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# Muon decays revisited: Effects of massive neutrinos, their mixings, and grand unification

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We have calculated the effects of massive Dirac and Majorana neutrinos and their mixings in the $e^+$ spectrum in $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$ ($\nu_\mu^c$) decays. Particular attention was paid to the three-neutrinos world with the mass of $\nu_\tau$ in the MeV/$c^2$ range. We also obtained electron-neutrinos correlations and indicated their use as a signature of Majorana neutrinos.

## I. INTRODUCTION

In this paper we present a detailed analysis of the effects of massive neutrinos and their subsequent mixings in the Michel spectrum of free-muon decay.[1] We analyze both cases of Dirac and Majorana neutrinos.[2] This is a classical problem whose previous analysis tacitly did not include mixing of neutrinos even when they were assumed to be massive.[3] Current theoretical interest in unification of electroweak interactions described by the SU(2) × U(1) gauge theory[4] and the color $SU_c(3)$ interactions between quarks has focused our attention on the possible existence of both very heavy Majorana and light Majorana neutrinos.[5] Furthermore, partial-unification schemes such as the $SU_L(2) \times SU_R(2) \times U(1)$ theory also contain massive Majorana neutrinos.[6] It is clear that the fundamental question of whether neutrinos, and we know of three species, are massive, and if so what are their masses, deserves study in its own right. Our analysis will be general and is independent of any particular gauge models. We shall only assume that neutrinos have masses and for simplicity take them to be nondegenerate. A predominantly left-handed weak-interaction Lagrangian will be assumed. Details of our assumptions will be spelled out in Sec. II.

It is also our purpose in this paper to examine some possible tests of whether the neutrinos $\nu_e$, $\nu_\mu$, and $\nu_\tau$ are Majorana. If so the concept of a conserved lepton number has to be abandoned in accordance with most grand unified theories.[5] On the other hand, if the neutrinos involved in $\mu$ decays are strictly Dirac particles then a conserved lepton quantum number $L$ will be called for. To date we have only paid attention to no-neutrino double-$\beta$-decay[7] as a signal of the existence of Majorana neutrinos. This of course is a test of the violation of $L$, through the virtual exchange of such leptons. Here we propose a different test involving measuring the correlation between the decay $e^+$ and $\nu_e$ (Ref. 8) in the reaction

$$\mu^+ \to e^+\nu_e\nu_\mu^c . \tag{1.1}$$

The particle $\nu_e$ is the usual left-handed neutrino partner to the $e_L^-$ in the usual SU(2) × U(1) lepton doublet, whereas $\nu_\mu^c$ is the right-handed neutrino which forms a doublet with $\mu_R^+$. Details are given in Sec. II. The existence of high-flux muon beams at meson factories can provide the necessary neutrino flux. The neutrino $\nu_e$ can be detected in a large-mass neutrino detector via the inverse-$\beta$-decay process

$$\nu_e + n \to e^- + p . \tag{1.2}$$

The two different correlations for Dirac and Majorana neutrinos are presented in Sec. IV. We mention here that the difference in correlation appears even when the neutrinos $\nu_e$ are very light (i.e., less than a few eV), but depends crucially on the mixings between various neutrino flavors.

The layout of our paper is as follows. In Sec. II we spell out our theoretical assumptions and calculate the Michel spectrum of the $e^+$ for the decay of reaction (1.1). Particular attention is given to the three-neutrino world. This is the most relevant phenomenological case. For completeness we also present a brief review of the theory of neutrino mass.[9]

In Sec. III we examine in detail the sensitivity of a careful measurement of the asymmetry parameter $\xi$ for polarized-muon decays to the mass and the mixing parameters of a heavy neutrino that mixes into the decay (1.1).

In Sec. IV we discuss the case of electron-neutrino correlation measurements. Here we focus on differentiating between Dirac and Majorana neutrinos even if they are light. Both energy and angular correlations of the $\nu_e$ and the $e^+$ are given.

Section V contains our conclusions. Here we note that no attempt will be made to compare with experiments[10] since we do not include radiative corrections in our analysis and certainly such corrections are important for massless neutrinos. Such an elaborate calculation will be left for a future publication.

## II. THEORY OF NEUTRINO MIXINGS

For simplicity we start by discussing a single lepton family in the standard $SU(2)\times U(1)$ guage model.[4] The left-handed lepton fields are given isodoublet assignment $(\nu, e^-)_L$, where $\nu_L \equiv \frac{1}{2}(1-\gamma_5)\nu$. The right-handed fields are then put in isosinglet, i.e., $e_R^-$ and $\nu_R$ with $\nu_R \equiv \frac{1}{2}(1+\gamma_5)\nu$.[11] In the standard model,[4] $\nu_R$ is explicitly excluded. The general form of the neutrino mass term in the Lagrangian for electroweak interactions is given by

$$\mathcal{L}_M = -(A\bar{\nu}_L^c\nu_L + B\bar{\nu}_R^c\nu_R + D\bar{\nu}_L\nu_R + \text{H.c.})\,. \tag{2.1}$$

We adopt the convention that the charge-conjugation matrix $C$ is given by $C\equiv i\gamma^2\gamma^0$. Hence

$$\psi^c \equiv C\tilde{\bar{\psi}} = i\gamma^2\psi^* \tag{2.2}$$

and

$$\psi_L^c \equiv (\psi_L)^c = \tfrac{1}{2}(1+\gamma_5)\psi^c = (\psi^c)_R\,. \tag{2.3}$$

