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Majorana neutrinos and magnetic fields

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It is stressed that if neutrinos are massive they are probably of "Majorana" type. This implies that their magnetic-moment form factor vanishes identically so that the previously discussed phenomenon of spin rotation in a magnetic field would not appear to take place. We point out that Majorana neutrinos can, however, have transition moments. This enables an inhomogeneous magnetic field to rotate both spin *and* "flavor" of a neutrino. In this case the spin rotation changes particle to antiparticle. The spin-flavor-rotation effect is worked out in detail. We also discuss the parametrization and calculation of the electromagnetic form factors of Majorana neutrinos. Our discussion takes into account the somewhat unusual quantum theory of massive Majorana particles.

I. INTRODUCTION

The growing presumption that neutrinos may actually be *massive* particles[1] makes it interesting to reexamine their properties, especially in the light of the modern gauge theories of weak interactions. The "classical" questions are: how do neutrinos interact with (i) electromagnetic fields, (ii) gravitational fields, and (iii) matter (absorption and coherent scattering). Here we shall be concerned with an aspect of case (i)—the interaction of neutrinos with magnetic fields. It has been realized for a long time[2] that the weak interactions should induce a neutrino magnetic moment of, roughly,

$$eG_F m_\nu , \tag{1.1}$$

where G_F is the Fermi constant, e is the proton charge, and m_ν is the neutrino mass. The effect[3] of a nonzero moment is, of course, to rotate the neutrino's spin when it encounters a magnetic field. There has been interesting speculation in the literature[3] about whether or not there exist somewhere in the universe (or laboratory) strong enough fields to give observable effects with moments as small as those given by (1.1). We shall not focus on this matter here. Our goal is to illustrate an interesting theoretical aspect of neutrino spin rotation, in the case wherein neutrinos are described by Majorana fields. The usual scenario is then not directly applicable, since a Majorana neutrino cannot have a magnetic moment. Historically the possibility of neutrinos being of Majorana type has generally been thought very exotic, even though it was recognized[4] that the interactions of a zero-mass Majorana neutrino are *indistinguishable* from those of a chirally projected Dirac particle. Even if the neutrino has small mass the differences are extremely difficult to detect. We would like to emphasize that the possibility of the usual neutrinos (assumed massive) being of Majorana type is, rather than an isolated curiosity, actually the most probable situation. There are two reasons for this. The first is that the usual neutrinos which emerge[5] from grand unified theories like SO(10) are of this nature. The second reason is perhaps more profound and is independent of which gauge theory turns out to be most nearly correct. It is simply that the most general description of spin-$\frac{1}{2}$ particles is in terms of two-component Weyl or van der Waerden spinors, which amounts to the Majorana description. In a fundamental weak-interaction theory it has come to be accepted that it is unaesthetic and unnecessary to make any *ad hoc* assumptions about the C, P, and T properties of the Lagrangian: the theory itself will decide on which (if any) symmetries of this type should emerge. A four-component Dirac description of a massive spin-$\frac{1}{2}$ particle can be thought of as an amalgamation of two two-component spinors in order to achieve a linear transformation property under parity. Therefore, it is unnatural to expect that the fundamental fields of the theory be Dirac fields: The Dirac theory is a special case of the theory of two massive Weyl fields.[6] (The Majorana description of a massive Weyl field amounts to halving the degrees of freedom of the Dirac theory by imposing a *constraint*.)

If one grants the force of the above argument it appears that neutrinos should not suffer any spin rotation in a magnetic field. However, this conclusion is not true. As we shall see it is possible for a pair of Majorana neutrinos to have a *transition moment*, roughly analogous to the Σ^0-Λ hyperon transition moment. This will enable a neutrino of a given spin to rotate into *another* neutrino of different spin in the presence of a magnetic field. For this to happen the magnetic field must be inhomogeneous either in space or time. There is another interesting difference between this spin rotation and the spin rotation for a Dirac neutrino. In the case of a massive four-component Dirac neutrino it is expected that lepton number is conserved. An antineutrino which, for example, has been produced in β decay will after spin rotation by a magnetic field lose some of its effectiveness for producing e^+'s, μ^+'s, etc., and not gain anything in return. [This is true in an $SU(2)\otimes U(a)$ gauge theory of massive Dirac neutrinos.] On the other hand in the present case the effect of a magnetic field on an antineutrino produced in β decay will be to enhance its strength for producing e^-'s, etc.: In the present case the magnetic field rotates particles into antiparticles. In fact our original motivation in examining this question arose from an attempt to upgrade a neutrino-to-antineutrino-oscillation "thought experiment" (which was shown[7] to be sensitive to new CP-violating phases) to the status of a process which could conceivably be observed in an astrophysical environment.

In Sec. II the kinematics of massive two-component neutrinos is briefly reviewed. The general electromagnetic form factor is presented and it is shown that the *diagonal* moment-type form factors vanish identically. More discussion on the theory of Majorana neutrinos is given in the Appendix.

