Supersymmetry and the scale of unification

S. Dimopoulos
Institute for Theoretical Physics, Stanford University, Stanford, California 94305, Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, and University of Michigan, Ann Arbor, Michigan 48109

S. Raby
Institute for Theoretical Physics, Stanford University, Stanford, California 94305 and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

Frank Wilczek
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

(Received 20 April 1981)

Unified theories which are supersymmetric down to energies $\sim 10^3$–10^5 GeV have been proposed as possible solutions to the gauge-hierarchy problem. The additional particles then required have significant effects on renormalization of coupling constants. The previous successful calculation of the weak mixing angle is only slightly changed, but the scale of unification is moved significantly higher, into the range of the Planck mass. This may be suggestive of an eventual unification including gravity, and markedly reduces the predicted rate of nucleon decay.

Apparently successful gauge theories of the strong, electromagnetic, and weak interactions have suggested that all these interactions are manifestations of a larger, encompassing gauge symmetry softly broken at a large mass scale. Models realizing this idea have been constructed and analyzed.[1] Three important features of the analysis are as follows:

(1) Three observed parameters—the strong, weak, and electromagnetic couplings—must be calculated from two parameters, namely, the scale at which the unified symmetry breaks and the single gauge coupling at this scale. In this way one new constraint is found among the low-energy couplings. This may be expressed as a prediction for the weak mixing angle, and appears to be remarkably successful.[2]

(2) The required scale of symmetry breakdown is found to be $\simeq 10^{15}$ GeV.[2] Vector bosons in the broken-symmetry directions, which mediate nucleon decay, acquire masses of this order—yielding the prediction of a nucleon lifetime $\tau \simeq 10^{31\pm 2}$ yr.[1]

(3) The enormous difference in scale between the breakdown of the unified symmetry at $\sim 10^{15}$ GeV and the weak-electromagnetic symmetry at $\sim 10^2$ GeV requires unexplained, and exceedingly accurate, cancellations among renormalized coupling constants. This aspect of the models, which many people find unattractive, is known as the gauge-hierarchy problem.[3]

Recently some attempts to exorcise the gauge-hierarchy problem have been based on the notion of supersymmetry.[4] The hierarchy problem may be restated in this form: why does the Higgs boson system which breaks $SU(2)\times U(1)$ symmetry at $\sim 10^2$ GeV have such a small mass parameter? Ordinarily mass parameters for scalar fields violate no symmetries and their smallness cannot be explained on symmetry grounds. In supersymmetric theories, where there are symmetry transformations mixing fermions and bosons, the situation is completely different however. Chiral symmetries which forbid fermion-mass terms will also forbid the corresponding boson-mass terms. The idea is that there is a chiral supersymmetry good down to $\simeq 10^3$–10^5 GeV, whose breakdown allows the Higgs-boson system to acquire a mass parameter of this general magnitude. Of course we are then left with the question of why the scales of unified symmetry breakdown at $\sim 10^{15}$ GeV and of weak-electromagnetic breakdown at $\sim 10^2$ GeV are so different—but this may be a less severe form of hierarchy problem.[5]

Supersymmetry may be attractive in a more general way, supplying a *raison d' etre* for the scalar fields which seem to be necessary to give a convincing mechanism of fermion-mass generation.[6]

If, motivated by supersymmetry or otherwise, we contemplate adding additional fields into a unified theory, we must consider not only their direct effects—e.g., if supersymmetry is restored at $\sim 10^3$ GeV, a rich spectrum of new particles at such energies is predicted—but also the effect of their virtual emission on the renormalization of couplings in unified theories. Let us briefly recall how this goes. The effective couplings of the $SU(3)\times SU(2)\times U(1)$ gauge symmetry change with energy as

$$\alpha_3^{-1}(M) = \alpha_G^{-1} + \frac{1}{6\pi}\ln(M/\mu)b_3\,,$$

$$\alpha_2^{-1}(\mu) = \alpha_G^{-1} + \frac{1}{6\pi}\ln(M/\mu)b_2\,, \tag{1}$$

$$\tfrac{3}{5}\alpha_1^{-1}(\mu) = \alpha_G^{-1} + \frac{1}{6\pi}\ln(M/\mu)b_0\,,$$

where the b_N are group-theoretical constants, depending on the number of fields participating in the renormalization. We have, for example,[7]

$$b_N = \begin{cases} 4f - 11N & \text{(ordinary theory)},\\ 6f - 9N & \text{(supersymmetric theory)},\end{cases} \tag{2}$$

where f is the number of families [e.g., a $\underline{5}$ and $\overline{\underline{10}}$ representation of SU(5)]. The difference between ordinary and supersymmetric theories is that in the latter the spin-$\frac{1}{2}$ fermion families have spin-0 partners and the spin-1 gauge bosons have spin-$\frac{1}{2}$ fermion partners which contribute to vacuum polarization.

