PHYSICAL REVIEW D VOLUME 24, NUMBER 7 1 OCTOBER 1981

Some consequences of extending the SU(5) gauge symmetry to the generation symmetry $SU(5)^e \times SU(5)^\mu \times SU(5)^\tau$

S. Rajpoot
Physics Department, Imperial College, London SW7 2BZ, England
(Received 17 November 1980)

The possibility that each fermion generation has its own flavor-color $SU(3)_c^i \times SU(2)_L^i \times U(1)^i$ ($i = e, \mu, \tau$) symmetry is entertained in the $SU(5)^e \times SU(5)^\mu \times SU(5)^\tau$ generation symmetry. It is shown that its subsequent descent to the low-energy symmetry $SU(3)_c \times SU(2)_L \times U(1)$ through several intermediate stages leads to (i) a renormalized weak angle $\sin^2\theta_W$ that lies within the presently allowed experimental range, and (ii) low intermediate mass scales that can fill the energy gap between 10^2 GeV and 10^{14}–10^{15} GeV encountered in the simple SU(5) theory.

I. INTRODUCTION

Present-day attempts at building a realistic and perhaps aesthetically appealing unified picture of the interactions that govern the realm of elementary particles hinge upon the hypotheses that (i) these interactions—at least the strong, weak, and electromagnetic—are described by a gauge symmetry that is *broken*, and (ii) the disparity in the strengths of these interactions is the consequence of the decoupling theorem of Appelquist and Carazzone.[1]

The precise mechanism of symmetry breaking is still unclear. It could be either spontaneous, needing elementary scalars, or dynamical, needing composite scalars formed from exotic fermions. An example of (i) that has empirical support is the electroweak theory of Glashow, Salam, and Weinberg[2] that partially unifies the weak and the electromagnetic forces. In this theory the spontaneous breakdown[3] of the $SU(2)_L \times U(1)$ gauge symmetry to $U(1)_{em}$ is manifested in the relation $(M_W/M_{Z^0}\cos\theta_W)^2 = 1$ and is in remarkable agreement with the experimental value[4]

$$(M_W/M_{Z^0}\cos\theta_W)^2 = 0.985 \pm 0.023\,. \tag{1.1}$$

Here M_W and M_{Z^0} are the masses of the charged and neutral gauge bosons and θ_W, better known in the form of $\sin^2\theta_W$, is the weak angle. That the application of (i) and (ii) does lead to a *nearly successful* synthesis of the strong, weak, and electromagnetic interactions was first demonstrated by Georgi, Quinn, and Weinberg[5] in the SU(5) grand unified theory of Georgi and Glashow.[6] For the hierarchy

$$SU(5) \xrightarrow[M_G]{} SU(3)_c \times SU(2)_L \times U(1) \xrightarrow[M_W \sim 10^2\,\mathrm{GeV}]{} SU(3)_c \times U(1)_{em}$$

they showed that the renormalized weak-angle ($\sin^2\theta_W$) expression at the renormalization point $\mu = M_W$ and the grand unifying mass M_G are given by[5]

$$\sin^2\theta_W(M_W) = \frac{1}{6} + \frac{5}{9}\,\frac{\alpha(M_W)}{\alpha_s(M_W)}, \tag{1.2}$$

$$\ln\frac{M_G}{M_W} = \frac{1}{6e^2K}\left(1 - \frac{8\alpha(M_W)}{3\alpha_s(M_W)}\right), \tag{1.3}$$

where α (α_s) is the electromagnetic (strong) fine-structure constant and $e^2K = 11\alpha(M_W)/6\pi$. For $\alpha(M_W) \simeq 1/128.5$, $\alpha_s(M_W) = 0.13$, Eq. (1.3) gives M_G of order 10^{14} to 10^{15} GeV which leads to the proton lifetime falling within the experimentally allowed value of $10^{30\pm 2}$ yr. For the same values of α and α_s, Eq. (1.2) gives $\sin^2\theta_W \simeq 0.20$, close to the world-average experimental value[7] $\sin^2\theta_W|_{exp} = 0.23 \pm 0.015$. However, this small discrepancy between the predicted and the experimental value of $\sin^2\theta_W$, the enormous energy gap between M_W ($\simeq 10^2$ GeV) and M_G ($\simeq 10^{14}$ to 10^{15} GeV), and the fact that there are three generations of fermions, each treated sequentially, suggests that the simple SU(5)-based elementary-particle synthesis is far from complete.

