PHYSICAL REVIEW D VOLUME 24, NUMBER 6 15 SEPTEMBER 1981

Four-fermion interaction in the mean-field expansion. Formation of Cooper pairs in four dimensions

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(Received 8 April 1980; revised manuscript received 19 February 1981)

The mean-field expansion of a four-fermion scalar-scalar interaction is reconsidered in two- and four-dimensional space-time. We study the model with positive and negative signs of the coupling constant. If the coupling constant is negative, the generating functionals allow the Laplace expansion after reparametrization with use of the collective boson variables in the fermion-fermion sector. If the coupling constant is positive, collective variables should be introduced in the fermion-antifermion sector. In two dimensions the theory is consistent for positive, coupling constant, in four dimensions for negative coupling constant. Even in four dimensions the theory is renormalizable by power counting.

I. INTRODUCTION

It has recently been recognized that a class of theories describing fermions with quartic self-interaction can be considered renormalizable by power counting even in four-dimensional space if the standard perturbation expansion in powers of the coupling constant is replaced by Hartree-type expansion techniques. The simplest four-dimensional model of this kind with scalar-scalar four-fermion interaction is

$$L=\bar{\psi}(i\not{\partial}-m)\psi-\tfrac{1}{2}g^2(\bar{\psi}\psi)^2\,, \tag{1.1}$$

and the positive coupling constant g^2 has been studied with the use of the $1/N$ expansion,[1,2] and a handful of results is already available. A similar technique turns out to be also useful as a tool for studying more complicated models such as nonpolynomial scalar interactions in four dimensions.[3]

In a smaller number of space-time dimensions these techniques, when applied to the model (1.1), yield results bearing no apparent marks of inconsistency.[4,5] In four dimensions the situation was far from being clarified because all existent findings exhibited ghost poles in propagators of the collective excitations. Even with unwanted poles, the model (1.1) was particularly appealing mainly because of its unexpected renormalizability in the framework of the new expansion technique.

In this paper we shall attempt to reconsider the theory in order to find the possible origin of unwanted poles. We shall use the integral version of the mean-field expansion as formulated in Ref. 6 and applied to fermionic theories in Refs. 2 and 5. This expansion, however very similar to, is more general and more elegant than the ordinary $1/N$ expansion. At least formally, it can be used even for single (or few) component fields which are indeed the case of final physical interest.

The common feature of the mean field and the $1/N$ expansion is the introduction of auxiliary collective fields which appear as intermediate bosons turning the original quartic coupling into the Yukawa-type coupling. In both methods auxiliary fields are introduced in such a way that the effective-action functional remains unchanged. Originally constant, the propagators of auxiliary fields acquire kinetic terms from radiative corrections. These kinetic terms are responsible for the change of the power-counting rules, as well as for the appearance of additional physical or unphysical poles in the theory.

If the number N of field components is large, then the sector in which collective fields should be introduced is fixed by combinatorics which picks out a class of diagrams contributing to the leading order in powers of $1/N$.

In the model with the interaction Lagrangian

$$\frac{1}{N}\sum_{i=1}^{N}(\bar{\psi}^i\psi^i)^2\,, \tag{1.2}$$

collective fields are introduced in the $(\bar{\psi}\psi)$ sector and the quartic interaction is replaced by the $\psi\psi\sigma$ Yukawa interaction. If the interaction Lagrangian has the form

$$\frac{1}{N}\sum_{j=1}^{N}\sum_{k=1}^{N}(\bar{\psi}^j\psi^k)^2\,, \tag{1.3}$$

then collective variables are defined in the $(\psi\psi)$ and $(\bar{\psi}\bar{\psi})$ sectors and $\psi\psi\sigma$ and $\sigma\bar{\psi}\bar{\psi}$ trilinear vertices replace the original quartic vertex.

Therefore, the insight gained from the analysis of the $1/N$-expanded theory may be misleading in the $N=1$ limit as, for $N\to\infty$, (1.2) and (1.3) describe completely different theories. Extrapolation of Green's functions from N large to the $N=1$

region is dangerous at least because for N large, order by order in $1/N$, different diagrams contribute to Green's functions of models (1.2) and (1.3). There are diagrams which contribute to the first order of the $1/N$ expansion applied to (1.2) and to the infinite order of the same expansion applied to (1.3), hence on the $N=1$ level, transition from (1.2) and (1.3) corresponds to the essential reordering of the (divergent) series.

In the mean-field expansion collective fields are introduced in the very same way but with no reference to combinatorical arguments. The ambiguity connected with the possibility of the free choice between two ways of introducing intermediate bosons was the main obstacle in understanding the significance of the expansion. In this paper we show that the mechanism creating neutral collective fields in the $(\bar\psi\psi)$ sector cannot compete with the mechanism which assembles $(\bar\psi\bar\psi)$ or $(\psi\psi)$ in double-charged collective modes. If one is consistent, the other is not. Which is the correct one depends on the sign of the coupling constant and the number of space-time dimensions. It has already been recognized[4] that the model (1.2) allows the $1/N$ expansion only in the case of the positive coupling while for (1.3) the expansion is correct only when the sign of the coupling is negative.[7] Otherwise, the reparametrized Lagrangian would be equivalent to the Lagrangian of the Yukawa model with infinitely heavy mesons and imaginary coupling constants. The above statement is independent of the dimensionality of the space-time but does not eliminate the ambiguity of the choice between reparametrizations corresponding to (1.2) and (1.3). In particular the Lagrangian of the free theory can be written in the form

$$L=\bar\psi(i\partial\!\!\!/-m)\psi+g^2(\bar\psi\psi)^2-g^2(\bar\psi\psi)^2\,. \tag{1.4}$$

Expanding the first interaction part with the $(\bar\psi\psi)$ collective field and the second with $(\psi\psi)$ and $(\bar\psi\bar\psi)$ modes we cannot obtain the complete mutual cancellation of terms unless all order contributions are evaluated and summed up. As a result, the theory remains ambiguous because contributions stemming from the expansion of the free theory written in the form (1.4) can be freely added to Green's functions obtained from (1.2) or (1.3).

There is one possible way out of this trouble. The expansion will be unambiguous if for some reason (1.2) or (1.3) will disallow the viable expansion in $1/N$ (or analogous mean-field expansion).

Searching for such an argument we have performed direct calculations in the first order to complete the existing results obtained in (1.2) and its mean-field analog.

In two dimensions the model is consistent if the coupling constant is positive and neutral collective $(\psi\psi)$ and $(\psi\bar\psi)$ fields are introduced.[5] We show that with negative coupling and double-charged collective $(\psi\psi)$ and $(\bar\psi\bar\psi)$ states the expansion fails because the ghost poles are produced in intermediate propagators. Then we pass to the four-dimensional case and find that the ghost pole disappears when the interaction Lagrangian enters with a negative sign and neutral bosons are replaced by double-charged collective states which resemble Cooper pairs in the theory of superconductivity.

II. THE MEAN-FIELD EXPANSION IN THE SECTOR OF COOPER PAIRS

We shall consider a theory of the spinor field with quartic scalar self-interaction given by the Lagrangian density

$$L_\psi=\bar\psi_\alpha(i\partial\!\!\!/_{\alpha\beta}-m\,\delta_{\alpha\beta})\psi_\beta+\tfrac{1}{2}\lambda(\bar\psi_\alpha\psi_\alpha)^2 \tag{2.1}$$

in two- and four-dimensional space-time and for both signs of the coupling constant λ. Our discussion is intended to complete the analysis of the model (2.1) which has already been extensively elaborated with use of the $1/N$ (Refs. 4 and 1) and mean-field[2,5] expansions, but only for positive values of the coupling constant.

We shall use the integral version[6] of the mean-field expansion which is a path-integral generalization of the Laplace method of the asymptotic evaluation of ordinary integrals. The vacuum functional of our model is

$$Z[\bar J,J]=\int[D\bar\psi][D\psi]\exp\left(i\int d^nx[\bar\psi_\alpha(i\partial\!\!\!/_{\alpha\beta}-m\,\delta_{\alpha\beta})\psi_\beta+\tfrac{1}{2}\lambda(\bar\psi_\alpha\psi_\alpha)^2+\bar J_\alpha\psi_\alpha+\bar\psi_\alpha J_\alpha]\right)\,. \tag{2.2}$$

Here and in the following we omit normalization factors but it should be understood that all integrals defining generating functionals are properly normalized.

To perform the mean-field expansion we introduce auxiliary Bose fields in such a way that the integration over fermionic degrees of freedom in (2.2) becomes Gaussian. This allows us to perform the integration over $\bar\psi$ and ψ variables explicitly and leaves us with integrals over auxiliary degrees of freedom. The integral over the Bose fields is not Gaussian but can be expanded with use of the Laplace method which yields a series of moments of Gaussian integrals which can be explicitly evaluated. The resulting perturbation expansion is essentially different from the expansion in powers of the coupling constant and its main virtue is that its renormalization properties are dramatically improved.

Auxiliary fields can be introduced in several ways; however, the requirements of consistency may exclude certain possibilities. This situation is well known from the $1/N$ expansion where intermediate fields are used as a convenient device which simplifies the $1/N$ power counting. For example, in the Gross-Neveu version of our model the interaction Lagrangian is

$$\tfrac{1}{2}\lambda \sum_{a=1}^{N} (\bar{\psi}_\alpha^a \psi_\alpha^a)^2 ,$$

where a enumerates the species of fermions and α their spinor indices. The $1/N$ power counting is simplified by introducing intermediate scalar collective field σ whose equation of motion is an equation of constraints of the form

$$\sigma = \lambda^{1/2} \bar{\psi}_\alpha^a \psi_\alpha^a .$$

This pattern of construction has been adapted to the mean-field expansion in Refs. 2 and 5. Ojima and Fukuda[7] have investigated a similar two-dimensional model with double-charged complex collective fields σ and $\sigma^\dagger$ satisfying

$$\sigma = \lambda^{1/2} \psi \bar{\psi}^C , \quad \sigma^\dagger = \lambda^{1/2} \bar{\psi} \psi^C ,$$

where the superscript C means charge conjugation. This choice is appropriate to the interaction Lagrangian

$$-\lambda \bar{\psi}_\alpha^a \psi_\alpha^b \bar{\psi}_\beta^a \psi_\beta^b .$$

As before, Latin indices enumerate fermions and Greek indices their components.