By manipulating the system of Eqs. (1) we can solve for the weak mixing angle and the scale; the results are

$$\tfrac{3}{5}\,\alpha_1^{-1} - \tfrac{3}{5}\,\alpha_2^{-1} - \tfrac{2}{5}\,\alpha_3^{-1} = \frac{12}{5}\,\frac{1}{6\pi}\ln(M/\mu)\times\begin{cases}11 & \text{(ordinary)},\\ 9 & \text{(supersymmetric)},\end{cases} \tag{3}$$

$$\sin^2\theta \equiv \frac{\alpha_1}{(\alpha_1^{\,2}+\alpha_2^{\,2})^{1/2}} = \frac{1}{6} + \frac{5}{9}\,\frac{\alpha}{\alpha_3}\,, \tag{4}$$

where $\alpha = \alpha_1\,\alpha_2/(\alpha_1^{\,2}+\alpha_2^{\,2})^{1/2}$ is the ordinary fine-structure constant and all couplings are expressed at some small scale μ. The notable features of these results are that the prediction of the angle is unmodified and that the difference in scales can be summarized simply as

$$(M/\mu)_{\text{supersymmetric}} = (M/\mu)_{\text{ordinary}}^{11/9}\,. \tag{5}$$

This change in scale has several important implications:

(1) In the ordinary theory $M \simeq 10^{15}$ GeV and, say, $M/\mu \simeq 10^{13}$ so the supersymmetric $M \simeq 10^{18}$ GeV. This is of the same order of magnitude as the Planck mass, hinting perhaps a larger unification including gravity as well.

(2) Since nucleon decay scales as M^{-4}, the lifetime of the proton due to gauge bosons in supersymmetric theories would be much longer than in ordinary theories, $\tau \sim 10^{45}$ yr, which is inaccessible experimentally. Of course one cannot, even in these theories, necessarily preclude the existence of unexpectedly light scalar bosons mediating the nucleon decay at a faster rate.

(3) The magnetic monopoles and perhaps even some gauge bosons of supersymmetric theories have $M > M_{\text{Planck}}$. Therefore, their compton radius is inside their Schwarzschild radius—they are black holes.

(4) In the extremely early Universe gauge bosons so massive will not be able to maintain thermal equilibrium in competition with the rapid expansion of the Universe. We will therefore have to determine the initial state from some as yet unknown dynamics, presumably involving particle production in strong gravitational fields. It of course remains the case that a universe beginning symmetrically between matter and antimatter will develop an asymmetry.

In addition to the change in scale of unification, additional matter fields affect the size of the coupling at the unification scale. With three families ($f=3$) Eq. (1) tells us that the SU(2) coupling α_2 is almost unrenormalized, so $\alpha_G^{-1} \simeq 30$. If we contemplate, inspired by heavy-color ideas or otherwise, adding many more SU(2) multiplets into the theory, we run into the possibility of the coupling becoming large at a scale smaller than the unification scale. In this case we could no longer compute renormalization effects perturbatively and our whole analysis (including the prediction for $\sin^2\theta$) would be problematic. In order to avoid this our theories should contain *no more than ten* supersymmetric chiral SU(2) doublets in addition to the three ordinary families. This suggests that there should not be more than five families.

If superheavy fermions come in complete SU(5) families, then the heavy-color (supercolor) group should not be bigger than SU(2). In this case it is difficult to understand why it becomes strong at 10^3–10^5 GeV.

This work was supported in part by the National Science Foundation Grant No. PHY77-27084. S.D. acknowledges support in part by NSF Grant No. PHY77-27084. S.R. was supported in part by NSF Grant No. PHY78-26817 and by DOE Grant No. DE-AC03-76SF00515.

[1]For a review and references, see P. Langacker, SLAC Report No. 2544, 1980 (unpublished).
[2]H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).
[3]E. Gildener, Phys. Rev. D 14, 1667 (1976); S. Weinberg, *ibid.* 19, 1277 (1979); L. Susskind, *ibid.* 20, 2619 (1979).
[4]S. Dimopoulos and S. Raby, ITP report (unpublished); E. Witten, Princeton report (unpublished); M. Dine, W. Fishler, and M. Srednicki, Princeton report (unpublished).
[5]See the first paper of Ref. 4. Theories of supercolor or supersymmetric heavy color do *not* suffer from this hierarchy problem.
[6]Attempts to do without fundamental scalars in this connection, e.g., extended heavy color, seem to have difficulties with K_L-K_S mixing, $\mu \rightarrow 3e$, etc.
[7]We are oversimplifying slightly here. The Higgs-boson system which ultimately breaks $SU(2)\times U(1) \rightarrow U(1)$ should be included; this is a small effect. Notice that the supersymmetric partners of ordinary fermions cannot be identified with this Higgs-boson system since the partners of the quarks are light ($\sim 10^3$ GeV) and would mediate nucleon decay at an unacceptably large rate if they coupled to light fermions.