In Sec. III we calculate the matrix element for a neutrino of one type to transform to a neutrino of opposite spin and different type in the presence of an inhomogeneous magnetic field. For illustration we adopt a particularly simple type of inhomogeneity—a space pulse—and construct an effective four-level Hamiltonian to approximate the system. This permits us to find the transition amplitudes after passage through the field region. There is a certain methodological interest in the construction of the effective Hamiltonian. This is because, as discussed in the text, one cannot write down a *first*-quantized description of a system involving massive Majorana particles in the ordinary way.

Section IV contains a brief discussion of the order of magnitude of the spin-flavor-rotation effect and the possibilities for enhancing it. We also show how the existing calculations of neutrino decays for massive Dirac neutrinos in $SU(2)_L\otimes U(1)$ gauge theory can be used to furnish the intermediate-boson loop contribution to the Majorana-neutrino transition form factor.

II. TRANSITION MAGNETIC MOMENT

First we write down the Lagrangian density for n free two-component (Weyl or van der Waerden) spinor fields ν_α of different masses m_α:

$$\mathcal{L}_{\text{free}} = -\sum_{\alpha=1}^{n} [i\nu_\alpha^\dagger \sigma_\mu \partial_\mu \nu_\alpha + \tfrac{1}{2}(\nu_\alpha^T \sigma_2 \nu_\alpha m_\alpha + \text{H.c.})]. \quad (2.1)$$

Here we are using

$$\sigma_\mu = (\vec{\sigma}, -i)\,, \quad (2.2)$$

and the m_α are taken to be real by *convention*. The $SL(2,C)$ covariance of (2.1) may be seen manifestly by introducing dotted and undotted spinor indices as follows:

$$\begin{aligned} &\nu \to \nu^{\dot a}, \quad \nu^\dagger \to (\nu^\dagger)^a\,, \\ &(\sigma_\mu)_{a\dot b}\partial_\mu \equiv \partial_{a\dot b}, \quad (\sigma_2)_{\dot a\dot b} = -i\epsilon_{\dot a\dot b}\,. \end{aligned} \quad (2.3)$$

Then (2.1) can be rewritten

$$\mathcal{L}_{\text{free}} = -\sum_{\alpha=1}^{n}\left[i(\nu_\alpha^\dagger)^a \partial_{a\dot b}\nu_\alpha^{\dot b} - \frac{i}{2}(\nu_\alpha^{\dot a}\epsilon_{\dot a\dot b}\nu_\alpha^{\dot b} + \text{H.c.})\right], \quad (2.1')$$

It may be noted from either (2.1) or (2.1′) that the mass terms of the free Lagrangian will vanish if the ν_α's are c numbers. We overcome this problem by treating the ν_α as quantized anticommuting fields from the beginning. A convenient field expansion, discussed in Sec. II and the Appendix of Ref. 6 is obtained in terms of the left-handed components of the ordinary massive Dirac wave functions $u^{(r)}$ and $v^{(r)}$. In this way, with a representation of the Dirac algebra where γ_5 is diagonal, one finds

$$\begin{pmatrix}\nu\\0\end{pmatrix} = \frac{1+\gamma_5}{2}\sum_{\vec p, r}\left(\frac{m}{E_p V}\right)^{1/2}[e^{ip\cdot x}u^{(r)}(\vec p)A_r(\vec p) + e^{-ip\cdot x}v^{(r)}(\vec p)A_r^\dagger(\vec p)]\,, \quad (2.4)$$

where $C\bar u^{(r)T} = v^{(r)}$, C being the charge-conjugation matrix. The quantization of the Majorana field discussed by Case[8] is a special case of the above wherein the spinors are taken to be helicity eigenstates (see the Appendix). It is also helpful to give the intermediate-boson term of the charged leptonic weak interaction. The explicit form this takes depends on the gauge theory of interest. If we restrict ourselves to $SU(2)_L\otimes U(1)$, the interaction form depends on how many of the ν_α belong to left-handed doublets. For simplicity let us assume that all n of them do (the more general case is discussed in detail in Ref. 6). Then the interaction term is

$$\mathcal{L}_{\text{int}} = ig2^{-1/2}W_\mu^- \sum_{\alpha,\beta=1}^{n} \bar e_{L\alpha}\gamma_\mu K_{\alpha\beta}\begin{pmatrix}\nu_\beta\\0\end{pmatrix} + \text{H.c.}\,, \quad (2.5)$$

where $K_{\alpha\beta}$ is the unitary mixing matrix. It is apparent from (2.4) and (2.5) that the usual theory is recovered when all neutrino masses vanish. In that case one of the $u^{(r)}$ and one of the $v^{(r)}$ do not contribute because of the $(1+\gamma_5)$ projection.

