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Class of ghost-free gravity Lagrangians with massive or massless propagating torsion

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A class of six-parameter ghost-free gravity Lagrangians are found, which propagate massive tordions with $J^P \leq 2^{\pm}$. One particular three-parameter ghost-free gravity Lagrangian is found which propagates a massless tordion with $J^P = 1^-$.

In an attempt to find a unitary and renormalizable theory of gravity, in a recent article[1] which we will refer to as I, the author and van Nieuwenhuizen considered a nine-parameter $(R+R^2)$-type Lagrangians for e^a_μ and ω^{ab}_μ (the vierbein and Cartan spin-connection fields, respectively) such that there are at most second derivatives. Employing the spin-projection operator formalism, we found the constraints on the parameters for ghost- and tachyon-free Lagrangians, assuming that all the mass parameters of the theory are nonzero and nondegenerate. In agreement with Neville's result,[2] we did not find a power-counting renormalizable and unitary Lagrangian in that class. However, relaxing the power-counting renormalizability criteria, in I, we found some solutions to the constraints on the parameters.

In this paper, we find all possible solutions to these constraints and investigate a large class of Lagrangians (obtained by certain choices of the nine parameters) to obtain a ghost-free theory with massless propagating torsion. It turns out that there are 12 six-parameter Lagrangians which propagate massive tordions of various J^P. For the massless case, we find that a large number of parameter choices are ruled out in order to eliminate the dipole ghosts and the usual ghosts (that arise due to the nondefiniteness of the Minkowski metric[3]). However, we do find one particular ghost-free Lagrangian which propagates a massless tordion of $J^P = 1^-$. This Lagrangian has an extra (i.e., in addition to the local Lorentz and general coordinate invariance) linearized 16-parameter gauge invariance of the form $\delta\omega_{cab} = \partial_{[a}\Lambda_{b]c}$. We now derive these results and comment on them.

The nine-parameter Lagrangian which was analyzed in I is

$$\begin{aligned}\mathcal{L} = &-\lambda R(\omega) + \tfrac{1}{12}(4a+b+3\lambda)(R_{abc})^2 + \tfrac{1}{6}(-2a+b-3\lambda)R_{abc}R^{bca} \\ &+ \tfrac{1}{3}(-a+2c-3\lambda)(R_{ab}{}^b)^2 + \tfrac{1}{6}(2p+q)(R_{abcd})^2 + \tfrac{1}{6}(2p+q-6r)R_{abcd}R^{cdab} \\ &+ \tfrac{2}{3}(p-q)R_{abcd}R^{acbd} + (s+t)(R_{ab})^2 + (s-t)R_{ab}R^{ba} + e_{\mu a}\Sigma^{\mu a} + \omega_{\mu ab}\tau^{\mu ab}\,. \end{aligned} \tag{1}$$

We use the same notation and conventions of I. The basic result of I is that this Lagrangian is ghost and tachyon free if the nine parameters satisfy the following constraints[4]:

Massless sector: $\lambda > 0$

$$\begin{aligned}&(2^-)\colon p<0,\ a>0;\quad (0^-)\colon q<0,\ b>0;\\ &(1^-)\colon p+s+t<0,\ ac(a+c)>0;\quad (1^+)\colon 2r+t>0,\ ab(a+b)<0;\\ &(2^+)\colon 2p-2r+s>0,\ a\lambda(a+\lambda)<0;\quad (0^+)\colon p-r+2s>0,\ c\lambda(c-\lambda)>0;\end{aligned} \tag{2}$$

where it is assumed that all the mass parameters are nonzero and nondegenerate. That it is not possible to propagate the torsion in all J^P sectors is clear from the conflict in the inequalities given in Eq. (2). However, elimination of some of the massive poles can make Eq. (2) consistent, since in a sector with no pole, the no-ghost and no-tachyon conditions need not be satisfied. In that case, for example, the 1^+ sector of Eq. (2) should be read as $[2r+t>0$ and $ab(a+b)<0]$ or $(2r+t=0)$ or $(a+b=0)$.

