PHYSICAL REVIEW D VOLUME 24, NUMBER 1 1 JULY 1981

Generalized mixing angles in gauge theories with natural flavor conservation

Arthur C. Rothman
Department of Physics, The Ohio State University, Columbus, Ohio 43210

Kyungsik Kang
Department of Physics, Brown University, Providence, Rhode Island 02912
(Received 10 December 1980)

A number of theorems relating natural flavor conservation and calculability are proven for general gauge models of the weak and electromagnetic interactions with an unbroken U(1) symmetry. The concept of "nontriviality"—a necessary condition that all naturally flavor-conserving gauge models must obey in order to have nontrivial mixing angles—is introduced. It is found that naturality groups guaranteeing natural flavor conservation cannot generate meaningful mixing angles in any guage model.

I. INTRODUCTION

The elevation of flavor conservation from the observed absence of strangeness-changing neutral currents to a general theoretical constraint for all flavors in gauge theories, presaged by a few authors,[1] was realized in a paper by Glashow and Weinberg.[2] There they stated the conditions for $SU(2)_L \times U(1)$, the "standard model," to "naturally" conserve all quark flavors in the neutral currents. These conditions are that all quarks of a given charge and helicity must have the same values of T_3 and $\vec{T}^2$, and that all quarks of a given charge receive their mass either through the couplings of precisely one neutral Higgs meson or through an SU(2)-invariant mass term but not by both mechanisms.

While the former condition is necessary and sufficient for natural flavor conservation (NFC), the latter (hereafter referred to as the sufficiency condition) is only sufficient. That is, it is possible to have an arbitrary number of Higgs scalars coupled to a given charge sector and yet ensure NFC by applying a further "horizontal" symmetry, called a "naturality group,"[3] to the Lagrangian. Flavor conservation is still natural in this case in the sense that it follows from the group structure and representation content of the theory, rather than depending on the particular values of the various parameters.

One reason why we are interested in introducing more Higgs fields into gauge theories is that recent works[4] dealing with the calculation of flavor mixing angles from the mass terms in the effective Lagrangian call for a much richer Higgs content. There are those willing to posit the existence of more Higgs fields and other group symmetries in order to obtain meaningful mixing angles in the theory.

The naturality group K then must serve a dual purpose. It must ensure NFC and also limit the Higgs-boson-quark couplings in such a way as to allow calculable mixing angles. The mixing angles obtained in this way are left unaltered by renormalization. In the standard model it has been shown[5] that calculability necessitates more than one Higgs scalar.

There has recently been much work on characterizing naturality groups[6] and their relation to the calculability of mixing angles in the standard model.[3,7-10] In this paper we prove a number of theorems which are, in the main, generalizations of these works that are valid for any gauge model of the weak and electromagnetic interactions with an unbroken U(1) subgroup. In the following, "gauge model" will refer only to models of this type. In particular, we show that for a gauge model, where flavor conservation is enforced by a naturality group, all mixing angles in the charged currents, be they left- or right-handed, are phenomenologically unacceptable or noncalculable.

The motivation for considering a general class of gauge model is twofold. Simply, it is possible that the standard model may *not* be the operative gauge group of the weak and electromagnetic interactions at higher energies [for example, neutrinos may turn out to be massive after all, in which case the $SU(2)_L \times SU(2)_R \times U(1)$ model would appear more attractive]. More importantly, we would like to show that the results obtained previously are not peculiar to the standard model; and the "clashing" of NFC and calculability is a property inherent in all gauge models.[3,7-10] This would then be a much deeper result.

The paper is organized as follows. Section II contains general conditions of NFC for arbitrary gauge models along with the definitions of the mass matrix and mixing angles. Section III deals with the mathematical formulation of NFC in the

Higgs-boson-exchange processes. In Sec. IV, a nontriviality theorem for the mixing angles is proven. Section V presents the results of applying naturality groups to gauge models. In the Appendix, some mathematical preliminaries, which are used in the main text, are given.

