PHYSICAL REVIEW D VOLUME 24, NUMBER 7 1 OCTOBER 1981

Chou-Yang model and predictions for high-energy proton-deuteron scattering

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(Received 18 August 1980)

The modified Chou-Yang model and fits to the experimental data on the proton and the deuteron form factors by analytic formulas are used to calculate the differential cross section and investigate the possible presence of dips and higher-order maxima in high-energy elastic *pd* scattering, neglecting real-part and spin effects. Calculated values of the cross section agree well with the existing Fermilab data in the small-$|t|$ region. For larger $|t|$ at available Fermilab energies no dip structure is expected to appear, only a shoulder. Dips and higher-order maxima are predicted only for very high energies. As compared to *pp* and πp scattering the dip structure is predicted to start at much higher energies and lower values of $|t|$.

I. INTRODUCTION

Out of many models proposed for hadron-hadron collision processes the Chou-Yang model[1] with Hayot-Sukhatme[2] modification has proved to be the most successful one for predictions in high-energy elastic scattering. During the last decade the predictive power of this model and its comparison with experimental data have been studied for *pp* scattering,[3-6] extensively, and for $\pi^{\pm}p$, $K^{\pm}p$, and $\pi\pi$ scattering,[7,8] to a certain extent. In one of the most recent calculations, Chou and Yang[6] have shown that multiple dip structures will appear for very high energies for *pp* and $\pi^{\pm}p$ scattering, the number of dips increasing with increase of total cross section (or equivalently the interaction strength). Recently Chan *et al.*[8] have also noted the possible presence of dip structure for $\pi^{\pm}p$ scattering at high energies. The reason why the multiple dip structures have not yet appeared in the high-energy data even in the case of *pp* scattering has been pointed[6] out to be due to the fact that such high energies have not yet been achieved by the accelerators at present.

Using analytic parametrizations[9,10] of the existing data on the proton and the deuteron form factors, we first fit the product of the two form factors as a sum of four exponentials and using this fit, then calculate differential cross sections, and positions of dips and higher-order maxima for elastic *pd* scattering at high energies using the Chou-Yang model and neglecting real-part and spin effects. Available Fermilab data in the small-$|t|$ region agree well with the predicted values of the differential cross section. As compared to the experimental observation of the position of a dip in *pp* scattering and the calculated position for $\pi^{\pm}p$ scattering,[6,8] in *pd* scattering dips are predicted to show up at much higher energies and much smaller values of $|t|$. At available Fermilab energies, however, only a shoulder is predicted to appear.

Originally the Chou-Yang hypothesis was proposed for high-energy elastic hadron-hadron collision processes. In the present case one of the colliding objects is a light nucleus consisting of two nucleons. If the future data agree reasonably well with our calculations for larger $|t|$ values, it will indicate the possibility that the Chou-Yang model might be applicable for hadron-nucleus elastic scattering at high energies. But, on the other hand, if there is serious disagreement between theoretical predictions and the experimental data, even after real-part and spin effects are taken into account, the idea of applying the Chou-Yang model to such collision processes should be given up.

The paper is planned in the following manner. In Sec. II we discuss theoretical aspects of the model and a convenient scheme for calculation. In Sec. III we discuss exponential parametrization of the product of two form factors by interpolation of the proton and deuteron form-factor data using analytic formulas. In Sec. IV we report our results of calculation. Section V is devoted to a brief discussion of the results and a statement of summary and conclusion.

II. THEORETICAL ASPECTS OF THE MODEL

Using eikonal approximation for very small wavelengths and ignoring spin and real-part effects the differential cross section for elastic hadron-hadron scattering can be expressed as[1]

$$\frac{d\sigma}{dt} = \pi |a(t)|^2 , \tag{1}$$

where

$$a(t) = \int \frac{d^2\vec{b}}{2\pi} [1 - S(\vec{b})] e^{i\vec{k}\cdot\vec{b}} \tag{2}$$

with

$$S(\vec{b}) = e^{-\Omega(\vec{b})} . \tag{3}$$

In Eqs. (1)–(3) $\vec{k}$ is the two-dimensional transverse-momentum vector with $\vec{k}^2 = -t$, t being the square of four-momentum transfer in hadron-hadron scattering. Here $\vec{b}$ is the two-dimensional impact parameter and $\Omega(\vec{b})$ is the blackness at impact parameter $\vec{b}$. As in other cases of hadron-hadron elastic scattering, the following assumption is made to relate the differential cross section of pd scattering with the electromagnetic form factors of the proton $F_p(t)$, and the deuteron $F_d(t)$,

$$\langle\Omega\rangle = \mu_{pd} F_p(t) F_d(t), \tag{4}$$

where $\langle\Omega\rangle$ is the two-dimensional Fourier transform of the blackness function and μ_{pd} is the interaction strength which is generally a function[2] of s. In fact the only s dependence of the cross section at very high energies, according to the modification of the model,[2] is through the interaction strength. Expanding $S(\vec{b})$ as a series in increasing order of the convolution product one obtains, using Eqs. (2)–(4),

$$a_{pd}(t) = \mu_{pd} F_p(t)F_d(t) - \frac{\mu_{pd}^2}{2!} F_p(t)F_d(t) * F_p(t)F_d(t) + \frac{\mu_{pd}^3}{3!} F_p(t)F_d(t) * F_p(t)F_d(t) * F_p(t)F_d(t) - \cdots . \tag{5}$$

