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Brief Reports

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Scaling in multiplicity distributions in hadron-hadron interactions at 2–1000 GeV/*N*

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We present the measurement of charged-particle multiplicity distributions produced by 200, 300, 400, and 1000 GeV/*c* proton, 200 and 16 GeV/*c* pion, and 1.85 GeV/nucleon argon in nuclear emulsions. We find that the secondary particles produced in p-p and (p,π,Ar)-emulsion collisions satisfy Koba, Nielsen, and Olesen scaling.

Starting from Feynman's scaling for all the many-particle inclusive cross sections and by neglecting quantities of the order of $1/\log s$, Koba, Nielsen, and Olesen[1] derived KNO scaling for multiparticle distribution at infinite energies. It says that the probability $[P(n)]$ of observing n charged particles from pp collision can be related to a scaling function ψ in either of the following two ways, i.e.,

$$P(n)=\sigma_n/\sigma_{\text{inel}}\xrightarrow[s\to\infty]{}\langle n\rangle^{-1}\psi(z) \tag{1}$$

or in terms of moments of the multiplicity distributions, i.e.,

$$C_q=\langle n^q\rangle/\langle n\rangle^q=\text{constant}, \tag{2}$$

where

$$\langle n^q\rangle=\sum_{n=2}^{\infty}n^q\sigma_n/\sigma_{\text{inel}}$$

is the qth moment of the multiplicity distribution (for $n=2$ we exclude the elastic contribution), σ_n is the partial cross section for producing n charged particles, σ_{inel} is the total inelastic cross section (excluding coherent events), $\langle n\rangle$ is the average charged-particle multiplicity, and ψ and C_q are energy-independent functions through the variable $z\cong n/\langle n\rangle$. The analyses performed by Slattery[2] and others have shown that Eqs. (1) and (2) are in good agreement with the data on p-p interactions for laboratory energy between 50 and 300 GeV. We have already proved for different parameters the similarity that exists between p-p and p-nucleus interactions[3] at 200–300 GeV and thus we extended the discussion of the KNO scaling law to p-nucleus interactions and found[4] an excellent fit. A necessary condition for Eq. (1) to be satisfied is that various moments given by Eq. (2) of the multiplicity distribution be independent of the energy. We found that this condition was satisfied in Ref. 3, where the values of different moments for p-p and p-nucleus interactions were practically the same.

In order to accommodate the data at low energy, a simple empirical modification of Eq. (1) was proposed to extend this type of scaling (KNO) at lower energy.[5] Explicitly, the new scaling law is of the form

$$P(n)=[1/(\langle n\rangle-\alpha)]\psi(Z'), \tag{3}$$

where $Z'\equiv(n-\alpha)/(\langle n\rangle-\alpha)$ and α is constant and is independent of energy but may depend upon the type of interaction. So in the modified scaling, the successive moments of the multiplicity distribution should satisfy equations of the form $C_q=\langle(n-\alpha)^q\rangle/\langle n-\alpha\rangle^q$ which should remain constant with energy. After looking at the low-energy data which fit well with the KNO scaling given by the above relations, one wonders if the very central assumption of the KNO derivation, namely the exact Feynman scaling at asymptotic energies which is very badly violated at such low energies, is needed at all. In fact, KNO scaling may be considered a consequence of certain properties of moments and correlations when the particles are produced in clusters.

In order to study the particle production by collision of 200, 300, and 400 GeV proton and 200 GeV/*c* pion, with another proton in nuclear emulsion,[3,4] we used small stacks of Ilford G-5 emulsions exposed at Fermi National Accelerator to different beams. The emulsions were scanned by the along-the-track scanning method.[6,7] In multiplicity distributions for argon, we used all the inelastic events except the events with pure projectile fragmentations. On the other hand, for 16 GeV pion[4] and cosmic rays[8] at ~ 1000 GeV, events were found by area scanning.