Here we are considering the interaction of a single-component field and the combinatorics does not point out any particular method of the reparametrization. The quartic term is $\bar{\psi}_\alpha \psi_\alpha \bar{\psi}_\beta \psi_\beta$ and auxiliary collective modes can be introduced in $\bar{\psi}_\alpha \psi_\alpha$, $\bar{\psi}_\alpha \bar{\psi}_\beta$ as well as in $\psi_\alpha \psi_\beta$ sectors. Although the problem requires the discussion of all possibilities, we shall confine ourselves to the latter case and only to the scalar intermediate modes.

Let us rewrite (2.2) in the form

$$Z[\bar{J},J] = \int [D\bar{\psi}][D\psi][D\tilde{\sigma}][D\sigma] \exp\Big(i \int d^n x [\bar{\psi}_\alpha (i\partial\!\!\!/_{\alpha\beta} - m\delta_{\alpha\beta})\psi_\beta - \tfrac{1}{4}\lambda \bar{\psi}_\alpha \bar{\psi}_\beta \psi_\alpha \psi_\beta + \bar{J}_\alpha \psi_\alpha + \bar{\psi}_\alpha J_\alpha]\Big) , \tag{2.2$'$}$$

or equivalently,

$$Z[\bar{J},J] = \int [D\bar{\psi}][D\psi][D\tilde{\sigma}][D\sigma] \exp\Big(i \int d^n x [\bar{\psi}_\alpha (i\partial\!\!\!/_{\alpha\beta} - m_{\alpha\beta})\psi_\beta + 2^{d/2-1} g \bar{\psi}_\alpha \bar{\psi}_\beta \sigma_{\alpha\beta}$$
$$+ 2^{d/2-1} g \psi_\alpha \psi_\beta \tilde{\sigma}_{\alpha\beta} - \tilde{\sigma}_{\alpha\beta}\sigma_{\alpha\beta} + \bar{J}_\alpha \psi_\alpha + \bar{\psi}_\alpha J_\alpha]\Big) . \tag{2.3}$$

Equation (2.3) was obtained from (2.2) by adding to it a Gaussian term

$$-[\tfrac{1}{2}(-\lambda)^{1/2} \bar{\psi}_\alpha \bar{\psi}_\beta - \tilde{\sigma}_{\alpha\beta}][\tfrac{1}{2}(-\lambda)^{1/2} \psi_\alpha \psi_\beta - \sigma_{\alpha\beta}] ,$$

and defining $g_0 = 2^{-d/2(-\lambda)^{1/2}}$ where d is the dimension of the space-time introduced here to compensate later the factor produced by the trace of the unit matrix.

It is obvious that consistency requires that $\sigma_{\alpha\beta}$ be antisymmetric in α, β and $\tilde{\sigma}_{\alpha\beta}\sigma_{\alpha\beta}$, $\bar{\psi}_\alpha \bar{\psi}_\beta \sigma_{\alpha\beta}$, as well as $\tilde{\sigma}_{\alpha\beta}\psi_\alpha \psi_\beta$ should be real. All these requirements are met if we impose that

$$\sigma_{\alpha\beta} = 2^{-d/2} C_{\alpha\beta} \sigma , \quad \tilde{\sigma}_{\alpha\beta} = 2^{-d/2} C^{-1}{}_{\alpha\beta} \sigma^\dagger , \tag{2.4}$$

where $C_{\alpha\beta}$ is the operator of charge conjugation and σ is a scalar field. Adding source terms of the auxiliary fields and rewriting the entries of (2.3) in the matrix form we obtain

$$Z[\bar{J},J,j,j^\dagger] = \int [D\bar{\psi}][D\psi][D\sigma][D\sigma^\dagger] \exp\left\{ \int \left[-\sigma^\dagger \sigma + j^\dagger \sigma + j\,\sigma^\dagger + \tfrac{1}{2}(\psi_\alpha, \bar{\psi}_\xi) \begin{pmatrix} g\sigma C_{\alpha\beta} & -K^T_{\xi\zeta} \\ K_{\alpha\beta} & g\sigma^\dagger C^{-1}{}_{\zeta\xi} \end{pmatrix} \begin{pmatrix} \psi_\beta \\ \bar{\psi}_\zeta \end{pmatrix} \right.\right.$$
$$\left.\left. + (\psi_\alpha, \bar{\psi}_\xi) \begin{pmatrix} 0 & -\delta_{\zeta\xi} \\ \delta_{\alpha\beta} & 0 \end{pmatrix} \begin{pmatrix} J_\beta \\ \bar{J}_\zeta \end{pmatrix} \right] \right\} , \tag{2.5a}$$

where T means transposition and $K_{\alpha\beta} = i\partial\!\!\!/_{\alpha\beta} - m_0 \delta_{\alpha\beta}$. The shorthand for (2.5a) is

$$Z[\bar{J},J;j,j^\dagger]=\int[D\bar{\psi}][D\psi][D\sigma][D\sigma^\dagger]\exp\left\{i\int\left[\tfrac{1}{2}(\psi,\bar{\psi})\mathcal{K}\begin{pmatrix}\psi\\ \bar{\psi}\end{pmatrix}+(\psi,\bar{\psi})A\begin{pmatrix}J\\ \bar{J}\end{pmatrix}-\sigma^\dagger\sigma+j^\dagger\sigma+j\sigma^\dagger\right]\right\}. \tag{2.5b}$$

In order to perform the integration over fermionic degrees of freedom we change variables to get rid of the linear term. Defining

$$\begin{pmatrix}\psi\\ \bar{\psi}\end{pmatrix}\Rightarrow\begin{pmatrix}\psi\\ \bar{\psi}\end{pmatrix}+\mathcal{K}^{-1}A\begin{pmatrix}J\\ \bar{J}\end{pmatrix},$$

we obtain

$$Z[\bar{J},J;j,j^\dagger]=\int[D\bar{\psi}][D\psi][D\sigma][D\sigma^\dagger]\exp\left\{i\int\left[\tfrac{1}{2}(J,\bar{J})A^T\mathcal{K}^{-1}A\begin{pmatrix}J\\ \bar{J}\end{pmatrix}+\tfrac{1}{2}(\psi,\bar{\psi})\mathcal{K}\begin{pmatrix}\psi\\ \bar{\psi}\end{pmatrix}-\sigma^\dagger\sigma+j^\dagger\sigma+j\sigma^\dagger\right]\right\}.$$

The inverse matrix $\mathcal{K}^{-1}$ is

$$\mathcal{K}^{-1}=\begin{pmatrix}g\sigma^\dagger C^{-1} & i\not{\partial}^T+m\\ -i\not{\partial}-m & g\sigma C\end{pmatrix}(g^2\sigma^\dagger\sigma+m^2-\Box)^{-1}.$$

We can verify this by inspection if we notice that $C_{\alpha\xi}\not{\partial}_{\xi\beta}=C_{\beta\xi}\not{\partial}_{\xi\alpha}$. Now the integral is Gaussian and can be explicitly evaluated. The result is

$$\begin{aligned}Z[\bar{J},J;j,j^\dagger]=\int[D\sigma][D\sigma^\dagger]\exp\Big[i\int d^nx\Big(&i\,\mathrm{tr}\ln(g^2\sigma^\dagger\sigma+m^2-\Box)-\sigma^\dagger\sigma+j^\dagger\sigma+j\sigma^\dagger\\ &-\tfrac{1}{2}\int d^ny[J(x)S_{11}(x,y)J(y)+J(x)S_{12}(x,y)\bar{J}(y)+\bar{J}(x)S_{21}(x,y)J(y)\\ &+\bar{J}(x)S_{22}(x,y)\bar{J}(y)]\Big)\Big].\end{aligned} \tag{2.6}$$

The S_{ik} functions will be identified as fermion propagators and they are 4×4 elements of the 8×8 $A^T\mathcal{K}^{-1}A$ matrix. Their exact form is

$$S_{11}(x-y)=g\sigma^\dagger(x)C^{-1}(\Box_x-m^2-g^2\sigma^\dagger(x)\sigma(x))^{-1}\delta(x-y),$$
$$S_{12}(x-y)=(-i\not{\partial}_x+m)(\Box_x-m^2-g^2\sigma^\dagger(x)\sigma(x))^{-1}\delta(x-y), \tag{2.7a}$$
$$S_{21}(x-y)=(i\not{\partial}_x^T-m)(\Box_x-m^2-g^2\sigma^\dagger(x)\sigma(x))^{-1}\delta(x-y),$$
$$S_{22}(x-y)=g\sigma C^{-1}(\Box_x-m^2-g^2\sigma^\dagger\sigma)^{-1},$$

or, in the momentum representation,

$$\begin{aligned}S_{11}(p)&=g\sigma^\dagger C^{-1}(p^2-m^2-g^2\sigma^\dagger\sigma)^{-1},\\ S_{12}(p)&=(\not{p}+m)(p^2-m^2-g^2\sigma^\dagger\sigma)^{-1},\\ S_{21}(p)&=(-\not{p}^T-m)(p^2-m^2-g^2\sigma^\dagger\sigma)^{-1},\\ S_{22}(p)&=g\sigma C(p^2-m^2-g^2\sigma^\dagger\sigma)^{-1}.\end{aligned} \tag{2.7b}$$

In diagrams S_{ik} will be represented in the form of single directed lines as shown in Fig. 1.

