# Do form factors at large $Q^2$ and structure functions at large $x$ necessarily involve far-off-mass-shell quarks?

Xavier Artru
*Laboratoire de Physique Théorique et Hautes Energies,* Bâtiment 211, Université Paris-Sud 91405, Orsay, France*
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We study the question of small or large virtuality of the partons involved in elastic form factors at large $q^2$ and in deep-inelastic lepton scattering at $x \to 1$. We consider two classes of models, both allowed by kinematics: the soft ones, small-virtuality models (SVM), where all partons are near the mass shell, and the hard ones, high-virtuality models (HVM), where at least one active parton is far off the mass shell. Taking the Bethe-Salpeter wave function as input, we compare the kinematical and analytical properties of SVM and HVM in the above processes, using successively covariant formalism and old-fashioned perturbation theory at infinite momentum. Both models lead to the Drell-Yan-West relation, but Bloom-Gilman duality looks more natural in SVM. We apply our analysis to the recent quantum-chromodynamic approaches of asymptotic form factors.

## I. INTRODUCTION

Hadron form factors at high $q^2$ have been thought for a long time to contain basic information on hadronic structure (compositeness) and on strong interactions at short distances. This is certainly true in nonrelativistic theory: there, the two-body bound-state form factor is

$$F(q^2) = \int e^{i\vec{q}\cdot\vec{r}} |\psi(\vec{r})|^2 d^3\vec{r} = \int \tilde{\psi}^*(\vec{k}+\vec{q})\tilde{\psi}(\vec{k}) d^3\vec{k} . \quad (1.1)$$

Its high-$q^2$ behavior is sensitive to the short-distance behavior of the wave function $\psi$, or, equivalently, to the large momentum components of $\tilde{\psi}$. This in turn is governed by the singularity of the potential at $\vec{r}=0$ (Ref. 1); the more singular the potential, the slower is the decrease of the form factor.

In relativistic theory, the connection is not clear:

(i) one cannot sum all possible Feynman diagrams, so one considers only a subset of them. Until recently, most of the studies were based on the covariant impulse approximation which is represented by diagram (a) of Fig. 1. Its expression is the relativistic generalization of (1.1):

$$F(q^2) = i \int d^4k\, \psi^*(k+q, p-k)[(p-k)^2 - \mu^2]\psi(k, p-k) , \quad (1.2)$$

where $\psi$ is the (unfortunately unknown) Bethe-Salpeter equation.

Diagrams such as 1(b) are not taken into account. Diagrams such as 1(c) can be included in the impulse approximation with a quark form factor.

(ii) The relativistic generalization of "short distance," or large $k$, is "large virtualities," i.e., at least one of the parton virtual mass squared

$$a^2 = k^2, \quad b^2 = (k+q)^2, \quad s^2 = (p-k)^2$$

must be large compared to the typical hadronic mass scale $\Lambda \sim 1$ GeV. But it is possible to have simultaneously $q^2$ large and $a^2$, $b^2$, and $s^2$ finite. For instance, in the timelike region $q^2 > 0$, $\mathrm{Im}F(q^2)$ receives a contribution from two-quark intermediate states. $a^2 = b^2 = \mu^2 \cdot s^2$ is the invariant squared momentum transfer in the "final-state interaction" quark + antiquark → hadron + hadron, and is small in the forward direction. This kinematical situation also occurs in the triangular graph

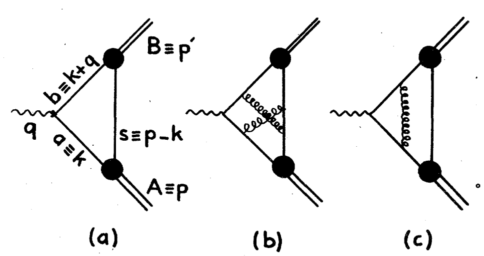


FIG. 1. Diagrams for the meson form factor: (a) impulse approximation, (b) gluon overlap, and (c) quark form factor.

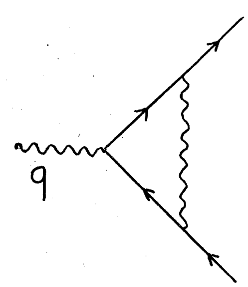


FIG. 2. Vertex correction in QED.

of QED (Fig. 2), and is responsible for the infrared divergence of the electron form factor[2]

$$\mathrm{Im}F_e(t)\simeq\frac{\alpha}{2}\left(\ln\frac{t}{m_\gamma^2}-\frac{3}{2}\right),\quad t\equiv q^2\gg m^2. \tag{1.3}$$

Thus, the asymptotic behavior of form factors may depend on the soft ("long-distance") part of the wave function as well as on the hard ("short-distance") part. In the case of the electron, considered as an electron-photon bound state, both regions, $s^2\sim m_\gamma^2$ and $s^2\sim t$, contribute to (1.3).

Dimensional counting rules (DCR) have been derived assuming that at least $a^2$ or $b^2$ is large.[3] For $n_s$ spectator partons, one has, modulo logarithmic factors,

$$F(t)\sim t^{-n_s},\quad t\to\infty \tag{1.4}$$

in agreement with available experiments. On the other hand, in the naive parton description of Feynman[4] all partons are nearly on the mass shell, the active quarks $a$ and $b$ taking nearly all the momenta of their parent hadrons and the spectator being soft. This is also the case in the massive quark model (MQM) of Preparata and co-workers[5] and in a similar approach by Einhorn and Fox.[6] (Actually, "massive" is not a convenient word since we know that quarks are light—at least inside the hadron—but it was chosen for historical reasons.) In these models, the asymptotic form factor is

$$F(t)\sim t^{\alpha_s(0)-1}, \tag{1.5}$$

where $\alpha_s$ is the assumed Regge trajectory of the spectator system. $\alpha_s(0)$ can be adjusted to fit the experiment. Thus, we distinguish two opposite kinds of models.

(1) High-virtuality models (HVM), used in the derivation of dimensional counting rules and in earlier works on form factors.[7] The recent calculations of asymptotic form factors in quantum chromodynamics[8] (QCD) are of this type.

(2) Small-virtuality models (SVM), such as the MQM and the Einhorn-Fox approach. In this category we include an approach by Licht and Pagnamenta[9] based on the Lorentz contraction of the bound-state wave function in the Breit frame, a two-dimensional QCD model of Einhorn[10] and a dispersion calculation, using two-quark unitarity, by Ghoroku and Kawabe.[11]

One may also have a combination of the two models, i.e., both short and long distances contributing to the asymptotic form factor, as in (1.3). Appelquist and Poggio have found that this is the case in $(\phi^3)_6$ theory.[12] For simplicity of the discussion we shall consider only pure SVM or pure HVM.

The same kinematical question can be stated about the exclusive limit of deep-inelastic lepton scattering, i.e., when one looks at the threshold behaviors of structure functions. Denoting by $x$ and by $\vec{k}_T$ the usual infinite-momentum parton variables, we have, inside a hadron of momentum $p$,

$$\frac{k^2+\vec{k}_T^{\,2}}{x}+\frac{(p-k)^2+\vec{k}_T^{\,2}}{1-x}=m_h^2. \tag{1.6}$$

Thus, when $x\to 1$, we have two possibilities: (1) in HVM, $(p-k)^2$ is finite, $k^2\to-\infty$ and (2) in SVM, $k^2$ is finite, $(p-k)^2\to-\vec{k}_T^{\,2}$, the last relation being allowed by the confinement of the spectator quark. Both models generally satisfy the Drell-Yan-West relation[13]

$$|F(t)|^2 \underset{t\to\infty}{\sim} t^{-\gamma} \Longleftrightarrow G(x) \underset{x\to 1}{\sim} (1-x)^{-1+\gamma}. \tag{1.7}$$

In view of the great popularity of dimensional counting rules, it is worthwhile to look more carefully at their theoretical basis and compare them to alternative models such as MQM. One might think that the recent QCD approaches of Efremov and Radyushkin, Brodsky and Lepage, Farrar and Jackson, Duncan and Mueller, and Pagels and Stokar have rendered this debate obsolete. In the case of the pion form factor, they justify the dimensional-counting-rule approach, apart from logarithmic corrections. This would be in favor of HVM. However, Duncan and Mueller[8] have raised some doubts about the validity of this approach in the case of the proton form factor, or the scalar form factor of the pion. For these reasons, we think the problem is still open.

