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Can quarks have integer charge?

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We construct a conservative model of quarks and gluons with the Han-Nambu charge assignment by slightly breaking the $SU(3)_{color} \times U(1)$ gauge symmetry observed at present energies. At very large masses, stable integrally charged colored quark states appear. The model is conservative in the sense that it differs as little as possible from conventional quark-gluon models at low energies. In particular, all conventional tests proposed to discriminate between fractional and integer quark charges fail. However, the model requires light charged Higgs scalars which should have been seen in $e^+e^- \to$ hadrons at present energies.

I. INTRODUCTION

There are two common assignments of electric charge to quarks. The more popular one is the Gell-Mann–Zweig (GMZ) assignment giving the quarks fractional charges depending on flavor and not on color.[1] The other is the Han-Nambu (HN) assignment, giving the quarks integer charge depending on both flavor and color.[2] The two can be related by

$$Q_{HN} = Q_{GMZ} + \frac{\lambda_8}{\sqrt{3}}, \tag{1.1}$$

where λ_8 is the eighth generator of color (not flavor) SU(3). [In the triplet representation, $\lambda_8 = \mathrm{diag}(1/\sqrt{3}, 1/\sqrt{3}, -2/\sqrt{3})$]. Lipkin has shown that most conventional tests to distinguish between integer and fractional charge fail at energies much below the threshold for producing colored states.[3] Here we examine whether it is possible to construct a viable model of quarks with the Han-Nambu charge assignment.

Our strategy will be to construct the most innocuous possible model: one in which the greatest amount of fractional-charge phenomenology is preserved, but in which finally at some very high energy, quarks give away their true integrally charged natures. We work in the context of conventional models with $SU(3)_{color} \times SU(2) \times U(1)$ gauge symmetry. Our quarks are assumed either to be stable or to have lifetimes long on the scale of ordinary hadronic phenomena and our conclusions may not apply to models with highly unstable quarks. At present energies, it appears the $SU(3)_{color} \times U(1)_{GMZ}$ gauge symmetry is exact. [$U(1)_{GMZ}$ is the symmetry generated by eQ_{GMZ}.] If $SU(3)_{color}$ is an exact local gauge symmetry and furthermore is absolutely confined, then quarks will appear fractionally charged at all energies. For quarks to display integer electric charges, $SU(3)_{color} \times U(1)_{GMZ}$ must be broken in such a way that a residual U(1) coupling to Q_{HN} remains unbroken. Then at scales where the symmetry breaking is negligible, quarks will appear fractionally charged, but where it is not they will appear integrally charged. A mechanism for "gently" breaking confined gauge symmetries was proposed in Ref. 4 in the context of the bag model and applied to $SU(3)_{color}$. By "gently" we mean that the gauge bosons are given tiny Lagrangian masses (of order μ) by a Higgs mechanism. In a model which confines exactly when $\mu = 0$, the effects of the symmetry breakdown are evident only at masses on the order of $M_c \sim M_H^2/\mu$, the threshold for producing colored states (where M_H is a typical hadron scale).[4] The authors of Ref. 4 called this phenomenon "quasiconfinement." Following Ref. 4 we construct a model of color-dependent electric charge by gently breaking $SU(3)_{color} \times U(1)_{GMZ}$. At energies well below color threshold, the phenomenology of this model differs from that of fractional charge by corrections of order μ^2/M_H^2. Since the color threshold is inversely proportional to μ, we can postpone the appearance of integer charge indefinitely by making μ very small. So far, so good. In the end, however, we find this most conservative course is not possible. This model requires light, colored, *charged* Higgs bosons which should have been seen in $e^+e^- \to$ hadrons well below present energies.

Okun, Voloshin, and Zakharov[5] have recently discussed and criticized a class of popular integer-charge models. Although their work has itself been criticized by Chetyrkin *et al.*,[6] we believe the framework they set up for the discussion of interger versus fractional charge is a useful one and wish to outline it here. Reference 5 begins with an $SU(3)_{color} \times U(1)_{GMZ}$ gauge symmetry, where $U(1)_{GMZ}$ is generated by $e'Q_{GMZ}$. They break the symmetry and, as in the Weinberg-Salam model,[7] there remains a U(1) gauge symmetry generated by eQ_{HN}. The associated massless gauge particle is identified as the photon and is a mixture of $SU(3)_{color}$ and $U(1)_{GMZ}$ with mixing angle θ (analogous to the weak mixing angle). The electric charge e equals $e'\cos\theta$. From the success of quantum-chromodynamics (QCD) sum rules at momentum transfer $|Q^2| \geq 1\ \text{GeV}^2$, they (and others) conclude that the original $SU(3)\times U(1)$ symmetry is effectively restored at such momenta, and therefore that the massive gauge particles must have masses $M \lesssim 1$ GeV. The effects of symmetry breaking are suppressed by powers of $M^2/|Q^2|$. For color singlets, the effective charge must change from $eQ_{HN}(=eQ_{GMZ})$ at low momenta to $e'Q_{GMZ}$ at high momenta.[5,8] Okun, Voloshin, and Zakharov claimed to show that an enhancement of electric charge by a factor $e'/e = \sec\theta$ could be ruled out by experiment. They also pointed out that since the eight gauge bosons orthogonal to the photon of this model are not seen freely, they are presumably confined at present energies. This seems unnatural since at least one of them has an Abelian part.