In order to perform the mean-field expansion we introduce the arbitrary expansion parameter ϵ into (2.6) which we now rewrite in the abridged form

$$Z_\epsilon[J,\bar{J};j,j^\dagger]=\int[D\sigma][D\sigma^\dagger]\times\exp\left(\frac{1}{i\epsilon}F[\sigma,\sigma^\dagger;\bar{J},J,j,j^\dagger]\right). \tag{2.8a}$$

Rotating to Euclidean space we obtain

$$Z_\epsilon[\bar{J},J;j,j^\dagger]=\int[D\sigma][D\sigma^\dagger]\times\exp\left(-\frac{1}{\epsilon}F[\sigma,\sigma^\dagger;\bar{J},J,j,j^\dagger]\right). \tag{2.8b}$$

In the end of calculations the expansion parameter is set equal to unity. It was introduced in the same way as the loop expansion parameter and in the

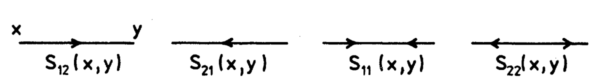

FIG. 1. Fermionic propagators.

diagrammatical language our expansion is actually the intermediate field-loop expansion. The nth order collects contributions stemming from graphs containing n loops involving at least one σ line.

The dominant contribution to the integral (2.8) comes from the path on which F_E assumes minimal value. Let such a minimum exist for $\sigma(x) = \sigma_0(x)$ and $\sigma^\dagger(x) = \sigma_0^\dagger(x)$. The mean fields $\sigma_0(x)$ and $\sigma_0^\dagger(x)$ are thus defined by conditions

$$\left.\frac{\delta F}{\delta\sigma(x)}\right|_{\sigma^\nu(x)=\sigma_0^\nu(x)} = 0\,, \qquad \left.\frac{\delta F}{\delta\sigma^\dagger(x)}\right|_{\sigma^\nu(x)=\sigma_0^\nu(x)} = 0\,. \tag{2.9}$$

We also require that either

$$A_{11}(x,y) \equiv \left.\frac{\delta^2 F}{\delta\sigma(x)\,\delta\sigma(y)}\right|_{\sigma^\nu=\sigma_0^\nu} > 0\,, \qquad A_{22}(x,y) \equiv \left.\frac{\delta^2 F}{\delta\sigma^\dagger(x)\,\delta\sigma^\dagger(y)}\right|_{\sigma^\nu=\sigma_0^\nu} > 0\,, \qquad \det(A_{ik}) > 0\,, \tag{2.10}$$

or

$$A_{11}(x,y) = A_{22}(x,y) = 0\,, \qquad A_{12}(x,y) \equiv \left.\frac{\delta^2 F}{\delta\sigma(x)\,\delta\sigma^\dagger(y)}\right|_{\sigma^\nu=\sigma_0^\nu} > 0\,. \tag{2.11}$$

Let us define

$$B_{ijk}(x,y,z) \equiv \left.\frac{\delta^3 F}{\delta\sigma_i^\nu(x)\,\delta\sigma_j^\nu(y)\,\delta\sigma_k^\nu(z)}\right|_{\sigma^\nu=\sigma_0^\nu}\,, \quad i,j,k=1,2 \tag{2.12}$$

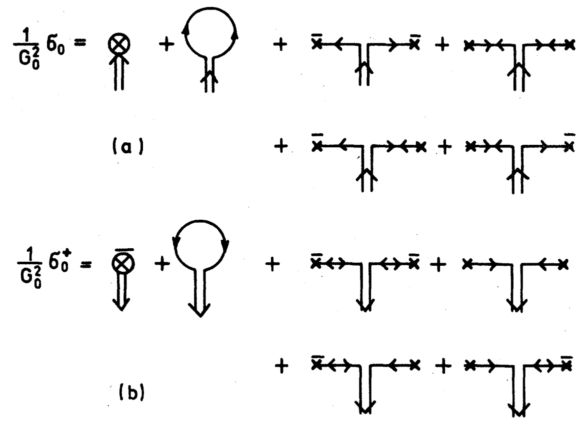

FIG. 2. Diagrammatical representation of the gap equation.

and

$$C_{ijkl}(x,y,z,w) \equiv \left.\frac{\delta^4 F}{\delta\sigma_i^\nu(x)\,\delta\sigma_j^\nu(y)\,\delta\sigma_k^\nu(z)\,\delta\sigma_l^\nu(w)}\right|_{\sigma^\nu=\sigma_0^\nu}\,, \tag{2.13}$$

where

$$\sigma_i^\nu(v) = \begin{cases} \sigma(v), & \text{if } i=1\,,\\ \sigma^\dagger(v), & \text{if } i=2\,,\end{cases}$$

e.g.,

$$B_{112}(x,y,z) = \delta^3 F/\delta\sigma_0(x)\,\delta\sigma_0(y)\,\delta\sigma_0^\dagger(z)\,.$$

Making a Laplace expansion of the functional integral (2.8b) and then returning to Minkowskian momenta we obtain

$$\begin{aligned} Z_\epsilon[\bar J,J,j,j^\dagger] \approx\ & \exp\left(\frac{i}{\epsilon}F[\sigma_0,\sigma_0^\dagger;\bar J,J,j,j^\dagger]\right) \\ & \times\exp\left(\frac{i}{2}\operatorname{tr}\ln D\right)\left\{1-\frac{\epsilon}{2}\int d^nx\,d^ny\,d^nz\,d^nw\,C_{ijkl}(x,y,z,w)(A^{-1})_{ij}(x,y)(A^{-1})_{kl}(z,w)\right. \\ & +\frac{\epsilon}{24}\int d^nx\,d^ny\,d^nz\,d^na\,d^nb\,d^nc\,B_{ijk}(x,y,z)B_{lmn}(a,b,c) \\ & \times[2(A^{-1})_{il}(x,a)(A^{-1})_{jm}(y,b)(A^{-1})_{kn}(z,c)+3(A^{-1})_{ij}(x,y)(A^{-1})_{kl}(z,a)(A^{-1})_{mn}(b,c)] \\ & \left.+O(\epsilon^2)\right\}\,, \end{aligned} \tag{2.14}$$

where $D=\det(A_{ik})$. The derivation of (2.14) follows the steps described in Ref. 6 with only minor modifications which are due to the presence of two intermediate fields.

Before we proceed to the calculations of the effective potential and several Green's functions, we find it convenient to redefine the auxiliary field and the coupling constant g so as to make g dimensionless and σ^ν to acquire the canonical dimension of the scalar field. We define

$$\sigma \Rightarrow M\sigma, \quad g \Rightarrow gM^{-1}, \quad g > 0 . \tag{2.15}$$

This redefinition does not change our previous formulas except for the free $\sigma^\dagger\sigma$ mass term which now gets a more conventional $M^2\sigma^\dagger\sigma$ form.

The mean-field conditions (2.9) yield the gap equations which are represented in Fig. 2. Their analytical form is

$$\begin{aligned}\left.\frac{\delta F}{\delta\sigma(x)}\right|_{\sigma_0^\dagger} = & -M^2\sigma_0^\dagger(x) + j^\dagger(x) - ig\,\mathrm{tr}CS_{11}^{(0)}(x,x) \\ & -\tfrac{1}{2}g\int d\zeta\, d\xi [J(\zeta)S_{11}^{(0)}(\zeta - x)CS_{11}^{(0)}(x-\xi)J(\xi) + J(\zeta)S_{11}^{(0)}(\zeta - x)CS_{12}^{(0)}(x-\xi)\bar{J}(\xi) \\ & + \bar{J}(\zeta)S_{21}^{(0)}(\zeta - x)CS_{11}^{(0)}(x-\xi)\bar{J}(\xi) + \bar{J}(\zeta)S_{21}^{(0)}(\zeta - x)CS_{12}^{(0)}(x-\xi)\bar{J}(\xi)] = 0\end{aligned} \tag{2.16a}$$

and

$$\begin{aligned}\left.\frac{\delta F}{\delta\sigma^\dagger(x)}\right|_{\sigma_0^\nu} = & -M^2\sigma_0(x) + j(x) - ig\,\mathrm{tr}C^{-1}S_{22}^{(0)}(x,x) \\ & -\tfrac{1}{2}g\int d\zeta\, d\xi [J(\zeta)S_{12}^{(0)}(\zeta - x)C^{-1}S_{21}^{(0)}(x-\xi)J(\xi) + J(\zeta)S_{12}^{(0)}(\zeta - x)C^{-1}S_{22}(x-\xi)\bar{J}(\xi) \\ & + \bar{J}(\zeta)S_{22}^{(0)}(\zeta - x)C^{-1}S_{21}(x-\xi)J(\xi) + \bar{J}(\zeta)S_{21}^{(0)}(\zeta - x)C^{-1}S_{12}(x-\xi)\bar{J}(\xi)] = 0 ,\end{aligned} \tag{2.16b}$$

where the propagators $S_{ik}^{(0)}$ are obtained by replacing σ and $\sigma^\dagger$ by σ_0 and $\sigma_0^\dagger$ in the expressions for S_{ik}. Spinor indices have not been shown explicitly. The summation over them follows the standard matrix convention [notice that in the definitions (2.7) certain matrices are transposed].