In this paper we shall review in parallel the applications of SVM and HVM to asymptotic form factors and threshold behaviors of structure and fragmentation functions, and emphasize the relevant theoretical and kinematical differences. We shall work in a nongauge theory with spinless hadrons and constituents, as if gauge and spin were "unessential complications" in the problem of high or low virtuality. The input assumptions which will characterize the two models will be about the Bethe-Salpeter wave function $\psi(a^2,s^2)$. We shall use successively the covariant formulation and old-fashioned perturbation theory in the infinite-momentum frame ($\mathrm{OFPT}_\infty$), and check the consistency of the results.

The rest of the paper is organized as follows: In Sec. II we consider the: asymptotic form factor in the covariant impulse approximation. In Sec. III we examine the structure function in the wave-function approach. In Sec. IV we discuss the structure and fragmentation functions in the leptoproduction approach. In Sec. V we consider form factors in $\mathrm{OFPT}_\infty$. In Sec. VI we present a summary and conclusion.

## II. ASYMPTOTIC FORM FACTOR IN THE COVARIANT IMPULSE APPROXIMATION

We assume that the asymptotic form factor is given by the "impulse" triangular diagram [Fig. 1(a)] We describe the internal structure of the hadron by a covariant hadron-parton vertex $V_A(a^2, s^2)$ or equivalently by a Bethe-Salpeter wave function

$$\psi_A(a^2,s^2)=\frac{i}{a^2-m_a{}^2}\,\frac{i}{s^2-m_s{}^2}\,V_A(a^2,s^2)\,. \tag{2.1}$$

The scalar form factor is given by (throughout this paper we omit factors of $2\pi$)

$$F(q^2)=i\int d^4s\;\psi_A(a^2,s^2)(s^2-m_s{}^2)\psi_B(b^2,s^2)\,, \tag{2.2}$$

with

$$\begin{aligned} q&=B-A=b-a\,,\\ s&=B-b=A-a\,. \end{aligned} \tag{2.3}$$

The generalization to the $n$-parton case is

$$F(q^2)=-i\int d^4s\,\frac{1}{a^2-m_b{}^2}\,\frac{1}{b^2-m_b{}^2}\times T_{-a,A\to -b,B}(a^2,b^2;s^2,q^2)\,, \tag{2.4}$$

where $T$ is the off-shell quark-hadron scattering amplitude. Diagrams 1(b) and 1(c) are taken into account. From now on, we shall consider the spacelike form factor ($q^2<0$). The integration over $s$ will be analyzed in the Breit frame (Fig. 3)

$$\begin{aligned} &q_0=0\,,\\ &q_x=-Q\equiv-(-q^2)^{1/2}\,,\\ &q_y=q_z=A_y=A_z=B_y=B_z=0\,. \end{aligned} \tag{2.5}$$

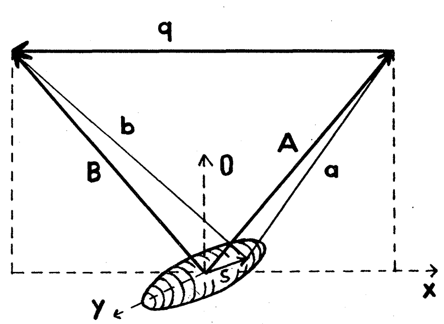


FIG. 3. Momentum diagram for the spacelike form factor in the Breit frame (2.5). In the small-virtuality model, $s$ must lie in the cigarlike domain.

We use the lightlike notations

$$\begin{aligned} &s^{\pm}=s_0\pm s_x\,,\\ &\vec{s}_T=(s_y,s_z)\,,\\ &x_s^+=s^+/A^+,\quad x_s^-=s^-/B^-\,, \end{aligned} \tag{2.6}$$

etc. For the active quarks, we write $x_a\equiv x_a^+$, $x_b\equiv x_b^-$. The virtual masses squared are given by

$$\begin{aligned} &s^2=s^+s^- -\vec{s}_T{}^2\,,\\ &a^2=(A^+-s^+)(A^- -s^-)-\vec{s}_T{}^2\,,\\ &b^2=(B^+-s^+)(B^- -s^-)-\vec{s}_T{}^2\,, \end{aligned} \tag{2.7}$$

with, at large $Q$,

$$\begin{aligned} &A^+\simeq B^-\simeq Q\,,\\ &A^-\sim B^+\simeq \frac{m_B{}^2}{Q}\,. \end{aligned} \tag{2.8}$$

### A. Small-virtuality model

As in MQM or in the covariant parton model of Landshoff and Polkinghorne[14] we assume (softness hypothesis) that $V(a^2,s^2)$, or $\psi(a^2,s^2)$, decreases very fast when $|a^2|+|s^2|\to\infty$ with a cutoff $\Lambda^2\sim 1\ \mathrm{GeV}^2$. This gives

$$\begin{aligned} &2A\cdot s\simeq Qs^-<\Lambda^2\,,\\ &2B\cdot s\simeq Qs^+<\Lambda^2\,,\\ &-s^2\simeq \vec{s}_T{}^2<\Lambda^2\,. \end{aligned} \tag{2.9}$$

As $Q^2$ increases, the effective domain for $s$ is a narrowing cigar of length $\sim\Lambda$, diameter $\sim\Lambda^2/Q$, and volume $\Lambda^6/Q^2$. Also, the integrand in (2.2) is invariant under the transformation

$$\begin{aligned} &(Q;A^+,B^-)\to\lambda(Q,A^+,B^-)\,,\\ &(s^{\pm},A^-,B^+)\to\lambda^{-1}(s^{\pm},A^-,B^+)\,,\\ &\vec{s}_T\to\vec{s}_T\,. \end{aligned} \tag{2.10}$$

Thus, one gets at large $t$

$$\begin{aligned} &F(\lambda t)\simeq\lambda^{-1}F(t)\,,\\ &F(t)\sim t^{-1}\,. \end{aligned} \tag{2.11}$$

This power law is *independent of the degree of softness of the theory*. The generalization to $n_s$ spectators gives

$$F(t)\sim t^{-n_s}\,. \tag{2.12}$$

We note in passing that

$$\begin{aligned} &1-x_a\equiv x_s^+<\Lambda^2/Q^2\,,\\ &1-x_b\equiv x_s^-<\Lambda^2/Q^2\,. \end{aligned} \tag{2.13}$$

We would like to calculate the normalization factor. Neglecting terms in $s^+s^-$ in (2.7), we can replace $d^4s$ by

$$d^4s=\tfrac{1}{2}d^2\vec{s}_T\,ds^+ds^-\simeq\tfrac{1}{2}d^2\vec{s}_T\frac{da^2db^2}{Q^2}\,. \qquad (2.14)$$

One thus gets, from (2.2),

$$F(t)=\frac{-i}{2|t|}d^2\vec{s}_T(\vec{s}_T^{\,2}+m_s^2)f_A(-\vec{s}_T^{\,2})f_B(-\vec{s}_T^{\,2})\,, \qquad (2.15)$$

with

$$f_A(s^2)\equiv\int_{-\infty}^{+\infty}\psi_A(a^2,s^2)da^2 \ \text{(similarly for } B). \qquad (2.16)$$

*Discussion*

Expression (2.15), which is also valid in the timelike region, has bad analytic properties; in particular, it gives a purely imaginary, instead of real, form factor in the spacelike region. In conventional field theory one can get rid of the factor $i$ by making a Wick rotation in the $s^0$ (or $s^+$, or $s^-$) complex plane. Instead, in a confining model like MQM (Refs. 5 and 6) one assumes that $\psi_A(a^2,s^2)$ is an entire function of $a^2$ and $s^2$ with an essential singularity at infinity, for instance, similar to $\exp[-(a^2)^2]$. Thus a Wick rotation is not possible. However, MQM takes field theory only as a guide; it starts from (2.4) but removes the poles of the quark propagators and takes a Reggeized form for $T$. In such an unconventional model, there is no reason to keep the factor $i$ in (1.2) anyway.

### B. High-virtuality model

Now we turn to a more conventional model where $\psi_A(a^2,s^2)$ is a decreasing function of $a^2$ and $s^2$ in all directions of the complex planes, with the usual poles and cuts of a nonconfining theory. The idea behind this is that, at short distances or large momenta, the quarks "do not know" that they are confined. Now, one cannot use the approximation (2.14). Had we made it, (2.15) would give us zero owing to the superconvergence sum rule

$$\int_{-\infty}^{+\infty}\psi(a^2,s^2)da^2=0\,, \qquad (2.17a)$$

which comes from analyticity in $a^2$ in the upper half plane and from a weak softness hypothesis:

$$|a^2|\to\infty \Longleftrightarrow |a^2\psi(a^2,s^2)|\to 0 \qquad (2.17b)$$

uniformly in all directions.

