PHYSICAL REVIEW D VOLUME 24, NUMBER 7 1 OCTOBER 1981

Present status of the pion-nucleon σ term

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(Received 23 September 1980)

The long-standing discrepancy between the predicted value of the πN σ term in the framework of quantum chromodynamics (QCD) and the value extracted from extrapolated on-mass-shell πN scattering data is critically reexamined. Assuming the validity of the Okubo-Zweig-Iizuka (OZI) rule at $t=0$ and using quark mass ratios extracted from the pseudoscalar-meson mass spectrum one obtains the canonical result $\sigma_{\pi N}\simeq 23\pm 5$ MeV. It is argued that the possibility of readjusting the quark mass ratios to give a σ term of 60–70 MeV so as to agree with some values extracted from πN data is most likely ruled out. In particular this would imply a huge violation of the nonrenormalization theorem in K_{13} decay. Since the OZI rule is unreliable at $t=0$, we have recalculated the matrix element $\langle p|\bar{s}s|p\rangle/\langle p|(1/2)(\bar{u}u+\bar{d}d)|p\rangle$ using the Goldstone-boson-pair mechanism and have found a σ term of 36 ± 8 MeV. Other evidence supporting a breakdown of the OZI rule at $t=0$ is also examined. Another theoretical uncertainty is the effect of higher-order terms in SU(3) breaking. Two recent calculations suggest that such terms could increase the σ term by another 10 MeV or more. Finally, the whole procedure of extracting a value of the σ term from πN scattering data is reconsidered. The usual evaluations do not include the uncertainties or their correlations in the πN amplitude or in the fixed-t dispersion relations. We perform several fits to the πN amplitude at $\nu=0$ that suggest that values of the σ term in the range 30–70 MeV are not ruled out by the existing data. Also, nonlinearities in the πN amplitude lead to another ± 10 MeV uncertainty in the comparison between theory and experiment. We therefore conclude that both the theoretical and experimental uncertainties in the determination of the σ term are sufficiently large for the apparent discrepancy to provide neither evidence against QCD nor against the canonical quark mass ratios that one obtains in the conventional $(3,\bar{3})$ model.

I. INTRODUCTION

The theoretical calculation of the pion-nucleon σ term $\sigma_{\pi N}$ as well as its extraction from extrapolated pion-nucleon scattering data has received a great deal of attention in the past.[1] The main reason for this effort is that since $\sigma_{\pi N}$ is a quantity of first order in chiral-symmetry breaking it provides, *a priori*, an important tool to test the detailed structure of that breaking. The importance of a reliable theoretical and experimental determination of $\sigma_{\pi N}$ has become even more crucial after the advent of quantum chromodynamics[2] (QCD) as a probable candidate for a field theory of the strong interactions. In fact, in the framework of QCD the only possible chiral-symmetry-breaking term is the quark bare mass term[3] which transforms according to the $(3,\bar{3})+(\bar{3},3)$ representation of SU(3) $\times$ SU(3). In other words, QCD is a concrete realization of the Gell-Mann–Oakes–Renner[4] (GMOR) model of chiral-symmetry breaking supplemented with a possible isospin-violating up-down-quark mass difference.[5] Therefore, the pion-nucleon σ term can be regarded, to a large extent, as a test of QCD. Unfortunately, it should be stressed that a knowledge of the representation content of the chiral-symmetry-breaking Hamiltonian is not enough to calculate $\sigma_{\pi N}$ completely; additional dynamical assumptions that go beyond symmetry are in fact needed and they necessarily introduce a certain degree of uncertainty. To be more specific let us consider the theoretical expression of $\sigma_{\pi N}$ in QCD terminology, neglecting isospin-breaking effects for the moment, i.e.,

$$\sigma_{\pi N}^{\rm thy}=\left(\frac{m_u+m_d}{2}\right)\langle N(p)\,|\bar{u}u+\bar{d}d\,|N(p)\rangle\,, \tag{1}$$

where m_u and m_d are the renormalized current up- and down-quark masses, respectively. Although the matrix element in Eq. (1) is not precisely known it can be written in terms of the SU(3)-octet operator $(\bar{u}u+\bar{d}d-2\bar{s}s)$ which generates the medium-strong mass differences, in which case Eq. (1) becomes

$$\begin{aligned}\sigma_{\pi N}^{\rm thy}&=\left(\frac{m_u+m_d}{2}\right)\langle N(p)\,|\bar{u}u+\bar{d}d-2\bar{s}s\,|N(p)\rangle\left(1+2\frac{\langle N(p)\,|\,\bar{s}s\,|\,N(p)\rangle}{\langle N(p)\,|\,\bar{u}u+\bar{d}d-2\bar{s}s\,|\,N(p)\rangle}\right)\\&=\frac{3}{2}\,\frac{m_u+m_d}{2m_s-(m_u+m_d)}(M_\Lambda+M_\Sigma-2M_N)\left(1+2\frac{\langle N(p)\,|\,\bar{s}s\,|\,N(p)\rangle}{\langle N(p)\,|\,\bar{u}u+\bar{d}d-2\bar{s}s\,|\,N(p)\rangle}\right),\end{aligned} \tag{2}$$

where the last line in Eq. (2) is valid to lowest order in SU(3) breaking. Since the value of $\langle N|\bar{s}s|N\rangle$ is not fixed by symmetry considerations Eq. (2) provides a test of QCD to the extent that the amount of strange-quark sea component in the nucleon is known from separate dynamical information. For example, it has been customary[6] to invoke the Okubo-Zweig-Iizuka[7] (OZI) rule, i.e.,

$$\langle N|\bar{s}s|N\rangle \simeq 0\,, \tag{3}$$

in which case $\sigma_{\pi N}$ becomes a function of the quark mass ratio

$$r_+ = \frac{m_u + m_d}{2m_s}\,, \tag{4}$$

which may be extracted from experimental data on various other processes. For instance, from the pseudoscalar-meson mass spectrum one has[4,5,8] $r_+ \simeq 0.034 \pm 0.008$ yielding $\sigma_{\pi N}^{\rm thy} = 23 \pm 5$ MeV.