II. PRELIMINARIES

Let us first generalize the results of Glashow and Weinberg to an arbitrary gauge model.

A first step has been taken in this direction in Ref. 11. We find that[12] the conditions for NFC in an arbitrary gauge group G with an unbroken U(1) symmetry are the following.

(1) All quarks of a given charge and helicity transform identically under G.

(2) All quarks of a given charge receive their mass either (a) through a G-invariant mass term or (b) through the couplings of neutral Higgs mesons, where the couplings of each Higgs meson are simultaneously diagonalizable in a natural manner with the mass matrix, but not by both mechanisms (a) and (b).

These conditions are necessary and sufficient to guarantee NFC in a gauge model. The case (2a) is trivial since it is often excluded by the gauge group and representation and, more importantly, because it is not possible to generage quark masses and calculable mixing angles in this case. "Diagonalizable in a natural manner" means that the couplings derive this property from a group symmetry or representation.

Consider now a gauge model based on the gauge group G which is, in general, a direct product of simple Lie groups. The theory will contain elementary left-handed fermions, right-handed fermions, and Higgs scalars in the multiplets $\psi_{iL}(x)$, $\psi_{jR}(x)$, and $\phi^k(x)$, respectively ($i=1,2,\ldots,n_L$; $j=1,2,\ldots,n_R$; $k=1,2,\ldots,m$). These multiplets will transform according to the arbitrary representations F_{iL}, F_{jR}, and B_k of the gauge group. The gauge-invariant Yukawa interaction for this theory will be

$$\mathcal{L}_Y=\bar{\psi}_{iL}\,\Gamma^{\alpha}_{ij}\,\phi^{\alpha}\,\psi_{jR}+\text{H.c.} \tag{1}$$

After the spontaneous symmetry breaking of G down to U(1), we obtain the mass term $\mathcal{L}_M$:

$$\mathcal{L}_M=\sum_Q \bar{q}^{\,Q}_{0L}\,M(Q)\,q^{Q}_{0R}+\text{H.c.}, \tag{2}$$

where

$$M(Q)=\sum_{\alpha}\Gamma^{\alpha}_{Q}\lambda^{\alpha}_{Q}, \tag{3}$$

and where $q^Q_{0L(R)}$ is the left- (right-) handed projection of a vector containing all bare quark fields of a given charge Q. $\lambda^{\alpha}_{Q}\equiv\langle\phi^{\alpha}\rangle_Q$ is the vacuum expectation value of a neutral Higgs scalar that contributes to the mass matrix for quarks of charge Q. Γ^{α}_{Q} is the associated coupling matrix. Of course, in general we may have $\lambda^{\alpha}_{Q}=\lambda^{\alpha'}_{Q'}$ (if there is an $\tilde{\phi}$ coupling) or $\Gamma^{\alpha}_{Q}=\Gamma^{\alpha}_{Q'}$, if the Q and Q' quarks are both contained in the same left-handed multiplet and the same right-handed multiplet.

For obvious phenomenological reasons, we shall assume throughout that $M(Q)$ is nonsingular and nondegenerate. We can diagonalize each $M(Q)$ with a biunitary transformation:

$$U^{Q}_{L}M(Q)U^{Q\dagger}_{R}=\hat{M}(Q), \tag{4}$$

where $\hat{M}(Q)$ is diagonal. This will define the physical quark fields $q^{Q}_{L(R)}$ by

$$q^{Q}_{L(R)}\equiv U^{Q}_{L(R)}\,q^{Q}_{0L(R)}. \tag{5}$$

Now, due to condition (1), the gauge fields will couple to all quarks of a given charge if they couple to any. From the coupling of quarks of charges Q and Q' in the multiplet of given helicity, the charged currents take the form

$$J^{\mu}_{L\,R}=\bar{q}^{\,Q}_{0L\,R}\,\gamma^{\mu}q^{Q'}_{0L\,R}. \tag{6}$$

That the currents take this form is a nontrivial consequence of condition (1) for NFC.