In Fig. 1(a) we plot $\langle n\rangle(\sigma_n/\sigma_{\text{inel}})$ vs $n/\langle n\rangle$ for white stars with even multiplicities (i.e., for quasi p-p interactions) for 200, 300, and 400 GeV proton beams. Our experimental points lie on a universal curve given by Slattery[2] which can be fitted with an empirical function

$$\psi(z=n/\langle n\rangle)=(3.79z+33.7z^3-6.64z^5+0.332z^7)\times\exp(-3.04z). \tag{4}$$

The values of χ^2 (~ 0.87 per degree of freedom) between Eq. (4) and the experimental data points for all beams are very close to one another and thus the agreement with scaling given by Eq. (1) is good. In Fig. 1(b) is shown the experimental data points for 200, 300, and 400 GeV proton and 200 GeV pion for all events with $N_h \geq 0$ having only the even number of produced charged multiplicity.

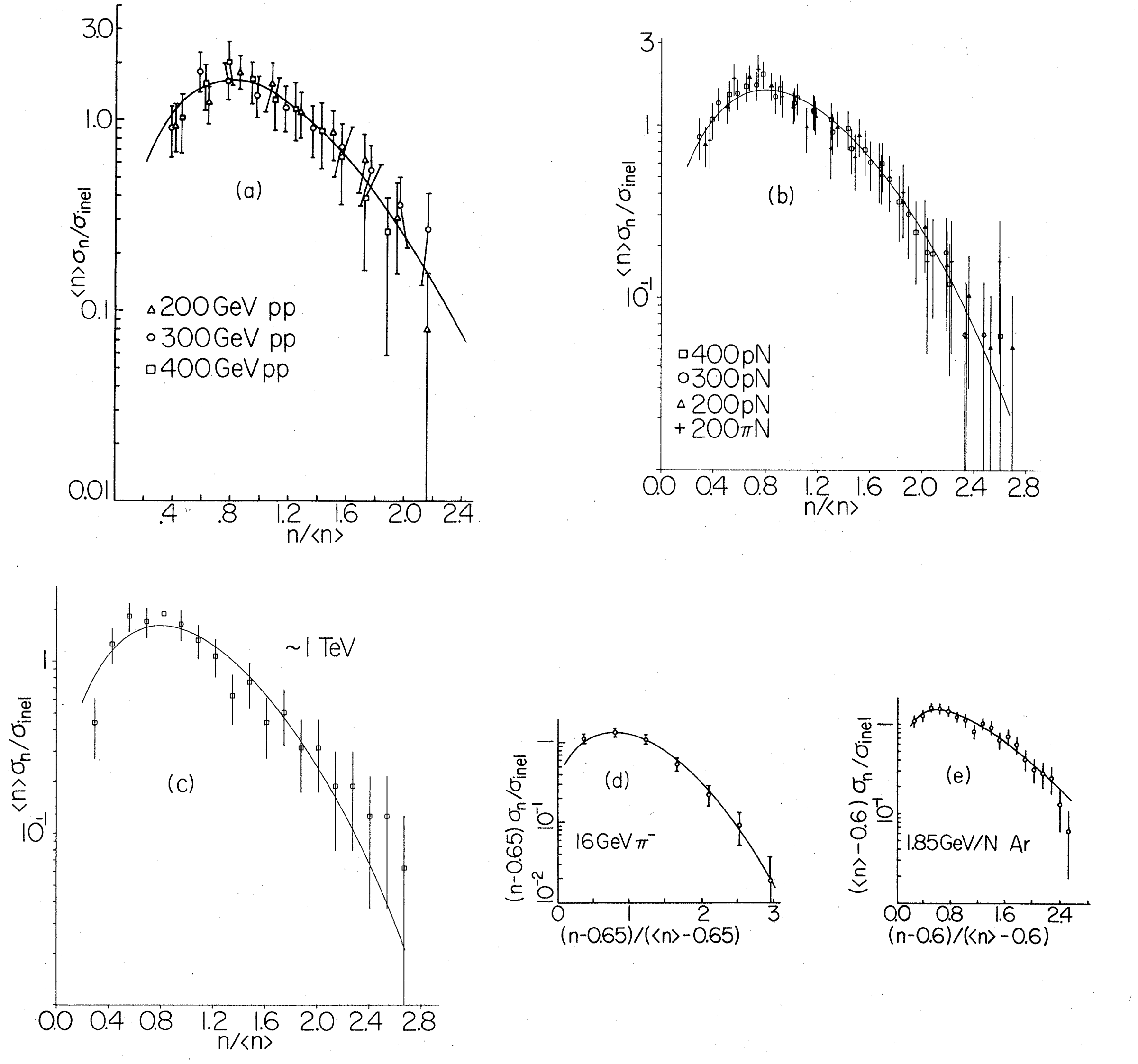

FIG. 1. (a) Plot of $\langle n\rangle\,(\sigma_n/\sigma_{\text{incl}})$ vs $n/\langle n\rangle$ for p-p interactions (white stars with $N_h=0,1$) with even multiplicities for 200, 300, and 400 GeV/c. The curve is an empirical fit to the data, the functional form of the curve is presented in the text, Eq. (4). (b) Plot for KNO scaling for 200, 300, and 400 GeV/c proton and 200 GeV/c pion, for all events with even multiplicity for $N_h \geq 0$. The curve is an empirical fit to the data, given by Eq. (4). (c) Plot for KNO scaling for cosmic-ray events of energy 1000 GeV with all multiplicities of shower tracks $n_s \leq 40$ and black prong $N_h \leq 6$. The curve is given by Eq. (4). (d) Plot for KNO scaling for 16 GeV/c pion events with all even multiplicities for $N_h \geq 0$. (e) Plot for KNO scaling for 1.85 GeV/N argon events with all multiplicities.

TABLE I. Experimental values for the ratio $C_q = \langle n^q \rangle / \langle n \rangle^q$ for 200, 300, 400, and 1000 GeV/c proton, 200 GeV/c pion, and 1.85 GeV/N argon interaction in emulsion (Em). The values for C_2, C_3, and C_4 are almost constant at different energies for different targets within their statistical limits.