Proceeding to solve Eq. (2), one discovers that at least three of the massive poles have to be eliminated. From a straightforward analysis of Eq. (2) is follows that there are 12 such solutions. They are listed in Table I, together with the particle content of the Lagrangians they give rise to. These propagating sectors must, of course, satisfy the relevant inequalities in Eq. (2). Several special cases can be obtained by eliminating fur-

TABLE I. Parameter choices for ghost- and tachyon-free gravity Lagrangians. The J^P of the propagating torsion sectors are listed under the particle content. These sectors must satisfy the corresponding inequalities given in Eq. (2). Special cases can be obtained by converting some of those inequalities into equalities consistently $[\lambda=(16\pi G)^{-1}$, $abc\neq 0$ always], therefore eliminating more massive poles.

Parameter choices	Particle content
(1) $p=0,\ a+b=0,\ s+t=0$	$2^+,0^+,0^-$
(2) $p=0,\ a+b=0,\ s-2r=0$	$1^-,0^+,0^-$
(3) $p=0,\ a+b=0,\ r-2s=0$	$2^+,1^-,0^-$
(4) $p=0,\ a+c=0,\ s-2r=0$	$1^+,0^+,0^-$
(5) $p=0,\ a+c=0,\ r-2s=0$	$2^+,1^+,0^-$
(6) $p=0,\ a+\lambda=0,\ s+t=0$	$1^+,0^+,0^-$
(7) $p=0,\ a+\lambda=0,\ r-2s=0$	$1^+,1^-,0^-$
(8) $q=0,\ a+b=0,\ 2p-2r+s=0$	$2^-,1^-,0^+$
(9) $q=0,\ a+c=0,\ 2p-2r+s=0$	$2^-,1^+,0^+$
(10) $2r+t=0,\ a+c=0,\ 2p-2r+s=0$	$2^-,0^+,0^-$
(11) $p=0,\ a+c=0,\ a+\lambda=0$	$1^+,0^-$
(12) $p=0,\ s+t=0,\ 2r+t=0$	$0^+,0^-$

ther some of the massive poles within this propagating sector. For example, for $a=-b=-c-\lambda$, where the torsion-squared terms are absent, only the $J^P=0^-$ tordion propagates and the only condition on the curvature-squared terms is that $q<0$ (in addition to the graviton with $\lambda>0$).

One of the unattractive aspects of these theories is the presence of as many as six independent coupling constants (some of which are dimensionless and some dimensionful). Furthermore, there are no extra gauge invariances for any choice of the parameters, as long as $a,b,c,\lambda\neq 0$. One helpful criterion for narrowing down the number of "acceptable" Lagrangians may be the Birkoff theorem[5] which states the following: For O(3) spherically symmetric space-time the unique solution of the Einstein equations *in vacuo* is the static Schwarzchild solution. Recently, Rauch and Nieh[6] have found two Lagrangians, within the class of Lagrangians considered here, which satisfy the Birkhoff theorem. They are given by the parameter choices (i) $p=q=0$, $s=-t=2r$, $a=-b=-c=-\lambda<0$, and (ii) $p=q=r=s=t=0$, $abc\neq 0$, $\lambda>0$. These are, of course, special cases of some of the solutions given in Table I. Note that in these two cases, torsion does not propagate but contact torsion forces exist. If one assumes that the torsion is parity invariant or the scalar curvature vanishes, then there are more solutions[6] which satisfy a weakened version of the Birkhoff theorem. Those solutions are special cases of the ones given in Table I and some of them propagate massive tordions as well as the massless graviton.

It is by no means necessary to choose these Lagrangians over the rest as viable ones, since torsion effects can be made arbitrarily small in a Lagrangian which does not obey the Birkhoff theorem, due to the arbitrariness in the torsion coupling constants. Thus, unless the coupling constants of the theory are related to each other due to extra symmetries and other considerations, even in a theory with spherically symmetric sources (and no Birkhoff theorem) the torsion effects may not conflict with the usual tests of general relativity which assume such sources. However, it is an elegant and enormously simplifying principle to demand the Birkhoff theorem in a theory with torsion. In that case the torsion effects may manifest themselves only in the presence of spherically nonsymmetric sources.