On the other hand, typical values for the σ term obtained from extrapolated πN scattering data are[9] $\sigma_{\pi N}^{\rm exp} \simeq 65$ MeV. This discrepancy of nearly a factor of 3 between theory and experiment is clearly alarming and in fact it has been the subject of much controversy in the past. In this paper we shall address ourselves to this issue and discuss critically some of the possible origins of this discrepancy including (i) the extraction of $\sigma_{\pi N}^{\rm exp}$ from experiment, (ii) the possibility that r_+ is much larger than the value obtained from the pseudoscalar-meson mass spectrum,[10] (iii) the OZI assumption, Eq. (3), (iv) higher-order [in SU(3) breaking] corrections to Eq. (2), and (v) nonlinear corrections to the formula used to obtain $\sigma_{\pi N}^{\rm exp}$ from the πN amplitude. First, the extraction of the σ term from experiment requires an extremely delicate double extrapolation of a πN amplitude, first to $\nu = 0$ and then to the timelike point $t = 2\mu_\pi^2$. We argue that the uncertainty in $\sigma_{\pi N}^{\rm exp}$ is much larger than the values that are usually quoted, both because of uncertainties in the t extrapolation and because the quoted error bars often do not include the uncertainties (or their correlations) in the πN data or in the extrapolation in ν. We perform various fits to the amplitude that suggest that values of $\sigma_{\pi N}^{\rm exp}$ of 30 MeV or even less are not in obvious conflict with the πN data.[11] Second, we claim that the large value of r_+ required to solve the discrepancy would imply an unacceptably huge violation of SU(3) and we discuss how the K_{13} decays can be used to rule out, in all probability, this alternative. Next, the theoretical value of the σ term $\sigma_{\pi N}^{\rm thy}$ must be evaluated at $t = 0$ where the OZI ansatz is unreliable. We present an updated estimate of the corrections to Eq. (3) based on the Goldstone-boson-pair mechanism[12] and discuss other indications of a possible violation of the OZI rule in the low-t domain. Our results here indicate that these effects could increase $\sigma_{\pi N}^{\rm thy}$ to 44 MeV (or even higher). Finally, we report on the recent calculations of Gasser[13] and Jaffe[14] which suggest that nonleading corrections to (2) probably increase $\sigma_{\pi N}^{\rm thy}(0)$ by an additional 10 MeV or more. Similarly, nonlinearities in the πN amplitude lead to another uncertainty of $\simeq$10 MeV in the experimental value with which $\sigma_{\pi N}^{\rm thy}$ should be compared.

Therefore, we conclude that both the theoretical and the experimental errors and uncertainties in the determination of $\sigma_{\pi N}$ are sufficiently large for the apparent discrepancy to provide neither evidence against QCD nor against the canonical GMOR value of r_+. Other issues, including the t value at which the σ term should be evaluated and the question of the kaon-nucleon σ commutator, are briefly discussed.

II. FORMALISM

We start with the definition[1] of $\sigma_{\pi N}^{\rm thy}$, i.e.,

$$\sigma_{\pi N}^{\rm thy} = \frac{1}{3} \sum_{\alpha=1,2,3} \langle N(p')|[{}^5Q^\alpha, [{}^5Q^\alpha, H(0)]]|N(p)\rangle\,, \tag{5}$$

where $t = (p - p')^2$, ${}^5Q^\alpha$ are the weak axial-vector charges of isospin index α, and $H(0)$ is the strong-interaction Hamiltonian

$$H = H_0 + H_{\rm SB} = H_0 + \epsilon_0 u_0 + \epsilon_8 u_8 + \epsilon_3 u_3\,. \tag{6}$$

In Eq. (6) the term H_0 commutes with the ${}^5Q^\alpha$ and therefore we shall be concerned with the chiral-symmetry-breaking piece $H_{\rm SB}$. The term $\epsilon_0 u_0$ breaks SU(3) $\times$ SU(3) down to SU(3) while $\epsilon_8 u_8$ and $\epsilon_3 u_3$ break SU(3) and SU(2), respectively. In QCD the $H_{\rm SB}$ is due entirely to the quark bare mass terms, so that

$$H_{\rm SB} = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s\,, \tag{7}$$

which transforms according to the $(3, \bar{3}) + (\bar{3}, 3)$ representation of SU(3) $\times$ SU(3).[4] From Eqs. (5) and (7) one has

$$\sigma_{\pi N}^{\rm thy}(0) = \left(\frac{m_u + m_d}{2}\right)\langle N(p)|\bar{u}u + \bar{d}d|N(p)\rangle - \left(\frac{m_d - m_u}{6}\right)\langle N(p)|\bar{u}u - \bar{d}d|N(p)\rangle\,. \tag{8}$$

The second term in Eq. (8), which is one-sixth of the nonelectromagnetic piece of the proton-neutron mass difference,[5] contributes less than -0.5 MeV to $\sigma_{\pi N}^{\rm thy}$ and therefore it will be neglected; in this case Eqs. (1) and (2) then follow.

In order to make contact with experiment one can relate the σ term to pion-nucleon scattering as follows. Let $T^{(+)}(\nu, \nu_B, q^2, q'^2)$ be the off-mass-shell, crossing-even, amplitude for $\pi(q) + p(p)$

$\to \pi(q')+p(p')$, where

$$\nu=\frac{s-u}{4M_N},$$

and (9)

$$\nu_B=\frac{-q\cdot q'}{2M_N}=\frac{t-q^2-q'^2}{4M_N}.$$

Using current algebra and the partially conserved axial-vector current (PCAC) hypothesis one finds[1]

$$T^{(+)}(0,0,0,0)=\frac{-\sigma_{\pi N}(0)}{f_\pi^2}, \quad (10)$$

where $f_\pi=93.24\pm0.09$ MeV is the pion decay constant. Next, $T^{(+)}(0,0,0,0)$ can be related[15] to the on-mass-shell pion-nucleon amplitude evaluated at the unphysical (Cheng-Dashen) point $\nu=0$, $t=2\mu_\pi^2$ ($\nu_B=0$) by

$$T^{(+)}(0,0,\mu_\pi^2,\mu_\pi^2)=-T^{(+)}(0,0,0,0)+O(\mu_\pi^4\ln\mu_\pi^2). \quad (11)$$

Equation (11) follows from combining a linearity assumption (in q^2 and q'^2) for $T^{(+)}$ with the Adler consistency condition[16]

$$T^{(+)}(0,0,0,\mu_\pi^2)=T^{(+)}(0,0,\mu_\pi^2,0)=0, \quad (12)$$

so that

$$T^{(+)}(0,0,q^2,q'^2)=T^{(+)}(0,0,0,0)\left(1-\frac{q^2+q'^2}{\mu_\pi^2}\right), \quad (13)$$

where it should be noted that $T^{(+)}(0,0,0,0)$ is itself of order $O(\mu_\pi^2)$. From Eqs. (10) and (11) one finally has

$$T^{(+)}(\nu=0,\ t=2\mu_\pi^2)=\frac{\sigma_{\pi N}(0)}{f_\pi^2}+O(\mu_\pi^4\ln\mu_\pi^2), \quad (14)$$

where $T^{(+)}(\nu,t)$ is now the on-mass-shell pion-nucleon amplitude.[17]

One possible source of corrections of order μ_π^4 to Eq. (14) involves the coupling of heavy pions (three-pion resonances) to ordinary off-mass-shell pions in the manner of extended PCAC (EPCAC).[18] Assuming linearity not in $T^{(+)}(\nu,\nu_B,q^2,q'^2)$ but in a slightly different off-mass-shell pion-nucleon amplitude and using current algebra and EPCAC one finds[18]

$$T^{(+)}(\nu=0,\ t=2\mu_\pi^2)=\frac{\sigma_{\pi N}(0)}{(1-\Delta_\pi)^2f_\pi^2}, \quad (15)$$

where $\Delta_\pi\simeq0.05\pm0.01$ is the Goldberger-Treiman discrepancy in SU(2) × SU(2) (Ref. 19) and $\Delta_\pi=O(\mu_\pi^2)$.

