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SO(10) model of fermion masses

S. M. Barr
Department of Physics, FM-15, University of Washington, Seattle, Washington 98195
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A model based on the grand unifying group SO(10) is described which leads to a realistic hierarchy of masses for the quarks and leptons. The SU(5) prediction $m_e \sim m_d$ is obviated in a simple way. Some simplification of the Higgs structure is achieved. Only Higgs representations appearing in the fermion-fermion operator are employed.

In this paper we wish to describe an SO(10) (Ref. 1) grand unified model in which a hierarchy[2] of fermion masses approximating that observed in nature emerges naturally. In this scheme the t, c, b, and τ masses are "tree-level" masses, the u, d, and e masses are one-loop radiative effects, and the μ and s masses may be either tree-level or one-loop in different versions of the model. The splitting of the e and d masses happens naturally in this model. Cabibbo mixing also occurs.

In Ref. 3 we described how, in grand unified models involving groups which break down to SU(5), a natural hierarchy among the masses of the light fermions may arise. [By "light fermions" we always mean those whose masses are $\lesssim 300$ GeV, as opposed to "ultraheavy fermions" whose masses are of the order of the grand unification (GU) breaking scale.] According to that idea the grand unified gauge symmetry itself is responsible for the masslessness of some fermions at tree level, and the breaking of the full symmetry down to SU(5) is responsible for the small, calculable masses they receive as radiative corrections. This can happen as follows. If the scalar fields whose vacuum expectation values (VEV's) break the $SU(2)_W$ of weak isospin (giving mass to the light fermions) represent the grand unified group G (which would not necessarily be the case were they composites as in so-called "hypercolor" models) then their Yukawa couplings to the fermions must respect the GU symmetry. In particular the GU symmetry may forbid (and in general it will) the presence of some Yukawa couplings necessary for certain light fermions to acquire mass at tree level. Other light fermions, however, may acquire tree-level masses from the $SU(2)_W$-breaking VEV's. Those light fermions which remained massless at tree level as a consequence of the GU symmetry will not stay massless to all orders, however, because the GU symmetry gets broken to SU(5). They will acquire radiative masses at one-loop (or higher) order. The diagrams which give them mass must "know" that the GU group is broken. They will if they contain, as virtual particles, particles which acquire ultralarge masses when G breaks down to SU(5). Typical diagrams are shown in Fig. 1. Since the light-fermion masses break $SU(2)_W$, these diagrams must contain an $SU(2)_W$-breaking VEV somewhere. [In Fig. 1(a), it couples to the virtual ultraheavy fermions; in Fig. 1(b) it couples to the virtual ultraheavy boson.] Thus the loop masses are proportional to $SU(2)_W$ breaking and in fact turn out to be roughly of $O((\alpha/\pi)m_f)$ where m_f is a tree-level light-fermion mass.

In this paper we will describe a realistic SO(10) model in which precisely this mechanism operates.

The usual SO(10) models contain three 16-dimensional complex SO(10) spinors of fermions. Let us supplement these by two real fermion representations: a 10 and a 45 of Weyl spinors. Under SU(5) the SO(10) fermion representations decom-

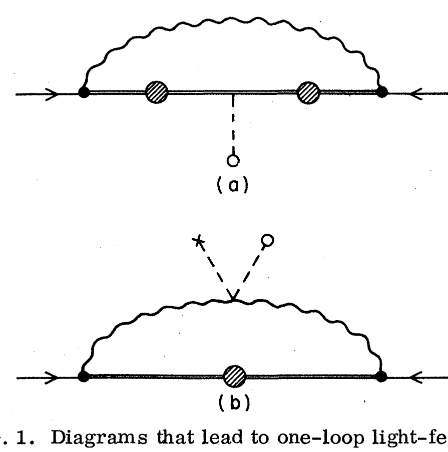

FIG. 1. Diagrams that lead to one-loop light-fermion masses. Single straight lines are light fermions ($m < 300$ GeV). Double straight lines are ultraheavy fermions ($m \sim 10^{15}$ GeV). Virtual bosons are ultraheavy. An open circle represents $SU(2)_W$-breaking VEV. A cross represents an ultralarge SO(10)-breaking VEV. Arrows on fermions refer to helicity.

pose as follows:

$$16^I_L \to 1 + 10 + 5^*, \quad I = 1, 2, 3$$
$$10_L \to 5 + 5^*, \qquad (1)$$
$$45_L \to 10 + 10^* + 24 + 1 .$$

By the "survival hypothesis"[4] what will remain at the SU(5) level is precisely three 10's and three 5*'s of fermions. (We choose to deal, as usual, only with left-handed Weyl spinors.) A $(10+10^*)$, a $(5+5^*)$, the 24, and the 1's will become ultraheavy because they are real representations of SU(5).