Let us first integrate (2.2) on $s^-$ at fixed $s^+$ and $\vec{s}_T$. More precisely we shall take the boost-invariant quantities $A^+s^-$ and $x_s^+$. In the $A^+s^-$ plane, the integrand has poles located at [see Eqs. (2.7)]

$$\sigma=\frac{\vec{s}_T^{\,2}+m_s^2-i\epsilon}{x_s^+}\,,$$
$$\alpha=m_A^2-\frac{\vec{s}_T^{\,2}+m_a^2-i\epsilon}{1-x_s^+}\,, \qquad (2.18)$$
$$\beta=Q^2-\frac{\vec{s}_T^{\,2}+m_b^2-i\epsilon}{x_B^+-x_s^+}$$

and cuts coming from the cuts of the vertex functions. Depending on $x_s^+$, we have four possible cut configurations (Fig. 4). In cases 1 and 4, one can close the contour in the $A^+s^-$ plane and the result is zero. In the other two cases, one can rotate the contour far away from the soft region [given by (2.9)] such that at least $|a^2|$ or $|b^2|$ is large. Thus, *the softer the theory, the faster is the decrease of* $F(t)$, in contrast to the SVM result (2.11). As an example, let us take

$$m_A=m_B=0,\quad m_a=m_b=m_s=\mu\sim 1\text{ GeV}\,,$$
$$\psi_A(u^2,v^2)=\psi_B(u^2,v^2) \qquad (2.19)$$
$$=(\mu^2-u^2)^{-1-\lambda}(\mu^2-v^2)^{-1-\lambda}\lambda>0\,.$$

Introducing $\tau\equiv\mu^2+\vec{s}_T^{\,2}$, $z\equiv\mu^2-s^2=\tau-s^+s^-$ and performing the Wick rotation, we rewrite (2.2) as

$$F(t)=\frac{i\pi}{2}\int_{\mu^2}^{\infty}d\tau\int_0^1\frac{dx_s^+}{x_s^+}\int_{R-i\infty}^{R+i\infty}dz\,z^{-1-2\lambda}\left(z+\frac{\tau-z}{x_s^+}\right)^{-1-\lambda}(z+Q^2x_s^+)^{-1-\lambda}\,. \qquad (2.20)$$

Choosing, for instance, $R=\tau/2$ in order to get away from the soft region, we can see that

$$F(-Q^2)\sim(Q^2)^{-1-\lambda}\ln Q^2 \qquad (2.21)$$

the main contribution coming from the region

$$z\sim\tau\sim\mu^2,\quad \frac{\mu^2}{Q^2}\lesssim x_s^+<1\,,$$

i.e.,

$$\vec{s}_T^{\,2},\ s^2,\ s^+s^-\sim\mu^2\,, \qquad (2.22a)$$

$$(1-x_a)(1-x_b)=x_s^+x_s^-\lesssim\mu^2/Q^2\,, \qquad (2.22b)$$

$$|a^2|,\ |b^2|\lesssim Q^2\,, \qquad (2.22c)$$

$$|a^2b^2|\lesssim\mu^2Q^2\,. \qquad (2.22d)$$

[This is compared to $a^2\sim b^2\sim\Lambda^2$ and (2.9) and (2.13) in SVM.] The "rapidity" of the spectator

$$Y_s=\tfrac{1}{2}\ln\frac{x_s^+}{x_s^-}\simeq\tfrac{1}{2}\ln\frac{a^2}{b^2} \qquad (2.23)$$

is uniformly distributed between $-\ln(Q/\mu)$ and

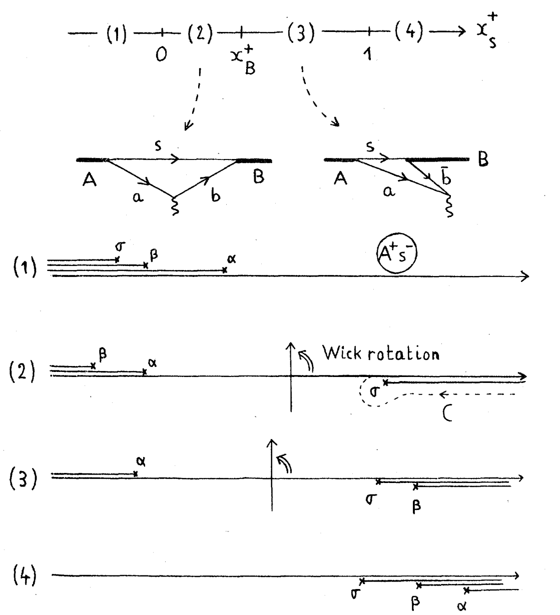


FIG. 4. Configurations of the cuts in the $A^+s^-$ plane, according to the $x_s^+$ intervals (shown in the upper figure). Poles and branch points are supposed to coincide. Cases 2 and 3 correspond to nonvanishing diagrams of old-fashioned perturbation theory in the infinite-momentum frame.

$+\ln(Q/\mu)$ (whence the logarithmic factor). The dimensional counting rule is obtained in the limit $\lambda \to 0$.

### C. Comparison of SVM and HVM approaches

In the derivation of the SVM result (2.11), it is implicitly assumed that there is no cancellation due to change of sign of the integrand, within the soft region (2.9). In the HVM, such a cancellation does occur due to the superconvergence sum rule (2.17) (in Fig. 4, the soft region is the neighborhood of $\alpha$). Thus the analyticity of $\psi$ spoils the naive expectation based on kinematics alone. On the other hand, the SVM assumes an essential singularity at infinity so that (2.17) does not hold. For the moment, both approaches seem to be internally consistent.

## III. STRUCTURE FUNCTION IN THE WAVE-FUNCTION APPROACH

In old-fashioned perturbation theory at infinite momentum the structure function of a two-parton bound state is proportional to the square of the wave function $\phi(x,k_T)$. Let us consider hadron $A$ of the preceding section. We shall use alternatively our previous notations and the more standard ones

$$x \equiv x_a^+ = 1 - x_s^+ ,\qquad P\equiv A,\quad k\equiv a,\quad p-k=s . \tag{3.1}$$

We normalize the one-parton states as

$$\langle x, \vec{k}_T | x', \vec{k}_T' \rangle = 2x\delta(x-x')\delta^{(2)}(\vec{k}_T - \vec{k}_T') \tag{3.2}$$

and define the wave function by

$$\langle A | x_a, \vec{a}_T; x_s, \vec{s}_T\rangle = 2\delta(1-x_a-x_s)\delta^{(2)}(\vec{a}_T+\vec{s}_T)\phi(x_a,\vec{a}_T) . \tag{3.3}$$

Then, the structure function at fixed $k_T$ is

$$G(x,k_T) = \frac{|\phi(x,\vec{k}_T)|^2}{2x(1-x)} . \tag{3.4}$$

(Here we consider only the spinless case.) We want to relate $\phi$ to the Bethe-Salpeter wave function $\psi_A(a^2,s^2)$. Let us compare OFPT$_\infty$ and covariant expressions for the process shown in Fig. 5 where $T_{a+s\to f}$ is a hard subprocess (for instance quark + antiquark $\to W$ boson), in which one can neglect the $a^2$ and $s^2$ dependence:

$$T_{a\to f} = \begin{cases} \displaystyle\int_0^1 \frac{dx}{2x(1-x)} d^2\vec{k}_T \phi(x,\vec{k}_T) T_{a+s\to f} , & (3.5\text{a}) \\ \displaystyle i\int d^4k\, \psi(k^2,(p-k)^2) T_{a+s\to f} . & (3.5\text{b}) \end{cases}$$

Identifying the two results, one gets

$$\phi(x,\vec{k}_T) = ix(1-x)p^+ \int_{-\infty}^{\infty} dk^- \psi(k^2,(p-k)^2) \tag{3.6}$$

with

$$a^2 = k^2 = xp^+k^- - \vec{k}_T{}^2 ,\qquad s^2 = (p-k)^2 = (1-x)(m_A{}^2 - p^+k^-) - \vec{k}_T{}^2 . \tag{3.7}$$

Expression (3.6) can also be obtained from the Bethe-Salpeter wave function in position space at equal $X^+$. (This is done, for instance, by Ida and Yabuki[15] for spin-$\frac{1}{2}$ partons.) $a^2$ and $s^2$ are related by the useful identity

$$\frac{a^2+\vec{k}_T{}^2}{x} + \frac{s^2+\vec{k}_T{}^2}{1-x} = m_A{}^2 , \tag{3.8}$$

which comes from $a^- + s^- = A^-$.