The straightforward embedding of the low-energy interaction symmetry $H = SU(3)_c \times SU(2)_L \times U(1)$ together with the generation symmetry $SU(2), SU(3)\ldots$ of the electron, muon, and τ families into a symmetry larger than SU(5) is uninteresting since such an extension leads to the same weak-angle and energy-gap predictions as in the simple SU(5) theory. Alternative ways of enlarging H are to consider the following three possibilities.

(i) Each family has a distinct flavor $SU(2)_L \times U(1)$ symmetry but shares a common color $SU(3)_c$ symmetry, i.e.,

$$H \to [SU(2)_L \times U(1)]^e \times [SU(2)_L \times U(1)]^\mu \times [SU(2)_L \times U(1)]^\tau \times SU(3)_c\,.$$

(ii) Each family has a distinct color $SU(3)_c$ symmetry but shares a common flavor $SU(2)_L \times U(1)$ symmetry, i.e.,

$H \to SU(2)_L \times U(1) \times SU(3)^e_c \times SU(3)^\mu_c \times SU(3)^\tau_c$.

(iii) Each family has a flavor and color symmetry of its own in which case

$H \to H^e \times H^\mu \times H^\tau$, where $H^i = SU(3)^i_c \times SU(2)^i_L \times U(1)^i$ $(i=e, \mu, \tau)$.

The last possibility, of which the former two are special cases, is realized in the generation gauge symmetry $G = SU(5)^e \times SU(5)^\mu \times SU(5)^\tau$ considered first by Elias and Salam[8] to explain the hierarchy of fermion masses between the different generations. In their work it was assumed that the descent of the intermediate stage $H^e \times H^\mu \times H^\tau$ to H involved a single mass scale and the prediction for the renormalized weak angle $\sin^2\theta_W$ turned out to be the same as in the simple SU(5) theory in Eq. (1.2). The present work is a generalization of their work (Ref. 8). It is shown that the subsequent descent of G to H can involve several intermediate mass scales and the renormalized weak angle $\sin^2\theta_W$ turns out to be a probe of the intermediate region between $H^e \times H^\mu \times H^\tau$ and H.

II. THE MODEL

Grand unification based on the generation gauge symmetry $G = SU(5)^e \times SU(5)^\mu \times SU(5)^\tau$ is a straightforward generalization of the simple SU(5) theory. In it the electron, the muon, and the τ families are each assigned an SU(5) gauge symmetry of their own. Thus the fermions of each family form the fundamental 5 (ψ^i_5) and the antisymmetric 10 (ψ^i_{10}) dimensional representations of their generic $SU(5)^i$ $(i=e, \mu, \tau)$ and anomaly cancellation proceeds just as in the simple SU(5) theory. A set of discrete symmetries $e \leftrightarrow \mu \leftrightarrow \tau$ is imposed on G for one coupling constant, i.e., $g_e = g_\mu = g_\tau$.

Let g be the common gauge coupling of the generation group G. The interaction of the fermions in ψ^i_5 and ψ^i_{10} $(i=e, \mu, \tau)$ with the gauge bosons of G is given by the Lagrangian L_I, where

$$L_I = ig\,\bar{\psi}^e_{5\alpha}\gamma_\mu\left(\frac{\lambda\cdot W^e_\mu}{2}\right)_{\alpha\beta}\psi^e_{5\beta} + ig\,\bar{\psi}^e_{10\alpha\beta}\gamma_\mu\left(\frac{\lambda\cdot W^e_\mu}{2}\right)_{\beta\gamma}\psi^e_{10\gamma\alpha} + (e\to\mu) + (\mu\to\tau)\,. \tag{2.1}$$