Higher derivatives of F can be immediately obtained from (2.15) with use of the identities

$$\begin{aligned}\frac{\delta}{\delta\sigma(z)} S_{ij}(x,y) &= gS_{i1}(x,z)\,CS_{1j}(z,y) , \\ \frac{\delta}{\delta\sigma^\dagger(z)} S_{ij}(x,y) &= gS_{i2}(x,z)\,C^{-1}S_{2j}(z,y) .\end{aligned} \tag{2.17}$$

Again, these identities can be derived immediately if we remember that $C_{\alpha\xi}\partial\!\!\!/_{\xi\beta}$ is symmetric.
For example, the leading-order contributions to the inverse σ propagators are

$$\begin{aligned}A_{11}(x-y) = & \frac{\delta^2 F}{\delta\sigma_0(x)\delta\sigma_0(y)} \\ = & -ig^2\,\mathrm{tr}\,CS_{11}^{(0)}(x-y)\,CS_{11}^{(0)}(y-x) - g^2\int d\zeta\, d\xi [J(\zeta)S_{11}^{(0)}(\zeta - x)\,CS_{11}^{(0)}(x-y)CS_{11}^{(0)}(y-\xi)J(\xi) \\ & + J(\zeta)S_{11}^{(0)}(\zeta - x)\,CS_{11}^{(0)}(x-y)\,CS_{12}^{(0)}(y-\xi)\bar{J}(\xi) \\ & + \bar{J}(\zeta)S_{21}^{(0)}(\zeta - x)\,CS_{11}^{(0)}(x-y)\,CS_{11}^{(0)}(y-\xi)J(\xi) \\ & + \bar{J}(\zeta)S_{21}^{(0)}(\zeta - x)\,CS_{11}^{(0)}(x-y)\,CS_{12}^{(0)}(y-\xi)\bar{J}(\xi)] ,\end{aligned} \tag{2.18a}$$

$$\begin{aligned}A_{12}(x-y) = & \frac{\delta^2 F}{\delta\sigma_0(x)\delta\sigma_0^\dagger(y)} \\ = & -M^2\delta(x-y) - ig^2\,\mathrm{tr}CS_{12}^{(0)}(x-y)C^{-1}S_{21}^{(0)}(y-x) \\ & -\tfrac{1}{2}g^2\int d\zeta\, d\xi [J(\zeta)S_{11}^{(0)}(\zeta - x)\,CS_{12}^{(0)}(x-y)C^{-1}S_{21}(y-\xi)J(\xi) \\ & + J(\zeta)S_{11}^{(0)}(\zeta - x)\,CS_{12}^{(0)}(x-y)C^{-1}S_{22}(y-\xi)\bar{J}(\xi) \\ & + \bar{J}(\zeta)S_{21}^{(0)}(\zeta - x)\,CS_{12}^{(0)}(x-y)C^{-1}S_{21}^{(0)}(y-\xi)J(\xi) \\ & + \bar{J}(\zeta)S_{21}^{(0)}(\zeta - x)\,CS_{12}^{(0)}(x-y)\,C^{-1}S_{22}^{(0)}(y-\xi)\bar{J}(\xi) + (\zeta \leftrightarrow \xi)] ,\end{aligned} \tag{2.18b}$$

$$
\begin{aligned}
A_{22}(x-y) &= \frac{\delta^2 F}{\delta\sigma_0^\dagger(x)\,\delta\sigma_0^\dagger(y)} \\
&= -ig^2 \operatorname{tr} C^{-1}S_{22}^{(0)}(x-y)\,C^{-1}S_{22}^{(0)}(y-x) \\
&\quad - g^2\int d\zeta\, d\xi \big[J(\zeta)\,S_{12}^{(0)}(\zeta-x)\,C^{-1}S_{22}(x-y)\,C^{-1}S_{21}(y-\xi)\,J(\xi) \\
&\qquad + J(\zeta)\,S_{12}^{(0)}(\zeta-x)\,C^{-1}S_{22}(x-y)\,C^{-1}S_{22}(y-\xi)\,\bar{J}(\xi) \\
&\qquad + \bar{J}(\zeta)\,S_{22}^{(0)}(\zeta-x)\,C^{-1}S_{22}(x-y)\,C^{-1}S_{21}(y-\xi)\,J(\xi) \\
&\qquad + \bar{J}(\zeta)\,S_{22}^{(0)}(\zeta-x)\,C^{-1}S_{22}(x-y)\,C^{-1}S_{22}(y-\xi)\,\bar{J}(\xi) \big]\,. \qquad (2.18c)
\end{aligned}
$$

In Fig. 3 we have translated the above formulas in diagrammatical language. It is straightforward to calculate higher derivatives with respect to σ and $\sigma^\dagger$. To represent the $(n+m)$th derivative $\delta^{n+m}F/\delta\sigma_0^n\delta\sigma^{\dagger m}$ graphically, we draw all possible $(n+m)$-sided polygons and $(n+m+1)$-sided open polygonal paths. Then, in all possible ways we attach Fermi sources and sinks to the ends of open polygons. To the $n+m$ apexes of each diagram we attach, also in all possible ways, m outgoing and n incoming double arrows representing differentiations. Finally, we orient Fermi propagators according to the orientation of arrows in apexes and species at the ends of open paths.

Having expressions for A, B, and C is trivial, however, space consuming to calculate the vacuum functional by direct substitution of these quantities into (2.14). The rules of the calculus are similar to those of Refs. 2 and 5, except for

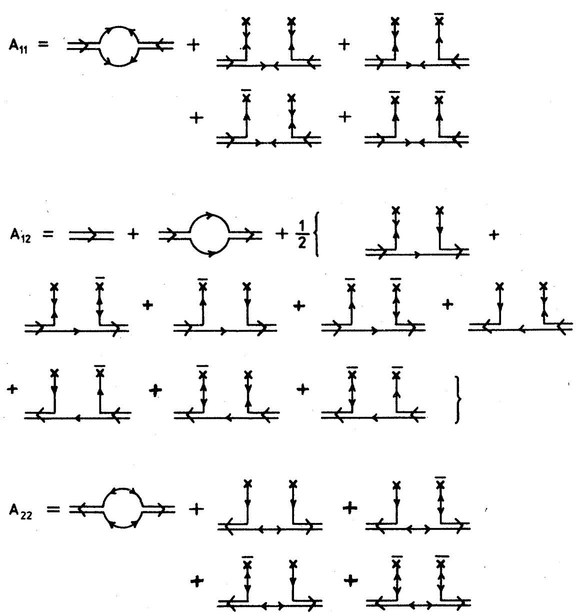

FIG. 3. Diagrams contributing to the σ two-point functions.

complications caused by the presence of three kinds of Fermi propagators and three kinds of intermediate propagators. Although $A^{-1}{}_{KK}$ and S_{KK} are nonvanishing functions, this must not mean that the charge (Fermi number) is not conserved in our model. All propagators used in our expansion are σ_0 dependent and σ_0 satisfies the mean-field conditions, thus it is manifestly source dependent. What we really need here is that $A^{-1}{}_{KK}$ and S_{KK} disappear when sources are switched off. This may take place if $\sigma_0^v(\bar{J}=J=j=j^\dagger=0)=0$.

III. TWO-DIMENSIONAL MODEL TO THE LOWEST ORDER

In the lowest order of the mean-field expansion the generating functional of connected Green's functions is

$$W_0 = F[\sigma_0^\dagger, \sigma_0; \bar{J}, J, j, j^\dagger]\,, \qquad (3.1)$$

with σ_0 and $\sigma_0^\dagger$ given by the mean-field conditions (2.16). Considerations of the preceding section have revealed that in general two-point S_{ii} and $A^{-1}{}_{ii}$ functions do not vanish. This implies the possibility of the charge nonconservation and casts doubt on the physical meaning of our model. Therefore, we should check whether the mean field develops nonvanishing values when sources are set equal to zero. Let us recall that the mean-field conditions are actually conditions for the minimum of $-F$. With sources off, at least to the lowest order, the mean-field conditions are identical to the conditions for the minimum of the effective potential. The lowest-order unrenormalized σ-effective potential is

$$V_0(\sigma_0, \sigma_0^\dagger) = M^2\sigma_0^\dagger\sigma_0 + i\int \frac{d^n k}{(2\pi)^n} \ln(g^2\sigma_0^\dagger\sigma_0 + m^2 - k^2)\,. \qquad (3.2)$$

Differentiating it with respect to σ_0 and $\sigma_0^\dagger$ we obtain

$$\frac{\partial V_0}{\partial \sigma_0} = M^2\sigma_0^\dagger - ig^2\sigma_0^\dagger \int \frac{d^n k}{(2\pi)^n} \frac{1}{m^2 + g^2\sigma_0^\dagger\sigma_0 - k^2}, \tag{3.3a}$$

$$\frac{\partial V_0}{\partial \sigma_0^\dagger} = M^2\sigma_0 - ig^2\sigma_0 \int \frac{d^n k}{(2\pi)^n} \frac{1}{m^2 + g^2\sigma_0^\dagger\sigma_0 - k^2}, \tag{3.3b}$$

so that the minimum may occur either for $\sigma_0 = \sigma_0^\dagger = 0$, or for

$$i\int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 - m^2 - g^2\sigma_0^\dagger\sigma_0} = \frac{2M^2}{g^2}. \tag{3.4}$$

The second derivatives are

$$\frac{\partial^2 V_0}{\partial \sigma_0^2} = ig^4\sigma_0^\dagger\sigma_0^\dagger \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - m^2 - g^2\sigma_0^\dagger\sigma_0)^2}, \tag{3.5a}$$

$$\frac{\partial^2 V_0}{\partial \sigma_0^{\dagger 2}} = ig^4\sigma_0\sigma_0 \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - m^2 - g^2\sigma_0^\dagger\sigma_0)^2}, \tag{3.5b}$$

and

$$\frac{\partial^2 V_0}{\partial \sigma_0 \partial \sigma_0^\dagger} = M^2 - ig^2 \int \frac{d^n k}{(2\pi)^n} \frac{1}{m^2 + g^2\sigma_0^\dagger\sigma_0 - k^2} + ig^4\sigma_0\sigma_0^\dagger \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - m^2 - g^2\sigma_0^\dagger\sigma_0)^2}. \tag{3.5c}$$