Extrapolating the experimental pion-nucleon amplitude to the Cheng-Dashen point and using Eq. (14) or Eq. (15) one can then extract $\sigma_{\pi N}^{\rm exp}$. In the following sections we shall generally quote results using both Eq. (14) and Eq. (15) to give an indication of the magnitude of the $O(\mu_\pi^4)$ uncertainties.

III. EXPERIMENTAL VALUE OF THE σ TERM

Many attempts have been made in the past to extract $\sigma_{\pi N}^{\rm exp}$ using various πN phase-shift analyses and extrapolation techniques.[1] We shall concentrate here on the most recent determinations by Höhler *et al.*[20] and by Langbein[21] who obtain $\sigma_{\pi N}^{\rm exp}=65\pm6$ and 69 ± 22 MeV, respectively. These values do not reflect the $O(\mu_\pi^4)$ corrections; using Eq. (15) they reduce to 59 ± 5 and 62 ± 20 MeV, respectively.

These determinations involve four separate ingredients, viz. (a) the pion-nucleon data, (b) a phase-shift or amplitude analysis, (c) the determination of $T^{(+)}(\nu=0,t)$ for spacelike values of t using fixed-t dispersion relations, and (d) an extrapolation in t to the Cheng-Dashen point $\nu=0$, $t=2\mu_\pi^2$. This extrapolation is further constrained by the π-π elastic phase shifts, which determine the phase of the crossed channel reaction $\pi\pi\to N\bar{N}$ in the timelike region $t>4\mu_\pi^2$.

We shall not discuss here steps (a) and (b) but refer the interested reader to the articles by Höhler *et al.*[20] and by Langein.[21] We do wish to elaborate, though, on the important observation made by Banerjee and Cammarata[11] that step (d), i.e., the extrapolation in t, is extremely delicate and that the quoted errors on $\sigma_{\pi N}^{\rm exp}$ may greatly underestimate the true uncertainties.

The most systematic analysis is that of Höhler *et al.*,[20] who incorporate all available πN and $\pi\pi$ data. They extrapolate to the Cheng-Dashen point using a fixed-ν dispersion relation for $T^{(+)}(0,t)$. The near parts of the left- and right-hand cuts are evaluated using the amplitudes extrapolated from the πN and $\pi\pi$ data and the distant parts of the cuts are represented by a polynomial (in t) discrepancy function with coefficients also obtained from the extrapolated amplitudes. We believe that the determinations of Höhler *et al.* are the most complete available, but wish to emphasize that the quoted errors reflect only the uncertainties in the discrepancy function. The errors in the extrapolated amplitude $T^{(+)}(0,t)$ for spacelike t are not known and the results of these uncertainties are either not included in the error quoted for $\sigma_{\pi N}^{\rm exp}$ or are only guessed at (this fact is clearly stated in Ref. 20 but generally forgotten when the results are quoted). The true uncertainties in $\sigma_{\pi N}^{\rm exp}$ may, therefore, be much larger than the nominal value of ±6 MeV.

It would, therefore, be desirable to perform a complete analysis including a proper treatment of the error matrix for the extrapolated amplitudes at $\nu=0$. This may not be an easy task since, as pointed out by Höhler *et al.*,[20] these errors are dominated by such effects as systematic discrepancies between experiments and uncertain charge dependent corrections in the first resonance region. As an alternative to such a complete analysis we would like to suggest that the dispersive calculations be repeated including as a theoretical constraint the values of $T^{(+)}(0,\ 2\mu_\pi^2)$ corresponding to $\sigma_{\pi N}=30$, 40, 50 MeV, etc. Such constrained fits might give some insight into how low $\sigma_{\pi N}$ would have to be before conflicting with the experimental data.

The type of analysis suggested above is far outside the scope of this paper. We therefore restrict our considerations to the simplified analysis of Langbein,[21] which has the advantage of having quoted errors for the extrapolated amplitudes $T^{(+)}(0,\ t)$. Since the accuracy of these errors has been questioned[20] we shall study at the end their impact on our conclusions. Langbein's values for $T^{(+)}(0,\ t)$ are shown in Fig. 1. His analysis is not based on exactly the same data base as that of Höhler *et al.*, but their results for $T^{(+)}(0,\ t)$, at the t values where they overlap, are in agreement.

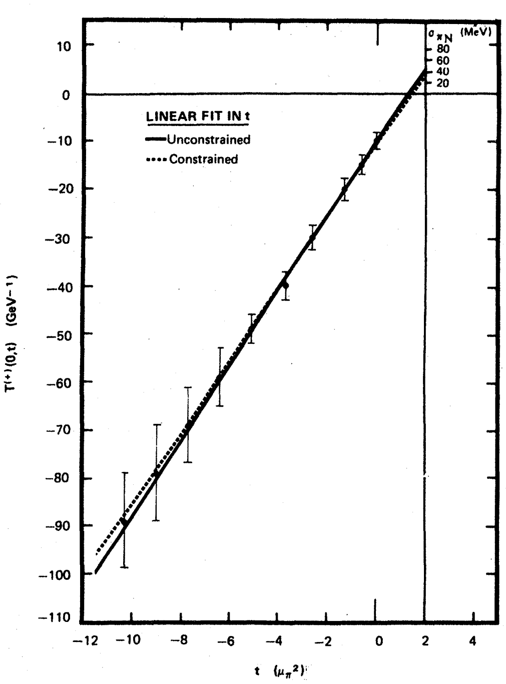

FIG. 1. Linear fits in t to $T^{(+)}(0,t)$. The value of $\sigma_{\pi N}$ in MeV is given by $\sigma_{\pi N}=8.69\ T^{(+)}(0,2\mu_\pi^2)$, with $T^{(+)}$ in GeV^{-1}. The solid curve is the result of an unconstrained fit while the broken line has a theoretical point of $T^{(+)}(0,2\mu_\pi^2)=3.45\pm1.15$ GeV^{-1} added to the data base. This value of $T^{(+)}$ that constrains the fit corresponds to $\sigma_{\pi N}=30\pm10$ MeV.