We have constructed a model which is not subject to the criticisms of Okun, Voloshin, and Zakharov, by confining only the non-Abelian fields. To implement this we use the MIT bag model[9] in which all colored fields are *apriori* confined to hadrons. The unconfined Abelian $U(1)_{GMZ}$ field mixes with the SU(3) fields only inside hadrons. Outside hadrons, where there are no colored fields, the $U(1)_{GMZ}$ itself must be identified with the photon. In the limit that the Higgs vacuum expectation value[10] which breaks the symmetry inside the hadron vanishes, we get back the ordinary fractionally charged quark model. Also in this model, the electric charge e equals e', the original $U(1)_{GMZ}$ coupling.[11]

In Sec. II, we first show how to break the local $SU(3)_{\text{color}} \times U(1)_{GMZ}$ gauge symmetry in the bag model to give integer-charge quarks. We will show that at energies much below color threshold, it may be impossible to distinguish between fractional and integer charge in conventional tests. In Sec. III we discuss the Higgs sector, and show that for light Higgs bosons a charged, color decimet is needed. Then we show that the required Higgs bosons are ruled out by present experiments. We conclude that it is probably not possible to construct a viable model of integrally charged quarks using present theoretical technology. In Appendix A we summarize various calculations for the color-decimet Higgs bosons including the calculation of colored-state masses. In Appendix B, we show that $\eta \to \gamma\gamma$ decay, which is widely believed to be a model-independent test of quark charge, gives similar results for fractional- and integral-charge quarks in our model.

II. THE MODEL AND ITS PHENOMENOLOGY

In the bag model,[12] all colored fields are confined to hadrons. This requires certain boundary conditions for the fields on the hadron surface. In particular, for the gluon field V_a ($a = 1, \ldots, 8$),

$$n_\mu F_a^{\mu\nu} = 0\,, \tag{2.1}$$

where n_μ is the local normal to the surface. Color-singlet fields are continuous across the boundary. Gauss's law holds in a theory with exact color gauge symmetry, and when combined with (2.1), tells us that all hadrons are color singlets. When the color gauge symmetry is broken, Gauss's law no longer holds and colored states are possible.[4]

The bag Lagrangian density for an $SU(3)\times U(1)$ Higgs model is[5]

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \theta(\bar{q}q)[-B - \tfrac{1}{4}F_a^{\mu\nu}F_{a\mu\nu} + \tfrac{1}{2}i\bar{q}\gamma^\mu\overleftrightarrow{\partial}_\mu q - \bar{q}\gamma^\mu(e'Q_{GMZ}A_\mu + \tfrac{1}{2}g\lambda_a V_{a\mu})q$$
$$+ |(i\partial_\mu - e'Q_{GMZ}A_\mu - \tfrac{1}{2}g\lambda_a V_{a\mu})\phi|^2 - P(\phi^\dagger, \phi)]\,, \tag{2.2}$$

where

$$\theta(\bar{q}q) = \begin{cases} 1 & \text{for } \bar{q}q > 0\,, \\ 0 & \text{for } \bar{q}q < 0\,, \end{cases}$$

B is the MIT bag constant ($B^{1/4} = 145$ MeV), A_μ is the Abelian gauge field associated with the flavor-dependent charge operator Q_{GMZ} with covariant curl $F_{\mu\nu}$, q is the quark field with color and flavor indices suppressed, ϕ is the Higgs field, and P is the Higgs potential invariant under $SU(3)_{\text{color}} \times U(1)_{GMZ}$. Since the Higgs fields carry color, they too are confined to the interior of hadrons. We are restricting ourselves to moderate energies where it can easily be shown that weak-interaction effects are small. When $SU(3)_{\text{color}} \times U(1)_{GMZ}$ is unbroken, the quark charges are of course $e'Q_{GMZ}$.

The symmetry is broken so as to leave a U(1) gauge symmetry generated by[13]

$$\mathcal{Q} = e'Q_{GMZ}\cos\theta + g\frac{\lambda_8}{2}\sin\theta \tag{2.3}$$

and an associated massless gauge boson

$$\mathcal{A} = A\cos\theta + V_8\sin\theta\,. \tag{2.4}$$

The orthogonal combination, $\mathcal{B} = -A\sin\theta + V_8\cos\theta$, is massive. Were there no confinement, we would identify $\mathcal{A}^\mu$ with the photon field and $\mathcal{Q}$ with the electric charge. To obtain integer quark charges we would identify $e=e'\cos\theta$ and $\tan\theta=2e'/\sqrt{3}g$. Because the V_a^μ and Higgs fields are confined, however, A^μ remains massless outside hadrons. Since A^μ is the only gauge field outside hadrons, it must be identified as the photon.

Now let us determine the charge eQ of some hadron. To do so, we measure the electric field flowing out through its surface,

$$eQ=\oint dS\, n_\mu F^{\mu\nu}\,, \tag{2.5}$$

where $F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu$. From the definition of $\mathcal{A}$,

$$eQ=\oint dS(\sec\theta\, n_\mu F_{\mathcal{A}}^{\mu\nu}-\tan\theta\, n_\mu F_8^{\mu\nu})\,, \tag{2.6}$$

and using the boundary condition (2.1), the second term in the integral is zero. Since $\mathcal{A}$ is massless inside the hadron, we may use Gauss's law and the remaining term is simply

$$eQ=\sec\theta\mathcal{Q}=e'Q_{\rm GMZ}+\tan\theta g\frac{\lambda_8}{2}\,. \tag{2.7}$$

To obtain integer charge we must identify $e=e'$ and $\tan\theta=2e/\sqrt{3}g$.