After these preliminaries[9] we will write the effective electromagnetic interaction of magnetic-moment type between a pair of neutrinos, say ν_1 and ν_2. The most general effective electromagnetic interaction involving ν_1 and ν_2 may be written[10] as

$$\mathcal{H}^{\mathrm{EM}} = \sum_{\alpha,\beta=1}^{2} \nu_\alpha^\dagger [j_{\alpha\beta}(\Box)(\sigma_\mu \Box - \sigma_\lambda \partial_\lambda \partial_\mu) A_\mu] \nu_\beta + \left[\frac{i}{2} \nu_2^T \sigma_\mu^T \sigma_2 \sigma_\nu \nu_1 h(\Box) F_{\mu\nu} + \mathrm{H.c.}\right], \tag{2.6}$$

where A_μ is the electromagnetic field, $F_{\mu\nu} = \partial_\mu A_\mu - \partial_\nu A_\mu$, and $\Box$ is the d'Alembertian. $j_{\alpha\beta}(\Box) = j^*_{\beta\alpha}(\Box)$ and $h(\Box)$, the form factors,[11] are *complex* functions of $\Box$. In checking the covariance of (2.6) it is helpful to use (2.3) above. The effective *low-energy* moment-type interaction is the last term of (2.6) with $h(\Box)$ replaced by $h \equiv h(0)$:

$$\begin{aligned}\mathcal{H}^{\mathrm{EM}} &\approx \frac{ih}{2} \nu_2^T \sigma_\mu^T \sigma_2 \sigma_\nu \nu_1 F_{\mu\nu} + \mathrm{H.c.} \\ &= h \nu_2^T \sigma_2 \vec{\sigma} \cdot (\vec{\mathrm{B}} + i\vec{\mathrm{E}}) \nu_1 + \mathrm{H.c.}, \end{aligned} \tag{2.7}$$

In (2.7), $\vec{\mathrm{B}}$ and $\vec{\mathrm{E}}$ are the magnetic and electric fields. *CP* invariance would require h to be real. This statement follows from the assumed *CP* transformation properties

$$\begin{aligned} &\vec{\mathrm{E}}(\vec{\mathrm{x}},t) \to \vec{\mathrm{E}}(-\vec{\mathrm{x}},t), \quad \vec{\mathrm{B}}(\vec{\mathrm{x}},t) \to -\vec{\mathrm{B}}(-\vec{\mathrm{x}},t), \\ &\nu_\alpha(\vec{\mathrm{x}},t) \to -i\sigma_2 \nu_\alpha^*(-\vec{\mathrm{x}},t). \end{aligned} \tag{2.8}$$

The last of these has a unique phase, required in order to keep the free Lagrangian (2.1) *CP* invariant with the convention of *real* m_α.

Finally, we can verify that the *diagonal* moments vanish since

$$\mathcal{O} = \sigma_\mu^T \sigma_2 \sigma_\nu F_{\mu\nu} = \mathcal{O}^T$$

and

$$\nu^T \mathcal{O} \nu = -\nu^T \mathcal{O}^T \nu = 0,$$

by virtue of the anticommuting property of the ν fields. This argument does not eliminate the off-diagonal moments, of course.

III. SPIN-FLAVOR ROTATIONS

As noted in the last section there is no "first-quantized" description of a massive two-component field in terms of c-number wave functions.[12] Thus it is better to use perturbative field theory directly, considering the S-matrix element for the interaction of ν_1 and ν_2 with an external magnetic field $\vec{\mathrm{B}} = \hat{\mathrm{n}} B(x)$. Using (2.7) and the field expansion (2.4) we find

$$\langle \nu_1, p', r' | S | \nu_2, p, r\rangle = \frac{|h|}{V} T_{r'r} \int d^4x\, B(x) e^{i(p-p')\cdot x}, \tag{3.1}$$

$$T_{r'r} = \left(\frac{m_1 m_2}{p_0 p_0'}\right)^{1/2} \bar{u}^{r'}(\vec{\mathrm{p}}') \begin{pmatrix} \vec{\mathrm{n}} \cdot \vec{\sigma} e^{i\lambda} & 0 \\ 0 & \vec{\mathrm{n}} \cdot \vec{\sigma} e^{-i\lambda} \end{pmatrix} u^r(\vec{\mathrm{p}}). \tag{3.2}$$

Here r' and p' are the spin and momentum labels of the final neutrino ν_1 and r and p the same for the initial neutrino ν_2. The phase λ is defined from $h \equiv |h| e^{i(\lambda - \pi/2)}$. The Dirac spinors above are to be considered in a γ_5-diagonal representation. If the external field B is constant in space and time the integral in (3.1) will be proportional to $\delta^4(p - p')$. This will vanish if the neutrino masses m_1 and m_2 are different. Thus we must consider a magnetic field with some space-time inhomogeneity in order to find an effect. The simplest choice is a "space pulse"

$$B(x,y,z,t) = B(0,0,z,0) = \begin{cases} 0, & |z| > L \\ B_0, & |z| < L \end{cases}. \tag{3.3}$$