Among the several difficulties we mention the following: (i) The values of $T^{(+)}(0,\ t)$ at different t points are determined by the same phase shifts. The error bars on different points are therefore correlated, but this effect has apparently not been included in Langbein's extrapolation. (ii) $T^{(+)}(0,\ t)$ has a zero[11] very close to the Cheng-Dashen point. Therefore, small uncertainties in the extrapolation lead to large uncertainties in the value obtained for $\sigma_{\pi N}^{\rm exp}$. It should be apparent from Fig. 1 that small values of $\sigma_{\pi N}^{\rm exp}$ are compatible with the existing data on $T^{(+)}(0,\ t)$. In order to make this more precise we have performed various fits to the Langbein's points, the results of which are shown in Figs. 1 and 2 and Table I and described in the following. In Fig. 1 the solid curve corresponds to a linear fit in t, i.e.,

$$T^{(+)}(0,t)=C_0+C_1t\,, \qquad (16)$$

which yields $\sigma_{\pi N}^{\rm exp}=45\pm2$ MeV with a χ^2 per degree of freedom of $\chi_F^2=0.04$ (Langbein's result for a similar fit is $\sigma_{\pi N}^{\rm exp}=46$ MeV with no error quoted). We interpret this abnormal value of χ_F^2 to mean that the data points are highly correlated, in which case the ±2 MeV uncertainty obtained from the fit is meaningless. In order to obtain more information on the sensitivity of the fit one should know the correlations among errors and study the error matrix for the original $T^{(+)}$ points. Since this in-

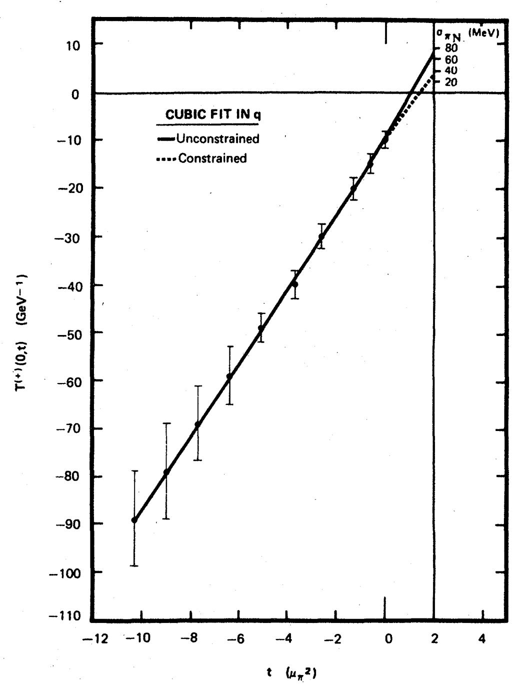

FIG. 2. Cubic fits in $q=(\mu_\pi^2-t/4)^{1/2}$. Solid and broken lines correspond to unconstrained and constrained fits, respectively, as in Fig. 1.

TABLE I. Results of the various least-squared fits to $T^{(+)}(0,t)$. The values of the σ term given in parentheses reflect the $O(\mu_\pi^4)$ corrections. The cubic in q fits have $C_1/C_0=-a_0^0$, where $a_0^0=0.28\pm0.05\ \mu_\pi^{-1}$ is the $I=J=0$ $\pi\pi$ scattering length.

Type of fit	C_0	C_1	C_2	C_3	χ_F^2	$T^{(+)}(0,2\mu_\pi^2)$ (GeV^{-1})	$\sigma_{\pi N}^{\text{exp}}(0)$ (MeV)
Linear in t (unconstrained)	-10.18 ± 0.18 GeV^{-1}	394.5 ± 3.1 GeV^{-3}			0.04	5.21	45 (41)
Linear in t (constrained)	-10.99 ± 0.16 GeV^{-1}	301.5 ± 3.2 GeV^{-1}			0.1	3.89	34 (31)
Cubic in q (unconstrained)	33.43 ± 0.23 μ_π^{-1}		2.12 ± 0.30 μ_π^{-3}	-1.43 ± 0.16 μ_π^{-4}	0.04	8.6	75 (67)
Cubic in q (constrained)	31.70 ± 0.14 μ_π^{-1}		4.25 ± 0.28 μ_π^{-3}	-2.52 ± 0.17 μ_π^{-4}	0.2	3.7	32 (29)

formation, which is extremely difficult to determine, is not provided in Refs. 20 and 21 we have proceeded to generate a theoretical point of $T^{(+)}(0,\ 2\mu_\pi^2)=3.45\pm1.15\ \text{GeV}^{-1}$ that corresponds to a σ term of $\sigma_{\pi N}=30\pm10$ MeV and added it to the data base on $T^{(+)}(0,\ t)$, repeating the linear fit in t. This value of $\sigma_{\pi N}$ is a reasonable compromise of the theoretical estimates to be discussed in the next section. We believe that this procedure should provide us with a reasonable indication of how sensitive the data on $T^{(+)}(0,\ t)$ is to a particular value of the σ term. The result of this constrained fit is shown as a broken line in Fig. 1 and it yields $\sigma_{\pi N}^{\text{exp}}=34\pm2$ MeV, with $\chi_F^2=0.1$, i.e., an equally good fit to the data with a much lower value of the σ term.

It has been argued that higher-order polynomials should be preferred over the linear function Eq. (16) on account of the behavior of $T^{(+)}$ near the t-channel threshold at $t=4\mu_\pi^2$. In particular, Langbein has performed a fit to $C^{(+)}(0,\ t)$ [see Eq. (16)] using a cubic polynomial in the real analytic threshold momentum $q(t)\equiv(\mu_\pi^2-t/4)^{1/2}$, i.e.,