The first two terms are the Majorana mass terms for the left- and right-handed neutrinos, respectively, and they explicitly violate lepton numbers by two units. The term $A$ can arise in the $SU(2)\times U(1)$ model via the introduction of Higgs triplets; whereas $B$ requires a Higgs singlet or a mass term. In general $A\neq B$. In the SO(10) model of grand unification[12] it is argued that $B \gg A$. The third term is the Dirac mass term and exists only if the right-handed field is not excluded by fiat. The mass matrix can be diagonalized by a linear combination of the two Majorana fields $\chi=\psi_L+\psi_L^c$ and $\eta=\psi_R+\psi_R^c$ yielding the mass eigenvalues

$$M_{1,2}=\tfrac{1}{2}\{(A+B)\pm[(A-B)^2+D^2]^{1/2}\}\,. \tag{2.4}$$

In the SO(10) model where $B$ is $\gtrsim 10^8$ GeV whereas $A$ and $D$ are ~GeV, then one obtains a very heavy and a light Majorana neutrino. Without loss of generality we divide our discussions into the two cases of pure Dirac neutrinos and pure Majorana neutrinos only.

### A. Dirac neutrinos

Here the fields $\nu_L$ and $\bar{\nu}_L$ are distinguishable by a conserved lepton number $L$. Conventionally $L=1$ for the particle and $L=-1$ for the antiparticle. We can easily generalize the above to $N$ lepton families. The weak neutrino eigenstates are denoted by $\nu_a$ where $a=e,\mu,\tau,\ldots$. The physical mass eigenstates are represented by $\nu_i$ where $i=1,2,\ldots,N$. The corresponding masses are given by $m_i$. They are related by an $N\times N$ unitary transformation $\nu_a = \sum_{i=1}^N U_{ai}\nu_i$. The Pontecorvo mixing parameters[13] satisfy the orthonormality conditions

$$\sum_j U_{aj}^\dagger U_{jb}=\delta_{ab} \tag{2.5}$$

and

$$\sum_a U_{ia}^\dagger U_{aj}=\delta_{ij}\,. \tag{2.5$'$}$$

In the special case of $A=B=0$, we have only a Dirac mass term and it is seen that this corresponds to the case of two degenerate Majorana leptons.[9] More importantly for Dirac neutrinos the concept of a conserved lepton number $L$ can be introduced; whereas for Majorana neutrinos this has to be taken as a broken symmetry. With a broken global symmetry one would have a massless Goldstone boson which can manifest itself in the Eötvos experiment. We note that *both* Dirac and Majorana mass terms flip helicity, and it will be difficult to tell the difference by helicity measurements.

For $N>2$, the $U_{ai}$'s are in general complex. At this stage we shall also take them to be constants which experiments are to determine. The effective four-fermion interaction Lagrangian relevant to the decay (1.1) is given by

$$\mathcal{L}=\frac{g^2}{8M_W^2}\sum_{i,j}U_{ei}U_{\mu j}^\dagger\bar{e}\gamma_\lambda(1-\gamma_5)\nu_i\bar{\nu}_j\gamma^\lambda(1-\gamma_5)\mu+\text{H.c.} \tag{2.6}$$

Neglecting the electron mass, the Michel spectrum for polarized $\mu^+$ can be obtained straightforwardly and is

$$\begin{aligned}\frac{dR}{dx\,dz_e}=\frac{G_F^2m_\mu^5}{96\pi^3}\sum_{i,j}|U_{ei}|^2|U_{\mu j}|^2\frac{x^2}{(1-x)^3}\lambda^{1/2}((1-x),\delta_i^2,\delta_j^2)\\ \times\Big\{[(1-x)^2+(1-x)(\delta_i^2+\delta_j^2)-2(\delta_i^2-\delta_j^2)^2]\Big[\Big(1-\frac{x}{2}\Big)+\frac{x}{2}\vec{S}_\mu\cdot\hat{P}_e\Big]\\ +\tfrac{1}{2}(1-x)[(1-x)^2-2(1-x)(\delta_i^2+\delta_j^2)+(\delta_i^2-\delta_j^2)^2](1-\vec{S}_\mu\cdot\hat{P}_e)\Big\}\,,\end{aligned} \tag{2.7}$$

where $x \equiv 2E_e/m_\mu$, $z_e \equiv \cos\theta_e$, $s_\mu$ is the spin vector of the muon, $\delta_i^2 = m_i^2/m_\mu^2$, and $\lambda(x,y,z) \equiv x^2+y^2+z^2 - 2xy - 2yz - 2zx$. The unit vector along the $e^+$ direction is given by $\hat{P}_e$. It is seen that the $e^+$ spectrum is actually a sum of nine different spectra for the three-nondegenerate-neutrinos case. In the limit that all neutrinos are massless or very light, Eq. (2.7) reduces to the usual spectrum with the aid of Eq. (2.5).