To do the various breakings let us have the following Higgs fields: One set of Higgs fields in the adjoint representation of SO(10), denoted by 45_H, and more than one (say k) SO(10)-spinor representations of Higgs fields, denoted by 16^K_H ($K = 1, \ldots, k > 1$). (The reason for having more than one 16_H will be seen later.) We will henceforth refer to a p-dimensional representation of SU(5) embedded in a q-dimensional representation of SO(10) as a $p(q)$. We assume that the $1(16^K_H)$ and $1(45_H)$ develop ultralarge ($>10^{15}$ GeV) VEV's that break SO(10) down to SU(5).[5] The $24(45_H)$ adjoint Higgs field is assumed to develop a superlarge ($\sim 10^{15}$ GeV) VEV of the usual form

$$\langle 24(45_H)\rangle = \omega \begin{bmatrix} \frac{1}{2} & & & & \\ & \frac{1}{2} & & & \\ & & -\frac{1}{3} & & \\ & & & -\frac{1}{3} & \\ & & & & -\frac{1}{3} \end{bmatrix} , \qquad (2)$$

which breaks SU(5) down to $SU(3)_c \times SU(2) \times U(1)$.[6] Finally, the several $5^*(16^K_H)$ develop VEV's of the form

$$\langle 5^*(16^K_H)\rangle = \begin{bmatrix} 0 \\ V^K \\ 0 \\ 0 \\ 0 \end{bmatrix} , \qquad (3)$$

which break $SU(2)_W \times U(1)_y$ down to $U(1)_{EM}$ at ~300 GeV. Of course this model has the notorious gauge hierarchy problem, a problem which we will ignore.

To understand the fermion masses we must look at the Yukawa couplings and Lagrangian fermion mass terms. They can be written schematically as

$$\mathcal{L}_{\text{Yukawa}} = \sum_{I=1}^{3}\sum_{K=1}^{k} a_{IK}(16^I_L 16^K_H 10_L) + \sum_{I=1}^{3}\sum_{K=1}^{k} b_{IK}(16^I_L 16^{K*}_H 45_L) + m_1(10_L 10_L) + m_2(45_L 45_L) + \text{H.c.} \qquad (4)$$

Altogether, in this model there are the following SU(5) representations of fermions:

$$\begin{aligned} &5(10_L),\ \ 5^*(10_L),\ \ 5^*(16^I_L),\\ &10^*(45_L),\ \ 10(45_L),\ \ 10(16^I_L),\ \ I=1,2,3\\ &1(45_L),\ 1(16^I_L),\\ &24(45_L). \end{aligned} \qquad (5)$$

We will look at these in turn to see which acquire ultraheavy mass and which remain light.

For the five-dimensional representations of SU(5) we have the ultralarge mass terms

$$\begin{aligned} &5(10_L) m_1 5^*(10_L),\\ &\sum_{I=1}^{3} 5(10_L)\left(\sum_{K=1}^{k} a_{IK}\langle 1(16^K_H)\rangle\right) 5^*(16^I_L). \end{aligned} \qquad (6)$$

Clearly the $5(10_L)$ and one linear combination of the four fields $5^*(10_L)$ and $5^*(16^I_L)$, $I=1,2,3$, become ultramassive, while the three orthogonal linear combinations of the $5^*(10_L)$ and $5^*(16^I_L)$ remain light.

This can be seen more clearly if we write Eq. (6) in matrix form:

$$(5^*(16^1_L), 5^*(16^2_L), 5^*(16^3_L), 5^*(10_L)) \times \begin{bmatrix} a_{1K}\langle 1(16^K_H)\rangle \\ a_{2K}\langle 1(16^K_H)\rangle \\ a_{3K}\langle 1(16^K_H)\rangle \\ m_1 \end{bmatrix} 5(10_L). \qquad (6')$$

This 4×1 mass matrix can be "diagonalized" to be brought to the form

$$\begin{bmatrix} m \\ 0 \\ 0 \\ 0 \end{bmatrix} .$$

Thus we see that three 5*'s are light and one is superheavy.