$$C^{(+)}(0,t)=\sum_{i=0}^{3}C_i q^i\,. \tag{17}$$

The validity of this expansion has been criticized due to the existence of a nearby singularity. We will use it only to illustrate the sensitivity to expansion uncertainties. In order to reduce the extrapolation errors Langbein has used the t-channel unitarity constraint $C_1/C_0=-a_0^0=-0.28\pm0.05\ \mu_\pi^{-1}$, where a_0^0 is the $I=J=0$ pion-pion scattering length.[22] We have performed a least-squares fit to Langbein's data using Eq. (17) together with the $\pi\pi$ scattering length constraint and found a σ term $\sigma_{\pi N}^{\text{exp}}=75\pm22$ MeV with $\chi_F^2=0.04$, in approximate agreement with Langbein's result of 69 ± 22 MeV. Repeating the fit with the added theoretical point at $t=2\mu_\pi^2$ we found $\sigma_{\pi N}^{\text{exp}}=32\pm10$ MeV with $\chi_F^2=0.18$, so that again the constrained fit, which gives a much smaller σ term, is perfectly compatible with the data. The error bar for the unconstrained fit includes a ±3-MeV uncertainty due to the quoted error in a_0^0. The value of $\sigma_{\pi N}^{\text{exp}}$ for the constrained fit is not sensitive to this uncertainty. These last fits are illustrated in Fig. 2 and Table I contains all the relevant numerical information. A recent determination of the $\pi\pi$ scattering length from the reaction $\pi^-p\to\pi^-\pi^+n$ near threshold gives[23] $a_0^0=0.177\pm0.017\ \mu_\pi^{-1}$. Using this value we obtained, in an unconstrained fit, $\sigma_{\pi N}^{\text{exp}}=67\pm24$ MeV with $\chi_F^2=0.04$, where the error in $\sigma_{\pi N}^{\text{exp}}$ includes a ±5 MeV uncertainty due to the quoted error in a_0^0. We take this as a further illustration of the uncertainties involved in the extraction of $\sigma_{\pi N}^{\text{exp}}$.

In summary, our simple analysis shows that errors on the σ term of the order of 10–20% often quoted in the literature are most likely underestimations of the true error so that small values of $\sigma_{\pi N}$ cannot be ruled out. It must be stressed, however, that we have not pretended to redo the phase-shift and amplitude analysis and thus we have taken Langbein's data and errors at face value. As already mentioned, Langbein's data[21] agrees with that of Höhler *et al.*,[20] but the latter do not quote errors on $T^{(+)}(0,\ t)$. Since the error determination might be the subject of a separate controversy we have done a simple check on its influence on our conclusions by rerunning our fits with the errors divided by a factor of 2 and by a factor of 4. In the case of the unconstrained fits one does not expect any changes except for an obvious increase in χ_F^2, and this is precisely what we found, viz., the σ term remained at the value of 75 MeV (for the cubic fits) with χ_F^2 increasing to 0.16 and 0.62 when the original errors were divided by 2 and by 4, respectively. For the constrained fits, changes are expected since the theoretical point added to the data base retains its original error. We found that the σ term increased, in the cubic fits, from its original value of 32 ± 12 MeV to 39 ± 16 and 52 ± 20 MeV with χ_F^2 increasing to 0.6 and 1.9 when the errors were divided by 2 and by 4, respectively (notice that the standard deviation is meaningful only in the latter case where $\chi_F^2 \geq 1$). This simple exercise shows then that our conclusions will remain largely unaltered even if the errors on $T^{(+)}(0, t)$ were highly overestimated.

We would like to make several additional comments. (a) The first concerns the constraints on $T^{(+)}(0,t)$ derived from right-hand cut ($\pi\pi \to N\bar{N}$) information. An example of this is the $\pi\pi$ scattering length included in our fits as discussed above, but additional constraints may be placed on $T^{(+)}(0, t)$ by considering phase shifts.[20] It should be noted, however, that this experimental information is available starting at $t \sim 16\mu_\pi^2$ and, therefore, its impact on the σ term is not very clear. For example, Durso, Jackson, and VerWest[24] discuss a model for $\pi\pi \to N\bar{N}$ that leads to a $T^{(+)}(0, t)$ in good agreement with the calculations of Höhler *et al.*[20] in the region $4\mu_\pi^2 \lesssim t \lesssim 35\mu_\pi^2$. The model, however, has exact chiral-symmetry built into it, i.e., the σ term is strictly zero. (b) Banerjee and Cammarata[11] have constructed a dynamical model for low-energy πN scattering, which is basically an extension of the Chew-Low model including nucleon recoil effects and seagull terms, in which the σ term is an adjustable parameter. They obtained a reasonable fit to the dispersion relation values of $T^{(+)}(0, t)$, at $t<0$, for $\sigma_{\pi N}=25.5$ MeV, concluding that a σ term of this size must be compatible with the data. It should be noted, however, that their model is not in good agreement with the πN s-wave amplitudes. (c) An additional source of uncertainty comes from the singularity in the vicinity of $t=4\mu_\pi^2$ which may introduce nonlinear effects of such a magnitude as to invalidate the Cheng-Dashen expansion (which assumes that the amplitudes are linear in this region). For example, according to Höhler, Koch, and Pietarinen[20] these nonlinear effects amount to some 13 MeV in the σ term, and when all uncertainties are taken into account they conclude that the error on the σ term is no more than a guess. (d) Olsson and Osypowski[25] have recently derived an alternate expression for $\sigma_{\pi N}$ which involves the πN amplitudes and their t derivatives evaluated at $t=0$. They obtain $\sigma_{\pi N}^{\rm exp}(0)=77\pm 9$ MeV, but an accurate determination of the σ term by this method requires precise values for the πN scattering lengths. The scattering lengths must be determined by extrapolating finite energy data[20] and therefore run into some of the same uncertainties as the method based on the Cheng-Dashen formula. In conclusion, the uncertainty in $\sigma_{\pi N}^{\rm exp}$ is simply not known and a low value is not obviously excluded.

IV. THEORETICAL VALUE OF THE σ TERM

We now discuss the theoretical determination of the σ term, including (a) the value of r_+ in Eq. (4), (b) the OZI ansatz, Eq. (3), and (c) higher-order corrections to Eqs. (2) and (14).

The quark mass ratio r_+ is determined from the pseudoscalar-meson mass spectrum. Neglecting SU(2) breaking relative to SU(3) breaking one has[4,5,8]

$$r_+=\frac{m_u+m_d}{2m_s}=\frac{\mu_\pi^2(1-\epsilon)}{2\mu_K^2-\mu_\pi^2(1-\epsilon)}, \tag{18}$$

where

$$1-\epsilon=\frac{f_\pi}{f_K}\frac{Z_K^{1/2}}{Z_\pi^{1/2}}. \tag{19}$$

In Eq. (20), f_K and f_π are the kaon and pion decay constants, respectively, and

$$\sqrt{2}Z_\pi^{1/2}\equiv i\langle 0|\bar{d}\gamma_5 u|\pi^+\rangle \tag{20}$$

is the pion wave-function renormalization constant; a similar definition holds for $Z_K^{1/2}$. The only unknown in Eq. (18) is the ratio $x\equiv Z_K^{1/2}/Z_\pi^{1/2}$. There is no reason for $Z_K^{1/2}$ and $Z_\pi^{1/2}$ to vanish in the chiral-symmetry limit, so x is not expected to deviate greatly from its SU(3) symmetric value of unity. For $x=1$ and $f_K/f_\pi=1.22$ one obtains[8] $r_+=0.031$.