Moving into the physical qurk basis defined by (5) we have

$$J^{\mu}_{L(R)}=\bar{q}^{\,Q}_{L(R)}\gamma^{\mu}U_{L(R)}q^{Q'}_{L(R)}, \tag{7}$$

where $U_{L(R)}$ is the generalized left- (right-) handed Cabibbo matrix given by

$$U_{L(R)}\equiv U^{Q}_{L(R)}(U^{Q'}_{L(R)})^{\dagger}. \tag{8}$$

III. NATURAL FLAVOR CONSERVATION

We would now like to mathematically formulate NFC condition (2b). One way to do this is to recognize that all the coupling matrices Γ^{α} in each charge sector are simultaneously diagonalizable.

Theorem 1. Condition (2b) is equivalent to requiring

$$U^{Q}_{L}\Gamma^{\alpha}_{Q}U^{Q\dagger}_{R}=\hat{\Gamma}^{\alpha}_{Q} \tag{9}$$

for all α and Q, where $\hat{\Gamma}^{\alpha}_{Q}$ is diagonal and $U^{Q}_{L(R)}$ is defined in (4).

Condition (2b) can also be formulated[7] in a basis-independent way.

Theorem 1′. Condition (2b) is equivalent to requiring the sets of matrices $\{\Gamma^{\alpha}_{Q}\Gamma^{\beta\dagger}_{Q}\}$ and $\{\Gamma^{\gamma\dagger}_{Q}\Gamma^{\delta}_{Q}\}$ to be Abelian for all α, β, γ, and δ in each charge sector.

One can find an even stronger constraint in the case of left-right-symmetric gauge models as follows.

Corollary 1. Condition (2b) for left-right-symmetric gauge models with real vacuum expectation values of the neutral Higgs scalars is equivalent to requiring

$$[\Gamma^{\alpha}_{Q}, \Gamma^{\beta}_{Q}] = 0 . \tag{10}$$

Proof. For these models the Lagrangian is invariant under left-right symmetry[13]:

$$\psi_{iL} \leftrightarrow \psi_{iR} \text{ and } \phi^{\alpha} \leftrightarrow \phi^{\alpha\dagger} . \tag{11}$$

It can then easily be shown that if λ_Q is real and Γ_Q is real (complex), then $M(Q)$ will be real symmetric (Hermitian) and so, of course, normal. Since a set of normal matrices commute if and only if they are simultaneously diagonalizable, Eq. (10) follows.

IV. NONTRIVIAL MIXING ANGLES

At this stage, without any reference to naturality groups, one can prove the following[3].

Theorem 2 (nontriviality). In a naturally flavor-conserving gauge model with an arbitrary number of Higgs multiplets, the necessary condition for nontrivial mixing angles (i.e., other than 0 or $\pi/2$) between any two charge sectors, Q and Q', is that the sets of coupling matrices $\{\Gamma^{\alpha}_{Q}\}$ and $\{\Gamma^{\beta}_{Q'}\}$ are not identical. [Suppose α, β, and c are such that $\Gamma^{\alpha}_{Q} \neq c\Gamma^{\beta}_{Q'}$ where c is a number.]

Proof. We prove the contrapositive. Suppose that $\{\Gamma^{\alpha}_{Q}\} = \{\Gamma^{\beta}_{Q'}\}$. Then from Theorem 1

$$U^{Q}_{L} M(Q) U^{Q\dagger}_{R} = \hat{M}(Q) = U^{Q'}_{L} M(Q) U^{Q'\dagger}_{R} . \tag{12}$$

This can be rewritten as

$$U_{L(R)} \mathfrak{M}_{Q} U^{\dagger}_{L(R)} = \mathfrak{M}_{Q} , \tag{13}$$

where $\mathfrak{M}_{Q} = \hat{M}(Q)\hat{M}^{\dagger}(Q) = \hat{M}^{\dagger}(Q)\hat{M}(Q)$, both of which are diagonal. From Corollary A1 in the Appendix we see that $U_{L(R)}$ must be diagonal unitary and so the mixing angles must be trivial.