We may set an upper limit on the size of the gauge-boson mass since colored states should appear at energies $\sim(\alpha'\mu)^{-1}$, where α' is the Regge slope (see Ref. 4 and Appendix A). A physical colored quark state would have fractional baryon number, so it would be distinguished by both high mass and absolute stability. If M_c is a lower limit on such objects, (say 10 GeV), then $\mu<(\alpha'M_c)^{-1}\sim 0.1$ GeV. There is presently no lower limit on μ, since for $\mu=0$ we have the conventional fractional-charge model. Throughout this paper, we shall assume that $\mu/M_H\ll 1$.

At this point it is obvious why it is difficult to distinguish between integer and fractional charge at energies below color threshold. With the exception of tiny gluon mass terms and a tiny mixing term

$$\tfrac{1}{2}\mu_{\mathcal{B}}^2\mathcal{B}^2=\tfrac{1}{2}\mu_{\mathcal{B}}^2(-A\sin\theta+V_8\cos\theta)^2 \tag{2.8}$$

only within the bag, the Lagrangian of the broken theory is identical to the Lagrangian of conventional fractionally charged QCD. Purely leptonic processes are unaffected by this model, since leptons interact with the photon A with coupling e as usual. In hadronic processes at ordinary energies, quarks appear to have fractional charge up to effects of order μ^2/M_H^2, where M_H is a typical hadron mass. If one insists on diagonalizing the mass matrix as in (2.8), the quarks will explicitly have integer charge in the Lagrangian, but this is a delusion since the quarks' coupling to $\mathcal{A}$ and $\mathcal{B}$ would add coherently in just such a way as to imitate fractional charge at all momenta much larger than the tiny gauge-boson mass. This is effectively gauge symmetry restoration. Even $\eta\to\gamma\gamma$ decay, which is widely regarded as a model-independent test of quark charge,[14,3] has only $O(\mu^2/M_H^2)$ corrections (see Appendix B). The reason other calculations have yielded finite differences between integer and fractional charge for $\eta\to\gamma\gamma$ was the tacit assumption that the gauge field coupling directly to the conserved current, $e'J_{\rm GMZ}\cos\theta+gJ_8\sin\theta$, is the photon field. That would identify $\mathcal{A}=A\cos\theta+V_8\sin\theta$ as the photon, independent of μ. In our model, the photon in the fractional-charge theory, A, is the same as the photon in the integer-charge theory in the $\mu\to 0$ limit.

It is conceivable that processes involving gauge bosons of momenta below apparent symmetry restoration, i.e., $\lesssim O(\mu)$, could distinguish between the two charge assignments if we understood the strong dynamics involved. However, for pure QCD processes the contributions of such small momenta (large wavelengths) would be cut off by the finite size of the hadron. For electromagnetic measurements, large wavelengths would only measure averages over the hadron, not individual quark charges.

So in tests of charge, the limit $\mu\to 0$ seems entirely smooth. If we continue to see fractionally charged quarks at higher and higher energies in more and more precise measurements, just around the corner it would seem lurks the possibility we will start seeing indications of integer charge.

There is a final further requirement before our model is guaranteed to liberate integer-charged quarks. To obtain integer charge the U(1) symmetry generated by $Q_{\rm HN}$ must remain unbroken, but to allow physical states with color not all generators of $\mathrm{SU(3)}_{\rm color}$ can remain unbroken. In the bag model of Ref. 4 any color generator which remains unbroken annihilates all physical states. Thus once we have chosen $Q_{\rm HN}=Q_{\rm GMZ}+\lambda_8/\sqrt{3}$, we must require the gauge symmetry generated by λ_8 to be broken. More precisely, we must require all unbroken color-symmetry generators to be orthogonal to λ_8.

III. THE HIGGS SECTOR

In the previous section, we showed that to get integer charge, $\mathcal{Q}=eQ_{\rm HN}\cos\theta$ must remain an unbroken gauge-symmetry generator, and no other unbroken color generators may depend on λ_8. In this section, we show that a Higgs sector can be chosen which breaks the symmetries in the way we require.

Since $\mathcal{Q}$ is unbroken, it is a symmetry of the vacuum. If v is the vacuum expectation value of ϕ, then $\mathcal{Q}v=0$, i.e.,

$$Q_{\text{GMZ}}v+\frac{\lambda_8}{\sqrt{3}}v=0\,. \tag{3.1}$$

Therefore, v must be an eigenvector of λ_8 in the representation of ϕ, *and the eigenvalue* $-Q_{\text{GMZ}}\sqrt{3}$ *must be nonzero* (or else λ_8 is unbroken.) Thus the Higgs scalars in our model must appear to have nonvanishing electric charge at energies where the symmetry breaking is negligible, *i.e.*, where the quarks look fractionally charged. It must be shown that the Higgs potential can break down in this way.