Such a choice can, of course, only be an approximation; it is similar to the field that a neutrino would see in traveling along the z axis through the center of the gap of a (gigantic) horseshoe magnet. With (3.3), the x and y components of the momentum as well as the energy of the neutrino will be conserved. The change in mass will be compensated by a change in z component of momentum. We shall adopt (3.3) for the purpose of illustrating the spin-flavor-rotation effect. Assigning to r the values of + helicity and − helicity and using the γ_5-diagonal-representation spinors of the Appendix, we find for the S-matrix element

$$\langle \nu_1, p' | S | \nu_2, p\rangle = \frac{2(2\pi)^3 |h| B_0}{V} \delta(p_x - p_x')\delta(p_y - p_y')\delta(p_0 - p_0') \frac{\sin[(p_z - p_z')L]}{p_z - p_z'} T, \tag{3.4}$$

$$T = \begin{pmatrix} T_{++} & T_{+-} \\ T_{-+} & T_{--} \end{pmatrix}, \tag{3.5}$$

$$\begin{aligned} T_{++} = -T_{--}^* &= \frac{n_z}{2}\left(\frac{m_1 m_2}{p_0 p_0'}\right)^{1/2} \left\{ e^{-i\lambda}\left[\frac{m_1(p_0 + p_z)}{m_2(p_0' + p_z')}\right]^{1/2} + e^{i\lambda}\left[\frac{m_2(p_0' + p_z')}{m_1(p_0 + p_z)}\right]^{1/2}\right\}. \\ T_{+-} = T_{-+}^* &= \frac{(n_x - in_y)}{2}\left(\frac{m_1 m_2}{p_0 p_0'}\right)^{1/2} \left\{ e^{-i\lambda}\left[\frac{m_1(p_0 - p_z)}{m_2(p_0' + p_z')}\right]^{1/2} + e^{i\lambda}\left[\frac{m_2(p_0' + p_z')}{m_1(p_0 - p_z)}\right]^{1/2}\right\}. \end{aligned} \tag{3.6}$$

The last two expressions simplify considerably if the neutrino masses m_1 and m_2 are small compared to the momenta:

$$T_{++} \approx \frac{n_z}{2p_0}(e^{-i\lambda}m_1 + e^{i\lambda}m_2)\,, \qquad T_{+-} \approx (n_x - in_y)e^{i\lambda}\,. \tag{3.7}$$

With $\lambda = 0$, (3.7) and (3.5) are of the same form as the spin-rotation Hamiltonian[13] in the Dirac theory, although the interpretation is different. Notice that T_{++} is generally negligible compared to T_{+-}. This shows that the components of magnetic field *perpendicular* to the neutrino's direction of motion are the effective ones for causing spin rotations. (This would also lessen the effects of the "fringing" field.) It is most convenient to orient the x and y axes so that

$$T_{+-} \approx \sin\theta\,, \tag{3.8}$$

θ being the angle between the direction of the beam and the magnetic field.

From the discussion above it is clear that there are nonzero matrix elements for a Majorana neutrino of one flavor to make a transition in the presence of an inhomogeneous magnetic field to a Majorana neutrino of different helicity and different flavor. In order to make this more vivid let us focus on the flavor and helicity labels describing a neutrino beam. Define an effective *four-level* Hamiltonian by

$$\langle \nu_\beta p' r' | S | \nu_\alpha p r\rangle = \frac{-i(2\pi)^3(2L)}{V}\delta(p_0 - p_0')\delta(p_x - p_x')\delta(p_y - p_y') \times \langle \nu_\beta r' | H_{\rm eff} | \nu_\alpha r\rangle\,. \tag{3.9}$$

The order of the labels is (1+, 1-, 2+, 2-) with the first label describing flavor and the second, helicity. Comparing this with (3.4) gives

$$H_{\rm eff} = |h| B_0 \left\{\frac{\sin[(p_z - p_z')L]}{(p_z - p_z')L}\right\}\begin{pmatrix} 0 & -iT \\ iT^\dagger & 0\end{pmatrix}\,, \tag{3.10}$$

where T is defined in (3.5) and (3.6) and approximated in (3.7) and (3.8). Note that $H_{\rm eff}$ is Hermitian. The factor in curly brackets is a geometrical factor describing the "shape" of the external magnetic field. As a consistency check we note that it vanishes as $L \to \infty$; in that limit the field becomes homogeneous and there is no rotation effect. On the other hand, for $(p_z - p_z')L \approx (m_1^2 - m_2^2)L/(2p_z)$, small compared to one, this shape factor is unity. In writing (3.10) we are implicitly assuming that the edge effects responsible for the transition can be "spread out" over the field region. This approximation is clearly more reasonable when $(p_z - p_z')L$ is small.

Using (3.10) as an effective four-level Hamiltonian we can easily discuss the time evolution of a relativistic neutrino beam entering the field region at time $t = 0$. If we neglect T_{++} in (3.7) and use (3.8) (we are here neglecting corrections due to the small neutrino masses), $H_{\rm eff}$ becomes

$$H_{\rm eff} = -i\Delta\begin{pmatrix} 0 & +\sigma_1 \\ -\sigma_1 & 0\end{pmatrix}, \qquad \Delta = |h| B_0 \sin\theta \left\{\frac{\sin[(p_z - p_z')L]}{(p_z - p_z')L}\right\}\,. \tag{3.11}$$

Equation (3.11) is easily diagonalized as follows:

$$V^\dagger H_{\rm eff} V = \Delta\,{\rm diag}(-,+,+,-)\,, \qquad V = \frac{1}{\sqrt{2}}\begin{pmatrix} U & -iU \\ -iU & U\end{pmatrix}, \quad U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1\end{pmatrix}. \tag{3.12}$$