Although the proof of this theorem is straightforward, the theorem itself is quite powerful in limiting the Higgs structure in gauge models. Specifically, in NFC $SU(2)_L \times SU(2)_R \times U(1)$ models, if one chooses the "standard" vacuum-expectation-value (VEV) structure $\left(\begin{smallmatrix} a & 0 \\ 0 & b \end{smallmatrix}\right)$ for the Higgs scalars, then there will only be trivial mixing angles in the theory, as explicitly demonstrated by Gatto, Morchio, and Strocchi.[3] Contrary to their results, however, an $SU(2)_L \times SU(2)_R \times SU(1)$ gauge model can give nontrivial mixing angles if the Higgs VEV'S are of the form $\left(\begin{smallmatrix} a & 0 \\ 0 & 0 \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} 0 & 0 \\ 0 & b \end{smallmatrix}\right)$. Gauge models with this choice of VEV structures can easily be shown to effect the same symmetry breaking as the "standard" ones, and at the same time contain no W_L-W_R mixing.

In general, this theorem states that NFC gauge theories whose Higgs multiplets contain more than one nonzero VEV will have corresponding trivial mixing angles, both left and right, between the two charge sectors.

Since we would like gauge theories to yield at least nontrivial mixing angles, we must require that for all Q and Q' in the same multiplet there exists a Γ^{α}_{Q} and a Γ^{β}_{Q}, such that $\Gamma^{\alpha}_{Q} \neq \Gamma^{\beta}_{Q'}$. We call this condition "nontriviality."

V. NATURALITY GROUPS AND MIXING ANGLES

We now introduce naturality groups[3,7-10] K. Assume that the elements of K commute with the elements of G; that is, assume that the set of symmetries $\{K_g\}$ acts on the irreducible multiplets of G. Then under K,

$$\begin{aligned} \psi_{iL} &\to (L_g)_{ik}\psi_{kL} , \\ \psi_{jR} &\to (R_g)_{jl}\psi_{lR} , \\ \phi^{\alpha} &\to (D_g)_{\alpha\beta}\phi^{\beta} , \end{aligned} \tag{14}$$

for all $g \in K$. L_g, R_g, and D_g are unitary matrices representing the transformation of the quark and scalar fields under K.

Invariance of $\mathfrak{L}$ (and so $\mathfrak{L}_Y$) under K requires

$$(D^{*}_{g})_{\beta\alpha}\Gamma^{\alpha} = L^{\dagger}_{g}\Gamma^{\beta}R_{g} \tag{15}$$

for all g and for each Q.

Transforming (15) into the physical quark basis using Theorem 1 we obtain

$$(D^{*}_{g})_{\beta\alpha}\hat{\Gamma}^{\alpha} = \hat{L}^{\dagger}_{g}\hat{\Gamma}^{\beta}\hat{R}_{g} , \tag{16}$$

where the transformed horizontal representations are defined as

$$\hat{L}^{Q}_{g} = U^{Q}_{L} L^{Q}_{g} U^{Q\dagger}_{L} \tag{17}$$

and

$$\hat{R}^{Q}_{g} \equiv U^{Q}_{R} R^{Q}_{g} U^{Q\dagger}_{R} .$$

Note that in gauge models whose left- and right-handed multiplets are not singlets, there will be elements of K degenerate for different values of Q since K acts on the same representations as does G. For example, in the standard model $L^{Q=2/3} = L^{Q'=-1/3}$ while for $SU(2)_L \times SU(2)_R \times U(1)$, $L^{Q=2/3} = L^{Q'=-1/3}$ and $R^{Q=2/3} = R^{Q'=-1/3}$. In general, the left- (right-) handed charged currents are defined between the quarks of two charge sectors Q and Q' contained in the same left- (right-) handed multiplet. This means that[10]