For a given value of μ_{pd}, obtainable from the total cross-section data, the series (5) can be used to compute the amplitude if a reasonably good estimation is available for the product of the two form factors. In Sec. III, we parametrize the product of the two form factors as a sum of four exponentials,

$$F_p(t)F_d(t) = \sum_{i=1}^{4} a_i e^{-b_i|t|} . \tag{6}$$

Using Eq. (6) the scattering amplitude can be written as

$$a_{pd}(t) = \mu_{pd} \sum_i a_i \exp(-b_i|t|) - \frac{\mu_{pd}^2}{2!} \sum_{i,j} \frac{a_i a_j}{2Y_{ij}} \exp\left(\frac{-b_i b_j |t|}{Y_{ij}}\right) + \frac{\mu_{pd}^3}{3!} \sum_{i,j,l} \frac{a_i a_j a_l}{2^2 Y_{ijl}} \exp\left(\frac{-b_i b_j b_l}{Y_{ijl}} |t|\right)$$
$$- \frac{\mu_{pd}^4}{4!} \sum_{i,j,l,m} \frac{a_i a_j a_l a_m}{2^3 Y_{ijlm}} \exp\left(\frac{-b_i b_j b_l b_m}{Y_{ijlm}} |t|\right) + \frac{\mu_{pd}^5}{5!} \sum_{i,j,l,m,n} \frac{a_i a_j a_l a_m a_n}{2^4 Y_{ijlmn}} \exp\left(\frac{-b_i b_j b_l b_m b_n}{Y_{ijlmn}} |t|\right) - \cdots , \tag{7}$$

where

$$\begin{aligned} Y_{ij} &= b_i + b_j , \\ Y_{ijl} &= Y_{ij} b_l + b_i b_j , \\ Y_{ijlm} &= Y_{ijl} b_m + b_i b_j b_l , \\ Y_{ijlmn} &= Y_{ijlm} b_n + b_i b_j b_l b_m . \end{aligned} \tag{8}$$

Using the optical theorem and the series (7), the total cross section can be related to the interaction strength,

$$\frac{\sigma_T^{(pd)}}{4\pi} = \mu_{pd} - \frac{\mu_{pd}^2}{2!} \sum_{i,j} \frac{a_i a_j}{2Y_{ij}} + \frac{\mu_{pd}^3}{3!} \sum_{i,j,l} \frac{a_i a_j a_l}{2^3 Y_{ijl}} - \cdots . \tag{9}$$

Because of the nonavailability of the data separately on the charge form factor of the deuteron, a certain approximation is made in Sec. III, which is expected to be very good if we confine the range of our prediction for $|t| \leq 2\ \text{GeV}^2$. Within this range we observe that only the first five terms in the series (7) contribute significantly to the amplitude.

III. INTERPOLATING ANALYTIC FORMULAS AND PARAMETRIZATION OF THE PRODUCT OF TWO FORM FACTORS

In order to use formula (7) for calculation it is necessary to fit the product function by Eq. (6) and find out the numerical values of the parameters a_i, b_i. But the experimental data on the product $F_p(t)F_d(t)$ have not been directly measured, although experimental data on the charge form factor of the proton and certain combinations of the form factors of the deuteron are separately available. To compute the data on $F_p(t)F_d(t)$ it is necessary that experimental data on individual factors are available for the same values of $|t|$. Examining the available data on $G_E^p(t)$ and $A(t)$ in the spacelike region, we find that although the number of computed data points for the function $G_E^p(t)[A(t)]^{1/2}$ are sufficient to impose constraint for a good fit, in the region $|t| \leq 1\ \text{GeV}^2$, their number is inadequate for $|t| > 1\ \text{GeV}^2$. Alternatively, a reasonably good estimate of the product function representing the data can be made if good fits to the individual form factors are available. We use analytic formulas as described below for the two form factors which have yielded excellent fits to the data and possess several other attractive features.[9,10]

A. Analytic parametrization of the proton form factor

The dipole formula for the proton form factor which has been used in some earlier cases for Chou-Yang-type prediction in πp and pp scattering is known to be a bad fit to the existing data.[9,11] In a critical assessment of the Chou-Yang hypothesis it has been pointed out[12] that for pp scattering the relationship between the proton blackness and the charge density is very sensitive both to the choice of the form factors and to the detailed parametrization of these form factors. Similar effects might be present also in other cases for which the Chou-Yang model has been used to calculate the differential cross section. To minimize discrepancies arising out of detailed parametrization, it is necessary to use a fit to the form-factor data with the lowest value of χ^2/DOF. Further, if the fit satisfies analyticity and convergence properties it is supposed to be more stable against extrapolations to the regions of $|t|$ for which experimental data are not available. In the physical region and outside it the analytic parametrization of the data satisfying the criterion of optimal convergence is the best available interpolating formula for the scattering amplitude or the form factor.[13]