With our level ordering the eigenvalues thus are $E_1 = E_4 = -\Delta$ and $E_2 = E_3 = +\Delta$. Let ψ_k denote the states of the incident beam $k = 1 \to 4$. The eigenstates of $H_{\rm eff}$, χ_k are related to these by

$$\chi = V^\dagger \psi\,. \tag{3.13}$$

Let us say that, at $t = 0$, the system is in the state ψ_k. After entering the field region, under the influence of $H_{\rm eff}$, ψ_k evolves to

$$\sum_l V_{kl}\chi_l e^{-iE_l t} = \sum_{l,j} V_{kl}V_{jl}^*\psi_j e^{-iE_l t}\,. \tag{3.14}$$

The amplitude for state k to go to state j at time $t = 2L$ is thus

$$\text{amp}(k \to j; t) = \sum_{l=1}^{4} V_{kl}V_{jl}^* e^{-iE_l t}\,, \tag{3.15}$$

where the E_l are given after (3.12). Equation (3.15) explicitly shows how an initial neutrino beam of a given type oscillates to the opposite flavor and spin. Taking $k = 1-$, for example, yields with the help of (3.12)

$$\begin{aligned} &\text{amp}(1- \to 1+) = \text{amp}(1- \to 2-) = 0\,, \\ &\text{amp}(1- \to 1-) = \cos\Delta t\,, \\ &\text{amp}(1- \to 2+) = -\sin\Delta t \end{aligned} \tag{3.16}$$

Notice that the labels 1 and 2 refer to neutrinos of definite mass. They are each mixtures of ν_e, ν_μ, etc. In this example the initial neutrino has dominant matrix elements for producing negative leptons by W exchange with matter. After passing through the magnetic field region there is amplitude $\sin 2\Delta L$ available for producing positive leptons.

This analysis can be immediately extended to three neutrino flavors, for example. Instead of the matrix (3.10) we would have the 6×6 matrix

$$\begin{pmatrix} 0 & -iT & -iT' \\ iT^{\dagger} & 0 & -iT'' \\ iT'^{\dagger} & iT'' & 0 \end{pmatrix}.$$

Here T' is the same as T except for an overall strength which characterizes the $\nu_1 \to \nu_3$ transition moment, etc.

IV. MAGNITUDES AND MOMENTS

The magnitude of the spin-flavor-rotation effect depends on the product of the transition moment h, the magnetic field B_0, and an effective time during which the neutrino beam transverses a varying field. There is also a shape factor, describing the field inhomogeneity. If the neutrino is a true elementary particle, rather than a composite, its transition moment will be very roughly given by (1.1). (In fact detailed calculations may give an additional suppression of 10^{-4} or so, depending on the Higgs-meson structure of the theory.) To appreciate how small the effect is we note that the product of (1.1) (assuming $m_\nu \approx 1$ eV) and a magnetic field of 10^4 G is about $10^{-8}\ \mathrm{sec}^{-1}$. Thus effective travel times of the order of 10^8 sec are needed for sizable particle to antiparticle conversion. The effect could be enhanced if (i) neutrinos were composite or if they had larger transition moments than (1.1) for other reasons, (ii) extremely strong rapidly changing (either in space or time) magnetic fields were present, or (iii) the neutrinos were to pass through the same B field again and again due to an orbiting motion. Although the present mechanism would probably be somewhat weaker than a comparable spin rotation for massive Dirac neutrinos, it seems more amusing. If, as seems likely, neutrinos are Majorana particles the effect described here is the only one.

Finally, we will comment on how much of the calculation of h from a gauge theory can be obtained from the existing literature, in which transition moments for massive *Dirac* neutrinos have in effect been calculated. Consider the intermediate-boson loop contribution to the transition moment. For definiteness in our formal argument imagine that we are using the U gauge (although this is a delicate one in practice). In addition to lepton-number-conserving diagrams like the one in Fig. 1(a) there will be corresponding lepton-number-violating diagrams like the one in Fig. 1(b). For Fig. 1(a) we use the interaction in (2.5) which we write out explicitly using a γ_5-diagonal representation of the Dirac algebra:

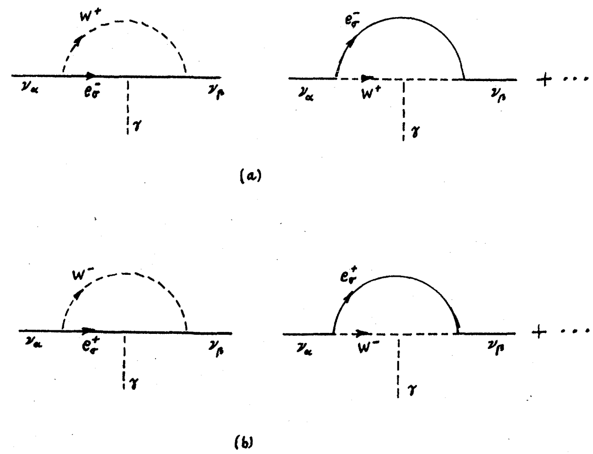

FIG. 1. Lepton-number-conserving (a) and lepton-number-violating (b) diagrams for the $\nu_\alpha \to \nu_\beta$ electromagnetic transition.