$$L^{Q}(R^{Q}) = L^{Q'}(R^{Q'}) . \tag{18}$$

Then using Eqs. (8) and (17) we obtain

$$\hat{L}_g^Q = U_L \hat{L}_g^{Q'} U_L^\dagger \tag{19}$$

and

$$\hat{R}_g^Q = U_R \hat{R}_g^{Q'} U_R^\dagger$$

for all values of Q and Q' for which a charged current is defined. Equations (15)–(19) shall be the main focus of our attention in proving the remaining theorems.

Theorem 3. [7]Natural flavor conservation for a general gauge theory necessitates that the matrices $\hat{L}_g^Q$ and $\hat{R}_g^Q$ be monomial (a matrix with exactly one nonzero entry in each row and column, that entry being of modulus one) and $|\hat{L}_g^Q| = |\hat{R}_g^Q|$ for all g and Q. ($|\hat{L}_g^Q|$ is just $\hat{L}_g^Q$ with all the phases set to zero.)

Proof. From the definition of $M(Q)$, (3), and from (15), we have

$$\lambda_\beta (D_g^*)_{\beta\alpha}\, \hat{\Gamma}^\alpha \equiv X = \hat{L}_g^\dagger \hat{M} \hat{R}_g\,, \tag{20}$$

where, of course, NFC demands $\hat{M}$ and X be diagonal. Multiplying (20) on either side by its adjoint, we have the equations

$$L_g^\dagger \mathfrak{M} \hat{L}_g = XX^\dagger \equiv D \tag{21}$$

and

$$\hat{R}_g^\dagger \mathfrak{M} \hat{R}_g = X^\dagger X \equiv D.$$

Now since these equations transform diagonal matrices into diagonal matrices and $\mathfrak{M}$ is nondegenerate, it follows from Theorem A1 in the Appendix that L_g and R_g are monomial for all g and Q. Also, from (21) it follows that

$$(\hat{R}_g \hat{L}_g^\dagger)\mathfrak{M}(\hat{R}_g \hat{L}_g^\dagger)^\dagger = \mathfrak{M}. \tag{22}$$

And so from Corollary A1, $\hat{R}_g \hat{L}_g^\dagger$ must be diagonal unitary, and this can only be true if $|\hat{L}_g| = |\hat{R}_g|$—if the permutation matrices underlying the monomial representations for L and R are the same for all g and Q.

Actually, one can prove more than this. The generalizations of the results obtained in Refs. 3 and 7–10 on the further characterizations of L, R, and D are easily obtained. For our purposes, we shall need only what we have proven.

When the sufficiency condition of Glashow and Weinberg is required, the following can be proven.[9]

Theorem 4. Any gauge theory that obeys the sufficiency condition will have trivial or undetermined mixing angles.

Proof. Assume that all quarks of a given charge receive their mass through precisely one neutral Higgs meson. Then Eqs. (21) take the form

$$\hat{L}_g^\dagger \mathfrak{M} \hat{L}_g = \mathfrak{M} \tag{23}$$

and

$$\hat{R}_g^\dagger \mathfrak{M} \hat{R}_g = \mathfrak{M}.$$

Again, since $\mathfrak{M}$ is nondegenerate, from Corollary A1 we see that $\hat{L}_g$ and $\hat{R}_g$ are diagonal unitary. Then from (19) $U_{L(R)}$ will just transform diagonals into diagonals. If $\hat{L}_g^Q$ ($\hat{R}_g^Q$) is nondegenerate, then from Theorem A1 one obtains the result that $U_{L(R)}$ is monomial, hence the mixing angles will be trivial.