Among the other criteria which may serve to reduce the number of arbitrary parameters of the theory are the appropriate Newtonian limit,[7] supersymmetrization,[8] and a better quantum behavior with regard to the ultraviolet divergences of the theory.

We now consider the massless torsion case. We will restrict our attention to the case where $a=b=c=0$, due to its simplicity. Then, the saturated propagator is simply[9]

$$\Pi=-\frac{1}{k^2}\tau\cdot\left[\frac{p(2^-)}{p}+\frac{p(0^-)}{q}+\frac{p_{11}(1^-)}{p+s+t}+\frac{p_{11}(1^+)}{2r+t}+\frac{p_{11}(2^+)}{2p-2r+s}+\frac{p_{11}(0^+)}{p-r+2s}\right]\cdot\tau$$
$$-\frac{1}{\lambda k^2}\Sigma\cdot\left[p_{22}(2^+)-\tfrac{1}{2}p_{22}(0^+)\right]\cdot\Sigma\,. \tag{3}$$

Note that the torsion propagator has decoupled from the graviton propagator (no $\tau-\Sigma$ mixing terms). It is also interesting to note that although the nonpresence of $P_{22}(1^{\pm})$ in the field equation does not imply a new gauge invariance, it does imply a new source constraint

$$\partial_c\tau_{cab}=0\,. \tag{4}$$

In searching for a ghost-free propagator, first the $1/k^4$ double poles must be eliminated. Using the completeness property of the spin-projection operators, it is easy to achieve such an elimination by simply choosing all the denominators in Eq. (3) equal, or some of them equal and the rest zero. However, the resulting saturated tordion propagator, which has the form

$$k^{-2}\tau_{cab}(\delta_{cc'}\delta_{aa'}\delta_{bb'})\tau_{c'a'b'}\,, \tag{5}$$

is not ghost free. To see this, let us decompose the Minkowski metric,[3] for $k^2=0$, as follows:

$$\eta_{ab}=\sum_{i=\pm1} e_a^i e_b^{-i}(-1)^i+(k\cdot \bar{k})^{-1}(k_a\bar{k}_b+\bar{k}_a k_b)\,, \tag{6}$$

where e_a^i are the usual two transversal polarization vectors while $\bar{k}_a$ is the time-reversed k_a. On the other hand, as we will show below, the elimination of the double poles with the way described above does not allow a linearized gauge invariance such that $\partial_b\tau_{cab}=0$. Therefore, the $k\bar{k}$ terms in Eq. (6), which are ghostly, cannot be eliminated. For the Lagrangian to have the desired gauge invariance, an inspection of the spin-projection operators given in I reveals that one has to eliminate the $J^P=2^+$, 1^+, 0^+ sectors. This can be done by choosing $p=r$, $s=0$, $2r+t=0$, which, on account of (20) and (21) of I, give rise to the following linearized gauge invariances and source constraints:

$$\delta\omega_{cab}=\partial_a\Lambda_{bc}-\partial_b\Lambda_{ac}\,, \tag{7a}$$

$$\partial_b\tau_{cab}=0\,. \tag{7b}$$

Equation (7b) not only eliminates all the k^{-4} poles but also the $k\bar{k}$ terms in the saturated propagator. However, now the torsion propagator is not in the form of Eq. (5), but instead it is in the following form:

$$\Pi(\tau)=\frac{-1}{k^2}\,\tau\cdot\left\{\frac{1}{p}\left[\tfrac{2}{3}(\eta\eta\eta)_{cab}-\tfrac{2}{3}(\eta\eta\eta)_{abc}-2\eta_{bc}\eta_{aa'}\eta_{b'c'}\right]+\frac{1}{q}\left[(\eta\eta\eta)_{cab}+\tfrac{2}{3}(\eta\eta\eta)_{abc}\right]\right\}\cdot\tau\,. \tag{8}$$