$$\mathcal{L}_{\mathrm{int}} = ig2^{-1/2}\sum(\bar{e}_\alpha\gamma_\mu K_{\alpha\beta}\nu_{\beta L}W^-_\mu + \bar{\nu}_{\beta L}\gamma_\mu K^*_{\alpha\beta}e_\alpha W^+_\mu)\,. \tag{2.5$'$}$$

Then, using the field expansion (2.4) we see that Fig. 1(a) will contribute a factor

$$\sum_\sigma K_{\sigma\alpha}K^*_{\sigma\beta}\bar{u}_{\beta L}\mathfrak{M}(\sigma)u_{\alpha L}\,, \tag{4.1}$$

where $\mathfrak{M}(\sigma)$ is a matrix, containing the "core" of the diagrams. While $\mathfrak{M}(\sigma)$ depends on the mass of the intermediate charged lepton σ it does not have any γ_5 factor. Equation (4.1) has the same momentum-space structure as what one would get from the massive Dirac theory. To calculate the diagrams of Fig. 1(b) imagine that we rewrite (2.5′) in terms of "charge-conjugate" fields (defined by $\psi^c = C\bar{\psi}^T$, for a fermion field ψ):

$$\mathcal{L}_{\mathrm{int}} = -ig2^{-1/2}\sum(\bar{\nu}^c_{\beta R}\gamma_\mu K_{\alpha\beta}e^c_\alpha W^-_\mu + \bar{e}^c_\alpha\gamma_\mu\nu^c_{\beta R}W^+_\mu)\,. \tag{4.2}$$

The expansion (2.4) may be equivalently presented as either

$$\nu \sim u_L A + v_L A^\dagger \tag{4.3}$$

or

$$\nu^c \sim v_R A^\dagger + u_R A\,,$$

in an evident shorthand. Comparing Fig. 1(a) computed using (2.5′) and Fig. 1(b) computed using the conjugate formulation (4.2) we see that there will be an overall difference of sign due to the electromagnetic interaction. Also, the mixing matrices will be complex-conjugated and right-handed spinors will appear resulting in

$$-\sum_\sigma K^*_{\sigma\alpha}K_{\sigma\beta}\bar{u}_{\beta R}\mathfrak{M}(\sigma)u_{\alpha R}\ . \tag{4.4}$$

The important point is that the same $\mathfrak{M}(\sigma)$ appears in (4.1) and (4.4). The total contribution of the diagrams involving gauge bosons is given by the sum of (4.1) and (4.4) which may be written as

$$i\sum_\sigma \mathrm{Im}(K_{\sigma\alpha}K^*_{\sigma\beta})\bar{u}_\beta\mathfrak{M}(\sigma)u_\alpha + \sum_\sigma \mathrm{Re}(K_{\sigma\alpha}K^*_{\sigma\beta})\bar{u}_\beta\mathfrak{M}(\sigma)\gamma_5 u_\alpha\ . \tag{4.5}$$

Now from the calculation of Petkov[14] (who was interested in the decay $\nu'_\alpha \to \nu_\beta + \gamma$) we can read off the *moment* parts of $\bar{u}_\beta\mathfrak{M}(\sigma)u_\alpha$ and $\bar{u}_\beta\mathfrak{M}(\sigma)\gamma_5 u_\alpha$:

$$\begin{aligned}\bar{u}_\beta\mathfrak{M}(\sigma)u_\alpha &= \frac{3eG_F(m_\alpha+m_\beta)}{32\pi^2}\left[\frac{m(l_\sigma)}{m(W)}\right]^2\bar{u}_\beta\epsilon_\mu\sigma_{\mu\nu}q_\nu u_\alpha\ ,\\ \bar{u}_\beta\mathfrak{M}(\sigma)\gamma_5 u_\alpha &= \frac{3eG_F(m_\beta-m_\alpha)}{32\pi^2\sqrt{2}}\left[\frac{m(l_\sigma)}{m(W)}\right]^2\bar{u}_\beta\epsilon_\mu\sigma_{\mu\nu}\gamma_5 q_\nu u_\alpha\ .\end{aligned} \tag{4.6}$$

Here q_μ is the four-momentum transfer and ϵ_μ the photon polarization. Still to be added are the contributions from the Higgs mesons. This will have the same general structure but there will be extra contributions characterizing the Higgs sector of the theory. Notice the additional suppression factor (mass of charged lepton divided by mass of intermediate boson)^2 compared to (1.1). It is important to check that the result in (4.5) and (4.6) gives zero for the diagonal moments (when $\alpha=\beta$): The $\sigma_{\mu\nu}\gamma_5$ term vanishes because $m_\alpha = m_\beta$ while the $\sigma_{\mu\nu}$ term vanishes because $\mathrm{Im}(K_{\sigma\alpha}K^*_{\sigma\alpha})=0$. Equation (4.5) shows that the coefficients of $\sigma_{\mu\nu}(1+\gamma_5)q_\nu$ and $\sigma_{\mu\nu}(1-\gamma_5)q_\nu$ are equal in magnitude, differing by a phase. This is the origin of the $e^{i\lambda}$ phase factor in our basic equation (3.2).