In an analytic parametrization[9] of the form factor using a modified N/D method, the effect of the two-pion cut has been taken into account by an effective-range-type formula by the D function, and, the effect of the three-pion cut has been taken into account by means of conformal mapping and optimized polynomial expansion[13] by the N function. The proposed formula has yielded an excellent fit to the experimental data in the range $0 \leqslant |t| \leqslant 25$ GeV^2. In addition the formula satisfies the general type of asymptotic behavior, yields a very good ρ signal, and matches with a Frascati datum point on extrapolation into the timelike region. The proposed formula for the proton form factor is[9]

$$F_p(t) = G_E^p(t) = \frac{g_0 + g_1 z + g_2 z^2}{g_0 + f_1 t + f_2 t^2 + f_3 t^3 + h(t) + m_\pi^2/\pi} \tag{10}$$

where

$$z(t) = \left\{ \ln\left[(-t/9m_\pi^2)^{1/2} + (-t/9m_\pi^2 + 1)^{1/2}\right]\right\}^2, \tag{11}$$

$$h(t) = \frac{2}{\pi}\frac{q^3}{\sqrt{t}} \ln\left[\left(\frac{t}{4m_\pi^2}\right)^{1/2} + \left(\frac{t}{4m_\pi^2} - 1\right)^{1/2}\right] - i\frac{q^3}{\sqrt{t}}, \tag{12}$$

with

$$q = \left(\frac{t}{4} - m_\pi^2\right)^{1/2}. \tag{13}$$

The values of the parameters occurring in Eq. (10) are

$$\begin{aligned} &g_0 = 0.537 \text{ GeV}^2, \quad f_1 = -1.605, \\ &g_1 = -0.035 \text{ GeV}^2, \quad f_2 = 0.675 \text{ GeV}^{-2}, \\ &g_2 = 0.005 \text{ GeV}^2, \quad f_3 = 0.0635 \text{ GeV}^{-4}. \end{aligned} \tag{14}$$

Besides representing the available data points for corresponding $|t|$ values, formula (10) is supposed to yield a better interpolation of the proton form factor for other $|t|$ values than any other existing fit.

B. Analytic parametrization of the deuteron form factor

In contrast to the proton case for which experimental data on the electric or magnetic form factor is available,[9] for the deuteron the data are available[10] on $A(t)$ which is a combination of the charge form factor $G_c(t)$, the magnetic form factor $G_M(t)$, and the quadrupole form factor $G_Q(t)$,

$$A(t) = G_C^2(t) + \frac{t^4}{18m^4} G_Q^2(t) - \frac{t}{6M^2}\left(1 - \frac{t}{4M^2}\right) G_M^2(t), \tag{15}$$

where M is the deuteron mass. These form factors are expected to fall off rapidly with increasing $|t|$ in the spacelike region. Because of the presence of kinematical factors occurring in the second and the third term in Eq. (15), we expect that up to a good approximation

$$G_C(t) \simeq [A(t)]^{1/2} \tag{16}$$

for those values of $|t|$ for which

$$|t| \ll 4M^2 \simeq 14 \text{ GeV}^2. \tag{17}$$

We confine the range of our calculation to be up to $|t| = 2$ GeV^2 and our predicted positions of the dips and maxima are seen to be confined within $|t| = 0.7$ GeV^2. Therefore, relation (16) is a good approximation in the entire range of our calculation and a very good approximation at least up to values of $|t|$ where dips and maxima are predicted to occur.

Using a similar technique of analytic parametrization as that adopted for proton,[9] the following formula[10] has been observed to yield an excellent fit to the existing data on the deuteron form factor in the range $0 \leqslant |t| \leqslant 6$ GeV^2:

$$A(t) = \frac{e_0 \exp[-\alpha z_a(t)]}{e_0 + e_1 t + e_2 t^2 + e_3 t^3 + h(t) + m_\pi^2/\pi}, \tag{18}$$

where

$$z_a(t) = \{\ln[(-t/t_a)^{1/2} + (-t/t_a + 1)^{1/2}]\}^2, \tag{19}$$

$$\begin{aligned} &\alpha = 0.931, \quad t_a = 0.033 \text{ GeV}^2, \\ &e_0 = 1.0 \text{ GeV}^2, \quad e_1 = -6.507, \\ &e_2 = 74.289 \text{ GeV}^{-2}, \quad e_3 = 0.192 \text{ GeV}^{-4}. \end{aligned} \tag{20}$$