If $\hat{L}_g^Q$ ($\hat{R}_g^Q$) is degenerate, then $U_{L(R)}$ will be undetermined in the subspace of the degeneracies.

We would now like to solve the problem for an arbitrary gauge model with an arbitrary number of Higgs scalars. We consider the case of irreducible representation first and prove the following.[8,10]

Theorem 5. Natural flavor conservation for an arbitrary gauge model with n generations of quark multiplets that transform irreducibly under a naturality group will induce generalized mixing matrices $U_{L(R)}$ that are either (1) monomial, corresponding to trivial mixing angles $(0, \pi/2)$, or (2) matrices with *every* element nonzero and having absolute value $1/\sqrt{n}$, corresponding to mixing angles $\geq 45°$.

Proof. As we have shown, an arbitrary gauge model with n generations of quark multiplets transforming under a naturality group will obey Eqs. (19). Then it follows from Theorem A2 in the Appendix that since the representations are irreducible, all nontrivial elements of $U_{L(R)}$ have the same absolute value.

Suppressing the subscripts on the generalized mixing matrices, define

$$(U_0)_{ij} \equiv |U_{ij}|\,. \tag{24}$$

We consider the two cases.

(1) $\det U_0 \neq 0$. In this case it is easy to see that U_0 must be a permutation matrix and so U will be monomial.

(2) $\det U_0 = 0$. For this case U_0 must be of the form

$$(U_0)_{ij} = \frac{1}{\sqrt{n}} \quad \text{for all } i, j\,, \tag{25}$$

which implies that every element of U is nonzero with absolute value $1/\sqrt{n}$.

From (19) one can show, using Schur's lemma, that these solutions are unique up to a multiple of the identity. For case (1) it has been shown[14] that radiative corrections do not alter the result. For case (2), the lower limit on the mixing angles is 45°.

For the case of *reducible representations*, we make use of Theorem 1 and the nonsingularity of M with arguments similar to Segré and Weldon.[10] The result is that U can be written as a direct sum of blocks U^α,

$$U=\sum_\alpha \oplus U^\alpha\,, \qquad (26)$$

such that every U^α is either (1) as in the irreducible case, i.e., monomial or satisfying $(U_0^\alpha)_{ij}=1$ for all i, j, or (2) undetermined.

We conclude from Theorem 5 and the above that, *for an arbitrary gauge model, it is not possible to guarantee NFC and obtain meaningful mixing angles from a naturality group.* This is a consequence of neither the specific gauge model nor the number and type of quark and Higgs fields. NFC through naturality groups and calculability are, therefore, inherently clashing notions.

We would not like to abandon either of these conditions as the alternatives present problems in themselves.[3,7-10] It is becoming increasingly clear that the answer lies beyond gauge theories—in the grand unified schemes of the strong, weak, and electromagnetic interactions.[15] Within that framework, the choices of Higgs-boson and quark representations may be enough in themselves to guarantee that both natural flavor conservation and calculability survive.

ACKNOWLEDGMENTS

One of us (A. C. R.) would like to thank Professor S. Glashow and Professor G. Segré for their helpful discussions. The other (K. K.) wishes to thank Professor R. Vinh Mau and other colleagues of Institut de Physique Nucléaire, Université Paris—Sud, for the kind hospitality extended to him. This work was supported in part by the U. S. Department of Energy under Contract No. De-AC02-76ER03130.A005 Task A and Contract No. EY-76-C-02-1545*000.

APPENDIX: MATHEMATICAL PRELIMINARIES

In order to expedite the proofs of the theorems in the text, we would like to state the following simple mathematical theorems.

Theorem A1. If two nondegenerate diagonal matrices A and B are related by a unitary transformation

$$UAU^\dagger=B\,, \qquad (A1)$$

then U will be a monomial matrix (a matrix with exactly one nonzero entry in each row and column, that entry being of modulus one).

A corollary to the above theorem when $A=B$ is also useful.