Next we focus our attention on the three-neutrinos world. We shall adopt the assumption that the masses $m_i$ are in ascending order of $\nu_i$; hence, $m_1 < m_2 < m_3$. Current experimental indications are that $16 < m_{\nu_e} < 40$ eV/$c^2$,[14] $m_{\nu_\mu} < 0.5$ MeV/$c^2$,[15] and $m_{\nu_\tau} < 250$ MeV/$c^2$.[16] If one assumes that $\nu_e$ is mostly $\nu_1$, $\nu_\mu$ is mostly $\nu_2$, etc., we have $\delta_1 \lesssim 4\times 10^{-7}$ and $\delta_2 < 4.8\times 10^{-3}$, and hence they can be neglected.[17] The only term that may contribute is $\delta_3$ if it is large. The analysis of $\pi_{e2}$ decays[18] have shown that if $m_3$ is in the few MeV/$c^2$ range, the Pontecorvo parameters $|U_{e3}|$ must be less than 0.02. However, with $m_3$ less than 0.5 MeV/$c^2$ there exist no good constraints.[19,20]

Keeping only $\delta_3$ we have

$$\frac{dR}{dx\,dz_e} = \frac{G_F{}^2 m_\mu{}^5}{48\pi^3}\left[(|U_{33}|^2 + |U_{13}|^2|U_{23}|^2)R_0 + (|U_{13}|^2 + |U_{23}|^2 - 2|U_{13}|^2|U_{23}|^2)R_3 + |U_{13}|^2|U_{23}|^2 R_{33}\right], \tag{2.8}$$

where

$$R_0 = \frac{x^2}{4}\left[3 - 2x - (1-2x)\vec{S}_\mu\cdot\hat{P}_e\right]\theta(1-x)\,, \tag{2.9a}$$

$$R_3 = \frac{x^2(1-x-\delta_3{}^2)}{4(1-x)^3}\left\{2[(1-x)^2 + \delta_3{}^2(1-x) - 2\delta_3{}^4]\left(1 - \frac{x}{2} + \frac{x}{2}\vec{S}_\mu\cdot\hat{P}_e\right) + (1-x)[(1-x)^2 - 2(1-x)\delta_3{}^2 + \delta_3{}^4](1 - \vec{S}_\mu\cdot\hat{P}_e)\right\}\theta(1-\delta_3{}^2)\,, \tag{2.9b}$$

and

$$R_{33} = \frac{x^2}{4}\frac{(1-x-4\delta_3{}^2)^{1/2}}{(1-x)^{3/2}}\left[2(1-x+2\delta_3{}^2)\left(1 - \frac{x}{2} - \frac{x}{2}\vec{S}_\mu\cdot\hat{P}_e\right) + (1-x)(1-x-4\delta_3{}^2)(1-\vec{S}_\mu\cdot\hat{P}_e)\right]\theta(1-4\delta_3{}^2)\,. \tag{2.9c}$$

The first term is the usual Michel spectrum with massless neutrinos.[21] It is amusing that the strength of this term is proportional to $|U_{33}|^2$. At this stage one must regard the mixing parameters as independent constants. It is reasonable to assume that in general $|U_{ii}| > |U_{ij}|$, i.e., the mixings are in general small. One takes a guide from the considerations of introducing discrete horizontal symmetries to estimate the quark mixing angles and naively takes them to be approximately correct for neutrino mixings. This we call hierarchical mixing, from which we obtain[22]

$$|U_{12}|^2 \simeq \frac{m_e}{m_\mu} = 0.005\,, \tag{2.10a}$$

$$|U_{13}|^2 \simeq \frac{m_e}{m_\tau} = 0.0003\,, \tag{2.10b}$$

$$|U_{23}|^2 \simeq \frac{m_\mu}{m_\tau} = 0.059\,, \tag{2.10c}$$

and

$$|U_{33}|^2 = 0.94\,. \tag{2.10d}$$

$R_{33}$ makes a negligible contribution and the deviation from the standard Michel spectrum is mainly due to $R_3$.

Furthermore the hierarchical mixings will give a very small probability of finding $\nu_3$ in $\pi\to\mu\nu$ decays.[18] The most recent search for secondary peaks in the muon spectrum performed at SIN limits the probability of finding a 6-to-11-MeV/$c^2$ neutrino in $\pi\to\mu\nu$ to less than about one percent. For hierarchical mixing one expects[23] about 6% probability. Hence, $m_3 < 6$ MeV/$c^2$ if $|U_{23}| > 0.06$. For $|U_{23}| < 0.06$ the data is too preliminary to set limits on $m_3$. The limit on the mass of $m_3$ will be determined by the energy resolution and statistics.

In Fig. 1(a) we display the difference of the mixing of a heavy $\nu_3$ into $\nu_1$ and $\nu_2$ to the full Michel spectrum. We see that the largest difference is to be found at the high-energy end of the positron. Figure 1b magnifies the $x\to 1$ region and one observes that with hierarchical mixing a 1-MeV/$c^2$ neutrino will show up only at $x > 0.99$. Hence, this is yet another way of finding the effects of a heavy $\nu_3$. All mixings are taken to be hierarchical [see Eqs. (2.10a)–(2.10d)].