In Eq. (19) t_a is the position of the anomalous cut in the t plane and the function $h(t)$ is the same as has been expressed in Eq. (12). To our knowledge formula (18) is so far the only available one in the literature describing a fit in the entire available $|t|$ range by minimizing total χ^2 value. Besides representing the actual data points for the available $|t|$ values in an excellent fashion, this formula is supposed to provide the best available interpolation[10, 13] of $A(t)$ for other values of $|t|$ for which data points do not exist for $|t| \leq 6$ GeV^2. Further, since the formula has been constructed to satisfy correct analyticity properties and, to a certain extent, the criterion of optimal convergence, it is supposed to be stable against extrapolations on to higher values of $|t|$ in the spacelike region and also on to the timelike region.[10,13] Therefore, it is not unreasonable to suppose that the formula represents a good interpolation to the deuteron form factor up to[14] $|t| = 20$ GeV^2 in the spacelike region.

C. Parametrization of the product function

Using Eqs. (10) and (18) we first obtain the curve for the function $G_E^p(t)\,[A(t)]^{1/2}$ as a function of $|t|$ in the spacelike region in the interval $0 \leq |t| \leq 20$ GeV^2. We then fit this curve as a sum of four exponentials,

$$G_E^p(t)[A(t)]^{1/2} = \sum_{i=1}^{4} a_i \exp(-b_i\,|t|) \tag{21}$$

by which the curve is found to be represented reasonably well. Addition of another exponential did not imporve the fit. For the best fit to the curve the parameters occurring in Eq. (21) have the following values,

$$\begin{aligned} &a_1 = 0.950, \quad b_1 = 16.4150 \text{ GeV}^{-2}, \\ &a_2 = 0.033, \quad b_2 = 3.1865 \text{ GeV}^{-2}, \\ &a_3 = 5.188\times 10^{-4}, \quad b_3 = 0.8420 \text{ GeV}^{-2}, \\ &a_4 = 7.700\times 10^{-6}, \quad b_4 = 0.2871 \text{ GeV}^{-2}. \end{aligned} \tag{22}$$

However, as stated in the beginning of this section, only for $|t| \ll 14$ GeV^2, $A(t)$ approximates well the deuteron-charge form factor $G_C(t)$. Hence within our region of investigation for pd scattering with $|t| \leq 2$ GeV^2, the sum of four exponentials in Eq. (21) with the parameters given in Eq. (22) is a good representation of the product of two charge form factors $G_E^p(t)G_C(t)$. A possibility that the left-hand-side of Eq. (21) represents the product of two charge form factors also for higher values of $|t|$ is not ruled out at present. If such a possibility can be shown to exist in future, the parameters in (21) can be used to obtain differential cross section for pd scattering even beyond $|t| = 6$ GeV^2.

Certain points of clarification are necessary at this stage on the method of parametrization adopted here. Since formulas (10) and (18) have the merits of satisfying correct analyticity properties and certain general types of asymptotic behaviors, whereas the sum of four exponentials in the right-hand side of Eq. (21) do not preserve these, one may question the importance given to the analytic parametrization of form factors by Eqs. (10) and (18). But here our purpose is to exploit the best available interpolating formulas for the form factors only in a limited part of the physical region rather than proposing a Chou-Yang-type model preserving analyticity properties. The exponential parametrization in (21) is expected to be a very good approximation only in the spacelike region up to $|t| = 6$ GeV^2 and may be somewhat less accurate in the spacelike region up to $|t| \sim 20$ GeV^2. In no case should Eq. (21) with the ansatz (10) and (18) be extended for timelike region or larger $|t|$ values in the spacelike region. Without using any interpolating formula, but only experimental data on $G_E^p(t)$ and $A(t)$, it is possible to obtain parameters necessary in (21) and (22) only for a limited range of $|t|$, namely, $|t| \leq 1$ GeV^2, since beyond this the number of computed values of data for the function $G_E^p(t)[A(t)]^{1/2}$ are inadequate to impose sufficient constraint on the parameters for a good fit. The method adopted here serves as a very good amalgamation of the two data sets and obtain a curve for the product function.

IV. PREDICTIONS OF DIFFERENTIAL CROSS SECTION AND POSITIONS OF DIPS AND MAXIMA

In order to calculate the differential cross section as a function of $|t|$ using formulas (1), (7), and (22), at first the interaction strength was calculated from the available data on total cross section[15] at various energies using Eq. (9). We have noted that the number of terms in the series (7) contributing significantly to the amplitude increases with the value of $|t|$. We have checked that even up to $|t| \sim 5$ GeV^2 the contribution of successive terms in (7) decreases rapidly, but because of the constraint imposed by the nonavailability of the deuteron charge form factor as discussed in the previous section, the approximation by Eq. (16) is expected to be very good for smaller $|t|$ values. Further, the positions of dips and maxima are seen to be confined within $|t| = 0.7$ GeV^2 for which the truncation of the series (7) after five terms appears to be a very good approximation. Thus while computing μ_{pd} numerically from the total-cross-section data,[15] we have retained only the first five terms in Eq. (9). Our results of calculation of μ_{pd} at different available laboratory momenta are listed in Table I. Calculated values of the differential cross section at these energies for the small-$|t|$ region for which experimental data are available[15] have been shown in Figs. 1(a) and 1(b). It can be seen that there is

TABLE I. Calculations of interaction strength from the total-cross-section data (Ref. 15) using formula (9) as described in the text.