Similarly, the ten-dimensional SU(5) representations of fermions have the ultralarge mass terms

$$\begin{aligned} &10^*(45_L) m_2 10(45_L),\\ &\sum_{I=1}^{3} 10^*(45_L)\left(\sum_{K=1}^{k} b_{IK}\langle 1(16^{K*}_H)\rangle\right) 10(16^I_L). \end{aligned} \qquad (7)$$

Again $10^*(45_L)$ and one linear combination of the four fields $10(45_L)$ and $10(16_L^K)$ become ultramassive, while the three orthogonal linear combinations of the $10(45_L)$ and $10(16_L^K)$ remain light.

As before we can write Eq. (7) in matrix form:

$$(10(16_L^1),10(16_L^2),10(16_L^3),10(45_L))\begin{pmatrix} b_{1K}\langle 1(16_L^{K*})\rangle \\ b_{2K}\langle 1(16_L^{K*})\rangle \\ b_{3K}\langle 1(16_L^{K*})\rangle \\ m_2 \end{pmatrix} 10^*(45_L)\ . \tag{7'}$$

Again this 4×1 mass matrix can be diagonalized to find the three linear combinations of the 10_L that remain light.

The question of the masses of the SU(5)-singlet fermions (some of which are right-handed neutrinos) is slightly more complicated. There are ultralarge mass terms

$$\begin{gathered} 1(45_L)m_2 1(45_L)\ , \\ \sum_{I=1}^{3} 1(45_L)\Big(\sum_{K=1}^{k} b_{IK}\langle 1(16_H^{K*})\rangle\Big)1(16_L^I)\ . \end{gathered} \tag{8}$$

Of the four SU(5)-singlet fermions, two thus get ultramassive at tree level. However, as shown in Fig. 2, there are two-loop mass terms that render the remaining SU(5)-singlet fermions supermassive as well.

And, finally, the $24(45_L)$ of fermions has a mass m_2 which is ultraheavy.

So, remaining as light fermions of the list in (5) are *three* 5_L^*, which are linear combinations of the four fields $5^*(10_L)$ and $5^*(16_L^I)$, $I=1,2,3$, and *three* 10_L which are linear combinations of the four fields $10(45_L)$ and $10(16_L^I)$, $I=1,2,3$.

Now let us investigate the light (<300 GeV) $SU(2)_W$-breaking masses of these light fermions. These masses come from $\langle 5^*(16_H^K)\rangle$. The relevant Yukawa couplings are of the form

$$\sum_{I=1}^{3}\sum_{K=1}^{k} a_{IK}10(16_L^I)5^*(16_H^K)5^*(10_L) + \sum_{I=1}^{3}\sum_{K=1}^{k} b_{IK}10(16_L^I)5(16_H^{K*})10(45_L) + \text{H.c.} \tag{9}$$

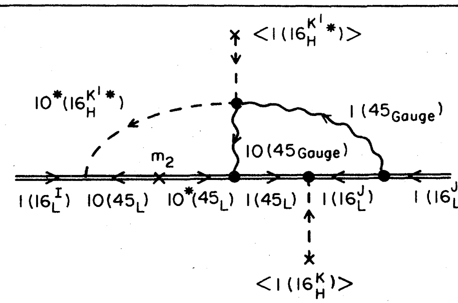

FIG. 2. A two-loop diagram leading to large masses for the SU(5)-singlet fermions. There are other such diagrams.

The first term leads to tree-level masses for the charged leptons and charge $-\frac{1}{3}$ quarks, which we will refer to as "down fermions." The second term leads to tree-level masses for the charge $+\frac{2}{3}$ quarks, which we will refer to as the "up quarks."[7] However, *not all* of the quarks and leptons acquire tree-level mass from these terms. This is not difficult to see. In the $10_L 5_H^* 5_L^*$ coupling, clearly only *one* linear combination of the light 5_L^* fermions is coupled to only *one* linear combination of the light 10_L fermions by the $\sim 10^2$ GeV mass resulting from $\langle 5^*(16_H^K)\rangle$. So the down leptons and the down quarks have mass matrices at tree level of the form

$$m_{\text{down}} = \begin{pmatrix} A & & \\ & 0 & \\ & & 0 \end{pmatrix}. \tag{10}$$

We can associate the quark and lepton which thus acquire tree-level masses with the b and τ.