Corollary A1. If a unitary matrix U commutes with a nondegenerate diagonal matrix A, i.e., $[U,A]=0$, then U is diagonal.

For completeness, we include here the following lemma due to the authors of Ref. 8.

Theorem A2. If X_g and Y_g are irreducible monomial representations of a group K and $X_g = UY_GU^\dagger$, then all the nontrivial elements of U have the same absolute value.

[1]K. Kang and J. E. Kim, Phys. Lett. 64B, 93 (1976).
[2]S. L. Glashow and S. Weinberg, Phys. Rev. D 15, 1958 (1977).
[3]R. Gatto, G. Morchio, and F. Strocchi, Phys. Lett. 80B, 265 (1979); R. Gatto, in *To Fulfill A Vision*, proceedings of the Einstein Centennial Symposium, Jerusalem, 1979, edited by Yuval Ne'eman (Addison-Wesley, Reading, Mass., 1980).
[4]S. Weinberg, in *Festschrift for I. I. Rabi*, edited by L. Motz (N. Y. Academy of Sciences, New York, 1977); A. De Rújula, H. Georgi, and S. L. Glashow, Ann. Phys. (N. Y.) 109, 258 (1977); F. Wilczek and A. Zee, Phys. Lett. 70B, 418 (1977); H. Fritzsch, *ibid.* 70B, 436 (1977); K. Kang and A. C. Rothman, Brown University Report No. HET-371, 1978 (unpublished); Ling-Fong Li, Phys. Lett. 84B, 461 (1979); A. Ebrahim, *ibid.* 72B, 457 (1978); 73B, 181 (1978); 76B, 605 (1978); T. Kitazoe and K. Tanaka, Phys. Rev. D 18, 3476 (1978); M.-A. deCrombrugghe, Phys. Lett. 80B, 365 (1979); A. C. Rothman and K. Kang, Phys. Rev. Lett. 43, 1548 (1979) in which further references can be found.
[5]R. Barbieri, R. Gatto, and F. Strocchi, Phys. Lett. 74B, 344 (1978); D. Wyler, Phys. Rev. D 19, 330 (1979).
[6]G. Sartori, Nuovo Cimento 55A, 377 (1980).
[7]G. Sartori, Phys. Lett. 82B, 255 (1979).
[8]R. Gatto, G. Morchio, G. Sartori, and F. Strocchi, Nucl. Phys. 163B, 221 (1980); R. Gatto, Université de Genevè Report No. UGVA-DPT 1979/08/209, 1979 (unpublished).
[9]R. Gatto, G. Morchio, and F. Strocchi, Phys. Lett. 83B, 348 (1979).
[10]G. Segre and H. A. Weldon, Ann. Phys. (N. Y.) 124, 37 (1980).
[11]M. S. Chanowtiz, J. Ellis, and M. K. Gaillard, Nucl. Phys. B128, 506 (1977).
[12]A. C. Rothman and K. Kang, Phys. Rev. D 23, 2657 (1981).
[13]It is possible to define an "unorthodox" left-right symmetry by

$$\psi_{iL}\leftrightarrow\psi_{iR} \text{ and } \phi^\alpha\leftrightarrow\tilde{\phi}^{\alpha\dagger}\,,$$

where $\tilde{\phi}^\alpha$ is the charge conjugate of ϕ^α. This is first discussed by R. H. Mohapatra, in *New Frontiers in High-Energy Physics*, proceedings of Orbis Scientiae 1978, Coral Gables, edited by Arnold Perlmutter and Linda F. Scott (Plenum, New York, 1978).
[14]G. Segre and H. A. Weldon, Phys. Lett. 86B, 291 (1979).
[15]See, for example, H. Georgi and C. Jarlskog, Phys. Lett. 86B, 297 (1979); H. Georgi and D. V. Nanopoulos, *ibid.* 82B, 392 (1979); Nucl. Phys. B155, 52 (1979).