The charge assignment of the fermions, i.e., fractional charges for the quarks and integer charges for the leptons, fixes the structure of the photon field A and the electric charge e to be

$$\frac{A}{e} = \frac{W^{0(e)}_L}{g^{(e)}_2} + \frac{W^{0(\mu)}_L}{g^{(\mu)}_2} + \frac{W^{0(\tau)}_L}{g^{(\tau)}_2} + \left(\frac{5}{3}\right)^{1/2}\left(\frac{B^{0(e)}}{g^{(e)}_5} + \frac{B^{0(\mu)}}{g^{(\mu)}_5} + \frac{B^{0(\tau)}}{g^{(\tau)}_5}\right); \tag{2.2}$$

$$\frac{1}{e^2} = \left(\frac{1}{g^{(e)2}_2} + \frac{1}{g^{(\mu)2}_2} + \frac{1}{g^{(\tau)2}_2}\right) + \frac{5}{3}\left(\frac{1}{g^{(e)2}_5} + \frac{1}{g^{(\mu)2}_5} + \frac{1}{g^{(\tau)2}_5}\right). \tag{2.3}$$

Thus the weak-isospin current of the e, μ, and τ families couples to the diagonally summed individual weak-isospin gauge fields $W^{0(i)}_L$ $(i=e, \mu, \tau)$ in Eq. (2.2) and the resultant SU(2) $[\equiv SU(2)^{e+\mu+\tau}]$ weak-isospin coupling $g_2(\mu)$ at the renormalization point μ is given by

$$\frac{1}{g_2{}^2(\mu)} = \frac{1}{g^{(e)2}_2(\mu)} + \frac{1}{g^{(\mu)2}_2(\mu)} + \frac{1}{g^{(\tau)2}_2(\mu)}\,. \tag{2.4}$$

Similarly, for the weak-hypercharge current the resultant U(1) $[\equiv U(1)^{e+\mu+\tau}]$ coupling $g_1(\mu)$ is given by

$$\frac{1}{g_1{}^2(\mu)} = \frac{5}{3}\left(\frac{1}{g^{(e)2}_5(\mu)} + \frac{1}{g^{(\mu)2}_5(\mu)} + \frac{1}{g^{(\tau)2}_5(\mu)}\right). \tag{2.5}$$

From Eqs. (2.3) and (2.4) and the usual definition of the weak angle,

$$\sin^2\theta_W(\mu) = e^2(\mu)/g_2{}^2(\mu)\,, \tag{2.6}$$

the bare value of the weak angle at energy scales where all couplings are equal is $\frac{3}{8}$, just as in the simple SU(5) theory. However, the renormalized weak angle turns out to be different since G descends to H through more than one intermediate symmetry stage. This and its consequences are discussed in what follows.

III. CONSEQUENCES

In the primary descent, each $SU(5)^i$ in the generation group G descends to its subgroup $H^i = SU(3)^i_c \times SU(2)^i_L \times U(1)^i$ $(i=e, \mu, \tau)$ through mass scales M^e_5, M^μ_5, and M^τ_5. In the secondary descent a possible scenario by which the surviving residual symmetry $H^e \times H^\mu \times H^\tau = SU(3)^e_c \times SU(3)^\mu_c \times SU(3)^\tau_c \times SU(2)^e_L \times SU(2)^\mu_L \times SU(2)^\tau_L \times U(1)^e \times U(1)^\mu \times U(1)^\tau$ descends to the observed low-energy symmetry $SU(3)^{e+\mu+\tau}_c \times SU(2)^{e+\mu+\tau}_L \times U(1)^{e+\mu+\tau} \equiv SU(3)_c \times SU(2)_L \times U(1)$ is the following: For energies below the intermediate mass scales $(M^e_5, M^\mu_5, M^\tau_5)$, each family has its own flavor $SU(2)^i_L \times U(1)^i$ and color $SU(3)^i_c$ $(i=e, \mu, \tau)$ symmetries. At energies equal to intermediate mass scales $(M^{\mu+\tau}_3, M^{\mu+\tau}_2, M^{\mu+\tau}_1)$ the flavor and color symmetries of the muon and the τ families combine into a common flavor weak-isospin $SU(2)^{\mu+\tau}$, weak-hypercharge $U(1)^{\mu+\tau}$, and color $SU(3)^{\mu+\tau}_c$. With further descent in energy, the electron, muon, and τ families evolve with $[SU(2)^e, SU(2)^{\mu+\tau}, U(1)^e, U(1)^{\mu+\tau}]$ flavor and $[SU(3)^e_c,$