If the Higgs bosons are light, one must insist that they may not bind with quarks to make color-singlet states with fractional baryon number (which would be absolutely stable), since such states are not seen. This is equivalent to requiring that the scalars be in an $SU(3)_{\text{color}}$ representation in the product of a triality zero set of quarks. [We are assuming that for the same reason that $q\bar{q}g$ states are not seen (where g is a gluon), it is possible that we will not see $q\bar{q}\phi$ states for octet Higgs bosons, etc.][15]

Going through color-SU(3) representations of low dimension, triplets or sextets are not allowed since they would permit unseen singlet hadrons. Octets will not permit unseen hadrons. However, it is impossible to leave $\mathcal{Q}$ unbroken without leaving other generators depending on λ_8 unbroken with only one complex octet (though two octets will work).

A decimet, with only the $\lambda_8/\sqrt{3}=-2$ component of v being nonzero, may work if the Higgs potential can break the symmetry that way. With $Q_{\text{GMZ}}=2$, Q_{HN} is unbroken, as well as λ_1, λ_2, and λ_3. This is easy to see from a weight diagram (Fig. 1). Using the methods of Li,[16] it can be shown that a renormalizable Higgs potential can be chosen to naturally break the symmetry in the necessary way. The spectrum of colored states may be calculated as in Ref. 4 (see Appendix A) and is governed by the mass operator (to lowest order in μ)

$$M=\frac{\sqrt{3}}{16\pi\alpha'\mu'}(\lambda_a\lambda_a-\tfrac{3}{4}\lambda_8{}^2)^{1/2}\,, \tag{3.2}$$

where $\lambda_a\lambda_a$ is the color Casimir operator and $\lambda_8/\sqrt{3}$ is the color hypercharge of the state, and μ' is the Lagrangian mass of gluons $a=4,5,6,7$. The Lagrangian mass of the $\mathfrak{B}$ boson is $\mu_{\mathfrak{B}}=2\sec\theta\,\mu'$ where it should be remembered that θ is very small (see Appendix A).

Although color SU(3) is broken in our model, nevertheless the Casimir operator $\lambda_a\lambda_a$ and λ_8 commute with the mass operator. Hence physical states can be classified by their color representation and, if it were necessary, by their λ_8 eigenvalue. However, since each SU(3) color representation has only one color isosinglet, and color isosinglets are the only allowed physical states, the classification by the eigenvalue of λ_8 is unnecessary.

The colored, charged Higgs bosons we have been forced to introduce would participate in deep-inelastic reactions as charged, scalar partons. The results of $e^+e^-\to$ hadrons experiments severely limit the number and charge of light scalar partons. The naive parton model predicts the angular distribution of jets to be

$$\frac{d\sigma}{d\Omega}\propto\sum_{\text{spin }0}\tfrac{1}{2}Q_{\text{GMZ}}{}^2(1-\cos^2\theta)+\sum_{\text{spin }1/2}Q_{\text{GMZ}}{}^2(1+\cos^2\theta)\,, \tag{3.3}$$

where θ is the angle between the jet axis and the incident positron momentum. The distribution is now experimentally known to be

$$\frac{d\sigma}{d\Omega}\propto 1+\alpha\cos^2\theta\,, \tag{3.4}$$

where[17] $\alpha=1.1\pm0.1$. $\sum_{\text{spin }1/2}Q_{\text{GMZ}}{}^2=\frac{11}{3}$ for the five known quark flavors, remembering there are three colors. One scalar with charge 2 would give $\alpha=\frac{5}{17}$. A decimet of such scalars certainly could not be tolerated.

So a decimet of light Higgs bosons will not work. It is easily shown that no light Higgs bosons will. Since the smallest eigenvalues of $\lambda_8/\sqrt{3}$ are $\pm\frac{1}{3}$, the minimum possible $|Q_{\text{GMZ}}|$ is $\frac{1}{3}$. For an n-plet,

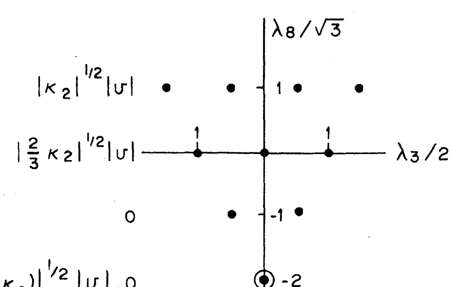

FIG. 1. The Higgs decimet. v is nonzero only in the circled component. Since v is a color isosinglet, λ_1, λ_2, and λ_3 remain symmetries. $\lambda_4+i\lambda_5$ and $\lambda_6+i\lambda_7$ raise v to the two $\lambda_8/\sqrt{3}=-1$ components, while $\lambda_4-i\lambda_5$ and $\lambda_6-i\lambda_7$ annihilate v. Therefore neither λ_4, λ_5, λ_6, λ_7 or any Hermitian linear combination of them is conserved. The masses of the Higgs bosons in each row are listed to the left of the row. The row $\lambda_8/\sqrt{3}=-1$ is eaten by V_4, V_5, V_6, and V_7, and the imaginary part of the $\lambda_8/\sqrt{3}=-2$ component is eaten by $\mathfrak{B}$.

this gives a lower limit on α:

$$\alpha = \frac{\frac{11}{3} - \frac{1}{2} n Q_{\rm GMZ}{}^2}{\frac{11}{3} + \frac{1}{2} n Q_{\rm GMZ}{}^2} \leq \frac{\frac{11}{3} - \frac{1}{18} n}{\frac{11}{3} + \frac{1}{18} n} . \qquad (3.5)$$

After the decimet, the next smallest allowed Higgs multiplet is two octets for which $\alpha \leq \frac{25}{41}$, which is far below experimental limits. Larger multiplets make α smaller and are therefore excluded. It should be noted that even the possibility of Higgs triplets should be testable by experiments in the near future. By our rules, one triplet of Higgs bosons would have $Q_{\rm GMZ} = \frac{2}{3}$ to give integer-charge quarks. However, with two Higgs triplets, $Q_{\rm GMZ} = \frac{1}{3}$ can be used, giving an α closer to unity, $\alpha = \frac{5}{6} \cong 0.83$.