P_{lab} (GeV/c)	$\sigma_T(pd)$ (mb)	μ_{pd} (GeV^{-2})
49	73.18	16.94
72	73.08	16.91
148	73.44	17.00
174	73.70	17.07
221	73.96	17.14
248	74.22	17.21
270	74.39	17.25
289	74.50	17.28
346	75.00	17.41
384	75.22	17.47

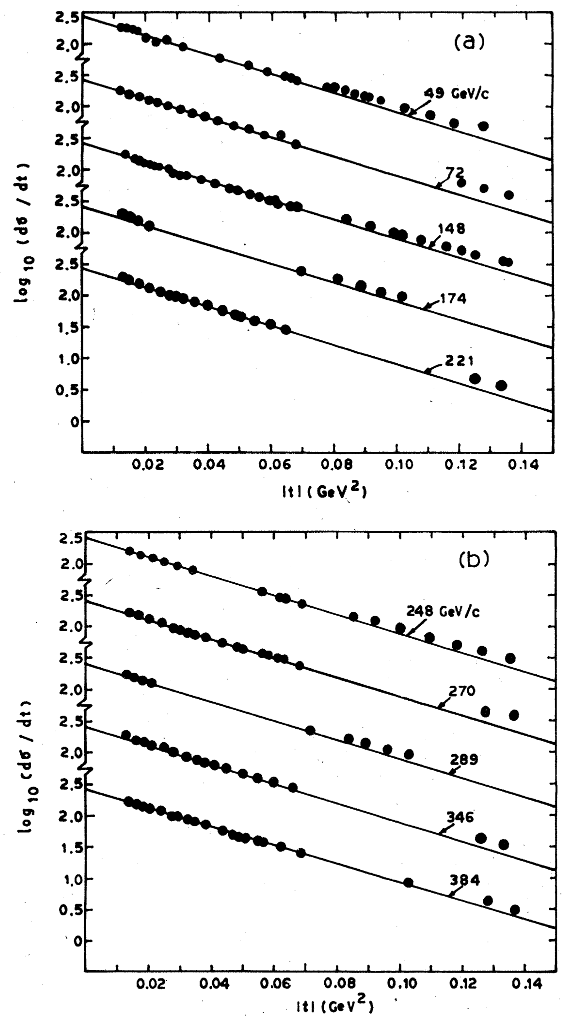

FIG. 1. Comparison of predicted values of the differential cross section $d\sigma/dt$ (mb/GeV^2), shown by solid lines, with the Fermilab data, represented by circles for (a) P_{lab}=49–221 GeV/c, (b) P_{lab}=248–384 GeV/c.

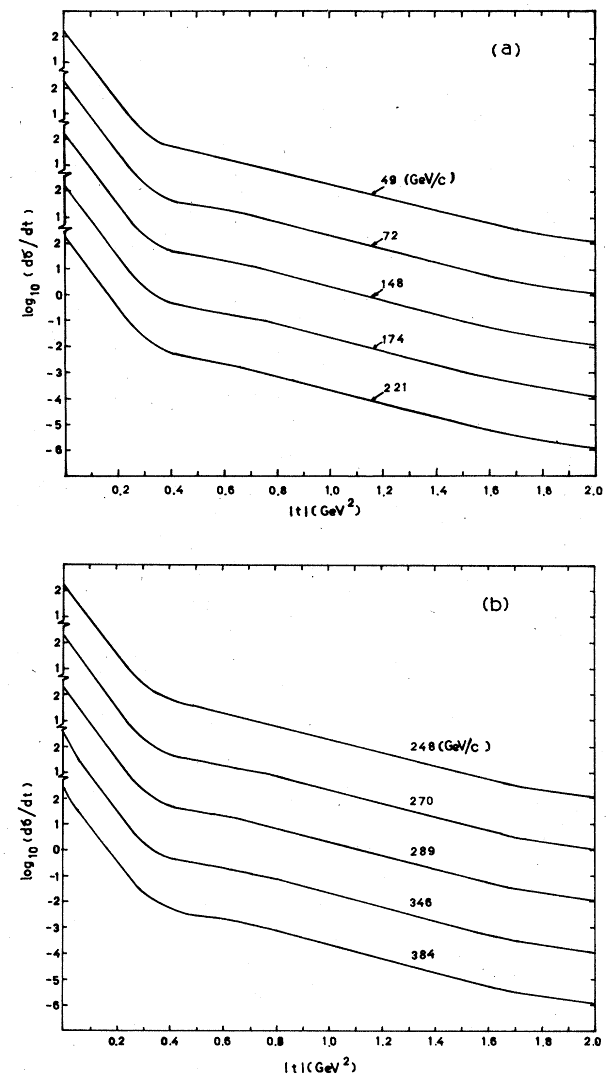

FIG. 2. Prediction of the differential cross section $d\sigma/dt$ (mb/GeV^2) for larger values of $|t|$ at Fermilab energies between (a) P_{lab}=49–221 GeV/c and (b) P_{lab}=248–384 GeV/c.