$SU(3)_c^{\mu+\tau}$] color symmetries until energies equal to intermediate mass scales $M_3^{e+\mu+\tau} M_2^{e+\mu+\tau} M_1^{e+\mu+\tau}$ are reached. At these mass scales the flavor and color symmetries of the electron, which up to now were evolving on their own, unite with the corresponding symmetries of the muon and τ families to give the observed $SU(3)_c^{e+\mu+\tau}$ of color and $SU(2)^{e+\mu+\tau} \times U(1)^{e+\mu+\tau}$ of flavor electroweak interactions. Thus the various subsymmetry stages in the secondary descent of $H^e \times H^\mu \times H^\tau$ to the observed symmetry $H = SU(3)_c \times SU(2)_L \times U(1)$ divide the energy scale into regions of energy with boundaries marked by the intermediate mass scales $(M_a^{\mu+\tau}, M_a^{e+\mu+\tau})$ $(a=1,2,3)$. The effects of various mass scales are summarized below:

$$SU(3)_c^e \times SU(3)_c^\mu \times SU(3)_c^\tau \xrightarrow[M_3^{\mu+\tau}]{} SU(3)_c^e \times SU(3)_c^{\mu+\tau} \xrightarrow[M_3^{e+\mu+\tau}]{} SU(3)_c^{e+\mu+\tau} \equiv SU(3)_c\,,$$

$$SU(2)^e \times SU(2)^\mu \times SU(2)^\tau \xrightarrow[M_2^{\mu+\tau}]{} SU(2)^e \times SU(2)^{\mu+\tau} \xrightarrow[M_2^{e+\mu+\tau}]{} SU(2)^{e+\mu+\tau} \equiv SU(2)_L\,,$$

$$U(1)^e \times U(1)^\mu \times U(1)^\tau \xrightarrow[M_1^{\mu+\tau}]{} U(1)^e \times U(1)^{\mu+\tau} \xrightarrow[M_1^{e+\mu+\tau}]{} U(1)^{e+\mu+\tau} \equiv U(1)\,.$$

Their magnitude characterizes energies at which new interactions are manifested with strengths given by the running coupling constants $g_a^e(\mu)$, $g_a^\mu(\mu)$, $g_a^\tau(\mu)$, $g_a^{\mu+\tau}(\mu)$, $g_a^{e+\mu+\tau}(\mu)$, where $a=1,2,3$ correspond to U(1), SU(2), and SU(3) groups, respectively. For instance, in weak interactions the three families share the same weak-isospin coupling constant $g_2^{e+\mu+\tau} \equiv g_2$ for energies below the mass scale $M_2^{e+\mu+\tau}$. At energies equal to $M_2^{e+\mu+\tau}$, g_2 splits into g_2^e of $SU(2)^e$ and $g_2^{\mu+\tau)}$ of the $SU(2)^{\mu+\tau}$. Thus $M_2^{e+\mu+\tau}$ signifies the mass scale at which the electron-muon *universality is lost.* Further ascent in energy splits $g_2^{\mu+\tau}$ into g_2^μ and g_2^τ at mass scale $M_2^{\mu+\tau}$. This "cascading" of the coupling constant is also believed to occur with $g_3^{e+\mu+\tau}$ $(\equiv g_3)$ and $g_1^{e+\mu+\tau}$ $(\equiv g_1)$. A special case is the situation in which the $SU(2)_L$ universality of the coupling g_2 between the families could be lost at one go[9] at a common mass scale $M_2^{e+\mu+\tau}$ instead of two stages at mass scales $M_2^{\mu+\tau}$ and $M_2^{e+\mu+\tau}$. However, due to a lack of knowledge at present as to how the universality breaks down, if at all, the discussion is left general.