Assuming the basic validity of a spontaneously broken chiral symmetry the only way to obtain a significantly different value for r_+ is to invoke an anomalously large breaking of SU(3) in the ratio x. For example, to obtain $\sigma_{\pi N}=65$ MeV with $\langle N|\bar{s}s|N\rangle=0$ one needs $x\simeq 2.7$ ($r_+\simeq 0.08$), representing a nearly 300% breaking of SU(3). It is convenient to rewrite Eq. (19) as

$$1-\epsilon=\frac{f_\pi f_+(0)}{f_K}\frac{x}{f_+(0)}, \tag{21}$$

where $f_+(0)$ is the K_{l3} form factor, because the combination $f_K/f_\pi f_+(0)=1.26\pm 0.02$ is experimentally known more accurately than f_K/f_π. One can now use Eqs. (21) and (18) together with (2) in order to relate the ratio $x/f_+(0)$ to $\sigma_{\pi N}^{\rm thy}$. In Fig. 3 we show the values of $x/f_+(0)$ that would be needed to obtain

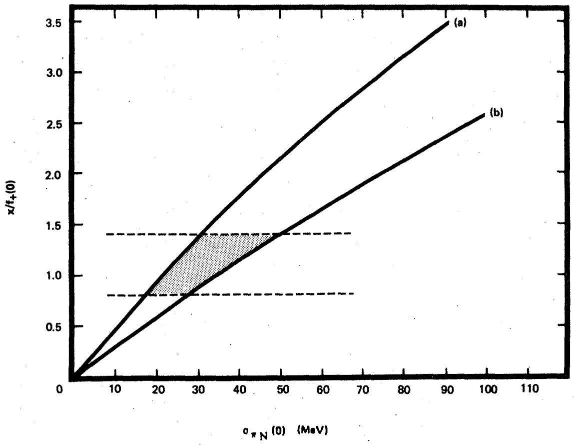

FIG. 3. The ratio $x/f_+(0)$ vs $\sigma_{\pi N}(0)$ for $\langle N|\bar{s}s|N\rangle = 0$, curve (a), and for $\langle N|ss|N\rangle/\langle N|\frac{1}{2}(\bar{u}u+\bar{d}d)|N\rangle = 0.36$, curve (b). The shaded region between the two horizontal broken lines represents the bound $0.8 \leq x/f_+(0) \leq 1.4$.

a given value of the σ term if (a) $y=0$ and (b) $y=0.36$, where y is defined as

$$y \equiv \frac{\langle N|\bar{s}s|N\rangle}{\frac{1}{2}\langle N|\bar{u}u+\bar{d}d|N\rangle}. \tag{22}$$

As seen in Fig. 3, for $y=0$ any large deviation of $\sigma_{\pi N}$ from 20–30 MeV would require a very large SU(3) violation in $x/f_+(0)$. This would be a disaster for the K_{13} decays, however, because $f_+(0)$ is very close to unity experimentally and any large SU(3) breaking in x would imply[26] large corrections to the nonrenormalization theorem for $f_+(0)$.[27] Furthermore, one can use[26] an analysis of Gaillard[28] to show that if kaon PCAC is violated at the 30% level [a value expected from the size of the Goldberger-Treiman discrepancy in SU(3) × SU(3)] then the rigorous bound $0.8 < x/f_+(0) < 1.4$ follows. Referring to Fig. 3 one sees that this bound would require $\sigma_{\pi N}$ to be in the range 18–32 MeV for $y=0$. We conclude, therefore, that a large value of the σ term is not likely to be accounted for by an *ad hoc* modification of the value of r_+ (for $y=0$).

Another possibility is that $\langle N|\bar{s}s|N\rangle \neq 0$. The usual justification for the OZI rule in QCD is that its violation requires the annihilation of quarks into gluons, and that the relevant diagrams are suppressed by powers of the strong coupling constant α_s. In fact, an estimate[29] of the lowest-order diagram leads, for small values of α_s, to a very small value of y. However, in the σ-term application $\langle N|\bar{s}s|N\rangle$ must be evaluated at zero momentum transfer, and the lowest-order diagrams involve light quarks only. Therefore, the effective coupling constant may be very large thus invalidating the perturbative result.

In Fig. 4 we show the values of y that would be needed to generate a given $\sigma_{\pi N}$ for the canonical value of r_+. For example, a σ term of 65 MeV would require $y=0.65\pm0.08$.

It was pointed out by Renner[12] that it is possible to estimate the value of $\langle N|\bar{s}s|N\rangle$ at zero momentum transfer in a nonperturbative fashion by using the Goldstone-boson-pair mechanism of Li and Pagels.[12] The general idea is to assume that matrix elements of bilinear quark operators between nucleons satisfy unsubtracted dispersion relations which can be saturated by Goldstone-boson pairs. The result of such a calculation gives

$$\begin{aligned} y &= \frac{\langle N|\bar{s}s|N\rangle}{\langle N|\frac{1}{2}(\bar{u}u+\bar{d}d)|N\rangle} \\ &= 2\,\frac{17\alpha^2+45(1-\alpha)^2-30\alpha(1-\alpha)}{43\alpha^2+63(1-\alpha)^2+30\alpha(1-\alpha)} \\ &\simeq 0.36\,, \end{aligned} \tag{23}$$

where the meson-baryon coupling $(f/d)_A=(1-\alpha)/\alpha$ and $\alpha=\frac{2}{3}$ has been used. From Eq. (2) one then finds

$$\sigma_{\pi N}^{\text{thy}}(0) = 36\pm8\ \text{MeV}\,, \tag{24}$$

to be compared with $\sigma_{\pi N}^{\text{thy}}(0)=23\pm5$ MeV for $y=0$. Considering the theoretical uncertainties involved in this calculation we find the result most encouraging. It would be important, however, to find independent tests of the assumption that $\langle N|\bar{s}s|N\rangle\neq0$ in the low-energy domain. One such possibility is offered by the ϕNN coupling which should be very small compared with, e.g., the ωNN coupling if the vector-current matrix element $\langle N|\bar{s}\gamma_\mu s|N\rangle$ is negligible. It has been known for quite some time from fits to the isoscalar Dirac form factor of the nucleon that the data seem to require a

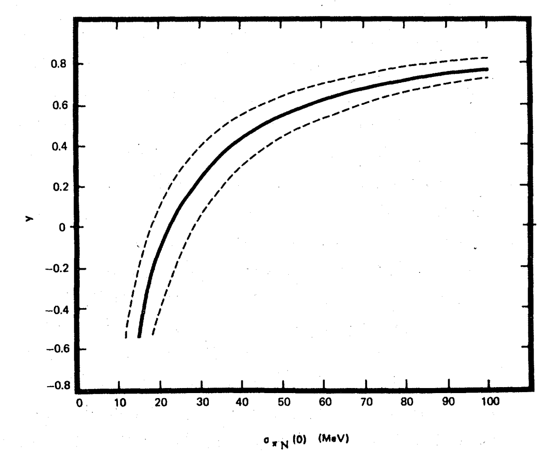

FIG. 4. The ratio $y \equiv \langle N|\bar{s}s|N\rangle/\langle N|\frac{1}{2}(\bar{u}u+\bar{d}d)|N\rangle$ vs $\sigma_{\pi N}(0)$ for $r_+=0.034\pm0.008$.

