P)\,G_{\mathrm{Coul}}(q^2)[(P\cdot q)q_\mu - q^2P_\mu] + i(m_l + m_{l^*})G_T(q^2)\,\epsilon_{\mu\nu\lambda\sigma}P^\nu q^\lambda \gamma^\sigma\gamma_5 \times \begin{Bmatrix} 1 \\ \gamma_5 \end{Bmatrix} u(p)$$

[J. D. Bjorken and J. D. Walecka, Ann. Phys. (N.Y.) 38, 35 (1966)]. The DHG sum rule places a restriction only on the transverse piece $G_T(0)$. The choice (3) in the text for the matrix element corresponds to the symmetric constraint $G_{\mathrm{Coul}}(q^2) = G_T(q^2)$. The results in part (3) of the text (pertaining to the decay $l^* \to l\gamma$) are *independent* of the assumed form (3), since this parametrization is completely general for real photons. The conclusions of parts (1) and (2) depend only on $G_{\rm Coul}$ not being grossly larger than G_T. Since most reasonable models of transition form factors of composite objects will give (for $|q^2| \sim m_{l^*}{}^2$ or less) $G_{\rm Coul} \approx G_T$, each being proportional to an overlap integral of parton wave functions of l and l^*, it is simplest for an order-of-magnitude estimate, to use the form (3) which (as stated above) enforces $G_{\rm Coul} = G_T$.

[9]The minus sign in Eq. (4) is not troublesome, since we are obtaining the contribution of a single l^* to $(\delta a^{\text{non-QED}})^2$. (It is analogous to the negative contribution of the $P_{11}(1450)$ to $a_{\rm nucleon}{}^2$). A complete theory would presumably give rise to enough helicity $\frac{3}{2}$ scattering to correctly give a positive $(\delta a_l^{\text{non-QED}})^2$. We do, however, take the absolute contribution in Eq. (4) as indicative of the magnitude of $(\delta a^{\text{non-QED}})^2$.

[10]The converse of this statement is that if an l^* of mass $\leq 10^3$ TeV is observed in $e^+e^- \to l^*\bar{l}$, then the separation $\delta a^{\rm QED} + \delta a^{\text{non-QED}}$ is not valid, and the dynamics of the l^* involves a coupling related to $\alpha_{\rm EM}$.