Application of the decoupling theorem of Applequist and Carazzone leads to the following effective coupling constants for various energy regimes:

$M_{3,2,1}^{\mu+\tau}, M_{3,2,1}^{e+\mu+\tau} < \mu < M_5^{e,\mu,\tau}$:

$$\frac{1}{g_a^{(i)2}(\mu)} = \frac{1}{g^2(M_5^i)} + A_a \ln\left(\frac{M_5^i}{\mu}\right), \quad i=e,\mu,\tau\,; \tag{3.1}$$

$M_{3,2,1}^{e+\mu+\tau} < \mu < M_{3,2,1}^{\mu+\tau}$:

$$\frac{1}{g_a^{(\mu+\tau)2}(\mu)} = \frac{1}{g_a^{(\mu)2}(M_a^{\mu+\tau})} + \frac{1}{g_a^{(\tau)2}(M_a^{\mu+\tau})} + B_a \ln\left(\frac{M_a^{\mu+\tau}}{\mu}\right); \tag{3.2}$$

$M_W < \mu < M_{3,2,1}^{e+\mu+\tau}$:

$$\frac{1}{g_a^{2}(\mu)} = \frac{1}{g_a^{(e)2}(M_a^{e+\mu+\tau})} + \frac{1}{g_a^{(\mu+\tau)2}(M_a^{e+\mu+\tau})} + C_a \ln\left(\frac{M_a^{e+\mu+\tau}}{\mu}\right), \quad a=1,2,3\,; \tag{3.3}$$

where $A_1=b$, $A_2=(b-2K)$, $A_3=b-3K$, $B_1=2b$, $B_2=2b-2K$, $B_3=2b-3K$, $C_1=3b$, $C_2=3b-2K$, $C_3=3b-3K$, $K=11/24\pi^2$, b is the total contribution from the fermion multiplets, $(1/12\pi^2)(\frac{1}{4})$ from the fundamental 5-dimensional representation and $(1/12\pi^2)(\frac{3}{4})$ from the antisymmetric 10-dimensional representation, and M_W (~100 GeV) is the mass of the charged gauge boson of $SU(2)_L^{e+\mu+\tau} \equiv SU(2)_L$.

From Eqs. (2.6), (3.1), (3.2), and (3.3) the renormalized weak-angle expression at the renormalization point μ equal to M_W is found to be

$$\sin^2\theta_W(M_W) = \frac{1}{6} + \frac{5\alpha(M_W)}{9\alpha_s(M_W)} + \frac{5}{3} e^2 K \ln \frac{M_2^{e+\mu+\tau} M_2^{\mu+\tau}}{M_3^{e+\mu+\tau} M_3^{\mu+\tau}} \tag{3.4}$$

and the intermediate mass scales $M_5^{e,\mu,\tau}$, $M_a^{\mu+\tau}$, $M_a^{e+\mu+\tau}$, M_W $(a=1,2,3)$ satisfy the relation

$$\ln\left(\frac{M_5^e M_5^\mu M_5^\tau}{M_W M_2^{\mu+\tau} M_2^{e+\mu+\tau}}\right) + \frac{4}{3}\ln\left(\frac{M_2^{\mu+\tau} M_2^{e+\mu+\tau}}{M_3^{\mu+\tau} M_3^{e+\mu+\tau}}\right) = \frac{1}{6e^2K}\left[1 - \left(\frac{8\alpha(M_W)}{3\alpha_s(M_W)}\right)\right]. \tag{3.5}$$

Comparing Eqs. (3.4) and (3.5) with (1.2) and (1.3) for the simple SU(5) theory, the logarithmic term in (3.4) makes up for the small discrepancy between the experimental and the predicted values of $\sin^2\theta_W$, while Eq. (3.5) gives a relation between the various mass scales instead of fixing one unifying mass scale as in Eq. (1.3). For $\alpha = 1/128.5$,

$\alpha_s = 0.15$, the ratios

$$R_1 = \frac{M_5^e M_5^\mu M_5^\tau}{M_W M_2^{\mu+\tau} M_2^{e+\mu+\tau}}$$

and

$$R_2 = \frac{M_2^{\mu+\tau} M_2^{e+\mu+\tau}}{M_3^{\mu+\tau} M_3^{e+\mu+\tau}}$$

of the mass scales for a range of values of $\sin^2\theta_W$ are

$\sin^2\theta_W$	R_1	R_2
0.21	$10^{12.6}$	$10^{0.8}$
0.215	$10^{12.4}$	$10^{1.2}$
0.22	$10^{11.9}$	$10^{1.4}$
0.225	$10^{11.5}$	$10^{1.7}$
0.23	$10^{11.1}$	10^{2}