Note added. After this paper was written we learned of a recent paper [Carnegie Mellon Report No. COO-3066-164, 1981 (unpublished)] by Lincoln Wolfenstein which also discusses transition magnetic moments between different Majorana neutrinos.

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APPENDIX

For convenience, the quantized-field expansion of the massive two-component spinor given in Eq. (2.4) uses the left-handed projections of ordinary Dirac wave functions. For the Dirac theory we adopt Pauli's conventions and use the explicit γ_5-diagonal representation

$$\vec{\gamma}=\begin{pmatrix}0 & -i\vec{\sigma}\\ i\vec{\sigma} & 0\end{pmatrix},\quad \gamma_4=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},$$
$$\gamma_5=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad C=\begin{pmatrix}-\sigma_2 & 0\\ 0 & \sigma_2\end{pmatrix}. \tag{A1}$$

Note that C obeys $-\gamma_\mu^T = C^{-1}\gamma_\mu C$. Furthermore, we adopt the phase convention

$$v^{(r)}(\vec{p}) = C\bar{u}^{(r)T}(\vec{p})\ . \tag{A2}$$

In (2.4) the index r labels any convenient set of two independent spin wave functions (see the discussion in the Appendix of Ref. 6). If we specialize r to the meaning of a helicity label

$$\vec{\sigma}\cdot\vec{p}u_L^{(\pm)}(\vec{p}) = \pm|\vec{p}|u_L^{(\pm)}(\vec{p})\ , \tag{A3}$$

we arrive at a quantization essentially equivalent to that of Case.[8] We used the γ_5-diagonal helicity spinors in computing (3.2); they are explicitly given by

$$u^{(\pm)}(\vec{p}) = \left[\frac{m}{2(E\pm|\vec{p}|)}\right]^{1/2}\begin{pmatrix}\chi^{(\pm)}\\ \frac{1}{m}(E\pm|\vec{p}|)\chi^{(\pm)}\end{pmatrix}, \tag{A4}$$

$$\chi^{(+)} = \frac{1}{[2(\hat{p}_z+1)]^{1/2}}\begin{pmatrix}1+\hat{p}_z\\ \hat{p}_x+i\hat{p}_y\end{pmatrix},$$
$$\chi^{(-)} = \frac{1}{[2(\hat{p}_z+1)]^{1/2}}\begin{pmatrix}-\hat{p}_x+i\hat{p}_y\\ 1+\hat{p}_z\end{pmatrix}. \tag{A5}$$

To check that, with helicity spinors, *both* terms of (2.4) obey *together* the equation of motion

$$-i\sigma_\mu\partial_\mu\nu = m\sigma_2\nu^*\ , \tag{A6}$$

for a given $\vec{p}$ and r, it is sufficient to use (A3) as well as [noting (A2)]

$$\vec{\sigma}\cdot\vec{p}v_L^{(\pm)}(\vec{p}) = \mp|\vec{p}|v_L^{(\pm)}(\vec{p})\ .$$

For example,

$$\begin{aligned}-i\sigma_\mu\partial_\mu[e^{ip\cdot x}u_L^{(+)}(\vec{p})A_+(\vec{p}) + e^{-ip\cdot x}v_L^{(+)}(\vec{p})A_+^\dagger(\vec{p})] &= (|\vec{p}|+E)e^{ip\cdot x}u_L^{(+)}(\vec{p})A_+(\vec{p})\\ &\quad + (|\vec{p}|-E)v_L^{(+)}(\vec{p})A_+^\dagger(\vec{p})\ ,\end{aligned} \tag{A7}$$

while

$$m\sigma_2[e^{-ip\cdot x}u_L^{(+)*}A_+^\dagger(\vec{p}) + e^{ip\cdot x}v_L^{(+)*}(\vec{p})A(\vec{p})]$$

is equal to (A7) since (A2) and (A3) imply

$$\sigma_2 v_L^{(+)*}=\frac{E+|\vec{p}|}{m}u_L^{(+)}\,,\quad \sigma_2 u_L^{(+)*}=\frac{|\vec{p}|-E}{m}v_L^{(+)}\,.$$

Notice that $u_L^{(-)}$ and $v_L^{(+)}$ *do not* vanish as the neutrino mass m goes to zero, while $u_L^{(+)}$ and $v_L^{(-)}$ do vanish. One arrives in this way at the old two-component massless-neutrino theory.

Finally, we give the *two* propagators (lepton-number violating and conserving) which follow from (2.4):

$$\langle 0|T\nu_a(x)\nu_b(y)|0\rangle = m(\sigma_2)_{ab}\Delta_F(x-y;m)$$

$$\langle 0|T\nu_a(x)\nu_b^\dagger(y)|0\rangle = i(\sigma_\mu^\dagger)_{ab}\partial_\mu\Delta_F(x-y;m)\,,$$

$$\Delta_F(x-y;m)=\frac{-i}{(2\pi)^4}\int d^4p\,\frac{e^{ip\cdot(x-y)}}{p^2+m^2-i\epsilon}\,. \qquad \text{(A8)}$$

[1]A review is furnished by S. M. Bilenky and B. Pontecorvo, Phys. Rep. 41C, 225 (1978). The *Proceedings of the Neutrino Mass Miniconference, Telemark, Wisconsin, 1980,* edited by V. Barger and D. Cline (University of Wisconsin, Madison, 1981), should enable one to trace more recent work.