Moment \ $p_{\rm lab}$ (GeV/c)	200 (p-p)	300 (p-p)	400 (p-p)	200 (p-Em)	300 (p-Em)	400 (p-Em)	≥1000 (p-Em)	1.85 GeV/N (^{40}Ar-Em)	200 (π-Em)
C_2	1.207 ±0.026	1.239 ±0.027	1.133 ±0.016	1.256 ±0.016	1.265 ±0.018	1.205 ±0.013	1.266 ±0.022	1.299 ±0.016	1.247 ±0.019
C_3	1.645 ±0.087	1.775 ±0.099	1.424 ±0.056	1.868 ±0.062	1.899 ±0.068	1.65 ±0.044	1.925 ±0.088	1.992 ±0.063	1.845 ±0.073
C_4	2.443 ±0.22	2.81 ±0.27	1.94 ±0.14	3.14 ±0.18	3.22 ±0.20	2.47 ±0.11	3.35 ±0.27	3.40 ±0.18	3.08 ±0.22

The data points are fitted with Eq. (4). The values of χ^2 (~ 0.94 per degree of freedom) between Eq. (4) and the experimental data points are very close to one another. Thus we see that a universal behavior of the multiplicity distribution in p-nucleus interactions is just of the same form as in nucleon-nucleon interactions.

As far as we know, KNO scaling has not been tested statistically at energy range ~ 1000 GeV. In Fig. 1(c), we present the experimental data points of cosmic-ray events of energy ~ 1 TeV. For multiplicity distribution, we selected events with shower multiplicity $(n_s) \leq 40$ and black-prong multiplicity $(N_h) \leq 6$. In such events, $\langle n_s \rangle = 15.18 \pm 0.51$. The experimental data points at cosmic-ray energy for all events also lie on a curve given by Eq. (4) with $\chi^2 = 15.8$ for 19 data points. Thus function $\psi(z)$ given by Eq. (1) seems to be universal. In order to take care of the data at low energies, it was pointed out earlier that Eqs. (1) and (2) are modified to Eq. (3). In order to check this new scaling law in pion-nucleus interactions, we used our data from 16.0-GeV π interactions in nuclear emulsion for even multiplicities with $N_h \geq 0$ and the results are shown in Fig. 1(d). We find that for $\alpha = 0.65$, the fit of the experimental data with the modified KNO scaling given[4] by the following equation is quite good, giving $\chi^2 \sim 1.17$ for seven data points, where