good agreement between the predicted results and experimental values. From Figs. 1(a) and 1(b) it can be noted that there is a small discrepancy between the theoretical prediction, shown by solid lines and experimental data for larger $|t|$ values, shown by solid circles, and there is a trend that this small discrepancy reduces further as energy increases. For example, in the case of the Fermilab data[15] at P_{lab} = 384 GeV/c, the deviation from the theoretical prediction appears to be negligible. Our calculations apply only for the contribution of the imaginary part and the possible additional contribution due to the real part might be the cause of such small disagreement for larger $|t|$ data at lower Fermilab energies.[15]

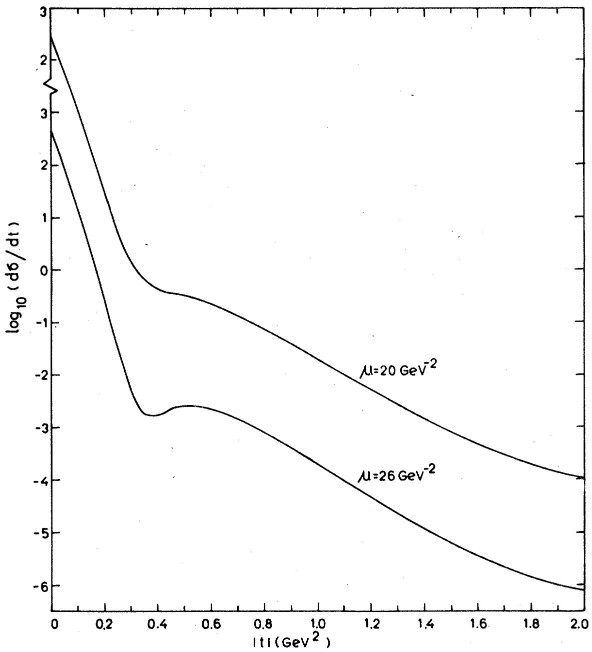

FIG. 3. Prediction of the differential cross section $d\sigma/dt$ (mb/GeV^2) and positions of dips and higher-order maxima for $\mu_{pd}=20$ and 26 GeV^{-2}.

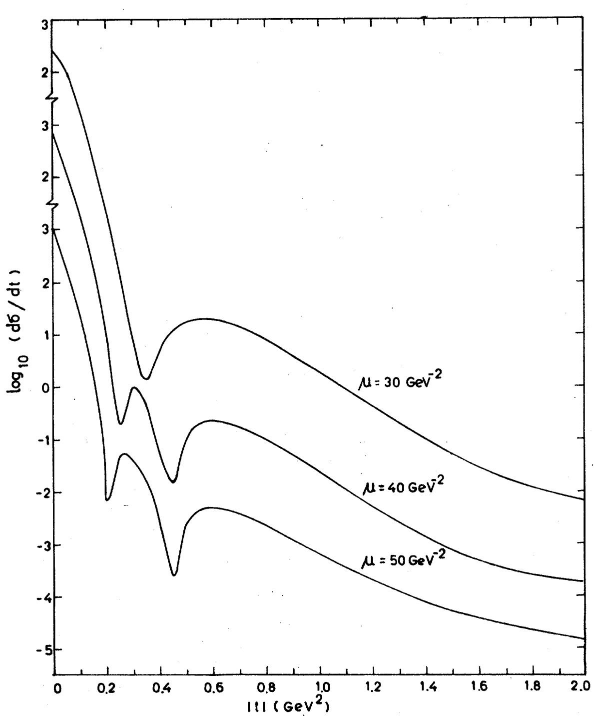

FIG. 4. Prediction of the differential cross section $d\sigma/dt$ (mb/GeV^2), positions of dips, and higher-order maxima for still higher values of $\mu_{pd}=30$, 40, and 50 GeV^{-2}.