Subtracting pole parts of dimensionally regularized integrals we obtain in two dimensions

$$\frac{\partial V_R}{\partial \sigma_0} = \sigma_0^\dagger \left(M^2 + \frac{1}{4\pi} g^2 \ln \frac{g_0^2\sigma_0^\dagger\sigma_0 + m^2}{\mu^2}\right), \tag{3.6a}$$

$$\frac{\partial V_R}{\partial \sigma_0^\dagger} = \sigma_0 \left(M^2 + \frac{1}{4\pi} g^2 \ln \frac{g^2\sigma_0^\dagger\sigma_0 + m^2}{\mu^2}\right), \tag{3.6b}$$

$$\frac{\partial^2 V_R}{\partial \sigma_0^2} = \frac{1}{4\pi} g^4 \frac{\sigma_0^\dagger\sigma_0^\dagger}{\sigma_0^\dagger\sigma_0 + m^2}, \tag{3.7a}$$

$$\frac{\partial^2 V_R}{\partial (\sigma_0^\dagger)^2} = \frac{1}{4\pi} g^4 \frac{\sigma_0\sigma_0}{\sigma_0^\dagger\sigma_0 + m^2}, \tag{3.7b}$$

$$\frac{\partial^2 V_R}{\partial \sigma_0 \partial \sigma_0^\dagger} = \left(M^2 + \frac{1}{4\pi} g^2 \ln \frac{g^2\sigma_0^\dagger\sigma_0 + m^2}{\mu^2}\right) + \frac{1}{4\pi} g^4 \frac{\sigma_0^\dagger\sigma_0}{\sigma_0^\dagger\sigma_0 + m^2}, \tag{3.7c}$$

where μ is an arbitrary scale parameter of the dimension of mass. In the massless ($m^2 = 0$) case the lowest-order effective potential has the symmetry-breaking, charge-nonconserving minimum at

$$\sigma_0^\dagger\sigma_0 = \mu^2 \exp(-8\pi M^2/g^2). \tag{3.8}$$

If fermions are massive from the beginning, we normalize (3.6) and (3.7) in the standard way:

$$\left.\frac{\partial V}{\partial \sigma_0}\right|_{\sigma_0 = 0, \sigma_0^\dagger = \tilde{\sigma}_0^\dagger} = M_R^2 \tilde{\sigma}_0^\dagger,$$

$$\left.\frac{\partial V}{\partial \sigma_0^\dagger}\right|_{\sigma_0^\dagger = 0, \sigma_0 = \tilde{\sigma}_0} = M_R^2 \tilde{\sigma}_0, \tag{3.9}$$

$$\left.\frac{\partial^2 V}{\partial \sigma_0 \partial \sigma_0^\dagger}\right|_{\sigma_0^\nu = 0} = M_R^2.$$

Setting $\mu^2 = m^2$ we find that V_R has a single minimum at $\sigma_0^\nu = 0$. First derivatives of V_R are positive for positive values of the classical field σ_0:

$$\frac{\partial V_R}{\partial \sigma_0} = \sigma_0^\dagger \left[M_R^2 + \frac{1}{4\pi} g^2 \ln\left(1 + g^2 \frac{\sigma_0^\dagger\sigma_0}{m^2}\right)\right], \tag{3.10a}$$

$$\frac{\partial V_R}{\partial \sigma_0^\dagger} = \sigma_0 \left[M_R^2 + \frac{1}{4\pi} g^2 \ln\left(1 + g^2 \frac{\sigma_0^\dagger\sigma_0}{m^2}\right)\right], \tag{3.10b}$$

and the effective potential is

$$V_R = M_R^2\sigma_0\sigma_0^\dagger + \frac{1}{4\pi}(m^2 + g^2\sigma_0^\dagger\sigma_0)\left[\ln\left(1 + g^2 \frac{\sigma_0^\dagger\sigma_0}{m^2}\right)\right]. \tag{3.11}$$

Now, except for the minimum at $\sigma_0^\nu = 0$, V_R has no other extremum and increases up to infinity. Second derivatives are everywhere positive hence the first derivatives are monotonic and the gap equations determine single-branch functions which are defined for all positive values of $\sigma_0\sigma_0^\dagger$. The effective action functional of the σ and $\sigma^\dagger$ fields is

$$\Gamma_0[\sigma_0, \sigma_0^\dagger] = W_0[\sigma_0, \sigma_0^\dagger; \bar{J}, J, j, j^\dagger] - \int j\sigma_0^\dagger - \int j^\dagger\sigma_0.$$

We define

$$J_1(x) \equiv J(x), \quad J_2(x) \equiv \bar{J}(x), \quad \psi_1^0(x) \equiv \bar{\psi}^0(x), \quad \psi_2^0(x) \equiv \psi^0(x), \tag{3.12}$$

where

$$\psi_i^0(x) = \int dy\, S_{ij}^0(x, y) J_j(y), \quad i, j = 1, 2. \tag{3.13}$$

Then

$$\Gamma_0(\sigma_0,\sigma_0^\dagger)=-M^2\sigma_0^\dagger\sigma_0+i\,\mathrm{tr}\ln(g^2\sigma_0^\dagger\sigma_0+m^2-\Box)$$
$$-\tfrac{1}{2}\psi_i^0(S^0)^{-1}{}_{ij}\psi_j^0\,, \tag{3.14}$$

so that the inverse Fermi propagator is $(S^0)^{-1}{}_{ij}$, as expected. With all sources off we have $\sigma_0=\langle\sigma_0\rangle=0$ and only the mixed $\bar\psi\psi$ lines persist in the diagrammatical expansion.

Setting $\sigma_0=0$ in (2.18) we find:

$$(D^0_{\sigma_0\sigma_0^\dagger})^{-1}(x,y)\equiv D_0^{-1}(x,y)$$
$$=-M^2\delta(x-y)$$
$$-ig^2\,\mathrm{tr}[S_{21}(x-y)CS_{12}(y-x)C^{-1}]\,. \tag{3.15}$$

Fourier transforming to the momentum space we obtain

$$\frac{\delta\Gamma_0}{\delta\sigma_0^\dagger\delta\sigma_0}=D_0^{-1}(p^2)=-M_0^2-ig^2\,\mathrm{tr}\int\frac{d^nk}{(2\pi)^n}S_{12}^{(0)}(k+p)C^{-1}S_{21}^{(0)}(k)\,C$$
$$=-M^2-ig^2\int\frac{d^nk}{(2\pi)^n}\,\frac{\mathrm{tr}[(\not p+\not k+m)\,C^{-1}(-\not k^T-m)\,C]}{[(k+p)^2-m^2](k^2-m^2)}\,. \tag{3.16a}$$

Only terms even in γ contribute. $\mathrm{tr}\,mCmC^{-1}=2^{n/2}$, but $\mathrm{tr}[(\not p+\not k)\,C^{-1}\not k^T\,C]$ contains the transposed γ matrices. We diminish this inconvenience using again the identities $C_{\mu\nu}\not p_{\nu\eta}=C_{\eta\nu}\not p_{\nu\mu}$ and $C_{\mu\nu}=-C_{\nu\mu}$. We have

$$(\not k+\not p)_{\mu\nu}\,C^{-1}{}_{\nu\eta}\not k_{\rho\eta}\,C_{\rho\mu}=-(\not p+\not k)_{\mu\nu}\,C^{-1}{}_{\nu\eta}\,C_{\mu\rho}\not k_{\rho\eta}$$
$$=-(\not p+\not k)_{\mu\nu}\,C^{-1}{}_{\nu\eta}C_{\eta\rho}\not k_{\rho\mu}$$
$$=-\mathrm{tr}[(\not k+\not p)\not k]=-2^{n/2}(kp+k^2)\,.$$

Hence

$$D_0^{-1}(p^2)=-M^2+ig^2\int\frac{d^nk}{(2\pi^2)^{n/2}}\,\frac{m^2-(kp)-k^2}{[(k+p)^2-m^2](k^2-m^2)}$$
$$\equiv-M^2+g^2\Pi(p^2,m^2)\,. \tag{3.16b}$$

The main difference between our model and the model with collective fields in the fermion-antifermion sector manifests itself in the numerator of the integrand in (3.16b). In the model with $(\bar\psi\psi)$ intermediate fields all terms in the numerator would have a positive sign. In the language of diagrams this difference is caused by the inversion of one fermion propagator forming the collective self-energy loop (see Fig. 4). Evaluating Π (p^2,m^2) in two-dimensional space and subtracting on zero momentum we obtain

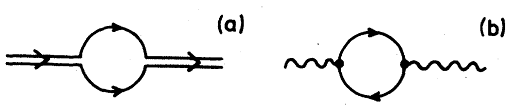

FIG. 4. One-loop diagrams contributing to the self-energy of the collective fields. (a) In the model with $(\psi\psi)$ and $\bar\psi\bar\psi$ collective fields. (b) In the model with the $\bar\psi\psi$ collective field.

$$\mathrm{sub}_0{}^1\Pi\,(p^2,m^2)=\frac{1}{2\pi}\,2\int_0^1 dx[1-m^2/p^2x(1-x)]^{-1}$$
$$+\int_0^1 dx\ln(1-p^2x(1-x)/m^2)\,. \tag{3.17}$$

Since

$$D_0^{-1}(p^2)=-M_0^2+g^2\Pi(0,m^2)$$
$$+g^2\,\mathrm{sub}_0{}^1\Pi(p^2,m^2)\,, \tag{3.18}$$

we define the renormalized constant M^2 by the equation

$$D_0^{-1}(0)=-M^2\,. \tag{3.19}$$

The integrability condition (2.11) requires that $M^2>0$, in agreement with our previous assumptions. We are now able to examine spectral properties of the renormalized intermediate propagator whose inverse is

$$D_0{}^{-1}(p^2)=-M^2+g^2\,\mathrm{sub}_0{}^1\Pi(p^2,m^2)\,. \tag{3.20}$$

The second integral in (3.17) equals $B(k^2/4m^2)-2$, where

$$B(x)=\begin{cases}2(1-1/x)^{1/2}\ln[(1-x)^{1/2}+(-x)^{1/2}] & \text{for } x\le 0\,,\\ 2(-1+1/x)^{1/2}\arctan[x/(1-x)]^{1/2} & \text{for } 0\le x\le 1\,,\\ (1-1/x)^{1/2}\{-i\pi+2\ln[(x-1)^{1/2}+x^{1/2}]\} & \text{for } x>1\,.\end{cases} \tag{3.21}$$

The first integral in (3.17) can be written in the form

$$\int_0^1 dx\,x^a(1-x)^b(1-ux)^{-c}(1-vx)^{-d}$$

and hence can be expressed in the form of the ${}_3F_1$-generalized hypergeometric function. For our purposes it is enough to observe that this integral is equal to zero for $p^2=0$ and tends to 1 in the limit $p^2\to\infty$. On the other hand, $B(0)=2$ and increases up to infinity as $p^2\to\infty$. It is then obvious that (3.20) assumes zero values somewhere between $p^2=0$ and $p^2=-\infty$ and therefore the propagator develops the tachyon pole. When $g^2\to 0$ the site of the ghost approaches minus infinity as $-m^2\exp(2\pi/g^2)$. It is worthwhile to note that the same kind of ghost would appear in the propagator of the collective field if we would blindly expand using the auxiliary field in the $(\bar\psi\psi)$ channel and the negative coupling constant.