[2]A review is given by A. O. Barut, in *Proceedings of the Neutrino Mass Miniconference, Telemark, Wisconsin, 1980* (Ref. 1).

[3]A. Cisneros, Astrophys. Space Sci. 10, 87 (1980); K. Fujikawa and R. E. Shrock, Phys. Rev. Lett. 45, 963 (1980); B. W. Lynn and G. Feinberg, Columbia University Report No. CU-TP 181, 1980 (unpublished).

[4]The history of Majorana neutrinos is discussed by S. P. Rosen, in *Proceedings of the Neutrino Mass Miniconference, Telemark, Wisconsin, 1980* (Ref. 1).

[5]For recent work on neutrino masses in SO(10) see, for example, R. Barbieri, CERN Report No. TH-2850, 1980 (unpublished); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315; L. Maiani, CERN Report No. TH-2846, 1980 (unpublished); H. Georgi, in *Particles and Fields—1974,* proceedings of the Meeting of the Division of Particles and Fields of APS, Williamsburg, Virginia, edited by Carl Carlson (AIP, New York, 1975); H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) 93, 177 (1975); P. Ramond, *Proceedings of First Workshop on Grand Unification*, edited by P. Frampton, S. L. Glashow, and A. Yildiz (Math Science Press, New Hampshire, 1980); Y. Tomozawa, University of Michigan Report No. HE-80-17, 1980 (unpublished); E. Witten, Phys. Lett. 91B, 81 (1980); S. Nandi, in *Proceedings of the Neutrino Mass Miniconference*, Ref. 1.

[6]See J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980); also J. Schechter, in *Proceedings of the Neutrino Mass Miniconference, Telemark, Wisconsin, 1980* (Ref. 1).

[7]J. Schechter and J. W. F. Valle, Phys. Rev. D 23, 1666 (1981).

[8]K. M. Case, Phys. Rev. 107, 307 (1957).

[9]Besides Ref. 6 above and references therein, recent discussions of neutrino "kinematics" include T. P. Cheng and L. F. Li, Phys. Rev. D 22, 2860 (1980); J. Harvey, P. Ramond, and D. Reiss, Caltech Report No. 68-800, 1980 (unpublished); P. Mannheim, University of Connecticut report, 1980 (unpublished); R. Mohapatra and G. Senjanovic, Phys. Rev. D 23, 165 (1980); L. Chang and N. Chang, Phys. Rev. Lett. 45, 1540 (1980).

[10]Taking the matrix element of (2.6) between neutrino states $A^\dagger|0\rangle$, one will find with the aid of (2.4) a result of the form (4.5) where four-component Dirac wave functions appear.

[11]Astrophysical bounds on Dirac-neutrino electromagnetic form factors have been suggested by J. Bernstein, M. Ruderman, and G. Feinberg, Phys. Rev. 132, 1227 (1963). For a review see A. Dolgov and Y. Zeldovich, Rev. Mod. Phys. 53, 1 (1981).

[12]This does not preclude a "pseudoclassical" description in terms of Grassman fields as in F. A. Berezin and M. S. Marinov, Ann. Phys. (N.Y.) 104, 336 (1977); A. Barducci, R. Casalbuoni, and L. Lusanna, Lett. Nuovo Cimento 19, 581 (1977).

[13]With $\lambda=0$ and $m_1=m_2=m$, T becomes

$$\begin{pmatrix} n_z m/p_0 & n_x-in_y \\ n_x+in_y & -n_z m/p_0\end{pmatrix},$$

which can also be derived directly for the massive Dirac theory. Diagonalizing this matrix gives eigenvalues and eigenvectors in agreement with Fujikawa and Shrock, Ref. 3 above. Notice also that $\lambda=0$ implies the hermiticity of the matrix T [as given exactly by Eqs. (3.5) and (3.6)] and thus permits (for any number n of "generations") the effective Hamiltonian corresponding to (3.10) to decouple as $H_{\rm eff}\propto\mu\otimes T$, where μ is an $n\times n$ skew-symmetric Hermitian matrix. For example, for three flavors it has the form

$$\begin{pmatrix} 0 & -ih_{12} & -ih_{13} \\ ih_{12} & 0 & -ih_{23} \\ ih_{13} & ih_{23} & 0\end{pmatrix},$$

where $h_{ab}=h_{ab}^*$ is the strength of the (ab) transition moment. Thus, for $\lambda=0$, the diagonalization of $H_{\rm eff}$ reduces to the diagonalization of the flavor-space matrix μ.

[14]S. T. Petkov, Yad. Fiz. 25, 641 (1977) [Sov. J. Nucl. Phys. 25, 340 (1977)]; *ibid.* 25, 698(E) (1977) [*ibid.* 25, 641 (1977)]; W. Marciano and A. I. Sanda, Phys. Lett. 67B, 303 (1977); B. W. Lee and R. Shrock, Phys. Rev. D 16, 1444 (1977); J. Kim, *ibid.* 14, 3000 (1976).