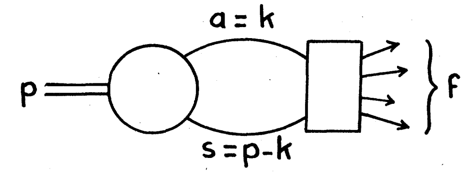


FIG. 5. Hard quark-antiquark process within a meson.

### A. Small-virtuality model

In SVM, expression (3.6) does not *a priori* vanish for $x$ outside $[0,1]$ so that it may not be possible to identify (3.5a) and (3.5b). Nevertheless, we shall keep this $\phi$, considering it as the equal-$X^+$ Bethe-Salpeter wave function. From softness hypothesis and (3.7), we get

$$k_T \lesssim \Lambda \tag{3.9}$$

for any $x\in[0,1]$. When $x\to 0$ or 1, (3.8) fixes the virtuality of the soft parton:

$$x\to 1,\quad s^2=(p-k)^2\to -\vec{k}_T^{\,2}. \tag{3.10}$$

Thus, we get for $x\to 1$

$$\begin{aligned}\phi(x,k_T)&\simeq i(1-x)f(-\vec{k}_T^{\,2}),\\ G(x,k_T)&\simeq(1-x)f^2(-\vec{k}_T^{\,2})\,/2,\end{aligned} \tag{3.11}$$

$f$ being defined by (2.16), provided again that the superconvergence sum rule (2.17) does not hold. The generalization to the case of $n_s$ spectators is

$$G(x,k_T)\sim(1-x)^{-1+2n_s}. \tag{3.12}$$

This result satisfies the Drell-Yan-West relation with (2.12).

### B. High-virtuality model

We return to expression (3.6) of $\phi(x,k_T)$. It must vanish for $x$ outside $[0,1]$ otherwise we cannot identify (3.5a) and (3.5b). The singularities of the integrand in the $p^+k^-$ plane are shown in Fig. 6.

In analogy with (2.18), we have two poles at

$$\begin{aligned}\alpha&=\frac{m_a^{\,2}+\vec{k}_T^{\,2}-i\epsilon}{x},\\ \sigma&=m_A^{\,2}-\frac{m_s^{\,2}+\vec{k}_T^{\,2}-i\epsilon}{1-x}\end{aligned} \tag{3.13}$$

and corresponding cuts. We see that for $x\notin[0,1]$

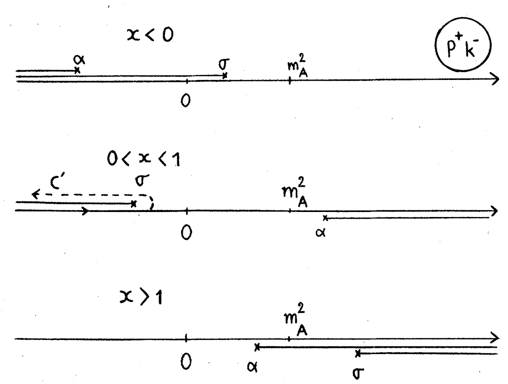


FIG. 6. Singularities of $\psi(k^2,(p-k)^2)$ at fixed $x$ and $k_T$ in the $p^+k^-$ plane. Poles and branch points are supposed to coincide.

we can close the contour and $\phi$ vanishes, as expected (see also Ref. 15). For $x\in[0,1]$ and $m_A > m_a+m_s$ ($A$ stable) the right- and left-hand singularities are separated by a gap

$$\alpha-\sigma=S-m_A^{\,2} \equiv\frac{m_a^{\,2}+\vec{k}_T^{\,2}}{x}+\frac{m_s^{\,2}+\vec{k}_T^{\,2}}{1-x}-m_A^{\,2}>0 \tag{3.14}$$

[$S-m_A^{\,2}$ is the energy denominator in $\mathrm{OFPT}_\infty$. We have the useful identity

$$\frac{a^2-m_a^{\,2}}{x}+\frac{s^2-m_s^{\,2}}{1-x}+S-m_A^{\,2}=0 \tag{3.15}$$

by combining (3.8) and (3.14).] Thus, one can make a Wick rotation around a point of the gap and get a real $\phi$. Alternatively, one can take a contour which encloses only the right-hand or the left-hand singularities. If one chooses, for instance $C'$, (3.6) becomes

$$\phi(x,k_T)=ix\int_{m_s^2}^{\infty}ds^2\,\mathrm{disc}_{s^2}\,\psi(a^2,s^2), \tag{3.16}$$

with $a^2$ and $s^2$ related by (3.8). In the case of a constant vertex $V(a^2,s^2)\equiv g$, one gets, omitting factors of $2\pi$,

$$\begin{aligned}\phi&=\frac{g}{S-m_A^{\,2}},\\ G(x,k_T)&=\frac{g^2}{2x(1-x)(S-m_A^{\,2})^2}.\end{aligned} \tag{3.17}$$

Let us now come to the limit $x\to 1$.

At finite $k_T$, there is a "soft region" in the $p^+k^-$ plane, given by $|p^+k^-|<\Lambda^2$, where both $k^2$ and $(p-k)^2$ are finite. This remains true in the limit $x\to 0$ or 1, although $\alpha$ or $\sigma\to\infty$. But in this case we can perform the Wick rotation far away from the soft region, for instance at

$$\mathrm{Re}\,p^+k^-=\frac{\alpha+\sigma}{2}. \tag{3.18}$$

Then,

$$\begin{aligned}|\mathrm{Re}\,a^2|&\sim\frac{m_s^{\,2}+\vec{k}_T^{\,2}}{2(1-x)},\\ |\mathrm{Re}\,s^2|&\sim\frac{m_a^{\,2}+\vec{k}_T^{\,2}}{2x}.\end{aligned} \tag{3.19}$$

Let us assume again that $\psi$ is given by (2.19), one gets (for $x\to 0$ or 1, and/or $k_T\to\infty$)

$$\phi(x,k_T)\sim x^{1+\lambda}(1-x)^{1+\lambda}(\mu^2+\vec{k}_T^{\,2})^{-1-2\lambda}, \tag{3.20}$$

$$G(x,k_T)\sim x^{1+2\lambda}(1-x)^{1+2\lambda}(\mu^2+\vec{k}_T^{\,2})^{-2-4\lambda}. \tag{3.21}$$

This result satisfies the Drell-Yan-West relation with (2.21), modulo the logarithmic factor. Again, DCR is obtained in the limit $\lambda\to 0$.

### C. Comparison

There is a deep similarity between the behavior of the structure function at $x \to 1$ and the asymptotic form factor. For both processes, we have, in SVM,

$$s^2 \simeq -\vec{s}_T^{\,2}, \quad a^2 \sim \Lambda \qquad (3.22)$$

and in HVM

$$s^2 \sim \mu^2, \quad a^2 \gg \mu^2. \qquad (3.23)$$

The Drell-Yan-West relation is a consequence of this similarity, in SVM as in HVM. The observed agreement of this relation with experiment does not favor particularly one of the two models.

However, Bloom-Gilman duality[16] implies a more precise connection between the two processes: assuming that $G(x)$ is built up of transition form factors, we have, for $m_A{}^2 \ll m_B{}^2 \ll |t|$,

$$m_B{}^2 \simeq |t|(1-x_a), \qquad (3.24a)$$

$$\int_{1-m_B{}^2/|t|}^{1} G_A(x)dx \simeq \sum_{m_{B'}<m_B} |F_{AB'}(t)|^2. \qquad (3.24b)$$

In the case $m_A \sim m_B \sim \Lambda$, we have

$$x_s^+ = 1 - x_a \sim \frac{\Lambda^2}{|t|}, \qquad (3.25)$$

which is the kinematical situation of SVM (2.13), but not of HVM (2.22): for instance, in the middle of the spectator rapidity plateau in HVM $[Y_s = 0$ in (2.23)], we have

$$x_s^+ \sim x_s^- \sim \frac{\mu}{Q}. \qquad (3.26)$$

Thus, Bloom-Gilman duality is better understood in SVM than in HVM. However, if, for some reasons, only the edges of the spectator rapidity plateau contribute to the asymptotic form factor in HVM, we have

$$\begin{Bmatrix} 1-x_a = x_s^+ \sim \dfrac{\mu^2}{Q^2} \\ x_b \not\to 1 \end{Bmatrix} \text{or} \begin{Bmatrix} 1-x_b = x_s^- \sim \dfrac{\mu^2}{Q^2} \\ x_a \not\to 1 \end{Bmatrix} \qquad (3.27)$$

and Bloom-Gilman duality is also possible in this case. We shall see in Sec. V that the kinematical situation (3.27) is assumed in the OFPT$_\infty$-QCD approach of the pion form factor by Efremov and Radyushkin, Brodsky and Lepage.[8]

## IV. STRUCTURE AND FRAGMENTATION FUNCTIONS IN THE LEPTOPRODUCTION APPROACH

Despite its simplicity, we may not be satisfied by the wave-function approach to the structure function, in particular for the following reasons:

(i) It ignores the final-state interactions between the spectator and the active quarks, responsible for the hadronization (confinement).