The possibility that the Higgs bosons are heavy can also be excluded. The Lagrangian mass of a gluon is $\sim g|v|$ while the Lagrangian mass of a Higgs scalar, $M_{\rm Higgs}$, is $\sim \sqrt{\kappa}|v|$, where κ is a typical quartic Higgs coupling. (For exact relations for a decimet of Higgs scalars, see Appendix A.) Here g is the strong coupling at typical hadronic scales. Since the gluon mass is $\sim(\alpha' M_c)^{-1}$ and $g \sim 1$,

$$\frac{M_{\rm Higgs}}{\sqrt{\kappa}} \sim |v| \sim \frac{\mu}{g} \sim \frac{1}{\alpha' M_c} \qquad (3.6)$$

or $\sqrt{\kappa} \sim \alpha' M_c M_{\rm Higgs}$. Taking a lower limit on M_c and $M_{\rm Higgs}$ of 10 GeV, we have $\sqrt{\kappa} \gtrsim 100$. For a κ this huge, perturbation theory breaks down and we no longer get something that looks like QCD. One might hope that perturbation theory could survive despite the huge κ because of the huge Higgs-boson masses in the denominators. Veltman, and Applequist and Shankar[18] have studied theories with a large Higgs-boson mass. They found that at energies much below the Higgs-boson mass, there are corrections in a series like

$$1 + g_G{}^2 \ln M_{\rm Higgs} + g_G{}^4 \frac{M_{\rm Higgs}{}^2}{M_{\rm gauge}{}^2} + g_G{}^6 \frac{M_{\rm Higgs}{}^4}{M_{\rm gauge}{}^4} + \cdots , \qquad (3.7)$$

where g_G is the coupling constant of the gauge group being broken and $M_{\rm gauge}$ is a typical (Lagrangian) mass for the gauge bosons. In a theory like the Weinberg-Salam model where $g_G{}^2$ is small, if we have $g_G{}^2 M_{\rm Higgs}{}^2 / M_{\rm gauge}{}^2 \sim \kappa \sim 1$, then the powers of $g_G{}^2$ shield the effect of the large Higgs-boson mass. However in our model, $g \sim 1$ and $\sqrt{\kappa} \sim 100$ and perturbation theory breaks down.

IV. CONCLUSIONS

We have presented a model of integer-charge quarks almost identical in phenomenology to that for fractional-charge quarks at moderate energies, the main differences being the production of integrally charged quark states at very high energy, and the necessity of light charged Higgs scalars. The model fails in the end because the Higgs bosons required cannot be tolerated by present data. However, if the gauge symmetry were broken dynamically without Higgs bosons, the model might once more be viable. If the model were resurrected it would be appropriate to go through an analysis similar to that of Ref. 4 for the phenomenology of colored states. If this model cannot be resurrected, it seems very unlikely that integer-charge quarks can be reconciled with our present understanding of hadronic phenomena.

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APPENDIX A

A decimet transforms under SU(3) as a totally symmetric tensor with three contravariant indices in a complex three-dimensional vector space. Let us denote the Higgs field by ψ^{abc} and its complex conjugate tensor by $\psi_{abc} \equiv (\psi^{abc})^*$. The ten independent components are ψ^{111}, ψ^{222}, ψ^{333}, ψ^{112}, ψ^{113}, ψ^{223}, ψ^{221}, ψ^{331}, ψ^{332}, and ψ^{123}. For a decimet, the most general polynomial invariant under SU(3) $\times$ U(1) is

$$P = -m^2 \psi^{abc}\psi_{abc} + \frac{\kappa_1}{2}(\psi^{abc}\psi_{abc})^2 + \frac{\kappa_2}{2}\psi^{abc}\psi_{abd}\psi^{efd}\psi_{efc} . \qquad (A1)$$

Define $X^c_d \equiv \psi^{abc}\psi_{abd}$ which is easily seen to be Hermitian and positive. We can diagonalize X^c_d to give $X_c\,\delta^c_d$ so that P can be rewritten as

$$P = -m^2 \sum_a X_a + \frac{\kappa_1}{2}\left(\sum_a X_a\right)^2 + \frac{\kappa_2}{2}\sum_a (X_a{}^2) . \qquad (A2)$$

For P to be positive for large $|\psi|$, we require $\kappa_1 > 0$ and $\kappa_2 > -\kappa_1$.