$$\psi(z') = 2.3(z' + 0.14) \times \exp(-0.058z' - 0.65z'^2).$$

In order to see how far we could extend the modified KNO scaling in the lower-energy range, we used 1.85 GeV/nucleon heavy ion of argon from the Berkeley Bevatron interacting with emulsion nuclei. The details of scanning of these pellicles is discussed elsewhere.[6,7] We used all events with $\langle n \rangle = 15.38 \pm 0.16$. The particles are produced in clusters[6] and are correlated. In Fig. 1(e) the data points for argon at 1.85 GeV/nucleon events with all multiplicities are shown. In order to evaluate α for making use of Eq. (3), we fitted the distributions using a simple parametrization for $\psi(z')$ and varied α in order to obtain the smallest overall χ^2. We find $\alpha = 0.6$ and

$$\psi(z') = 6.1(z' + 0.01) \times \exp(-1.53z' - 0.11z'^2).$$

In Fig. 1(e) the function $\psi(z')$ as given by the above equation has been plotted against z' with $\alpha = 0.6$. The value of χ^2 between the above equation and the data points is 9.1 for 20 data points and the agreement with scaling Eq. (3) is good.

We stated earlier that KNO scaling described by Eq. (1) can also be expressed in terms of moments given by Eq. (2), i.e., $C_q = \langle n^q \rangle / \langle n \rangle^q$ where the constants C_q are independent of energy. In Table I, we present the experimental values of the ratio $\langle n^q \rangle / \langle n \rangle^q$ for $q = 2$, 3, and 4, for each value of beam momentum. The ratios are consistent with being constants within their statistical limits, for particles with energies between 2 and 1000 GeV/nucleon. In Fig. 2(a) we plot D as a function of $\langle n \rangle$ for all interactions observed in the present experiment. Thus for D a linear dependence on $\langle n \rangle$ is observed. An equation for the fitted line is given by $D = 0.62\langle n \rangle - 1.169$. For higher moments, the same kind of results are obtained. The observed values of α_q are almost the same for different values of q ($\alpha_q = 1.17$). This means that we can use $z' = (n - \alpha)/(\langle n \rangle - \alpha)$ rather than $z = n/\langle n \rangle$

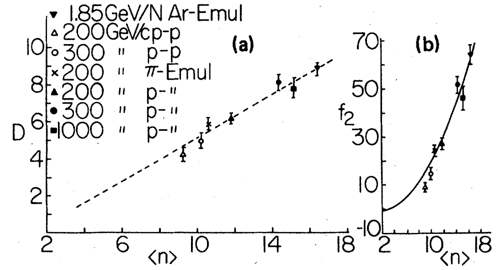

FIG. 2. (a) Plot for D vs $\langle n \rangle$ for all interactions from 2 to 1000 GeV/N. Equation for the least-squares fit for seven points is given by $D = 0.620\langle n \rangle - 1.169$ (dashed line). (b) Plot of the first Mueller's correlation parameter (f_2) vs $\langle n \rangle$ for all interactions from 2 to 1000 GeV/N. Solid curve is a plot of $f_2 = -\langle n \rangle (1 - 0.294\langle n \rangle)$.

as the proper scaling variable for the particles with the energies shown in this experiment. The f_q moments of the multiplicity distribution are called Mueller moments[9] and in Fig. 2(b) we have plotted f_2 moments[9] for all beams. The solid line is the prediction of plot $f_2 = -\langle n\rangle(1-\gamma_2\langle n\rangle)$, where $\gamma_2 = 0.2941$. Equation (2) for $q = 2$ implies that $f_2 = (C_2 - 1)\langle n\rangle^2 - \langle n\rangle$ and hence asymptotically this correlation moment will increase not as $\langle n\rangle$ but as $\langle n\rangle^2$.

From the above experiment, we conclude that the KNO scaling behaviors of the multiplicity distribution are well satisfied by all the produced hadrons at different energies. Early scaling seems to indicate that certain factors in the production mechanism are already stabilized. But, at the present time, an apparent success of KNO scaling should be viewed as an empirical fact, since the original assumptions on which KNO scaling was based are not convincing in the energy region considered here. Thus, we strongly urge that a different theoretical approach should be made for the derivation of the functional form as given by KNO scaling.

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