Only the ratio M_5^e/M_W is relevant for proton decay. Fixing $M_5^e/M_W \simeq R_1$ with $M_W \simeq 10^2$ GeV gives values of M_5^e that fall within the range $10^{14.5\pm1}$ GeV required for the proton lifetime to be of order $10^{30\pm2}$ yr. Taking $\sin^2\theta_W \simeq 0.23$ and $M_5^e \sim 10^{14}$ GeV fixes $M_5^\mu M_5^\tau / M_2^{\mu+\tau} M_2^{e+\mu+\tau}$ to be of order unity and $R_2 \sim 10^2$. These constraints are satisfied if the mass scales are chosen to be $M_5^\mu/M_W = M_2^{\mu+\tau}/M_W \simeq 10^{10}$, $M_5^\tau/M_W = M_2^{e+\mu+\tau}/M_W \sim 10^5$, $M_3^{\mu+\tau}/M_W \sim 10^9$, and $M_3^{e+\mu+\tau}/M_W \simeq 10^4$.

Thus not only is the value of $\sin^2\theta_W$ brought within the lower end of the experimentally allowed range but also the energy gap between M_W and M_5^e is filled with low-intermediate mass scales. The other mass scales $M_1^e, M_1^\mu, M_1^\tau, M_1^{\mu+\tau}, M_1^{e+\mu+\tau}$ dictate the structure of the U(1)'s operating at energies relevant to these mass scales and are not constrained by Eqs. (3.4) and (3.5). A possible handle on their magnitudes can be provided by future high-energy neutral-current phenomenology.

In passing it is to be noted that values of $\sin^2\theta_W > 0.23$ cause M_5^e to decrease rapidly from order 10^{14} GeV, making the proton lifetime considerably shorter than 10^{30} yr. One possibility that remedies this defect is that the fine-structure constant $\alpha(M_W)$ is less than the currently favored value[10] of 1/128.5. Then values of $\sin^2\theta_W > 0.23$ and $M_5^e \sim 10^{14}$ to 10^{15} GeV are simultaneously allowed (Table I).

The other possibility is a radical departure from $SU(3)_c \times SU(2)_L \times U(1)$ as the low-energy symmetry of strong, weak, and electromagnetic interactions. It is conceivable that right-handed

TABLE I. Constraints on the ratios $R_1 = M_5^e M_5^\mu M_5^\tau / M_W M_2^{\mu+\tau} M_2^{e+\mu+\tau}$ and $R_2 = M_2^{\mu+\tau} M_2^{e+\mu+\tau} / M_3^{\mu+\tau} M_3^{e+\mu+\tau}$ for a range of values of α, α_s, and $\sin^2\theta_W$.