For a ghost-free propagator it must be shown that the residue of Eq. (8) at $k^2=0$ is positive definite, for proper choice of p and q. To this end, we substitute Eq. (6) into Eq. (8) and after some algebra obtain

$$\Pi(\tau)=\frac{-1}{pk^2}\left[\tau_{cab}(-k)e_{ijk}^{cab}(-k)\right]\left[e_{ijk}^{c'a'b'}(k)\tau_{c'a'b'}(k)\right]\,, \tag{9}$$

where

$$e_{ijk}^{cab}(k)=\mp\tfrac{2}{3}e_i^c e_j^a e_k^b\pm\tfrac{1}{3}(e_j^c e_k^a e_i^b-e_k^c e_j^a e_i^b)-(-1)^n\delta_{i,-j}e_k^b\sum_{n=\mp1}e_n^c e_{-n}^a\,. \tag{10}$$

[Note that the $1/q$ term has dropped out in Eq. (9) because it is totally antisymmetric in a,b,c.[10]] Defining $\tau_{ijk}\equiv\tau_{cab}(k)e_{ijk}^{cab}(k)$, Eq. (9) implies a positive-definite residue at $k^2=0$, for $p>0$ [in Bjorken and Drell conventions, $\Pi=-\text{residue }(k^2=0)/k^2$]:

$$\frac{1}{p}|\tau_{ijk}|^2>0,\quad p>0\,. \tag{11}$$

From Eq. (10) it follows that the only nonvanishing components of the polarization tensor e_{ijk}^{cab} are e_{++-}^{cab} and e_{--+}^{cab}. These are clearly the two physical helicity states of the $J^P=1^-$ massless tordion.

In conclusion, we have found a class of ghost- and tachyon-free gravity Lagrangians [Eq. (2) and Table I] which propagate massive tordions with various J^P. Furthermore, we have found one particular ghost-free Lagrangian, propagating the $J^P=1^-$ tordion, which is given by

$$\mathcal{L}=-\lambda R(\omega(e))-24g_1R_{[ab]}R^{[ab]}+(g_1+g_2)R_{abcd}R^{abcd}+(-4g_1+g_2)R_{abcd}R^{cdab}+4(g_1-g_2)R_{abcd}R^{acbd}\,, \tag{12}$$

where $\lambda>0$, $g_1>0$, and g_2 is arbitrary. This Lagrangian has an extra linearized gauge invariance of the form $\delta\omega_{cab}=\partial_{[a}\Lambda_{b]c}$. One of the open questions is the consequences of the covariant form of this transformation for the full theory.[11] Another unsolved problem is the renormalizability properties of these Lagrangians. In the case of massive-tordion Lagrangians, as $k^2\to\infty$, the graviton propagator goes as k^{-2} while the tordion propagator goes as constant. In the case of a massless $J^P=1^-$ tordion Lagrangian, the graviton propagator is identical to the usual one while the tordion propagator is of the form k^{-2}. However, it should be noted that, as has been pointed out by Neville,[12] the tordion vertices are better behaved than the graviton vertices.

As far as coupling torsion to matter is concerned, the well-known difficulties[13,14] are present in our Lagrangians too. However, a new feature arises in the case of our massless $J^P=1^-$ tordion Lagrangian: The minimal coupling of the tordion to the Dirac field which is of the form $\bar{\psi}\{\gamma^c,\sigma^{ab}\}\psi\omega_{cab}$ vanishes, due to the fact that $J^P=1^-$ tordion field is symmetrical in two indices. It is interesting, however, that the minimal coupling to the Rarita-Schwinger field, which is of the form $\epsilon^{\lambda\rho\mu\nu}\epsilon_\mu^{abc}\bar{\psi}_\lambda\gamma_c\psi_\rho\omega_{\nu ab}$, does not vanish. Should sensible couplings of the $J^P=1^-$ tordion of the present model exist, the fact that the exchange of this tordion is of long-range nature makes it more interesting to investigate its astrophysical consequences.

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