In Fig. 2 we show the dependence of the $e^+$ spectrum on the Pontecorvo mixings for $m_3 = 5$ MeV/$c^2$. The sensitivity of the spectrum to $U_{ij}$ is not very

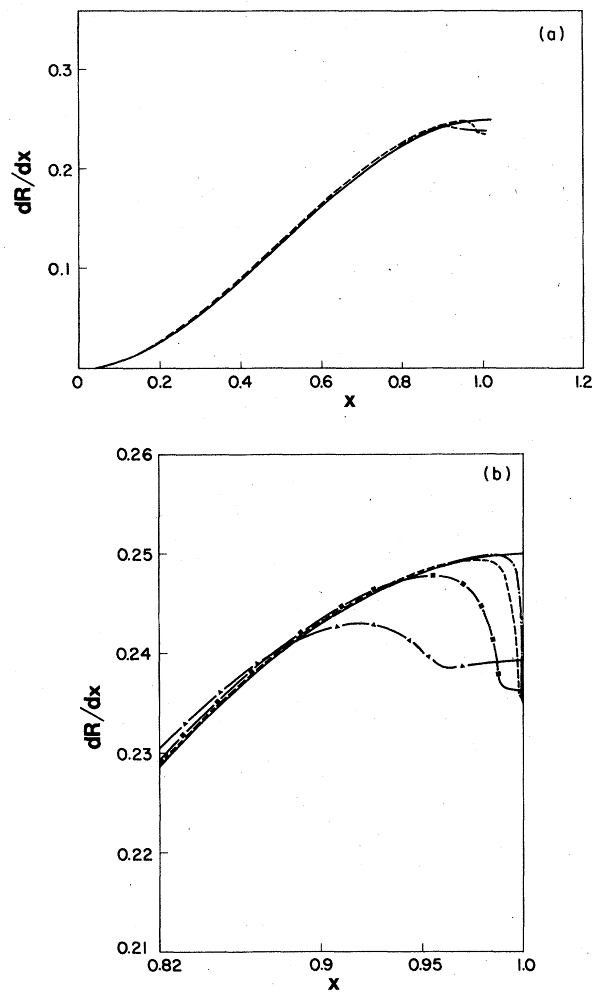


FIG. 1. (a) The Michel spectrum for unpolarized $\mu^+$ decay. The solid curve is for neutrinos all massless. The dashed curve is for $m_3 = 10$ MeV/$c^2$. The dashed-dot curve is for $m_3 = 20$ MeV/$c^2$. In both cases $m_1 = m_2 = 0$. Overall normalization is arbitrary. (b) High-energy end of the Michel spectrum with hierachical mixings of $\nu_3$ (see text). The solid curve is the massless-neutrinos case. The triangle-line curve is for $m_3 = 20$ MeV/$c^2$; the square-line curve is for $m^3 = 10$ MeV/$c^2$; the dashed curve is for $m^3 = 5$ MeV/$c^2$, and the dash-dot curve is for $m^3 = 3$ MeV/$c^2$. Normalization is as in Fig. 1(a).

pronounced until $x > 0.95$. We have used the hierarchical values and the extreme values of $U_{23} = 0.8$ and 0.5 but keeping $|U_{13}| \simeq 0$ as illustrations. Figure 3 illustrates the effects of $|U_{13}| = |U_{23}|$. As expected the difference is not very pronounced.

Currently there are two experiments[24,25] in progress to measure the $\vec{S}_\mu \cdot \hat{P}_e$ in the forward directions, i.e., the term with $\vec{S}_\mu \cdot \hat{P}_e = -1$ to 0.1% accuracy. As seen from Eq. (2.8a), the term

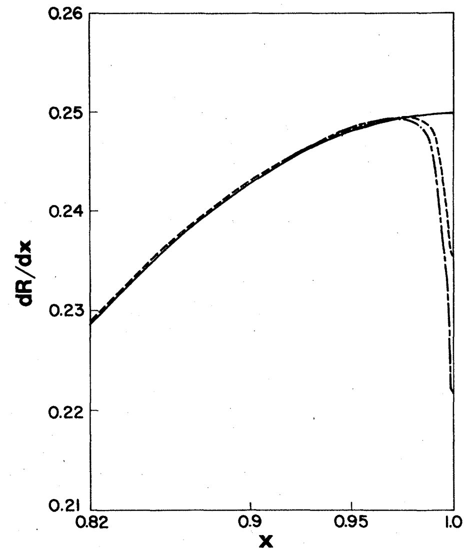


FIG. 2. Sensitivity of the $x \to 1$ end of the Michel spectrum to the mixing parameter $|U_{23}|$ for $m_3 = 5$ MeV/$c^2$. The dashed curve denotes $|U_{23}|^2 = 0.059$; the solid curve is for $|U_{23}|^2 = 0.2$, and the dash-dot curve is for $|U_{23}|^2 = 0.5$. The mixing $U_{13} \approx 0$ for all the above cases. All curves are normalized with respect to each other.