To find the minima of P we set the first derivative to 0:

$$0 = \frac{\partial P}{\partial \psi_{ijk}} = -m^2 \psi^{ijk} + \kappa_1\, \psi^{abc}\psi_{abc}\psi^{ijk} + \kappa_2\, \psi^{abk}\psi_{abd}\psi^{ijd} \qquad (A3)$$

Now replace $\psi^{abc}\psi_{abd}$ by X^c_d and diagonalize X^c_d to give

$$0 = -m^2 \psi^{ijk} + \kappa_1\left(\sum_a X_a\right)\psi^{ijk} + \kappa_2 X_k\, \psi^{ijk} \qquad (A4)$$

where these ψ^{ijk} are now in the "frame" where X

is diagonal. If there exists a $\psi^{ijk}\neq 0$ for a particular value of k, then

$$0=-m^2+\kappa_1\sum_a X_a+\kappa_2 X_k. \qquad (A5)$$

There are now four types of possible minima: the obvious one is where all $\psi^{ijk}=0$ and all $X_k=0$, the unbroken symmetry case. The other possible minima types are those with one, two, or three nonzero eigenvalues of X. Solving Eq. (A5) for these three cases and defining n as the number of nonzero X eigenvalues, the nonzero X eigenvalues are $m^2/(n\kappa_1+\kappa_2)$. The value of P at these X is $P_n=-\frac{1}{2}m^4/(\kappa_1+\kappa_2/n)$, so P takes its minimum when κ_2/n is minimized. Therefore for $\kappa_2>0$, the minimum of P_n is at $n=3$, i.e., $X_{1,2,3}=m^2/(3\kappa_1+\kappa_2)$; and for $\kappa_2<0$, the minimum is at $n=1$, *e.g.*, $X_3=m^2/(\kappa_1+\kappa_2)$, $X_1=X_2=0$. We desire that only ψ^{333} (the component of the decimet with $\lambda_8/\sqrt{3}=-2$) have a vacuum expectation value, and must therefore have $\kappa_2<0$.

The Higgs-boson masses are obtained from the second derivatives of P at the minimum $v=m/(\kappa_1+\kappa_2)^{1/2}$ (in the ψ^{333} component). The decimet has 20 real components, five of which are eaten by the gauge bosons to make them heavy. We are left with eight real scalars with mass $|\kappa_2|^{1/2}|v|$ (remember $\kappa_1>-\kappa_2>0$), six with mass $|\frac{2}{3}\kappa_2|^{1/2}|v|$, and one with mass $|2(\kappa_1+\kappa_2)|^{1/2}|v|$.

The gauge-boson Lagrangian masses in the bag are generated by the Lagrangian term $|[eQ_{\rm GMZ}A+g(\lambda_a/2)V_a]v|^2$. V_1, V_2, V_3, and $\mathcal{A}$ remain massless. The mass of the V_4, V_5, V_6, and V_7 is $\mu'=\sqrt{\frac{3}{2}}g|v|$ and the $\mathcal{B}$-boson mass is $\mu_{\mathcal{B}}=2\sec\theta\mu'$.

The masses of colored states can be calculated as in Sec. III of Ref. 4 with a slight complication due to the mixing of SU(3) and U(1). The derivation of Ref. 4 was not entirely correct and although the results do not change, we shall present a more careful derivation here. We approximate the bag as a static cavity of volume V. The massive pure color gauge fields satisfy

$$\vec{\nabla}\cdot\vec{\rm E}_a+\mu'^2V_a^0=g\rho_a \quad (a=4,5,6,7), \qquad (A6)$$

where ρ_a is the color charge density (including that due to gluons) and $\vec{\rm E}_a$ is the color electric field. Taking the volume integral over the bag, we get

$$\oint dS\,\vec{\rm n}\cdot\vec{\rm E}_a+\mu'^2\int_{\rm bag}d^3x\,V_a^0=gQ_a, \qquad (A7)$$

where $Q_a=\lambda_a/2$ is the charge of the state. The first term on the left-hand side is zero due to the bag boundary condition $\vec{\rm n}\cdot\vec{\rm E}_a=0$. For nonzero Q_a, V_a^0 contains a term which diverges as $\mu'^2\to 0$. To isolate the divergent term we define

$$V_a^0(x)\equiv\frac{gQ_a}{\mu'^2V}+\bar{\phi}_a(x). \qquad (A8)$$

$\bar{\phi}_a$ is easily shown to be well behaved as $\mu'^2\to 0$. $\mathcal{B}$ and $\mathcal{A}$ satisfy

$$\vec{\nabla}\cdot\vec{\rm E}_{\mathcal{B}}+\mu_{\mathcal{B}}^2\mathcal{B}^0=g\rho_8\cos\theta-e\rho_{\rm GMZ}\sin\theta, \qquad (A9)$$

$$\vec{\nabla}\cdot\vec{\rm E}_{\mathcal{A}}=g\rho_8\sin\theta+e\rho_{\rm GMZ}\cos\theta. \qquad (A10)$$

Multiplying (A9) by $\cos\theta$ and (A10) by $\sin\theta$, and adding them together, we get

$$\vec{\nabla}\cdot\vec{E}_8+\mu_{\mathcal{B}}^2\cos\theta\,\mathcal{B}^0=g\rho_8. \qquad (A11)$$