		$\alpha = \frac{1}{128.5}$		$\alpha = \frac{1}{132}$		$\alpha = \frac{1}{137}$	
$\sin^2\theta_W$	α_s	R_1	R_2	R_1	R_2	R_1	R_2
0.21	0.1	$10^{12.6}$	$10^{0.23}$	$10^{13.0}$	$10^{0.3}$	$10^{13.5}$	$10^{0.4}$
	0.13		$10^{0.58}$		$10^{0.6}$		$10^{0.7}$
	0.15		$10^{0.83}$		$10^{0.9}$		$10^{1.0}$
	0.17		$10^{1.0}$		$10^{1.1}$		$10^{1.2}$
	0.20		$10^{1.24}$		$10^{1.3}$		$10^{1.4}$
0.215	0.1	$10^{12.4}$	$10^{0.52}$	$10^{12.6}$	$10^{0.6}$	$10^{13.1}$	$10^{0.7}$
	0.13		$10^{0.86}$		$10^{0.8}$		$10^{1.0}$
	0.15		$10^{1.12}$		$10^{1.1}$		$10^{1.3}$
	0.17		$10^{1.31}$		$10^{1.4}$		$10^{1.5}$
	0.20		$10^{1.53}$		$10^{1.6}$		$10^{1.7}$
0.22	0.10	$10^{11.9}$	$10^{0.81}$	$10^{12.2}$	$10^{0.9}$	$10^{12.6}$	$10^{1.0}$
	0.13		$10^{1.15}$		$10^{1.2}$		$10^{1.4}$
	0.15		$10^{1.41}$		$10^{1.5}$		$10^{1.6}$
	0.17		$10^{1.60}$		$10^{1.7}$		$10^{1.8}$
	0.20		$10^{1.82}$		$10^{1.9}$		$10^{2.0}$
0.225	0.10	$10^{11.5}$	$10^{1.1}$	$10^{11.8}$	$10^{1.2}$	$10^{12.3}$	$10^{1.3}$
	0.13		$10^{1.44}$		$10^{1.5}$		$10^{1.7}$
	0.15		$10^{1.7}$		$10^{1.8}$		$10^{1.9}$
	0.17		$10^{1.9}$		$10^{2.0}$		$10^{2.1}$
	0.20		$10^{2.1}$		$10^{2.2}$		$10^{2.3}$
0.23	0.10	$10^{11.1}$	$10^{1.4}$	$10^{11.4}$	$10^{1.5}$	$10^{11.8}$	$10^{1.6}$
	0.13		$10^{1.7}$		$10^{1.8}$		$10^{2.0}$
	0.15		$10^{2.0}$		$10^{2.1}$		$10^{2.2}$
	0.17		$10^{2.2}$		$10^{2.3}$		$10^{2.4}$
	0.20		$10^{2.4}$		$10^{2.5}$		$10^{2.6}$

currents exist and the structure of the low-energy symmetry is $SU(2)_L \times SU(2)_R \times U(1)_{L+R} \times SU(3)_c$, which is contained in the Pati-Salam group $SU(2)_L \times SU(2)_R \times SU(4)_c$. In this case each family is assigned an $SU(2)_L \times SU(2)_R \times SU(4)_c$ symmetry of its own and performing the same analysis as before on the $SO(10)^e \times SO(10)^\mu \times SO(10)^\tau$ generation symmetry leads to the renormalized value of the weak angle that covers the entire experimentally allowed range $\sin^2\theta_W = 0.23 \pm 0.015$ and admits low-intermediate mass scales in the energy gap at the same time. Some details of this are presented in Ref. 8.

ACKNOWLEDGMENTS

The author would like to thank Professor Abdus Salam and Dr. V. Elias for helpful discussions.

[1]T. Appelquist and J. Carazzone, Phys. Rev. D 11, 2856 (1975).

[2]S. L. Glashow, Nucl. Phys. B 22, 579 (1961); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[3]Breaking the $SU(2)_L \times U(1)_{L+R}$ gauge symmetry dynamically also leads to the same relation. For details see S. Weinberg, Phys. Rev. D 19, 1277 (1979).

[4]P. Langacker *et al.*, in *Neutrino 79*, proceedings of the International Conference on Neutrinos, Weak Interactions and Cosmology, Bergen, Norway, edited by A. Haatuft and C. Jarlskog (Univ. of Bergen, Bergen, 1980), Vol. 1, p. 276; M. Roos and I. Liede, Phys. Lett. 82B, 89 (1978).

[5]H. Georgi, H. R. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).

[6]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[7]Langacker *et al.* (Ref. 4).

[8]A. Salam, in *Proceedings of the European Physical Society International Conference on High Energy Physics, Geneva, 1979*, edited by A. Zichichi (CERN, Geneva, 1980), footnote 41 therein; V. Elias, Trieste report (unpublished).

[9]B. Deo, J. C. Pati, S. Rajpoot, and A. Salam (in preparation). Such an effect has been called the "squeezing" or "pinching" of couplings and has been considered in a gauge symmetry $SU(2)_L^q \times SU(2)_R^q \times SU(4)_c^p \times \tilde{U}(1)$, where p and q correspond to the number of distinct color and flavor symmetries.

[10]J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. B176, 61 (1980).