rather large ϕ to ω ratio, viz.,[30]

$$\left|\frac{g^V_{\phi NN}}{g^V_{\omega NN}}\right| \simeq 0.58. \tag{25}$$

This result has been confirmed later by dispersion-relation analyses which give[31]

$$\left|\frac{g^V_{\phi NN}}{g^V_{\omega NN}}\right| \simeq 0.47. \tag{26}$$

Though this supports our assumption it does not provide compelling evidence since the observed enhancements could be attributed to the coupling of an ω-type radial excitation with a mass around 1200 MeV. This state, however, has not yet been seen. If the above results are taken at face value and if we assume that y is equal to the corresponding ratio of vector-current matrix elements, then one obtains $y \simeq 0.61-0.81$, or $\sigma^{\text{thy}}_{\pi N}(0) \simeq 60-100$ MeV.

Another source of information on the ratio of ϕ to ω coupling constants is the nucleon-nucleon interaction at intermediate energies. Here, there are also some indications[32] of a large ratio with values in the range 0.3–0.4. It should be pointed out, however, that in this case the uncertainties are much more severe than in the electromagnetic case.

We therefore conclude that the OZI-rule ansatz for y is very questionable in the kinematic region relevant to the σ term and that values of $\sigma^{\text{thy}}_{\pi N}(0)$ considerably larger than 23 ± 5 MeV may be possible.

We comment now on the possibility of higher-order corrections to Eqs. (2) and (14). Gasser[13] has recently completed a detailed analysis of the leading nonanalytic and kinematic corrections to the meson and baryon mass formulas and to the formula (2) for $\sigma^{\text{thy}}_{\pi N}$. He concludes that these corrections increase the theoretical prediction (for $y=0$) by about 10 MeV. Similarly, Jaffe,[14] in the context of a specific chiral bag model, has suggested that nonlinear (in the quark masses) corrections to the matrix elements in (2) could increase the value of $\sigma^{\text{thy}}_{\pi N}(0)$ by as much as a factor of 2. A more mundane estimate of the effects of higher-order SU(3) breaking contributions to (2) is that $\sigma^{\text{thy}}_{\pi N}$ would be reduced by 6% if the combination $2(M_\Xi - M_\Lambda)$ were substituted for $(M_\Sigma + M_\Lambda - 2M_N)$. We conclude that the higher-order corrections to Eq. (2) are uncertain and model dependent, but an increase in $\sigma^{\text{thy}}_{\pi N}(0)$ by $\lesssim 10$ MeV (for $y=0$) is likely.

Another uncertainty comes from possible corrections to the Cheng-Dashen expansion as suggested by nonlinear effects due to the singularity at $t=(4\mu_\pi^2 - \mu_\pi^4/M^2) \approx 3.98\mu_\pi^2$. Clearly, it is important for the nonlinearity to remain small since otherwise the connection between the σ term and the pion-nucleon amplitude would be lost. According to Ref. 20 these nonlinear effects could be $\gtrsim 20\%$ although the results of Ref. 24 seem to indicate that they might be less important. It should also be repeated that if the nonlinear corrections to Eq. (14) are handled in the manner of EPCAC[17] then $\sigma^{\text{exp}}_{\pi N}(0)$ gets reduced by about 10%.

Finally, we wish to outline the status of the kaon-nucleon σ term, which is defined as[1]

$$\sigma^{\text{thy}}_{KN}(0) = \frac{1}{2}\sum_{\alpha=4,5} \langle N(p)|[{}^5Q^\alpha, [{}^5Q^\alpha, H(0)]]|N(p)\rangle . \tag{27}$$

Two things are important here, (i) the isospin-breaking contribution cannot be neglected as in $\sigma^{\text{thy}}_{\pi N}(0)$, and (ii) the impact of the OZI-rule ansatz is more pronounced. The isospin-breaking piece, measured by the ratio

$$r_- = \frac{m_d - m_u}{2m_s} , \tag{28}$$

can be extracted from, e.g., the $\Delta I = 1$ baryon mass differences, the $\eta \to 3\pi$ decays, and the ρ-ω mixing.[5] All these independent extractions give consistent values of $r_- \simeq 0.01$. We find then that

$$\sigma^{\text{thy}}_{KN}(0) = \begin{cases} 210 \text{ MeV} & (y=0), \\ 440 \text{ MeV} & (y=0.36). \end{cases} \tag{29}$$

Regarding the determination of $\sigma^{\text{exp}}_{\pi KN}(0)$ from experimental kaon-nucleon scattering data, the situation is highly unclear. For a discussion of the nature of the uncertainties and a recent analysis we refer the reader to the articles by DiClaudio, Rodriguez-Vargas, and Violini. These authors find that $\sigma^{\text{exp}}_{\pi KN}(0)$ lies in the range 493–687 MeV, although the statistical uncertainties are of about 717 MeV. We must add that the corrections of $O(\mu_K^4)$ to the current algebra and PCAC relation are clearly going to be more important than in the pion-nucleon case. Using EPCAC we would expect a reduction in $\sigma^{\text{exp}}_{\pi KN}(0)$ of about a factor of 2.0 ± 0.9, corresponding to a Goldberger-Treiman discrepancy in SU(3) × SU(3) of 0.30 ± 0.15.[17]

V. CONCLUSION

We have argued that the true uncertainty in $\sigma^{\text{exp}}_{\pi N}(0) \simeq 65$ MeV is not known and that small values (such as 30 MeV) for $\sigma^{\text{exp}}_{\pi N}$ cannot be definitively ruled out at the present time. In particular, the nominal error of ±6 MeV does not take into account the experimental errors in $T^{(+)}(v,t)$ or in the extrapolation to $\nu = 0$. At this point, it must be reiterated that our simpleminded fits to the πN amplitude results of Langbein (including the $\pi\pi$ scattering length constraint) are in no way an alternative to more complete analyses that make use of all available data. Our only purpose here has been to see whether or not a small value of the σ term was in obvious conflict with the data. Our fits indicate that a value $\sigma_{\pi N} \simeq 30$ MeV cannot be excluded, although a more thorough analysis will be needed before taking this result seriously. One clear weakness of our fits is the lack of $\pi\pi$ information apart from the $I=0$ scattering length. However, it must be pointed out that at the present time the primary data for the right-hand cut starts at $t \sim 16\mu_\pi^2$, i.e., very far away from the σ-term point $t=2\mu_\pi^2$. That the extrapolation of the data to $t=2\mu_\pi^2$ involves a great deal of model dependence is best illustrated by comparing the Durso-Jackson-VerWest analysis[24] of $\pi\pi \to N\bar{N}$ with that of Höhler *et al.*[20] Although they both agree fairly well in the region $4\mu_\pi^2 < t < 35\mu_\pi^2$ the first one assumes $\sigma_{\pi N}=0$, while the latter gives (after including the πN data) $\sigma_{\pi N} \simeq 65$ MeV.