(ii) It seems difficult to generalize it to the crossed process quark → quark + hadron (fragmentation).

Thus, we have to consider a more complete description of the leptoproduction process. We choose the planar diagram (a) of Fig. 7, where the active quark is a valence quark, not only for simplicity but because the valence contribution dominates at $x \simeq 1$. In a nonconfining model, diagram 7(b) will be sufficient; at least it solves difficulty (ii). The unitarity diagrams corresponding to Figs. 7(a) and 7(b) are, respectively, the "cat's ears" and the "hand-bag" diagrams of Figs. 8(a) and 8(b).

For simplicity, we shall consider all particles, including the photon, as scalar. We can express the structure function directly in terms of the virtual photon-hadron inelastic amplitudes or (using unitarity) in terms of the off-shell forward Compton amplitude

$$G(x) = -q^2 \sum_X \delta^4(A+q-X)\left|\frac{1}{e}T_{q+A\to X}\right|^2 \qquad (4.1a)$$

$$= \frac{-2q^2}{e^2}\,\mathrm{Im}\,T_{q+A\to q+A}, \qquad (4.1b)$$

where $e$ is the parton charge. Now $G$ does not depend on $k_T$, since this is not an observable quantity, at least in confining models. Equation (4.1)

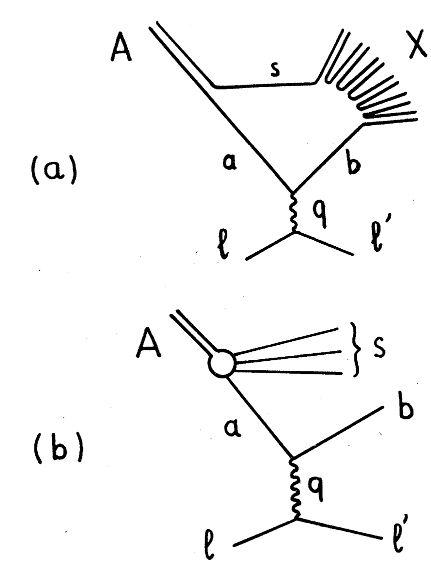


FIG. 7. Electroproduction diagram (a) confining model: planar duality diagram, (b) nonconfining model. These diagrams also describe the crossed process $e^+e^- \to A + X$.

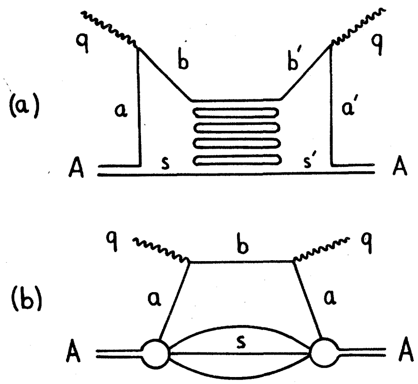


FIG. 8. Diagram representing the imaginary part of the off-shell Compton scattering amplitude, Eq. (41b): (a) confining model, (b) nonconfining model.

also applies to nonconfining models [Fig. 7(b)], with $X=\{b+s\}$. Similarly, the quark fragmentation function is

$$D(z)=zq^2\sum_X \delta^4(q-A-X)\left|\frac{1}{e}T_{q\to A+X}\right|^2. \tag{4.2}$$

The comparison of (4.1) and (4.2) gives formally the crossing relation (for spin-zero partons)

$$G(x)=-\frac{D(z)}{z}\,,\quad x=\frac{1}{z}\,. \tag{4.3}$$

### A. General kinematics

For deep-inelastic scattering, as for spacelike elastic form factors, we shall use the Breit frame (Fig. 3) replacing $B$ by $X$. We define

$$\begin{aligned} x &\equiv \frac{-q^2}{2A\cdot q}\simeq\frac{-q^+}{A^+}\,,\\ M_X{}^2 &\equiv W^2=(b+s)^2\simeq -q^2(1/x-1)+m_A{}^2\,. \end{aligned} \tag{4.4}$$

Both in HVM and SVM, $b^2$ and $s^2$ are finite. In the limit $-q^2\gg m^2+\vec{k}_T{}^2$ ($\vec{k}_T$ is the common value of $\vec{a}_T$, $\vec{b}_T$, and $-\vec{s}_T$), we have

$$x_a^+\simeq x\left(1+\frac{b^2+\vec{k}_T{}^2}{-q^2}\right)\simeq x \tag{4.5}$$

and the usual relationship

$$\frac{a^2+\vec{k}_T{}^2}{x}+\frac{s^2+\vec{k}_T{}^2}{1-x}\simeq m_A{}^2\,. \tag{4.6}$$

For $e^+e^-$ annihilation into hadrons, we use the center-of-mass frame, with $A_T=0$. Formulas (4.4)–(4.6) remain valid with

$$\begin{aligned} &x\Rightarrow\frac{1}{z}\,,\\ &A\Rightarrow -A\,,\quad a\Rightarrow -a\,. \end{aligned} \tag{4.7}$$

### B. Small-virtuality model

As we have already seen, confinement plays an important role in SVM, so we consider diagram (a) of Figs. 7 and 8. $|a^2|$, $|b^2|$, $|s^2|$, and $\vec{k}_T{}^2$ are cutoff at $\Lambda^2$, as in the case of elastic form factors. Using unitarity for quark-antiquark amplitudes, one gets

$$G(x)=\frac{1-x}{2x}\int d^2\vec{k}_T\,da^2\psi(a^2,s^2)\int d^2\vec{k}_T'\,da'^2\psi^*(a'^2,s'^2)\int\frac{db^2}{b^2-m_b{}^2}\,\frac{db'^2}{b'^2-m_b{}^2}\,\frac{\mathrm{Im}T_{b+s\to b'+s'}}{W^2}\,, \tag{4.8}$$

$s^2$ being given by (4.6). $\mathrm{Im}T_{b+s\to b'+s'}$ is a function of the virtual mass squared $b^2, s^2, b'^2, s'^2$, and of the Mandelstam invariants $W^2$ and

$$t_{bb'}\simeq-(\vec{k}_T-\vec{k}_T')^2\,. \tag{4.9}$$

Confinement implies that the poles of the propagators in $b^2$ and $b'^2$ are canceled by zeros of $\mathrm{Im}T$. $D(z)$ is obtained from $G(x)$ by use of the crossing relation (4.3).

#### *Scaling in SVM*

In order that expression (4.11) does not depend on $q^2$, we must have

$$\mathrm{Im}T_{b+s\to b'+s'}\sim W^2 f(b^2,b'^2,s^2,s'^2,t_{bb'})\,. \tag{4.10}$$

A Regge behavior such as

$$\mathrm{Im}T_{b+s\to b'+s'}=\beta(b^2,s^2,t_{bb'})W^{2\alpha(t_{bb'})}\beta(b'^2,s'^2,t_{bb'})\,, \tag{4.11}$$

with $\alpha(0)=1$ but $\alpha'(0)\neq 0$ gives an approximate scaling (i.e., up to logarithmic factors).

#### *Threshold behavior*

The limit $x\to 1$ or $z\to 1$ is obtained by putting $s^2=-\vec{k}_T{}^2$ in (4.8) [see Eq. (4.6)]. Assuming (4.10), one gets

$$\begin{aligned} G(x)&\simeq c(1-x)\,,\\ D(z)&\simeq c(1-z)\,, \end{aligned} \tag{4.12}$$

with the same coefficient $c$. This is the same power law as that obtained in the wave-function approach.