$\vec{S}_\mu \cdot \hat{P}_e$ receives contribution from $R_0$, $R_3$, and $R_{33}$. If all neutrinos are very light one would obtain a $(1-x)$ distribution for

$$\frac{(1-x)^3 dR}{dx\, dz_e} \Big/ x^2$$

at $z_e = -1$. In Fig. 4 we show the deviations from

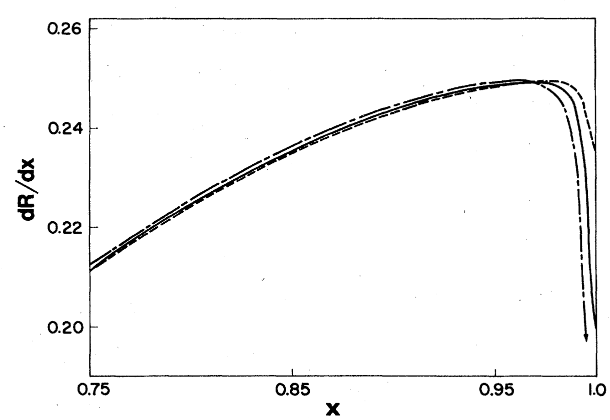


FIG. 3. Illustration of the effects of $U_{13}$ on the Michel spectrum for $m_3 = 5$ MeV/$c^2$ and $|U_{23}|^2 = 0.059$. The solid curve is the standard massless-neutrinos case. The dashed curve is for $|U_{13}|^2 = 3 \times 10^{-4}$ and the dash-dot curve is for $|U_{13}|^2 = 0.059$. The curves are all normalized to each other.

$(1-x)$ falloff due to a 5-MeV/$c^2$ $\nu_3$ with 0.1 mixing and maximal mixing of $|U_{e3}|=|U_{\mu 3}|=\frac{1}{3}$. The curves are displaced from each other so as to illustrate their differences. It is clear that only the case of maximal mixing can be distinguished from the massless-neutrino case.

The kinks in the maximal-mixing case in Fig. 4 are due to the kinematic turning off of $R_{33}$ at $x=0.9909$ for $m_3=5$ MeV/$c^2$ and then later at $x=0.9977$ when $R_3$ is switched off [see Eqs. (2.8)–(2.9c)]. For $|U_{13}|^2=|U_{23}|^2=0.1$ the corresponding deviation from $1-x$ behavior is hardly discernable. For $m_3=3$ MeV/$c$, the first kink will take place at $x=0.9967$ and the second one at $x=0.9992$. The first one will be measurable if the mixing is large and the second one will be outside the range currently planned in experiments.[24,25] Since such large mixing for an intermediate mass $\nu_3$ is disfavored by $\pi^+\to\mu^+\nu$, we conclude that massive neutrinos with predominantly left-handed couplings are unlikely to cause the measurement of the $\xi$ parameter to deviate from the $1-x$ behavior to within the sensitivity of currently planned experiments.

### B. Majorana neutrinos

Since we assume that the effective weak interaction at low energies is predominantly left handed[26] we rewrite (2.6) for Majorana neutrinos

$$\mathcal{L}=\frac{g^2}{8M_w^2}\sum_{i,j} V_{ei}V_{\mu j}^{\dagger}\,\bar{e}\gamma_\lambda(1-\gamma_5)\nu_i^c\bar{\nu}_j\gamma^\lambda(1-\gamma_5)\mu+\text{H.c.} \tag{2.11}$$

The mixing matrix $V$ is different from the Dirac case[9] and the sums over $i,j$ are taken over all light Majorana neutrinos only. Light right-handed Majorana neutrinos are omitted since its effective coupling is shown to be at least $10^{-4}$ times weaker than the left-handed ones[26] or of the order of $(m_\nu/E_\nu)^2$.

The Michel spectrum is then given by

$$\begin{aligned}\frac{dR}{dx\,dz_e}=\frac{G_F^2m_\mu^5}{96\pi^3}\Bigg\{&\sum_i\sum_{\substack{j\\ i\neq j}}|V_{ei}|^2|V_{\mu j}|^2\lambda^{1/2}((1-x),\delta_i^2,\delta_j^2)x^2(1-x)^{-3}\\
&\times\Big[[(1-x)^2+(1-x)(\delta_i^2+\delta_j^2)-2(\delta_i^2-\delta_j^2)^2]\Big(1-\frac{x}{2}+\frac{x}{2}\vec{S}_\mu\cdot\hat{P}_e\Big)\\
&\quad+\tfrac{1}{2}(1-x)[(1-x)^2-2(1-x)(\delta_i^2+\delta_j^2)+(\delta_i^2-\delta_j^2)^2](1-\vec{S}_\mu\cdot\hat{P}_e)\Big]\\
&+\frac{x^2}{2}(1-x)^{-3/2}\sum_{i=1}|V_{ei}|^2|V_{\mu i}|^2(1-x-4\delta_i^2)^{1/2}\Big[2(1-x+2\delta_i^2)\Big(1-\frac{x}{2}+\frac{x}{2}\vec{S}_\mu\cdot\hat{P}_e\Big)\\
&\quad+(1-x)(1-x-10\delta_i^2)(1-\vec{S}_\mu\cdot\hat{P}_e)\Big]\Bigg\}.\end{aligned} \tag{2.12}$$

The second term is due to the antisymmetrization required for Majorana neutrinos. Again when $\delta_i\simeq 0$, Eq. (2.12) reduces to the same usual Michel spectrum as the Dirac case.