Our calculations of differential cross sections for larger-$|t|$ regions up to $|t|=2$ GeV^2 have been shown in Figs. 2(a), 2(b), 3 and 4. It is clear from Figs. 2(a) and 2(b) that even at the highest available energy at Fermilab[15] ($P_{\rm lab}=384$ GeV/c) no dip has appeared in the differential cross section within $|t|=2$ GeV^2. Although the present approximation by Eq. (16) becomes worse to a certain extent for larger values of $|t|$, we have checked that no dip appears in the differential cross section even within $|t|=5$ GeV^2 in the available energy range.[15] However, a shoulder appears for $|t|=0.3$–0.4 GeV^2

We have investigated whether dips would appear in future high-energy experiments. As energy increases the total cross section and hence μ_{pd} are expected to increase. At least such a trend is expected from the data available at the present time on total cross section.[15] Using higher values of μ_{pd} in Eq. (7) we also found that only a shoulder but no dip structure, appears in the differential cross section within $|t|=2$ GeV^2 even up to $\mu_{pd} \simeq 25$ GeV^{-2}. But as μ_{pd} is increased further, the shoulder develops into a shallow dip at $|t|=0.375$ GeV^2 and a secondary maximum at $|t|=0.55$ GeV^2. For still higher values of interaction strength the dip sharpens and both the dip and the secondary maximum move slowly towards the forward direction. For sufficiently large values of μ_{pd}, a second dip and a third maximum appear in the differential cross section as displayed in Fig. 4 for $\mu_{pd}=40$ and 50 GeV^{-2}. The calculated positions of shoulders, dips, and maxima for different values of interaction strength have been listed in Table II. Although in Figs. 1–4 we have displayed the presence of various structures until $\mu_{pd}=50$ GeV^{-2}, we have checked that even up to $\mu_{pd}=60$ GeV^{-2} only the two-dip structure persists within $|t|=2$ GeV^2. Although the approximation to the charge form factor by Eq. (16) becomes worse to a certain extent, we have checked that no other dip structure appears in the differential cross section up to $|t|\simeq 5$ GeV^2 for $\mu_{pd} \leq 60$ GeV^{-2}.

The dips observed in the present case occur when, for a sufficiently large value of the interaction strength <60 GeV^{-2} and certain small value of $|t| < 0.6$ GeV^2, the positive sum of the odd terms in series (7) nearly cancels with the negative sum due to the even terms, the first of these latter terms being dominant. The product of the two form factors decreases rapidly with increasing $|t|$ and for $|t| > 0.7$ GeV^2 the magnitude of one sum continues to dominate over the other until $|t|=5$ GeV^2. Since $G_C(t)$ is supposed to be a continuously decreasing function of $|t|$ in the spacelike region, we hope this dominance continues up to higher values of $|t|$ which rules out the appear-

TABLE II. Positions of shoulders, dips, and higher-order maxima along the $|t|$ axis (in GeV^2) as a function of interaction strength μ_{pd} for pd scattering.

μ_{pd} (GeV^{-2})	Shoulder	First dip	Second dip	Second maximum	Third maximum
16.94–25	0.3–0.4				
26		0.375		0.55	
30		0.35		0.55	
40		0.25	0.45	0.30	0.60
50		0.20	0.45	0.27	0.58

ance of any further dips for $\mu_{pd} \leq 60$ GeV^{-2}. For certain large values of $\mu_{pd} > 60$ GeV^{-2} there is the possibility that many other dips may appear for both smaller- and larger-$|t|$ values.

Experimental data for pp scattering[16] confirm the presence of a sharp dip at $|t|=1.5$ GeV^2 and a second maximum for $P_{1ab}=200$ GeV/c ($\mu_{pp} \sim 9.5$ GeV^{-2}). Although experimentally no dip structure has yet been observed for πp scattering, calculation by Chan *et al.*[8] predicts the presence of a shallow dip at $|t| \simeq 5.2$ GeV^2 for $P_{1ab}=175$ GeV/c ($\mu_{\pi p} \sim 5.78$ GeV^{-2}). With increase of energy, the total cross section and the interaction strength, the dip sharpens and shifts to $|t| \simeq 3.8$ GeV^2 for $\mu_{\pi p} \sim 7.7$ GeV^{-2} [$\sigma_T(\pi p) \simeq 30$ mb]. For still higher energies and values of πp interaction strength, more recent calculations by Chou and Yang[6] predict the presence of two dips at $|t| \simeq 2$ and 4 GeV^2 for $\sigma_T(\pi p)=40$ mb. Both for pp and πp scattering Chou and Yang show the appearance of multiple dip structures for increasing values of interaction strength.

But our calculation for pd scattering predicts the presence of a dip structure only for $\mu_{pd} \geq 26$ GeV^{-2} and such high values of interaction strength can be achieved at energies much larger than 200 GeV/c for which dips have been observed (predicted) for pp (πp) scattering. Also the position of a dip starts from a value of $|t|$ much smaller than the observed (predicted) starting position for pp (πp) scattering.

For pp scattering the forward diffraction peak at high energies occurs within $|t| \simeq 1.5$ GeV^2 which corresponds to the first dip position. From the present analysis we learn that the first shoulder and/or the dip would occur for pd scattering at $|t|=0.3-0.4$ GeV^2. Hence the present analysis predicts that for Fermilab and higher energies the first diffraction peak is confined within $|t|=0.4$ GeV^2.