*Remarks*

The results (4.12) imply that the superconvergence sum rule (2.17) does not hold (otherwise $c=0$). Thus, confinement is relevant for two reasons: (i) (kinematics) to allow $s^2 \simeq -\vec{k}_T^2$, and (ii) (analyticity) to give $\psi$ an essential singularity at infinity. Similarly, one must have

$$\int \frac{db^2}{b^2-m_b^2}\; T_{bs\to b's'}(b^2, b'^2, -\vec{s}_T^{\,2}, -\vec{s}_T^{\,\prime 2}, W^2, t_{bb'}) \neq 0\,. \tag{4.13}$$

In (4.8) we have lost the symmetry $x \leftrightarrow 1-x$ that we had in the wave-function approach (3.6). Anyway, none of these formulas apply to small $x$ but the Regge mechanism:

$$G^{\text{val}}(x\to 0) \sim x^{-\alpha_R(0)}\,. \tag{4.14}$$

### C. High-virtuality model

In the spirit of HVM (and also of asymptotic freedom), we ignore confinement and consider now diagram (b) in Figs. 7 and 8.[14] In (4.1a) we replace $X$ by $\{b+s\}$. In the general case where $s$ is itself a multiparticle state as in Fig. 8(b) one gets

$$G(x,k_T)=\frac{x}{1-x}\int_{m_s^2}^{\infty} ds^2(a^2-m_a^2)^{-2}\,\text{Im}T_{-a,A\to -a,A}\,, \tag{4.15}$$

$a^2$ and $s^2$ being related by (4.6). In the one-spectator case, one has

$$G(x,k_T)=\frac{1}{2x(1-x)}\left(\frac{V(a^2,m_s^2)}{S-m_A^2}\right)^2 \tag{4.16}$$

with $S$ defined by (3.14). In the limit of a constant vertex one gets the same result as in the wave-function approach, (3.17), but this is not generally true. As in SVM there is no $x \leftrightarrow 1-x$ symmetry.

The fragmentation function $D(z,A_T)$ can be obtained from (4.15) or (4.16) by a generalization of the crossing relation (4.3):

$$D(z,A_T)=-\frac{1}{z}\,G\!\left(x=\frac{1}{z},\; a_T=\frac{A_T}{z}\right). \tag{4.17}$$

*Threshold behavior*

Assuming, in analogy with (2.19),

$$V(a^2,m_s^2)\sim (a^2)^{-\lambda} \tag{4.18}$$

one gets for $x\to 1$ and/or $k_T\to\infty$

$$G(x,k_T)\sim (1-x)^{1+2\lambda}(m_s^2+\vec{k}_T^2)^{-2-2\lambda}\,. \tag{4.19}$$

The $x$ dependence is the same as in the wave-function approach (3.21), but not the $k_T$ dependence; this is because, at large $k_T$, only one parton is far off the mass shell instead of two. The Drell-Yan-West relation is satisfied.

### D. Discussion

We see that, within each model, the wave-function approach and the multiproduction approach are in qualitative agreement about the threshold behaviors. In SVM, however, this agreement rests on assumption (4.10) or (4.11) which is necessary to get scaling.

SVM takes hadronization completely into account. In HVM the hand-bag diagram of Fig. 8(b) corresponds to "parton+core" decomposition of the hadronic state. At small $x$, the squared mass of the core is, from (4.6),

$$s^2 \sim \left|\frac{a^2+\vec{k}_T^2}{x}\right| \sim \frac{1}{x}\,.$$

Thus, we can identify the multiparticle state $s$ as the hadronic plateau of rapidity length $\ln(1/x)$. But, there remains a rapidity gap of length $\ln Q^2$ in the current region, which, in the real world, must be filled by the confinement mechanism.

Expression (4.15) has the advantage, over other approaches, to guarantee the probability sum rule for a valence quark

$$\int_0^1 dx \int d^2\vec{k}_T G^{\text{val}}_{a/A}(x,k_T)=1\,. \tag{4.20}$$

If, indeed, one calculates the elastic form factor (2.4) at $q=0$ by performing the $s^-$ integration along contour $c$ in Fig. 4, one obtains

$$1=F(0)$$

$$=\int d^2\vec{k}_T\int_0^1 \frac{x\,dx}{1-x}\int_{m_s^2}^{\infty} ds^2(a^2-m_a^2)^{-2}\,\text{Im}T_{-a,A\to -a,A} \tag{4.21}$$

(we put an extra $x$ factor because we consider the vector form factor). This is in fact an alternative method to derive (4.15).

Let us finally mention a recent model proposed by Azcoiti, Alonso, and Cruz[17] which is of SVM type, but retains the hand-bag diagram [Fig. 8(b)]. Confinement is ensured by forbidding pole and cuts in $a^2$ only. The result is a nonvanishing limit of the structure function at $x=1$.

## V. FORM FACTOR IN OFPT$_\infty$

Up to now, we have calculated the form factor and the structure function with covariant Feynman diagrams. But at least the spacelike form factor is often analyzed in old-fashioned perturbation theory,[18] in an infinite-momentum frame where $q$ is purely transverse. This frame may be obtained from the Breit frame introduced in Sec. II [(2.5)] by a boost in the $y$ direction. Also, we define

$$y_s \equiv \frac{s_0+s_y}{A_0+A_y} = 1-y_a = 1-y_b\,, \qquad \vec{s}_\perp = (S_x, S_z)\,. \tag{5.1}$$

The initial and final relative transverse momenta are

$$\vec{k}_\perp = \vec{s}_\perp + y_s\frac{\vec{q}}{2}\,, \qquad \vec{k}'_\perp = \vec{s}_\perp - y_s\frac{\vec{q}}{2}\,. \tag{5.2}$$

These new variables may be expressed in terms of the old lightlike variables of Sec. II [(2.6)]. Neglecting terms of relative size $m_A{}^2/Q^2$, $m_B{}^2/Q^2$ we have

$$\begin{aligned} y_s &= x_s^+ + x_s^- + \frac{2s_y}{Q}\,,\\ k_x &= -s_y - x_s^- Q\,,\\ k'_x &= s_y - x_s^+ Q\,,\\ k_z &= k'_z = s_z\,. \end{aligned} \tag{5.3}$$

In the OFPT$_\infty$ impulse approximation, the vector form factor is

$$F(-\vec{q}^{\,2}) = \int_0^1 \frac{dy_s}{2y_s(1-y_s)} \int d^2\vec{k}\, \phi(y_s, \vec{k}_\perp)\phi^*(y_s, \vec{k}_\perp - y_s\vec{q})\,. \tag{5.4}$$

The asymptotic behavior of $F$ depends on the behavior or $\phi$ at $k_\perp \to \infty$ and/or $y_s \to 0$. Suppose that

$$\phi(y_s, \vec{k}_\perp) \sim y_s^\alpha(\mu^2 + \vec{k}_\perp{}^2)^{-\beta}, \quad y_s \to 0 \text{ and/or } k_\perp \to \infty\,, \tag{5.5a}$$

whence

$$G(x, k_\perp) \sim (1-x)^{2\alpha-1}(\mu^2+\vec{k}_\perp{}^2)^{-2\beta}\,. \tag{5.5b}$$

Following Soper,[19] we distinguish three cases:

(1) If $\alpha<\beta$, i.e., the strongest damping is in $k_\perp$, then

$$k_\perp, k'_\perp \sim \mu\,, \qquad y_s \sim \frac{\mu}{Q}\,, \qquad F(-Q^2) \sim Q^{-2\alpha} \tag{5.6}$$

and the Drell-Yan-West relation holds.

(2) If $\alpha=\beta$ (limiting case), then

$$\left.\begin{aligned} k_\perp &\sim \mu\\ k'_\perp &\sim y_s Q \end{aligned}\right\} \text{ or vice versa}\,, \tag{5.7a}$$

$$F(-Q^2) \sim \int_{\mu/Q}^1 \frac{dy_s}{y_s} Q^{-2\alpha} \sim Q^{-2\alpha}\ln Q\,. \tag{5.7b}$$

The Drell-Yan-West relation is still satisfied, modulo logarithmic factors.

(3) If $\alpha>\beta$ (strongest damping in $y_s$), then

$$y_s \sim \tfrac{1}{2}\,,$$

$$\left.\begin{aligned} k_\perp &\sim \mu\\ k'_\perp &\sim Q \end{aligned}\right\} \text{ or vice versa}\,, \tag{5.8a}$$

$$F(-Q^2) \sim Q^{-2\beta} \tag{5.8b}$$

and the Drell-Yan-West relation is not satisfied.