As with $a=4,5,6,7$, we get

$$\mathcal{B}^0(x)=\frac{gQ_8}{\mu_{\mathcal{B}}^2\cos\theta V}+\bar{\phi}_{\mathcal{B}}(x), \qquad (A12)$$

where $\bar{\phi}$ is well behaved as $\mu_{\mathcal{B}}^2\to 0$. For small gauge-boson mass, the leading contribution to the field energy within the bag comes from the $0(\mu^{-2})$ term in V^0 and $\mathcal{B}^0$,

$$E_f\cong\int d^3x\left(\tfrac{1}{2}\mu'^2\sum_{a=4}^{7}(V_a^0)^2+\tfrac{1}{2}\mu_{\mathcal{B}}^2\mathcal{B}^{02}\right)$$

$$\cong\frac{1}{2}\frac{g^2}{\mu'^2V}(Q_aQ_a-\tfrac{3}{4}Q_8^2), \qquad (A13)$$

where, for attractiveness, we have added in Q_1^2, Q_2^2, and Q_3^2 which are 0 since their symmetries remain unbroken. Other terms in the QCD Hamiltonian which appear to diverge with μ' can be shown, in fact, to be finite and/or positive. The total energy is $E=E_f+BV$. For the static cavity approximation, a nonlinear bag boundary condition[4] requires the energy to be minimized with respect to V, whence to leading order,

$$E=\frac{\sqrt{3}}{16\pi\alpha'\mu'}(\lambda_a\lambda_a-\tfrac{3}{4}\lambda_8^2)^{1/2} \qquad (A14)$$

and $V=E(2B)^{-1}$, where we have replaced Q_a by $\lambda_a/2$, and for convenience we have introduced the bag expression for the Regge slope,[19]

$$\alpha'=\frac{1}{8\pi}\left(\frac{3}{2g^2B}\right)^{1/2}. \qquad (A15)$$

APPENDIX B

In the world of strange, down, and up quarks (in that order), $Q_{\rm GMZ}=-\lambda_8^f/\sqrt{3}$, where λ_8^f is the eighth generator of flavor (not color) SU(3), and $Q_{\rm HN}=-\lambda_8^f/\sqrt{3}+\lambda_8/\sqrt{3}$. In first-order electromagnetic transitions between color-singlet states, we cannot distinguish between the two charge assignments[3] because

$$\langle f|J_{\rm GMZ}|i\rangle=\langle f|J_{\rm HN}|i\rangle, \qquad (B1)$$

since $\langle f|J_8|i\rangle=0$ if $|i\rangle$ and $|f\rangle$ are color-singlet hadronic states. Certain second-order electromagnetic transitions between color-singlet states are generally believed to be model-independent tests of quark charge. However, as we have argued in Sec. II, the gauge symmetry is effectively restored at momenta above the tiny gauge-boson masses, since the contributions due to mass eigenstates add coherently. In this section, as an example the amplitude for $\eta_1 \to \gamma\gamma$ which is widely regarded as a model independent and $\mu_{\mathcal{B}}$ independent test of quark charge[14,3] is calculated. η_1 is the flavor-singlet part of the η or η'. As usual, we use the chiral anomaly[20] to calculate this amplitude.

We examine the graph in Fig. 2, where the large circle denotes the bag boundary. The triangle part of the graph should not be affected much by the bag since it probes short distances. The wiggly lines inside the bag are either $\mathcal{A}$ or $\mathcal{B}$ lines, both of which contain A. The relevant Green's function for the S-matrix element is

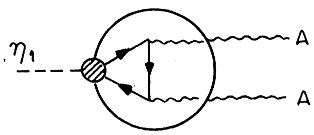

FIG. 2. η_1 decay into two photons.

$$G_{\mu\nu}(x_1, x_2, y) \equiv T\langle 0 | A_\mu(x_1) A_\nu(x_2) \partial \cdot J_{ax}(y) | 0\rangle , \tag{B2}$$

where J_{ax} is the flavor-singlet component of the axial-vector current, and x_1 and x_2 are points outside the hadron. Using the interaction representation, we have to lowest order in e,

$$\begin{aligned} G_{\mu\nu}(x_1, x_2, y) = -\tfrac{1}{2}\int_{\text{bag}} d^4z_1\, d^4z_2\, T\langle 0 | A^I_\mu(x_1) A^I_\nu(x_2) [J^I_{\mathcal{A}}(z_1)\cdot \mathcal{A}^1(z_1) + J^I_{\mathcal{B}}(z_1)\cdot\mathcal{B}^I(z_1)] \\ \times [J^I_{\mathcal{A}}(z_2)\cdot\mathcal{A}^I(z_2) + J^I_{\mathcal{B}}(z_2)\cdot\mathcal{B}^I(z_2)]\, \partial\cdot J_{ax}(y)|0\rangle_{\text{connected}} \end{aligned} \tag{B3}$$

$$\begin{aligned} = -\frac{1}{2}\int_{\text{bag}} d^4z_1\, d^4z_2\, T\langle 0 | A^I_\mu(x_1) A^I_\nu(x_2) [eJ^1_{\text{GMZ}}(z_1)\cdot A^I(z_1) + gJ^I_8(z_1)\cdot V^I_8(z_1)] \\ \times [eJ^I_{\text{GMZ}}(z_2)\cdot A^I(z_2) + gJ^I_8(z_2)\cdot V^I_8(z_2)]\, \partial\cdot J_{ax}(y)|0\rangle_{\text{connected}}, \end{aligned} \tag{B4}$$