Our result shows that in two-dimensional space-time the model (2.1) is inconsistent when forces between fermions are repulsive.

The analysis of the mean-field expansion applied to the model with positive coupling is already available. Without going into detail, let us only mention that neither ghost pole nor other apparent mark of inconsistency occurs in the theory.

IV. FOUR-DIMENSIONAL CASE

In four-dimensional space, after the dimensional regularization, the Eqs. (3.3) and (3.5) yield

$$\frac{\partial V}{\partial\sigma_0}=\sigma_0^\dagger\left[M^2-ig^2\int\frac{d^{4-\epsilon}}{4\pi^4}\,\frac{1}{m^2+g^2\sigma_0^\dagger\sigma_0-k^2}\right]$$

$$=\sigma_0^\dagger\left[M^2-\frac{g^2}{4\pi^2}\left(-\frac{2}{\epsilon}+\gamma-1\right)(m^2+g^2\sigma_0^\dagger\sigma_0)\left(1-\frac{\epsilon}{2}\ln\frac{m^2+g^2\sigma_0^\dagger\sigma_0}{\mu^2}\right)\right], \quad (4.1a)$$

$$\frac{\partial V}{\partial\sigma_0^\dagger}=\sigma_0\left[M^2-\frac{g^2}{4\pi^2}\left(-\frac{2}{\epsilon}+\gamma-1\right)(m^2+g^2\sigma_0^\dagger\sigma_0)\left(1-\frac{\epsilon}{2}\ln\frac{m^2+g^2\sigma_0^\dagger\sigma_0}{\mu^2}\right)\right], \quad (4.1b)$$

$$\frac{\partial^2 V}{\partial{\sigma_0}^2}=ig^4\sigma_0\sigma_0^\dagger\int\frac{d^{4-\epsilon}k}{4\pi^4}\,\frac{1}{(k^2-m^2-g^2\sigma_0^\dagger\sigma_0)^2}$$

$$=\frac{1}{4\pi^2}\,g^4\sigma_0^\dagger\sigma_0^\dagger\left[\left(\frac{2}{\epsilon}-\gamma\right)\left(1-\frac{\epsilon}{2}\ln\frac{m^2+g^2\sigma_0^\dagger\sigma_0}{\mu^2}\right)\right], \quad (4.2a)$$

$$\frac{\partial^2 V}{\partial\sigma_0^{\dagger 2}}=\frac{1}{4\pi^2}\,g^4\sigma_0\sigma_0\left[\left(\frac{2}{\epsilon}-\gamma\right)\left(1-\frac{\epsilon}{2}\ln\frac{m^2+g^2\sigma_0^\dagger\sigma_0}{\mu^2}\right)\right], \quad (4.2b)$$

$$\frac{\partial^2 V}{\partial\sigma_0\partial\sigma_0^\dagger}=M^2-ig^2\int\frac{d^{4-\epsilon}k}{4\pi^4}\,\frac{1}{m^2+g^2\sigma_0^\dagger\sigma_0-k^2}+ig^4\sigma_0^\dagger\sigma_0\int\frac{d^{4-\epsilon}k}{4\pi^4}\,\frac{1}{(k^2-m^2-g^2\sigma_0^\dagger\sigma_0)^2}$$

$$=M^2-\frac{g^2}{4\pi^2}\left(-\frac{2}{\epsilon}+\gamma-1\right)(m^2+g^2\sigma_0^\dagger\sigma_0)\left(1-\frac{\epsilon}{2}\ln\frac{m^2+g^2\sigma_0^\dagger\sigma_0}{\mu^2}\right), \quad (4.2c)$$

where γ is the Euler constant and μ^2 is arbitrary. We have written down all relevant terms of the expansion to make apparent the fact that there is no way to tame pole parts by adding counterterms to the existing parameters M^2 and g. We could add an extra parameter corresponding to the $(\sigma^\dagger\sigma)^2$ coupling (the fourth derivative of the effective potential is also divergent) and then proceed formally as in the perturbative Yukawa theory. When dealing with Green's functions we would then be forced to also introduce the σ-field renormalization constant. This way, for all it seems to be tempting, would render us directly or indirectly, with methodological problems which are typical for the nonrenormalizable theories in standard perturbative approaches.

We must remember that we are in the first order of the mean-field expansion and we do not have any finite "free" or "tree" theory below this order which could offer us normalization conditions for Green's functions in the σ sector. As (2.11) told, all higher-order Green's functions are generated by the leading-order A_{ik} functions used as nonlocal propagators and B, C, and other functions of this type used as nonlocal vertices. A, B, and C are divergent, so is $F[\sigma_0^\dagger,\sigma_0,\bar J,J,j,j^\dagger]$, thus we should make an attempt to regularize them, subtract divergent parts using a procedure which possibly keeps g^2, M^2, and m^2 parameters being fixed numbers to the first order of the expansion.

We can accomplish this by allowing the arbitrary μ^2 parameter to be the ϵ-dependent quantity. All divergences present in the leading order cancel if we define

$$\mu^2=\kappa^2\exp(-2/\epsilon)\,, \tag{4.3}$$

with arbitrary κ^2. There is nothing mysterious in this fact. In the standard approach the changes of the (then finite) scale parameter μ^2 can be compensated by appropriate redefinitions of the redefinitions of the renormalized parameters (including the $\sigma^\dagger\sigma^\dagger\sigma\sigma$ coupling). There is no reason which would *a priori* forbid us to repeat the trick also for the pole parts in order to see that subtraction procedures for all simply divergent diagrams are in fact connected and can be reduced to the singular rescaling of all dimensional quantities which are expanded in powers of the regularization parameter ϵ.

We do not need this property to persist in higher orders when overlapping divergences will occur. What is crucial for us is that we have reduced the number of arbitrary constants in the leading order to one (κ^2) keeping the parameters of the Lagrangian being fixed numbers of the lowest order of the expansion. The finite parts of Eqs. (4.1) and (4.2) are

$$\frac{\partial V}{\partial\sigma_0}=\sigma_0^\dagger\left[M^2-\frac{g^2}{4\pi^2}(m^2+g^2\sigma_0^\dagger\sigma_0)\ln\frac{m^2+g^2\sigma_0^\dagger\sigma_0}{\kappa^2}\right], \tag{4.4a}$$

$$\frac{\partial V}{\partial\sigma_0^\dagger}=\sigma_0\left[M^2-\frac{g^2}{4\pi^2}(m^2+g^2\sigma_0^\dagger\sigma_0)\ln\frac{m^2+g^2\sigma_0^\dagger\sigma_0}{\kappa^2}\right], \tag{4.4b}$$

$$\frac{\partial^2 V}{\partial\sigma_0{}^2}=-\frac{g^4}{4\pi^2}\sigma_0^\dagger\sigma_0^\dagger\ln\frac{m^2+g^2\sigma_0^\dagger\sigma_0}{\kappa^2}, \tag{4.5a}$$

$$\frac{\partial^2 V}{\partial\sigma_0^{\dagger 2}}=-\frac{g^4}{4\pi^2}\sigma_0\sigma_0\ln\frac{m^2+g^2\sigma_0^\dagger\sigma_0}{\kappa^2}, \tag{4.5b}$$

$$\frac{\partial^2 V}{\partial\sigma_0^\dagger\partial\sigma_0}=M^2-\frac{g^2}{4\pi^2}(m^2+g^2\sigma_0^\dagger\sigma_0)\ln\frac{m^2+g^2\sigma_0^\dagger\sigma_0}{\kappa^2} -\frac{g^4}{4\pi^2}\sigma_0^\dagger\sigma_0\ln\frac{m_0{}^2+g^2\sigma_0^\dagger\sigma_0}{\kappa^2}\,. \tag{4.5c}$$

In the following we shall argue that $\kappa^2=m^2$, thus in both massive and massless cases we find only one minimum at $\sigma_0^\nu=0$ and we need not worry about the possibility of the charge (fermion-number) nonconservation. Integrating the above equations we obtain:

$$V=M\sigma_0^\dagger\sigma_0 -\frac{1}{8\pi^2}(m^2+g^2\sigma_0^\dagger\sigma_0)^2\left(\ln\frac{m^2+g^2\sigma_0^\dagger\sigma_0}{m^2}-\frac{1}{2}\right). \tag{4.6}$$

In 4-ϵ dimensions (3.16b) equals

$$D_0{}^{-1}(p^2)=-M^2-\frac{g^2}{4\pi^2}\int_0^1 dx\left[m^2\left(-\frac{2}{\epsilon}+\gamma-1\right)+p^2x\left(\frac{6}{\epsilon}-3\gamma+1\right)-p^2x^2\left(\frac{6}{\epsilon}-3\gamma+1\right)\right] \times\left(1-\frac{\epsilon}{2}\ln\frac{m^2-p^2x+p^2x^2}{\mu^2}\right). \tag{4.7}$$

Using (4.3) we obtain

$$D_0{}^{-1}(p^2)=-M^2-\frac{g^2}{4\pi^2}\int_0^1 dx(m^2-3p^2x+3p^2x^2) \times\ln\frac{m^2-p^2x+p^2x^2}{\kappa^2}. \tag{4.8}$$

The integrations performed, we find that

$$D_0{}^{-1}(p^2)=M^2-\frac{g^2m^2}{4\pi^2}\ln(m^2/\kappa)^2 +\frac{g^2}{8\pi^2}p^2[J(p^2/4m^2)-2+\ln(m^2/\kappa^2)]\,. \tag{4.9}$$

This expression for the propagator still depends on the undetermined parameter κ^2 which should be eliminated by use of the suitable normalization condition. As we have already explained we do not want to impose any condition which would not refer to parameters of the original Lagrangian. In particular, we avoid assuming that the propagator (4.9) has the "traditional" form

$$D_0{}^{-1}(p^2)=-M_1{}^2+p^2+g^2(\text{radiative corrections})\,, \tag{4.10}$$

and then fixing κ^2 appropriately. This procedure, however "natural," cannot be justified by any argument referring to the original quartic interaction.