In the $10_L 5_H 10_L$ coupling the situation is almost as simple. The mass term that results from $5(16_H^{K*})$ couples a linear combination of the light 10_L fermions to some other[7] (but *not orthogonal*) linear combination of the light 10_L fermions. [This mass term couples a 10_L to another 10_L which *is* orthogonal in the four-space spanned by $10(45_L)$ and $10(16_L^I)$, but their projections into the three-dimensional subspace of light 10_L fermions are not orthogonal.] So the up-quark mass matrix at tree level has the form

$$m_{\text{up}} = \begin{pmatrix} B & C & \\ C & 0 & \\ & & 0 \end{pmatrix}. \tag{11}$$

Thus two up quarks have mass at tree level, the t and the c. If $B>C$, then $m_t/m_c \cong B^2/C^2$ which can nicely account for the large ratio of their masses.

For the down fermions there are one-gauge-loop diagrams that contribute to the mass matrix terms of order α. One of these is exhibited in Fig. 3. The down-fermion mass matrices therefore have the form, after diagonalizing,

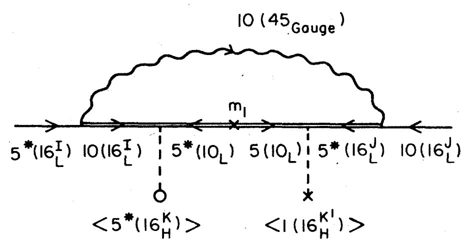

FIG. 3. Diagram leading to one-gauge-boson-loop down-fermion mass.

$$m_{\rm down}=\begin{pmatrix} A & & \\ & O(\alpha)A & \\ & & O(\alpha)A\end{pmatrix}. \tag{12}$$

The μ, s, e, and d masses, therefore, in this version of the model are all of similar magnitude. (We will describe a modified version shortly in which the μ and s masses arise at tree level.)

There are also one-Higgs-boson loop diagrams which contribute to the down-fermion mass matrices. One is shown in Fig. 4. One might expect these to be smaller than the one-gauge-loop diagrams. However, this is not the case. The diagram in Fig. 3 goes like

$$m(\text{Fig. 3}) \sim g^2 m_1 a\langle 1\rangle a\langle 5^*\rangle \frac{1}{m_X{}^2}\ln\left(\frac{m_F}{m_X}\right)^2,$$

where a is a Yukawa coupling, m_F is an ultraheavy-fermion mass, and $m_X{}^2$ is the ultraheavy-gauge-boson mass^2 $\sim g^2\langle 1\rangle^2$. So

$$m(\text{Fig. 3}) \sim \left(a^2\frac{m_1}{a\langle 1\rangle}\ln m_X{}^2\right)\times a\langle 5^*\rangle .$$

Figure 4 goes (crudely) as

$$m(\text{Fig. 4}) \sim a^2\lambda m_1\langle 1\rangle\langle 5^*\rangle \frac{1}{m_H{}^2}\ln\left(\frac{m_F}{m_H}\right)^2,$$

where λ is a quartic Higgs-boson coupling, and $m_H{}^2$ is an ultraheavy-Higgs-boson mass^2 $\sim\lambda\langle 1\rangle^2$. So

$$m(\text{Fig. 4}) \sim \left(a^2\frac{m_1}{a\langle 1\rangle}\ln m_H{}^2\right)\times a\langle 5^*\rangle .$$

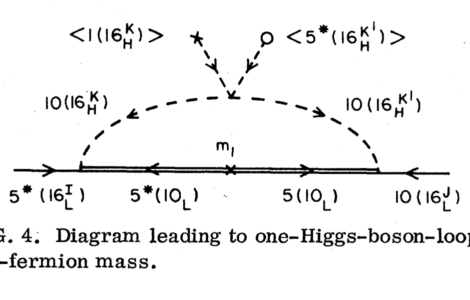

FIG. 4. Diagram leading to one-Higgs-boson-loop down-fermion mass.

The gauge-boson loops and Higgs-boson loops are obviously comparable. (If $\lambda \ll a^2$, so that m_H is less than the ultraheavy-fermion mass, the Higgs-boson loop diagram goes not as $1/m_H{}^2$ but as $1/m_F{}^2$. Then the Higgs-boson loop diagrams may indeed be smaller than the gauge-boson loop diagrams.)

Among the up quarks, two (t and c) had tree-level masses and one (u) was massless at tree level [see (11)]. The u acquires a one-loop mass from both a gauge-boson loop diagram (Fig. 5) and a Higgs-boson loop diagram (Fig. 6).