Specializing to the three neutrino world with $m_3$ being the only significant mass gives the following distribution for unpolarized muon

$$\begin{aligned}\frac{dR}{dx\,dz}=(\tau_\mu^0)^{-1}\{&(|V_{11}|^2+|V_{12}|^2)(|V_{21}|^2+|V_{22}|^2)R_0\\
&+[|V_{13}|^2(|V_{21}|^2+|V_{22}|^2)+|V_{23}|^2(|V_{11}|^2+|V_{12}|^2)]R_3+|V_{13}|^2|V_{23}|^2R_{3'}\},\end{aligned} \tag{2.13}$$

where

$$R_3'=\frac{x^2}{4(1-x)^{3/2}}(1-x-4\delta_3^2)^{1/2}[(1-x)(3-2x)+2\delta_3^2(-3+4x)]\,\theta(1-4\delta_3^2) \tag{2.14}$$

with $R_0$ and $R_3$ given in Eq. (2.9a) and (2.9b). Comparing Eqs. (2.13) and (2.14) and (2.9a), (2.9b) shows that the Dirac and Majorana neutrinos show very little difference in the Michel spectrum if the mixings are not too dissimilar. The only difference resides in $R_3$ and $R_3'$ and is multiplied by $\delta_3^2$. Unless $m_3$ is greater than several MeV/$c^2$ our previous conclusions on the distribution will continue to hold. This is a reflection of the left-handedness of weak interactions at these low en-

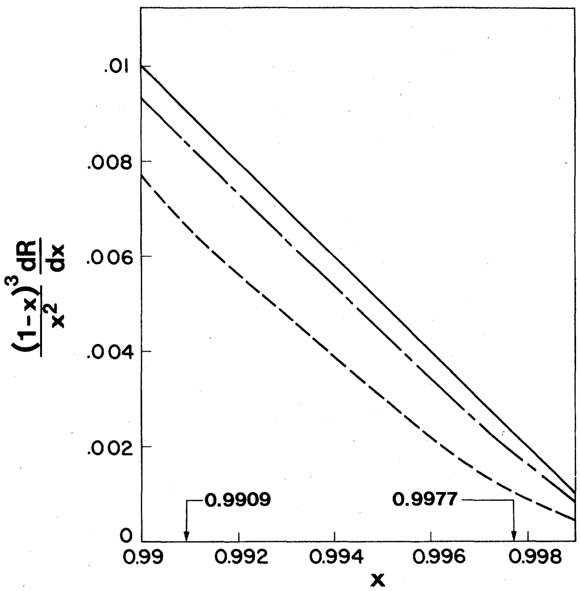


FIG. 4. Plot of $[(1-x)^3/x^2]dR/dx\,dz_e$ as a function of $x$ for $\vec{S}_\mu\cdot\hat{P}_e=-1$. The solid line is the $1-x$ behavior of massless neutrinos. The dash-dot curve is for $|U_{13}|^2=|U_{23}|^2=0.1$ mixing and the dashed curve is for maximal mixings of $|U_{13}|^2=|U_{23}|^2=\frac{1}{3}$. The curves are arbitrarily normalized so as to display their different $x$ dependence. The mass of $m_3$ is taken to be 5 MeV/$c^2$.

ergies. At the high-energy end a deviation from the standard case comes about because massive neutrinos are no longer helicity eigenstates for both Dirac and Majorana neutrinos. For $m_3 \ll m_\mu$ the disparity of the two cases due to the antisymmetrization of Majorana wave functions is diminished by the factor of $\delta_3^2$. Thus, it will not be fruitful to look for their difference in the $e^+$ spectrum unless the mass of $\nu_3$ or $\nu_\tau$ is in the tens of MeV range and the mixings are large. We deemed this to be unlikely.

## III. ELECTRON-NEUTRINO CORRELATIONS

Electron-neutrino correlations measurements were suggested previously as a test for possible admixture of $V+A$ currents into the standard $V-A$ current $\times$ current structure.[8,27] We explore here the possibility of using this correlation to probe the difference between Dirac and Majorana neutrinos. This difference should show up more prominantly than in the measurement of the $e^+$ spectrum alone, since one is actually observing the neutrino. In particular we focus on the coincidence of the two-step process of (1.1) and (1.2). We choose $\nu_e$ since it interacts via the charged current with a neutron, whereas $\nu_\mu$ will have only neutral-current interactions at these energies. Neutral-current interactions in general will occur for all species of neutrinos and thus do not seem suitable for use in selecting a particular neutrino.

Since it is reasonable to expect that $|U_{ei}|(V_{ei})$ and $|U_{\mu 2}|(V_{\mu 2})$ are the dominant coefficients, out of $N^2$ branches, the branch $\mu^+\to e^+\nu_1\bar{\nu}_2$ for Dirac neutrinos or $\mu^+\to e^+\nu_1\nu_2^c$ for Majorana neutrinos is the dominant branch to be measured in $e\nu_e$ correlation experiments. For Dirac neutrinos, the oscillations of $\bar{\nu}_2$ will not affect the rate for correlation experiments. If the neutrino is at a distance $d$ from the point of reaction (1.1), some fraction of $\nu_1$ will be depleted due to oscillations, i.e., $|\langle\nu_e|\nu_1\rangle|^2 \leq 1$ at $d$, but this will be compensated for by the oscillations into $\nu_e$ from the nondominant branches of the decay. For example, the number of $\nu_e$ at the detector will also have contribution proportional to $|\langle\nu_e|\nu_2\rangle|^2$ where $\nu_2$ comes from the branches $\mu^+\to e^+\nu_2\bar{\nu}_1$, $e^+\nu_2\bar{\nu}_2$, etc. Unless one is at a minimum for $\nu_e$ oscillations, in general the flux for $\nu_e$ Dirac neutrino will not be seriously affected by oscillations.