Let us now look for the connection between this classification and the SVM/HVM one.

### A. Small-virtuality model

SVM is clearly of type (1): $\alpha=1$, $\beta\sim\infty$. Putting expression (3.11) into (5.4) leads to

$$F(-Q^2) \simeq \frac{1}{2\pi Q^2}\left[\int d^2\vec{k} f(-\vec{k}_\perp{}^2)\right]^2. \tag{5.9}$$

Comparing with the covariant result (2.15), we have the same power law but a different normalization.

### B. High-virtuality model

One might take for $\phi$ the symmetrical expression (3.6) and for $\psi(a^2, s^2)$ the asymptotic behavior (2.19). Then, we have from (3.20) $\alpha=1+\lambda$, $\beta=1+2\lambda$, so we are again in case (1). On the other hand, the multiproduction approach to $G(x,k_T)$ gives $\alpha=\beta=1+\lambda$ [see Eq. (4.19)], i.e., we are in case (2). The fully covariant approach chooses the second case. The asymptotic forms (2.21) and (5.7b) are the same, and one can see by using formulas (5.3) that the effective kinematical domains (2.22) and (5.7a) are corresponding; for $x_s^+ > x_s^-$ (or $|a^2|<|b^2|$), we have

$$\begin{aligned} y_s &\simeq x_s^+\,,\\ k_\perp &\text{ finite}\,,\\ k'_\perp &\sim y_s Q\,,\\ \ln y_s Q &= \tfrac{1}{2}\ln\frac{x_s^+}{x_s^-} = |Y_s|\,. \end{aligned} \tag{5.10}$$

For $x_s^- < x_s^+$ we have the symmetrical situation.

To understand why the covariant approach corresponds to $\alpha=\beta$, let us evaluate (2.4) in the new infinite-momentum frame defined by (5.1). The following analysis is close to the one done by Hughes[20] using a slightly different $P_\infty$ frame, where $q_+=A_T=0$.

The singularities of the integrand in the $A^+s^-$ plane are again given by Fig. 4, the third configuration being absent ($A^+=B^+$), and with different expressions for $\alpha$, $\beta$, and $\sigma$. Integrating along contour $C$, we get, for the *vector* form factor

$$F(q^2)=-\int d^2\vec{s}\int_0^1\frac{(1-y_s)dy_s}{y_s}\times\int_{m_s^2}^\infty ds^2(a^2-m_a^2)^{-1}\times(b^2-m_b^2)^{-1}\operatorname{Im}T_{-a,A\to b,B}\tag{5.11}$$

with $a^2$ and $b^2$ depending on $y_s$, $\vec{s}_\perp$, and $s^2$. If one takes only the one-parton intermediate state of $\operatorname{Im}T$,

$$\operatorname{Im}(T^{\text{pole}})=-\pi\delta(s^2-m_s^2)V(a^2,m_s^2)V(b^2,m_s^2),\tag{5.12}$$

one gets the OFPT$_\infty$ formula (5.4) with the three-dimensional wave function

$$\phi_A(x,k_T)=\frac{V(a^2,m_s^2)}{S-m_A^2}.\tag{5.13}$$

This unsymmetrical wave function is the square root of $G(x)$ calculated in the leptoproduction approach (4.16). Thus, covariant impulse diagram + spectator pole approximation correspond to case (2) of OFPT$_\infty$, i.e., $\alpha=\beta$. In the general case, when one takes into account higher $s^2$ intermediate states, (5.11) is not equivalent to (5.4), due to time ordering in OFPT$_\infty$ (Fig. 9). (This has been discussed by Fishbane and Muzinich[7].) But the pole term will likely represent a sizable part of the asymptotic form factor.

Whatever model we take, we have $\alpha\leq\beta$, i.e., the damping at $y_s\to 0$ is not stronger than at $k_T\to\infty$. This ensures the Drell-Yan-West relation. The slow spectator region is dominant both in SVM ($y_s\sim 1/Q$) and in HVM [see Eq. (5.7b)].

We point out that Lepage, Brodsky, Efremov, and Radyushkin in their QCD-OFPT$_\infty$ approach neglect the small-$y_s$ region ($y_s$ is defined in Fig. 10), in contradiction with the above result. In our formulation this implies $\alpha>\beta$ and the violation of the Drell-Yan-West relation. In fact, they get the dimensional counting rule (modulo logarithm) for the pion form factor

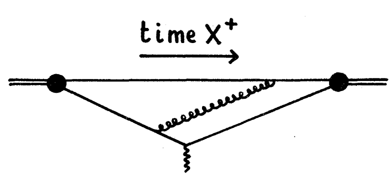


FIG. 9. Time-ordered diagram which is not included in the OFPT$_\infty$ impulse approximation, but in the covariant impulse approximation.

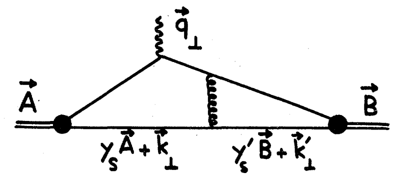


FIG. 10. Form-factor diagram used in the derivation of the dimensional counting rule in the QCD-OFPT$_\infty$ approach.

$$F_\pi(q^2)\sim\frac{1}{Q^2},\ \text{i.e.,}\quad\beta=1,\tag{5.14}$$

but take a "softer" threshold behavior for the pion structure function

$$G_\pi(x)\sim(1-x)^2,\ \text{i.e.,}\quad\alpha=\tfrac{3}{2}.\tag{5.15a}$$

This deviation from the dimensional counting rule, first proposed by Ezawa,[7] indicates that, in fact, spin does matter in HVM. The inclusive-exclusive connection (3.24b) is nevertheless fulfilled by adding a nonscaling piece[21] to $G_\pi(x)$:

$$G_\pi(x,Q^2)\underset{x\to1}{\simeq}C(1-x)^2+\frac{C'}{Q^2}.\tag{5.15b}$$

## VI. SUMMARY AND CONCLUSIONS

We have investigated the degree of quark virtuality in elastic form factors at high $q^2$, structure and fragmentation functions near threshold, in different frameworks: (1) $F(t)$ the in covariant impulse approximation, (2) $G(x,k_T)$ as the square of the OFPT$_\infty$ wave function, (3) $G(x)$ and $D(z)$ in the leptoproduction approach, and (4) $F(t)$ in the OFPT$_\infty$ approach.

We considered scalar "quarks" and assumed the existence of a Bethe-Salpeter wave function $\psi(a^2,b^2)$ for which we considered two possible softness assumptions:

(i) small-virtuality model: $\psi$ has a very strong damping in $a^2$ and $b^2$, no poles nor cuts, and, consequently, an essential singularity at infinity, or

(ii) high-virtuality model: $\psi$ decreases like a power in $a^2$ and $b^2$ and has the usual poles and cuts.

SVM emphasizes the confinement of the quarks, whereas HVM relies on asymptotic freedom. Within each model, approaches (1) and (4) for

TABLE I. Kinematical situation: $a, b, s, A, B, q$ are the four-momenta indicated in Figs. 1 and 7. $\vec{a} \simeq x_a\vec{A} + \vec{k}_T$, $\vec{b} \simeq x_b\vec{B} + \vec{k}_T$ in the Breit frame. $\vec{s} = y_s\vec{A} + \vec{k}_\perp = y_s\vec{B} + \vec{k}'_\perp$ in the infinite-momentum frame ($\vec{q} = \vec{q}_\perp$, $q_0 = 0$). $t \equiv q^2$, $Q \equiv |t|^{1/2}$.

| | SVM | HVM | |
|---|---|---|---|
| Elastic form factor | $a^2, b^2, s^2, \vec{k}_T^2$ finite | $s^2, \vec{k}_T^2$ finite | |
| $F(t)$, $t \to \infty$ | $s^2 \simeq -\vec{k}_T^2$ | $a^2 b^2 \sim t$ | |
| | $1 - x_a \sim t^{-1}$ | $(1-x_a)(1-x_b) \sim t^{-1}$ | |
| | $1 - x_b \sim t^{-1}$ | | |
| | $y_s \sim Q^{-1}$ | $Q^{-1} \lesssim y_s < 1$ ↑ | |
| | $k_\perp, k'_\perp$ finite | $\frac{dy_s}{y_s}$ behavior | |
| | | If $1 - x_a > 1 - x_b$: | If not: |
| | | $y_s \sim 1 - x_a$ | |
| | | $k_\perp$ finite | $a \longleftrightarrow b$ |
| | | $k'_\perp \sim y_s Q$ | $k \longleftrightarrow k'$ |
| Structure function | $a^2$ finite | $a^2 \sim \frac{1}{1-x}$ | |
| $G(x, k_T)$, $x \to 1$ | $s^2 \simeq -\vec{k}_T^2$ | $s^2$ finite | |