Another tempting possibility is to follow the pattern of Ref. 2 and use the gap equation. This method would be inconclusive here because of the lack of the symmetry breaking, but we shall devote some interest here because, slightly modified, it will offer us a certain consistency condition for the renormalization procedure we use.

The gap equations with Fermi sources off and constant meson sources are

$$M^2\sigma_0(x)=j(x)-ig\,\mathrm{tr}\,C^{-1}S_{22}^{(0)}(x,x)\,, \tag{4.11a}$$

$$M^2\sigma_0^\dagger(x)=j^\dagger(x)-ig\,\mathrm{tr}\,CS_{11}^{(2)}(x,x)\,. \tag{4.11b}$$

Defining

$$f(\sigma_0,\sigma_0^\dagger)\equiv -ig\frac{1}{\sigma_0^\dagger}\mathrm{tr}\,CS_{11}^{(0)}(x,x)$$
$$\equiv -ig\frac{1}{\sigma_0}\mathrm{tr}\,C^{-1}S_{22}^{(0)}(x,x)\,, \tag{4.12}$$

or, in the momentum space,

$$f(\sigma_0,\sigma_0^\dagger)=ig^2\int\frac{d^np}{(2\pi^2)^{n/2}}\frac{1}{m^2+g^2\sigma_0^\dagger\sigma_0-p^2}\,. \tag{4.13}$$

We rewrite (4.11) in the form

$$M^2\sigma_0=\sigma_0 f(\sigma_0,\sigma_0^\dagger)+j\,, \tag{4.14a}$$

$$M^2\sigma_0^\dagger=\sigma_0^\dagger f(\sigma_0,\sigma_0^\dagger)+j^\dagger\,. \tag{4.14b}$$

Differentiating (4.14a) with respect to σ_0 we obtain

$$M^2=f(\sigma_0,\sigma_0^\dagger)+\sigma_0\frac{\partial}{\partial\sigma_0}f(\sigma_0,\sigma_0^\dagger)+\partial j/\partial\sigma_0\,. \tag{4.15}$$

Using the relation

$$\frac{\partial^{n+m}F(\sigma_0(j,j^\dagger),\sigma_0^\dagger(j,j^\dagger);j,j^\dagger)}{\partial^n\sigma_0(j,j^\dagger)\partial^m\sigma_0^\dagger(j,j^\dagger)}$$
$$=\int d^4x_1\cdots d^4x_n d^4y_1\cdots d^4y_m\left.\frac{\delta^{n+m}F[\sigma_0(z;j(z),j^\dagger(z)),\sigma_0^\dagger(z;j(z),j^\dagger(z))]}{\prod_{i=1}^{n}\prod_{k=1}^{m}\delta\sigma_0(x_i;j(x_i),j^\dagger(x_i))\delta\sigma_0^\dagger(y_k;j(y_k),j^\dagger(y_k))}\right|_{\sigma_0^\nu(z;j(z),j^\dagger(z))=\sigma_0^\nu(j,j^\dagger)}\,. \tag{4.16}$$

For $F=j$ we find that $\partial j/\partial\sigma_0=-D_0^{-1}(0)$. Thus (4.15) gives

$$f(\sigma_0,\sigma_0^\dagger)=M^2+D_0^{-1}(0)-\sigma_0\frac{\partial}{\partial\sigma_0}f(\sigma_0,\sigma_0^\dagger)\,.$$

If the symmetry would be broken we could now use Eq. (4.14a) directly to conclude that with sources off

$$\Delta_0^{-1}(0)=\sigma_0\frac{\partial}{\partial\sigma_0}f(\sigma_0,\sigma_0^\dagger)\,. \tag{4.17}$$

Examining the derivatives of (4.17) with respect to σ_0^ν we would be able, as in Ref. 2, to find relations between the bare parameters and masses due to symmetry breaking, but now, with sources off $\sigma_0(j=0)=0$, and not necessarily $f(\sigma_0,\sigma_0^\dagger)_{j=0}=M^2$. Now, with sources off, we have

$$f(0,0)=M^2+D_0^{-1}(0)=g^2\Pi(0,m^2)\,. \tag{4.18}$$

Formally (4.18) was an identity, the integrals defining $f(\sigma_0^\dagger=\sigma=0)$ and $g^2\Pi(p^2=0,m^2)$ are identical, though divergent. When regularized, they of course yield the same pole parts. The finite parts depend on the arbitrary scale parameter, thus (4.18) is the consistency condition which tells us that the scales used in the regularized gap equation and Green's functions should be equal. If we impose the normalization condition on the propagator, e.g., fixing its value on zero external momentum, we are able to satisfy this condition by fixing κ^2. This will also determine the scale in the gap equation which does not depend on any momentum. This is not strange, the gap equation coincides with the condition for the minimum of the effective potential which is the generating functional of Green's functions on zero external momentum and thus must satisfy the same renormalization conditions as Green's functions.

The fermionic propagator remains unchanged due to the lack of the symmetry breaking and leading-order radiative corrections. The direct quartic coupling between fermions has been replaced by the exchange of the intermediate boson and the only reasonable condition fixing[2] is that which relates the four-fermion amplitude to the original quartic coupling constant $\lambda=-g^2/M^2$. If we do it on zero momentum, this implies that

$$g^2D_0(p^2=0)=-g^2/M^2\,. \tag{4.19}$$

This is satisfied if

$$\kappa^2=m^2; \tag{4.20}$$

thus $\Pi(0,m^2)=0$, and in view of (4.18) $f(0,0)=0$. The substitution (4.20) should also be done in the formulas for the renormalized effective potential and finite part of the regularized gap equation. The normalized inverse propagator (4.9) is

$$D_0^{-1}(p^2)=-M^2+\frac{g^2}{8\pi^2}p^2[B(p^2/4m^2)-\tfrac{5}{3}]\,. \tag{4.21}$$

For $p^2=0$ $B=0$ and increases as $p^2\to-\infty$ thus (4.21) is free of tachyon poles, in contradistinc-

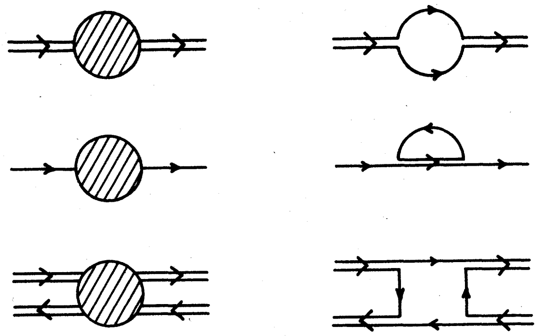

FIG. 5. Classes of diagrams which are divergent in four dimensions and lowest-order one-particle-irreducible graphs belonging to these classes.

tion to the theory with positive coupling and neutral intermediate collective modes.

Changing the sector in which collective fields occur and improving spectral properties we have not spoiled the improvement of the renormalization properties which have been found in the $(\overline{\psi}\psi)$ sector. Also in our case the fermion propagator has the usual high-momentum behavior $S_0(p) \sim \not{p}/p^2$, while the intermediate propagator behaves as $\overline{D}_0(p^2) \sim 1/p^2 \ln p^2$, thus we encounter only a finite number of kinds of superficially divergent diagrams. All of them are shown in Fig. 5. Notice that the conservation of charge exculdes diagrams with three external meson lines and that graphs with four external Fermi lines are superficially finite (see Fig. 6). This means that the original four-Fermi vertex which was eliminated in the very first step of our construction does not reappear in the result of the renormalization procedure. The same holds also in the model with $(\overline{\psi}\psi)$ modes but here we get something more: it requires particular emphasis that in our model the three-point $(\sigma\psi\psi)$ and $(\sigma^\dagger\psi\psi)$ effective vertices are superficially convergent. The reason is that if the charge is conserved then the vertex can join an intermediate line with two fermion lines or two antifermion lines but not with one fermion and one antifermion line. In such a theory it is topologically impossible to construct triangle loops consisting of one bosonic and two fermionic

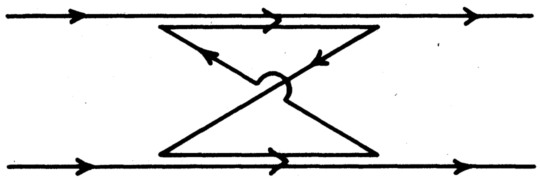

FIG. 6. Lowest-order diagram contributing to the superficially finite four-fermion vertex function.