In general, for Majorana neutrinos, neutrino oscillations can alter the flux of $\nu_e$ differently from the situation of Dirac neutrinos. The complications can arise from additional $SU(2)_L$-singlet right-handed neutrinos which may lead to a rectangular rather than square transformation matrix $V_{ij}$. If one assumes that $V$ is a square matrix, then one has a similar case as Dirac neutrinos. The dominant branch is still $\mu^+\to e^+\nu_1\nu_2^c$ with $\nu_1$ and $\nu_2^c$ left-handed and right-handed neutrinos, respectively. Again assuming $|V_{ei}|$ and $|V_{\mu 2}|$ to be the largest element of the mixing matrix, the flux of $\nu_e$ is not expected to be substantially altered at a distance $l$ after decay (1.1).

We can calculate the positron-neutrino correlation using the effective Lagrangian of Eq. (2.6). For Dirac neutrinos the double-differential distribution is given by

$$\frac{dR}{dx\,d\cos\theta_{e\nu}}=\frac{G_F{}^2 m_\mu{}^5}{32\pi^3}|U_{e1}|^2\frac{x^2(1-x)}{(1-x\sin^2\theta_{e\nu}/2)^4}\left\{\cos^2\frac{\theta_{e\nu}}{2}+\sum_{j=1}^{N}|U_{2j}|^2\delta_j{}^2\left[(1-x)\sin^2\frac{\theta_{e\nu}}{2}-2\cos^2\frac{\theta_{e\nu}}{2}\right]\right\}, \tag{3.1}$$

where $\theta_{e\nu}$ is the opening angle between the $e^+$ and $\nu_e$ which is to be detected. The second term gives the contribution of the $j$th massive neutrino. Equation (3.1) is only good to $O(\delta_j{}^2)$. If there are more than one massive neutrino contributing then a sum of $j$ is taken. For the three-neutrino world, we have one $\delta_3{}^2$ and the contribution of the second term is negligible. We have also explicitly assumed that $\delta_1$ is small in accordance with the measurement of Ref. 14. If all neutrinos are light then we have just the first term and all mixings have been summed over. In this case only an overall factor $|U_{ei}|^2$ enters. The limit of no mixing is obtained by setting $|U_{ei}|^2=1$, and $\delta_j=0$ in Eq. (3.1).

With the same assumptions as above we can calculate the distribution for Majorana neutrinos. The following result is obtained:

$$\frac{dR}{dx\,d\cos\theta_{e\nu}}=\frac{G_F^2 m_\mu^5}{32\pi^3}|V_{l1}|^2\frac{x^2(1-x)^2}{(1-x\sin^2\theta_{e\nu}/2)^4}\left\{\cos^2\frac{\theta_{e\nu}}{2}-\tfrac{1}{2}|V_{\mu 1}|^2\left[\cos\theta_{e\nu}+x(2-x)\sin^4\frac{\theta_{e\nu}}{2}\right]+O(\delta_j^2)\right\}. \quad (3.2)$$

Since Eqs. (3.2) and (3.1) are correlations involving a very light neutrino, namely, $\nu_1$ or $\nu_e$, it is not surprising that the difference is proportional to $|V_{21}|^2$. In the Majorana case this term does not combine fully with the rest due to antisymmetrization of the wave function. It is clear from Eq. (3.2) that the difference is larger if the mixing is substantial. In Fig. 5 we show the difference in the double-differential distribution for a Dirac and a Majorana neutrino at the point $x=\frac{1}{2}$ as a function of $\sin^2\frac{1}{2}\theta_{e\nu}$. The value of $|U_{21}|=0.1$ is used. If the mixing $|U_{21}|\sim 0.1$, then the two kinds of neutrinos can be distinguished if a 5% correlation experiment can be done. Certainly for maximal mixing of $|U_{12}|=\frac{1}{3}$ a much larger difference can be seen.

Another correlation will be the distribution in $x$ and $y$ where $y\equiv 2E_\nu/m_\mu$. For the Dirac case we have

$$\frac{dR}{dx\,dy}=\frac{G_F^2 m_\mu^5}{32\pi^3}|U_{e1}|^2[y^2(1-y)+O(\delta_j^2)], \quad (3.3)$$

whereas Majorana neutrinos induce a distribution given by

$$\frac{dR}{dx\,dy}=\frac{G_F^2 m_\mu^5}{32\pi^3}\{|V_{e1}|^2 y^2(1-y)-\tfrac{1}{2}|V_{\mu 1}|^2[y(1-y)-(2-x-y)\times(1-x-y)]+O(\delta_j^2)\}. \quad (3.4)$$

As before the difference is proportional to $|V_{\mu 1}|^2$. Furthermore, at the end point of $y=1$ the Majorana neutrino distribution does not vanish, unlike that of Eq. (3.3). This shows yet another difference in the two cases. Figure 6 displays a more pronounced difference between Dirac and Majorana neutrinos. As seen in Eq. (3.3) for Dirac neutrinos one has a flat distribution in $x$ for all values of $y$. On the other hand, Majorana neutrinos give a quadratic dependence on $x$ for any given value of $y$. We have illustrated this for the point $y=0.25$. Experimentally it will be more difficult to measure the energy correlations since it will require the