$F(t)$, and (2) and (3) for $G(x)$ give consistent results. The kinematical situation is summarized in Table I. In SVM, one can predict the power-law behavior at $t \to \infty$, $x$ or $z \to 1$ from naive kinematical arguments. $n_s$ being the number of spectators, one has (for spin-0 partons)

$$F(t) \sim t^{-n_s},$$

$$G(x) \sim (1-x)^{-1+2n_s},$$

$$D(z) \sim (1-z)^{-1+2n_s}.$$

In HVM, the naive kinematical arguments fail, due to superconvergence relations in $a^2$ and $b^2$. Assuming $\psi(a^2, b^2) \sim (\mu^2 + |a^2|)^{-1-\lambda}(\mu^2 + |b^2|)^{-1-\lambda}$ at large $|a^2|$ and/or large $|b^2|$, one gets, for $n_s = 1$,

$$F(t) \sim t^{-1-\lambda} \ln(-t),$$

$$G(x) \sim (1-x)^{1+2\lambda},$$

$$D(z) \sim (1-z)^{1+2\lambda},$$

dimensional counting rules: $\lambda = +0$.

From the theoretical point of view, SVM is more questionable than HVM. One must introduce wave functions of the momenta with essential singularities at infinity. One must also drop a factor $i$ from the conventional expression of $F(t)$ in the covariant impulse approximation. On the other hand, HVM wave functions have parton poles which actually do not exist, due to confinement, so the soft integration region, i.e., $s^2, a^2, b^2$ finite, is certainly not well taken into account. Thus, neither HVM nor SVM are theoretically adequate. Besides, if one comes to a gauge theory of strong interactions, our diagrams, e.g., impulse approximation, become gauge dependent (an exception is the two-dimensional QCD model of Einhorn, which is of SVM type).

From the phenomenological point of view, both models lead to the Drell-Yan-West relation. Bloom-Gilman duality looks more natural in SVM, due to $1 - x_a \sim 1/Q^2$, than in HVM. But in both models, the asymptotic form factor is sensitive to the small-$y_s$ region. The opposite assumption is made in the recent QCD approach.

Dimensional counting rules are obtained by accident, for spin-0 partons, in SVM and in the limit $\lambda = 0$ in HVM. At fixed $x$, the high-$k_T$ parton spectrum is cut off exponentially in SVM, whereas in HVM one has

$$G(x, k_T) \sim k_T^{-4-4\lambda}.$$

Only spin-0 partons have been treated. Will spin modify the power-law behaviors? In HVM, in principle, DCR are spin independent, although helicity conservation *increases* in some cases the exponent of $(1-x)$ or $1/Q$, for instance in the pion structure function. In SVM, there should be a systematic dependence on the spectator spin (see Eq. 1.5) which may *decrease* the exponent of $(1-x)$ or $1/Q$. The same conclusion was made in "old" HVM by Ciafaloni.[7] This question of spin dependence, which we have neglected in this paper to concentrate only on the kinematics, seems to us very important to clarify.

To conclude, we have not given an answer to the initial question, i.e., does the form factor at high $q^2$ involve at least one far-off-mass-shell parton? We have shown that both possibilities are kinematically allowed; it depends on the assumed analytic properties of the Bethe-Salpeter wave function, which must choose between confinement and asymptotic freedom. In any case, no Bethe-Salpeter wave function can describe both "soft" and "hard" regions of internal momenta for use in the impulse approximation. Concerning the recent QCD approaches, we think that they give the contribution of the hard region alone, but that one cannot exclude an equal importance or a dominance of the soft region; one must wait for a more precise knowledge of confinement. Meanwhile, we hope to have clarified the distinctions between the two alternatives.

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*Associé au Centre National de Recherche Scientifique.


[1]C. Alabiso and G. Schierholz, Phys. Rev. D 10, 960 (1974).

[2]T. Appelquist and J. R. Primack, Phys. Rev. D 1, 1144 (1970).

[3]V. A. Matveev, R. M. Muradyan, and A. N. Tavkhelidze, Lett. Nuovo Cimento 7, 719 (1973); S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. 31, 1153 (1973); Phys. Rev. D 11, 1309 (1975); D. Sivers, S. J. Brodsky, and R. Blankenbecler, Phys. Rep. 23C, 1 (1976).

[4]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972), p. 140.

[5]G. Preparata, in *Lepton and Hadron Structure*, proceedings of the International School of Subnuclear Physics, Erice, 1974, edited by A. Zichichi (Academic, New York, 1975), p. 54; R. Gatto and G. Preparata, Nucl. Phys. B67, 362 (1973); Riv. Nuovo Cimento 4, 445 (1974).

[6]M. B. Einhorn and G. C. Fox, Nucl. Phys. B89, 45 (1975).

[7]D. Amati, L. Caneschi, and R. Jengo, Nuovo Cimento 58A, 783 (1968); J. S. Ball and F. Zachariasen, Phys. Rev. 170, 1541 (1968); M. Ciafaloni and P. Menotti, *ibid.* 173, 1575 (1968); M. Ciafaloni, *ibid.* 176, 1898 (1968); M. G. Schmidt, Phys. Rev. D 9, 408 (1973); P. M. Fishbane and I. J. Muzinich, *ibid.* 8, 4015 (1973); Z. F. Ezawa, Nuovo Cimento 23A, 271 (1974).

[8]A. V. Efremov and A. V. Radyushkin, JINR Report Nos. E2-11983, 1978 and E2-12384, 1979 (unpublished); Phys. Lett. 94B, 245 (1980); G. P. Lepage and S. J. Brodsky, Phys. Lett. 87B, 359 (1979); G. D. Farrar and D. R. Jackson, Phys. Rev. Lett. 43, 246 (1979); S. J. Brodsky, Y. Frishman, G. P. Lepage, and C. Sachrajda, *ibid.* 91B, 239 (1980); A. Duncan and A. H. Mueller, Phys. Rev. D 21, 1636 (1980); H. Pagels and S. Stokar, *ibid.* 20, 2947 (1979).

[9]A. L. Licht and A. Pagnamenta, Phys. Rev. D 2, 1150 (1970); 2, 1156 (1970).

[10]M. B. Einhorn, Phys. Rev. D 14, 3451 (1976).

[11]K. Ghoroku and T. Kawabe, Prog. Theor. Phys. 54, 1753 (1975).

[12]T. Appelquist and E. Poggio, Phys. Rev. D 10, 3280 (1974).

[13]S. D. Drell and T. M. Yan, Phys. Rev. Lett. 24, 181 (1970); G. B. West, *ibid.* 24, 1206 (1970).

[14]P. V. Landshoff and J. C. Polkinghorne, Phys. Rep. 5C, 1 (1971); P. V. Landshoff, J. C. Polkinghorne, and R. D. Short, Nucl. Phys. B28, 225 (1971).

[15]M. Ida and H. Yabuki, Prog. Theor. Phys. 55, 1606 (1976).

[16]E. D. Bloom and F. J. Gilman, Phys. Rev. Lett. 25, 1140 (1970); Phys. Rev. D 4, 2901 (1971).

[17]V. Azcoiti, J. L. Alonso, and A. Cruz, Nucl. Phys. B167, 525 (1980); V. Azcoiti and J. L. Alonso, Phys. Rev. Lett. 45, 1537 (1980).

[18]R. Blankenbecler, S. J. Brodsky, and J. F. Gunion, Phys. Rev. D 8, 287 (1973); F. E. Close, *An Introduction to Quarks and Partons* (Academic, New York, 1979), p. 199.

[19]D. E. Soper, Phys. Rev. D 15, 1141 (1977).

[20]R. P. Hughes, Nucl. Phys. B66, 189 (1973).

[21]G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. 35, 1416 (1975); A. I. Vainshtein and V. I. Zakharov, Phys. Lett. 72B, 368 (1978); E. L. Berger and S. J. Brodsky, Phys. Rev. Lett. 42, 940 (1979); G. P. Lepage and S. J. Brodsky, Phys. Rev. D 22, 2157 (1980).