where the various J's are various quark currents in an obvious notation. Although $\mathcal{A}$ and $\mathcal{B}$ are the mass eigenstates inside the bag, it is actually more appropriate to use A and V_8 in this calculation. $\mathcal{A}$ and $\mathcal{B}$ are not mass eigenstates outside the bag. A and V_8 are the mass eigenstates with masses 0 and M_8, respectively, where M_8 is huge since V_8 is quasiconfined. Outside the bag, $\mathcal{A}$ and $\mathcal{B}$ mix strongly, the mixing term being $\frac{1}{2}M_8{}^2V_8{}^2 = \frac{1}{2}M_8{}^2(\mathcal{A}\sin\theta + \mathcal{B}\cos\theta)^2$. Even for x and y in the bag, $T\langle 0|\mathcal{A}(x)\mathcal{B}(y)|0\rangle$ would not be 0 because $\mathcal{A}$ and $\mathcal{B}$ are mixed strongly by the boundary conditions. A and V_8 also mix but only because of a tiny mixing term only inside the bag, $\frac{1}{2}\mu_{\mathcal{B}}{}^2\mathcal{B}^2 = \frac{1}{2}\mu_{\mathcal{B}}{}^2(-A\sin\theta + V_8\cos\theta)^2$, and they satisfy simpler boundary conditions than $\mathcal{A}$ and $\mathcal{B}$. Aside from $\mu_{\mathcal{B}}{}^2$, the relevant scales, the η mass and the hadron size, are typical hadron scales, and the mixing term may be treated as a perturbation. (B4) is then equal to (dropping I's)

$$\begin{aligned} G_{\mu\nu}(x_1, x_2, y) = -\int_{\text{bag}} d^4z_1 d^4z_2 T\langle 0 | A_\mu(x_1) A_\rho(z_1)|0\rangle T\langle 0|A_\nu(x_2)A_\sigma(z_2)|0\rangle T\langle 0|\partial\cdot J_{ax}(y) e^2 J^\rho_{\text{GMZ}}(z_1) J^\sigma_{\text{GMZ}}(z_2)|0\rangle \\ - \int_{\text{bag}} d^4z_1 d^4z_2 T\langle 0|A_\mu(x_1)V_{8\rho}(z_1)|0\rangle\, T\langle 0|A_\nu(x_2)V_{8\sigma}(z_2)|0\rangle T\langle 0|\partial\cdot J_{ax}(y) g^2 J^\rho_8(z_1) J^\sigma_8(z_2)|0\rangle, \end{aligned} \tag{B5}$$

where the propagator $T\langle 0|A(x_1)V_8(z_1)|0\rangle$ is not zero but small because the mixing term A and V_8 inside the bag is tiny. As $\mu_{\mathcal{B}} \to 0$, the mixing goes away:

$$\lim_{\mu_{\mathcal{B}}{}^2 \to 0} T\langle 0|A^\mu(x_1)V^\rho_8(z_1)|0\rangle = -i\mu_{\mathcal{B}}{}^2 \sin\theta\cos\theta \int_{\text{bag}} d^4w\, T\langle 0|A^\mu(x_1)A_\lambda(w)|0\rangle\, T\langle 0|V^\lambda_8(w)V^\rho_8(z_1)|0\rangle , \tag{B6}$$

which smoothly goes to zero, and $T\langle 0|A(x_1)A(z_1)|0\rangle$ also smoothly goes to its value in the fractional-charge model. Therefore, for sufficiently small $\mu_{\mathcal{B}}{}^2$, there is no essential difference in $\eta \to \gamma\gamma$ between our integer-charge model and fractional charge.

When a finite "model-independent" difference was obtained between fractional- and integral-charge assignments, the tacit assumption had been made that the photon field couples directly to the current J_{HN}. That would identify the photon as $\mathcal{A}$. Had we substituted $\mathcal{A}$ for our bag-exterior photon field A, in Eq. (B3), we would have only needed to study $T\langle 0 | \partial \cdot J_{ax}(y) J_{em}(z_1) J_{em}(z_2) | 0 \rangle$ to determine the difference between integer and fractional charge. However, in our model we must study the entire $\eta \to \gamma\gamma$ Green's function.

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[10]Since the required Higgs bosons are colored, they exist only inside hadrons. We use the term "Higgs-field vacuum expectation value" loosely to denote the mean value of the Higgs field inside hadrons. See the second paper of Ref. 4 for a discussion.

[11]To our knowledge, J. Pati and A. Salam [Phys. Rev. D 8, 1240 (1973); 10, 275 (1975)] first proposed the idea that the local symmetry SU(3) color $\times$ $U(1)_{GMZ}$ is broken spontaneously to $U(1)_{HN}$ so as to assign intercharge to quarks, and proposed the idea of partial confinement [in *Proceedings of the International Neutrino Conference, Aachen, 1976,* edited by H. Faissner, H. Reithler, and P. Zerwas (Vieweg, Braunschweig, West Germany, 1977)]. J. Pati [in *Proceedings of the Seoul Symposium on Elementary Particle Physics in Memory of Benjamin W. Lee*, edited by J. Kim, P. Y. Pac, and H. S. Song (Seoul National University Press, Seoul, 1979)] emphasized the idea of softly breaking SU(3) $\times$ U(1) using a very small Higgs-field vacuum expectation value. This model is of the type discussed by Okun *et al.* (Ref. 5) and Chetyrkin *et al.* (Ref. 8).

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