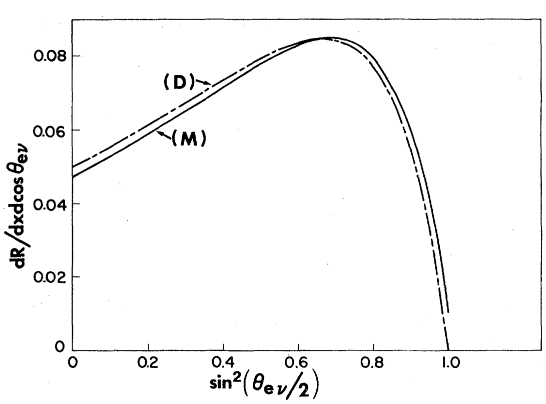


FIG. 5. The double-differential cross section $dR/dx\,d\cos\theta_{e\nu}$ is plotted as a function of $\sin^2(\theta_{e\nu}/2)$ for Dirac neutrinos (*D*) and Majorana neutrinos (*M*). The value of $x=\frac{1}{2}$ is chosen and the mixing $|U_{21}|^2=0.1$ is used.

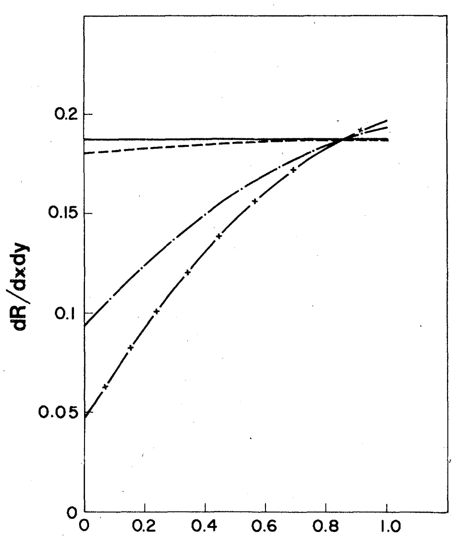


FIG. 6. Energy correlations between $e^+$ and $\nu_e$ for $y=\frac{1}{4}$ (see text). The solid line is obtained for Dirac neutrinos; the rest are for Majorana neutrinos. The dashed curve is for hierarchical mixing, the dash-dot curve is for Cabibbo mixing, and the dash-cross curve is for $|V_{21}|^2=0.1$. The normalizations are arbitrary.

measurement of both the energies of the final-state electron and proton of reaction (1.2).

We realize that such $e^+\nu_e$ correlations are notoriously difficult; however the issue of Majorana versus Dirac neutrinos is central to current unification models and thus deserves more attention. Using the currently available surface $\mu^+$ flux at meson factories and a 100-ton $4\pi$ water detector one obtains several tens of events per day for the correlation measurements. The difficulty will be in detecting very low-energy electrons and protons. Perhaps muon decay in flight will be more suitable.

## IV. CONCLUSIONS

We have examined the effects of massive Dirac and Majorana neutrinos and their mixings in free-muon decays under the assumption that the weak interactions responsible are predominantly left-handed. It is found that in general massive neutrinos change the high-energy end of the $e^+$ spectrum more significantly then the rest of the spectrum. This is as expected since for a massive neutrino, helicity conservation is no longer as good as with massless neutrinos. In particular in the three-neutrino world a 5-MeV/$c^2$ $\nu_3$ with less than 0.1 mixing into $\nu_\mu$ or $\nu_e$ will not disturb the Michel spectrum too strongly. Secondly, the difference between Dirac and Majorana neutrinos is negligible in the Michel spectrum unless the neutrino under consideration is relatively heavy and has a large mixing. A heavy neutrino with large mixing is unlikely since its presence can be inferred from accurate measurements of $\mu^+$ spectrum in $\pi^+\to\mu^+\nu$ decays which give no such indications.

Turning to precise measurement of the $\vec{S}_\mu\cdot\hat{P}_e$ term in the Michel spectrum, we found that heavy neutrinos less than 5 MeV/$c^2$ do not change the $1-x$ behavior substantially unless their mixing is large. Again no discernable distinction between Dirac and Majorana neutrinos is obtained.

We have also examined the dependence of positron-spin terms in the Michel spectrum on neutrino masses and their mixings and found that these terms are very insensitive to the neutrino masses. They are typically very small terms and owing to current sensitivity of $e^+$ spin measurements we did not pursue them further.

To test for light Majorana neutrinos in $\mu^+$ decays we propose that $e^+$ and $\nu_e$ correlation experiments be closely examined since this touches upon the important issue of the existence of conserved lepton number. We found differences that are directly proportional to the mixing of the two principal neutrinos $\nu_1$ and $\nu_2$ [mostly $\nu_e$ and $\nu_\mu$ in the usual $SU(2)_L\times U(1)$ three-lepton-families scenario]. Presently, we do not have a firm phenomenological guide on the value of this mixing; however, new neutrino-oscillation experiments both at nuclear reactors and the meson factories should provide valuable information. This will certainly help determine whether the type of correlation experiments we are considering are feasible. If so then one can look into whether it is better to have the neutrino detector separated from the $\mu^+$ decay region or to enclose the decay region so that one has a $4\pi$ solid-angle coverage. We feel that the above possibility deserves further investigation.

*Note added.* While this manuscript is being prepared we received a paper by R. Shrock [Phys. Rev. D 24, 1275 (1981)] which contained some of our conclusions. However, our approach is different and he does not address the question of $e\nu_1$ correlations.

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