We hope that these considerations will dramatize the urgent need for a renewed and concerted effort to pin down the πN and $\pi\pi$ low-energy parameters. In this regard, new experimental information to fill in the gaps in the πN and $\pi\pi$ data is a matter whose importance must be strongly emphasized.

The theoretical value for the σ term is also very uncertain. To leading order in SU(3) breaking and assuming the OZI ansatz: $\langle N|s\bar{s}|N\rangle = 0$ at $t=0$, one has $\sigma_{\pi N}^{\text{thy}}(0) = 23 \pm 5$ MeV. However, the OZI rule may be unreliable at $t=0$ because the effective QCD coupling is large. A simple estimate of the OZI rule violation by the Goldstone-boson-pair mechanism increases the prediction to $\sigma_{\pi N}^{\text{thy}}(0) = 36 \pm 8$ MeV, and even larger violations are possible. Furthermore, SU(3)-breaking effects possibly increase $\sigma_{\pi N}^{\text{thy}}$ by an additional 10 MeV (or more). Finally, nonlinear corrections to the Cheng-Dashen formula introduce an uncertainty of about 20% in the comparison between $\sigma_{\pi N}^{\text{thy}}$ and $\sigma_{\pi N}^{\text{exp}}$. On the other hand, a large deviation of the quark mass ratio $r_+ = (m_u + m_d)/2m_s$ from its canonical value is highly unlikely because of constraints from the K_{l3} system.

As a closing remark we wish to comment briefly on the status of other quantities which are also of first order in chiral-symmetry breaking and, therefore, provide additional tests of the underlying chiral structure of QCD. These quantities are the deviations from the Goldberger-Treiman and Adler-Weisberger relations and certain combinations of $\pi\pi$ and πN scattering lengths. Despite some minor disagreement between theory and experiment,[19] the Goldberger-Treiman discrepancy is certainly consistent with being of $O(\mu_\pi^2)$. Using the relevant combination of πN amplitudes as determined by Höhler *et al.*,[20] together with an axial-vector coupling $g_A \sim 1.23-1.25$, one finds that the deviation from the Adler-Weisberger relation is also of the same order. Next, a calculation of the $I=0$ and $I=2$ $\pi\pi$ scattering lengths using PCAC or EPCAC yields the unique prediction of their ratio

$$\frac{a_0^{\pi\pi}}{a_2^{\pi\pi}} = -\frac{7}{2}, \tag{30}$$

which is independent of the EPCAC corrections. The K_{l4} data[22] gives, instead, -9.3 ± 4.4, while the new information from pion production[23] gives, -3.34 ± 0.55, in remarkable agreement with Eq. (30). Since the $I=0$ scattering length is used as a constraint on the extraction of the σ term it is important to compare it by itself with experiment. The PCAC prediction is $a_0^{\pi\pi} = 0.156\ \mu_\pi^{-1}$, while in EPCAC one has $0.173 \pm 0.008\ \mu_\pi^{-1}$, to be compared with the K_{l4} result[22] of $0.26 \pm 0.05\ \mu_\pi^{-1}$ and the pion-production[23] value of $0.177 \pm 0.017\ \mu_\pi^{-1}$. It is important to point out that the latter value of the scattering length favors a smaller value of the σ term in our fits.

Finally, a calculation of the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ πN scattering lengths using PCAC (EPCAC) gives $a_{1/2}^{\pi N} = 0.155\ \mu_\pi^{-1}$ $(0.172 \pm 0.008\ \mu_\pi^{-1})$ and $a_{3/2}^{\pi N} = -0.078\ \mu_\pi^{-1}$ $(-0.087 \pm 0.004\ \mu_\pi^{-1})$, to be compared with the determinations of Höhler *et al*: $a_{1/2}^{\pi N} = 0.171 \pm 0.004\ \mu_\pi^{-1}$ and $a_{3/2}^{\pi N} = -0.105 \pm 0.003\ \mu_\pi^{-1}$. The disagreement of more than 5 standard deviations in $a_{3/2}^{\pi N}$ is clearly alarming. To draw the connection with the σ term one can evaluate the ratio of the so-called even and odd combinations, viz.,

$$\frac{a^{(+)}}{a^{(-)}} \equiv \frac{a_{1/2}^{\pi N} + 2a_{3/2}^{\pi N}}{a_{3/2}^{\pi N} - a_{1/2}^{\pi N}}, \tag{31}$$

which is proportional to $\sigma_{\pi N}$ and therefore, expected to be consistent with the other quantities of $O(\mu_\pi^2)$ discussed above. Using the values of Höhler *et al.*, one finds, instead, the large ratio $a^{(+)}/a^{(-)} = 0.14 \pm 0.03$, which reflects the large value of $\sigma_{\pi N}$ in that analysis. On the other hand, values of $\sigma_{\pi N}$ in the range 25–35 MeV would bring the ratio $a^{(+)}/a^{(-)}$ in line with the other tests.

In summary, our current ideas on chiral-symmetry breaking as implied by QCD seem well supported by such tests as the deviations from the Goldberger-Treiman and Adler-Weisberger relations and the $\pi\pi$ scattering lengths (when compared to the new pion-production data). A problem of consistency remains, though, in connection with the $I=\frac{3}{2}$ πN scattering length and the σ term. In view of all the uncertainties, both theoretical and experimental, involved in the extraction of $\sigma_{\pi N}$ we feel that this inconsistency does not provide, at the present time, compelling evidence against QCD. However, it should be reiterated that a renewed effort to improve the theoretical and experimental determinations of the relevant πN and $\pi\pi$ amplitudes is of the utmost importance.

ACKNOWLEDGMENTS

We would like to thank M. K. Banerjee, D. V. Bugg, T. P. Cheng, R. Dashen, G. Höhler, W. Langbein, H. Leutwyler, M. G. Olsson, H. Pagels, and B. J. VerWest for very useful discussions and correspondence. This work was supported in part by the U. S. Department of Energy; the work of C.A.D. under Contract No. DE-AS05-76ER05223 and the work of P.L. under Contract No. EY-76-02-3071.

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*On leave of absence from Centro de Investigación del Instituto Politécnico Nacional, Mexico.

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