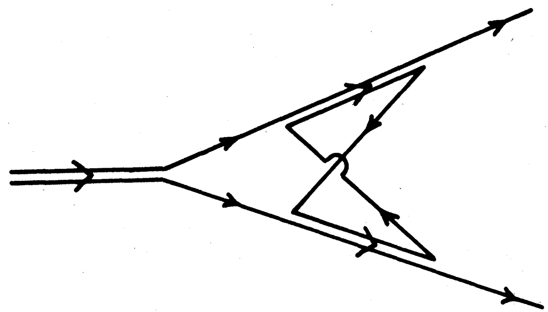

FIG. 7. Diagram contributing th the $(\sigma\overline{\psi}\,\overline{\psi})$ vertex function.

propagators. Integrations corresponding to such loops would be slightly divergent because

$$\int_0^\Lambda \frac{d^8p}{p^8 \ln p^2} \sim \ln\ln\Lambda\ .$$

This kind of divergence is typical for the theory with a neutral intermediate $(\overline{\psi}\psi)$ state which allows us to construct triangle loops. In our case the simplest one-particle-irreducible graphs contributing to the three-point $(\sigma\overline{\psi}\overline{\psi})$ function has two loops and is nonplanar, as shown in Fig. 7. Subintegrations corresponding to both loops are manifestly finite, the overall degree of divergence is zero, but the integration is superficially convergent because the diagram contains two intermediate propagators. Each of these propagators contribute a $p^2 \ln p^2$ term in the denominator. Introducing the cutoff Λ we find that the contribution from the upper limit of the integration behaves as

$$\int_0^\Lambda \frac{d^8p}{p^8 \ln^2 p^2} \sim \frac{\Lambda}{\ln\Lambda}.$$

Thus the diagram of Fig. 7 is free of overlapping divergences and does not require renormalization. The general topological rule which eliminates certain kinds of graphs is the following: Each loop containing an odd number of internal fermion lines contains an odd number of lines of the intermediate collective field. If the number of fermion lines is even, then the number of intermediate lines is also even (or zero). Notice that this condition restricts the number of lines in a loop, not in a diagram. In the model with neutral intermediate particles the number of lines of a given species in a loop was unrestricted.

The quartic $(\sigma^\dagger\sigma)^2$ vertex function is superficially divergent, but again, in consequence of our condition there are only two superficially divergent diagrams which contribute to this function. One has been represented on the bottom of Fig. 5, another will contain one self-energy insertion on the fermion line. The diagram with two intermediate internal lines will be superficially convergent for the same reason as the diagram of Fig. 7.

The coupling strength corresponding to this divergent function, which we define as the finite part of the $(\sigma^\dagger\sigma)^2$ amplitude on zero external momentum is fixed by conditions (4.20) and (4.15). Differentiating (4.15) twice, with respect to σ_0 and $\sigma_0^\dagger$ we obtain

$$\frac{\partial^3 j}{(\partial\sigma_0)^2\partial\sigma_0^\dagger} = -2\frac{\partial^2}{\partial\sigma_0\partial\sigma_0^\dagger}f(\sigma_0,\sigma_0^\dagger) - \sigma_0\frac{\partial^3}{(\partial\sigma_0)^2\partial\sigma_0^\dagger}f(\sigma_0,\sigma_0^\dagger)\,. \tag{4.22}$$

With sources off, $\partial^3 j/(\partial\sigma_0)^2\partial\sigma_0^\dagger = \Gamma^{2\sigma,2\sigma^\dagger}(0,0,0)$, the last term in (4.22) vanishes as $\sigma_0=0$ and the first gives

$$2i\int\frac{d^n p}{(m^2-p^2)^2}\,.$$

The pole part of this integral cancels if we substitute (4.3). According to (4.20) we should set $\kappa^2=m^2$ and then the finite part also vanishes, thus

$$\Gamma^{2\sigma,2\sigma^\dagger}(0,0,0)=0\,. \tag{4.23}$$

We should remember that (4.23) is the consequence of the normalization condition (4.19) imposed on the intermediate propagator.

V. FINAL REMARKS

Our results show that the mean-field-expanded quartic fermion interaction is as consistent in four-dimensional space-time as it was in two dimensions. There is no need to search for arguments justifying the occurrence of tachyon poles in auxiliary propagators. Tachyons disappear when the sign of the coupling constant is changed and intermediate collective fields suitably redefined. This fact together with the observation made in the Introduction that the expansion would be ambiguous if it could be used for both signs of the coupling constant leads to the unique conclusion: In two dimensions the model is consistent for $g^2/M^2>0$; in four dimensions for $g^2/M^2<0$. In two dimensions collective states are formed in the $(\bar\psi\psi)$ sector while in four dimensions Cooper-type $(\psi\psi)$ pairs are binded. The signs of the parameters of the theory were crucial for the argument of consistency. The sign of the coupling constant in the Lagrangian was important when we were introducing intermediate fields—the wrong choice would lead to the imaginary coupling at the vertex joining the intermediate field to fermions. It was also important in the study of the spectral properties of the propagator of the σ field. We have used the renormalization procedure which has kept the parameters of the Lagrangian as fixed numbers so that we were able to combine both conditions. We have used rather unusual renormalization procedures in which pole parts of divergent integrals were compensated by suitable redefinition of the scale parameter rather than by introducing counterterms to all couplings corresponding to divergent Green's functions. This has allowed us to treat all signs seriously as all parameters were fixed numbers of the first order of the expansion.

The use of this procedure requires some comments. As we have already stressed before we are using the mean-field expansion as the alternative procedure for evaluating the path integral defining the effective-action functional. Our expansion reduces the problem to the workable series of integrals being just Gaussian moments. The same can be done with use of the standard perturbation expansion but then the asymptotic expansion would be completely different. In the conventional approach the quadratic term entering in the Gaussian integrals would be just free part of the Lagrangian while the moments would be given by the expansion of the exponent involving the interaction part of the Lagrangian. First order of such a series does not contain any divergent integration over internal momenta. Higher-order divergences can be eliminated by suitable subtractions and then normalization which must refer to finite parameters present in lowest orders. The renormalization fails for obvious reasons if the coupling constant has the dimensionality of the inverse power of mass.

In the mean-field expansion the point vertex which in four dimensions would have the dimension of mass^{-2} is replaced by the intermediate propagator of the same dimensionality, but now the mass^2 term in the denominator appears together with the $p^2\ln p^2$ term gained from the radiative corrections. Now dimensionless polynomials built from such propagators, powers of external momenta, and fermion mass cannot have positive power-momentum behavior and the traditional argument stating nonrenormalizability does not work. Of course, the term "radiative corrections" was applied here by abuse of language. As long as we are in the first order of the expansion we cannot add corrections to anything and this is the most dangerous point of the method we have used. In the mean-field expansion divergent integrals occur in the leading order. In two dimensions we have only one logarithmically divergent integral; theory is renormalizable anyway so that we were able to use the standard pattern of the renormalization although we could avoid this, as we explain below.

In four dimensions the situation is more complicated. We need several subtractions, several normalization conditions, and the σ-field renormalization constant. The fact that we do not have any finite "free" approximation could mean that as a matter of fact we are *defining* our four-fermion theory to be just the Yukawa theory and, as long as we are using standard renormalization prescriptions, such suspicion is more than justifiable. This is why we have tried to get rid of the leading-order divergences by introducing a singular factor in the scale parameter rather than by adding counterterms: This is justified by the fact that the scale parameter should be introduced even before we write down diagrams because dimensional quantities occur in the argument of the tr ln term in the effective action from the very beginning. For example, when writing the effective potential (3.2) it should be understood that its actual form is

$$V=M^2\sigma_0^\dagger\sigma_0-\int d^nk\ln\frac{g^2\sigma_0^\dagger\sigma_0+m^2+k^2}{\mu^2}. \qquad (5.1)$$

[We have performed the notation to the Euclidean space and neglected the $(2\pi)^{-n}$ factor which is inessential for the following argument.] The integral in (5.1) is divergent, we have calculated its finite part by subtracting pole parts from the divergent derivatives of (5.1) with respect to σ^ν and then integrating the result over σ^ν. Doing so we remained in agreement with the traditional approach though it was not necessary. We could as well rewrite the integral in (5.1) in the form

$$\int d^nk\ln\frac{g^2\sigma_0^\dagger\sigma_0+m^2+k^2}{\mu^2} =\mu^n\int^{(g^2\sigma_0^\dagger\sigma_0+m^2)/\mu^2}d\alpha^2\int d^n\tau\frac{1}{\tau^2+\alpha^2}, \qquad (5.2)$$

with α and τ being dimensionless. Evaluating the integral over τ we obtain

$$\mu^n\Gamma\left(1-\frac{n}{2}\right)\pi^{n/2}\int^{(g^2\sigma_0^\dagger\sigma_0+m^2)/\mu^2}d\alpha^2(\alpha^2)^{n/2-1}. \qquad (5.3)$$

Expanding in powers of $\epsilon=2-n$ in two dimensions and $\epsilon=4-n$ in four dimensions, again neglecting the π factor we obtain

$$\left(\frac{2}{\epsilon}-\gamma\right)(g^2\sigma_0^\dagger\sigma_0+m^2)\left(1-\frac{\epsilon}{2}\ln\frac{g^2\sigma_0^\dagger\sigma_0+m^2}{\mu^2}\right)\quad\text{for } d=2$$

and

$$\frac{1}{2}\left(-\frac{2}{\epsilon}+\gamma-1\right)(g^2\sigma_0^\dagger\sigma_0+m^2)^2 \times\left(1-\frac{\epsilon}{2}\ln\frac{g^2\sigma_0^\dagger\sigma_0+m^2}{\mu^2}\right)\quad\text{for } d=4. \qquad (5.4)$$

In two dimensions we can renormalize in the conventional way because (5.1) contains the $M^2\sigma_0^\dagger\sigma_0$ term. In four dimensions we cannot because (5.1) does not involve quartic coupling. In both cases we are able to get rid of the pole part by assigning

$$\mu^2=\kappa^2e^{-2/\epsilon}. \qquad (5.5)$$

Now we can see that (5.5) can be given clear interpretation. Returning to (5.1) we find that we have just subtracted the infinite constant from the effective potential which can be interpreted as the energy density of the ground state. The essential singularity of the exponent in (5.5) at $\epsilon=0$ is not of great importance because μ^2 occurs only as the argument of the logarithm.

ACKNOWLEDGMENT

I acknowledge Dick Haymaker for valuable discussions while completing the final version of this paper.

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