Altogether, then, this model has the t, c, b, and τ getting tree masses and the u, s, μ, d, and e getting loop masses.

A variant of the above model that has certain advantages can be obtained by having *two*, instead of merely one, 10-dimensional SO(10) representations of fermions. The important new feature is that in addition to the $10(16_L)5^*(16_H)5^*(10_L)$ Yukawa interaction shown in (9) there is another: $10_L(16_L)5^*(16_H)5^*(10_L')$ where $10_L'$ is the new additional fermion representation. Then the tree-level down-fermion mass matrices are of the form, after diagonalization,

$$m_{\rm down}=\begin{pmatrix} A & & \\ & A' & \\ & & 0\end{pmatrix}. \tag{13}$$

In this variant of the model the μ and s masses arise at tree level.

In this second version of the model the 45 of Higgs bosons can couple to the fermions via a term of the form $c10_L 45_H 10_L'$. In this case the above picture is somewhat modified. It remains true that three families of fermions, each consisting of an SU(5)10_L and 5_L^* remain light after ultraheavy symmetry breaking [SO(10) $\to$ SU(5) $\to$ SU(3) $\times$ SU(2) $\times$ U(1)]. Also the two-tier hierarchy we have described still arises. However, the VEV of the $24(45_H)$ shown in Eq. (2) breaks SU(5). Therefore, the relations $m_e \approx m_d$, $m_\mu \approx m_s$, and $m_\tau \approx m_b$ are disturbed. This effect comes from three sources.

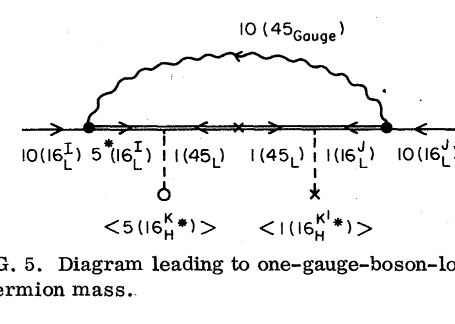

FIG. 5. Diagram leading to one-gauge-boson-loop up-fermion mass.

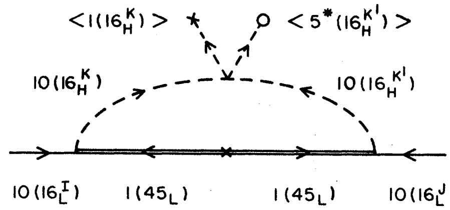

FIG. 6. Diagram leading to one-Higgs-boson-loop up-fermion mass.

(i) The ultraheavy mass terms are changed. In (6), there is an additional term of the form $c5(10_L)24(45_H)5^*(10'_L)$. (Also m_1 is now a 2×2 matrix.) So the linear combinations of fields that remain as light fermions are modified. For example, b and τ no longer belong to a single 5^* of SU(5).

(ii) The ultraheavy gauge bosons and Higgs bosons in Figs. 3 and 4 have masses which are not SU(5) invariant.

(iii) In Figs. 3 and 4, there are contributions from diagrams in which the mass insertion m_1 is replaced by $c\langle 24(45_H)\rangle$.

These effects can explain the deviations of m_e/m_d and m_μ/m_s from the values predicted in the simplest SU(5) model. Because m_e and m_d are one-loop effects the relation $m_e \approx m_d$ is modified by effect (iii), whereas the relations $m_\mu \approx m_s$ and $m_\tau \approx m_b$ are not.

In summary, we have exhibited a model in which broken SO(10) gauge symmetry is responsible for producing a hierarchy among light-fermion masses. The resulting pattern of masses looks fairly realistic: The t, c, b, and τ are heavy, while the u, d, and e are light and arise as $O(\alpha)$ radiative effects. The μ and s can have tree-level or radiative masses in different versions of the model. A fairly simple explanation of $m_e \neq m_d$ is possible in this model. The model is fairly economical, requiring few Higgs fields, which belong to representations appearing in the fermion-fermion operator.

Certainly some questions remain. For example, the model predicts that Cabibbo mixing will occur, but seems not to require the mixing angles to be small.

It would also be desirable to achieve a greater degree of unification, and explain the choice of fermion and Higgs representations in the context of some larger group.

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[7]The reason for having more than one 16 of Higgs bosons is that with only a single such Higgs 16 it will happen that the SU(2)-breaking mass term will only give tree-level mass to a single light up quark.