V. SUMMARY, DISCUSSION, AND CONCLUSION

We have used the Chou-Yang[1] model with the Hayot-Sukhatme[2] modification to calculate differential cross section in pd scattering and investigate whether dips and higher-order maxima would appear at very high energies. For this purpose we have used interpolating fomulas[9,10] for the proton and the deuteron form factors which have exploited analyticity properties and yielded excellent fits to the existing data. Owing to the want of actual data on the charge form factor separately for the deuteron, our predictions on differential cross sections are valid with good approximation for $|t| \leq 2$ GeV^2 and our predicted positions of dips and maxima are valid with very good approximation. Calculated results on differential cross section agree well with the available data at Fermilab energies[15] for $0 \leq |t| \leq 0.12$ GeV^2 except for small deviations at larger-$|t|$ values which seem to be less at the highest available energy ($P_{1ab}=384$ GeV/c). The present calculations indicate that the Chou-Yang approximation to the differential cross section with the only imaginary part may be applicable for both smaller and larger $|t|$ values in the diffraction-peak region for certain higher energies with $P_{1ab} \gg 384$ GeV/c. The small deviations at lower Fermilab energies may be due to real-part and/or spin effects. It is also likely that at currently available Fermilab energies but values of $|t| > 0.1$ GeV^2, the assumption that the charge form factor of the deuteron approximates its matter form factor might not be holding well. With a very good approximation the calculated values of differential cross sections at higher values of $|t| \leq 2$ GeV^2 show the absence of any dip at Fermilab energies, but only a shoulder may appear for $|t|=0.3-0.4$ GeV^2 if real-part and/or spin effects are negligible. The absence of any dip structure continues up to $\mu_{pd} \sim 25$ GeV^{-2}. It is interesting to note that even though no real-part and spin effects have been taken into account, a shoulder continues to appear for energies for which a sharp dip has emerged in other cases.[6,8,16] With relatively less accurate approximation we have also verified the absence of any dip structure within $|t| \simeq 5$ GeV^2 for $\mu_{pd} \leq 25$ GeV^{-2}. However, a shallow dip at $|t|=0.375$ GeV^2 starts to appear with a seond maximum at $|t|=0.55$ GeV^2 when μ_{pd} reaches the value of 26 GeV^{-2}. When the pd interaction strength is increased further ($\mu_{pd} \sim 30$ GeV^{-2})

with increase of energy, the dip sharpens and moves slowly nearer to the forward direction. For still higher values of μ_{pd}($\sim$40 GeV^{-2}) two dips, a second and a third maximum appear within $|t|$ <0.7 GeV^2. We have checked that the two-dip structure persists even up to $\mu_{pd}\sim 60$ GeV^{-2} and even within $|t| \leq 5$ GeV^2.

Experimental data for pp scattering[16] confirm the presence of a sharp dip at $t=-1.5$ GeV^2 and $P_{\rm lab}=200$ GeV/c. Calculations by Chan *et al.*[8] for πp scattering predict the presence of a dip at $|t|=4$–5 GeV^2 around the same energy. For higher energies recent calculations by Chou and Yang[6] predict double dip structure at $|t|\simeq 2$ and 4 GeV^2, the number of the dips increasing with energy. For pd scattering the interaction strength of μ_{pd} $\simeq 26$ GeV^{-2} corresponding to the appearance of a dip will be achieved at much higher energies than available at Fermilab at present for pd scattering. As compared to pp and πp scattering a dip in pd scattering is predicted to show up at much higher values of energy and lower value of $|t|$. The present analysis predicts that the first diffraction peak in pd scattering is confined within $|t|\sim 0.4$ GeV^2 even at the available Fermilab energies. It is generally believed that for a diffractive process the contribution of the real part, which might mask the presence of a dip structure, increases for increasing values of $|t|$. Although the detection of a dip structure may be difficult from the point of view of the availability of a larger-incident-energy proton, it will be easier to detect such a dip once the high-energy incident proton is available, since the real-part effects are supposed to be less important at smaller $|t|$ values as compared to those in pp and πp scattering.

The Chou-Yang hypothesis[1] was originally developed for hadron-hadron elastic scattering. In the present case one of the colliding objects is a light nucleus which is a bound state of two nucleons and calculations according to the modified model[2] have been carried out with the conjecture that at ultrahigh energies, the matter form factor of the deuteron may be the same as its charge form factor, at least within a limited $|t|$ range. Such a conjecture seems to hold even at available Fermilab energies up to $|t|\sim 0.1$ GeV^2. If the ultrahigh-energy experimental results in future for pd scattering agree reasonably well with the present predictions, this will indicate the possibility of applying the Chou-Yang model for hadron-nucleus elastic scattering. On the other hand, if large discrepancies are found between future experimental data and the present predictions, even after real-part and spin effects are taken account, one should give up the idea of using the model for such processes.

ACKNOWLEDGMENTS

The authors are thankful to the University Grants Commission, New Delhi for supporting a research project and acknowledge the use of some computational facilities from the Computer Center, Utkal University.

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