Confinement and pair creation in strongly coupled quantum electrodynamics

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We study the Breit equation for two massless spin-1/2 particles interacting through an attractive Coulomb interaction $V(r) = -\alpha/r$. We find that pair production occurs for $\alpha > \alpha_{\text{critical}} = 2$, with a probability W_P given by $[\lambda = (\alpha^2/4-1)^{1/2}]$ $W_P = 1 - e^{-2\pi\lambda}\sinh\pi(\alpha/2-\lambda)/\sinh\pi(\alpha/2+\lambda)$. We briefly discuss the unobservability of free one-particle states in our simple model.

Several authors[1,2] recently studied strongly coupled quantum electrodynamics as a model for quark confinement. They find that there exists a critical coupling strength above which the ordinary vacuum becomes unstable with respect to creation of particle-antiparticle pairs. For supercritical coupling strengths, a free particle state becomes an excited state of some particle-antiparticle condensate (vacuum).[1,2] It has been argued[3] that an analogous mechanism might also take place in quantum chromodynamics, thus providing a possible explanation for the nonexistence of free quarks: upon being produced, these quarks would subsequently decay into a multiquark condensate because the quark-antiquark interaction would be supercritical.

The purpose of this work is to perform an explicit calculation of the probability of pair creation in a model similar to that considered in Refs. 1 and 2. We consider the Breit equation for two spin-$\frac{1}{2}$ massless particles interacting through a strongly attractive Coulomb interaction. Reasons for using the Breit equation rather than other two-body equations are discussed in Refs. 4 and 5. In particular, the Breit equation correctly reduces to the Dirac equation when the mass of one of the particles becomes infinite. We find that pair creation occurs for $\alpha > \alpha_{cr} \equiv 2$; furthermore, the probability for pair creation approaches 1 already for $\alpha = 2.5$, so that values of α reasonably close to the critical one would indeed lead to confinement.

We start with the Breit equation for two massless particles of spin $\frac{1}{2}$ in the center-of-mass system[5]:

$$[E - \vec{\alpha}^{(1)}\cdot\vec{p} + \vec{\alpha}^{(2)}\cdot\vec{p} - V(r)]\psi(\vec{r}) = 0 . \tag{1}$$

In Eq. (1), E is the total energy of particles 1 and 2, while $V(r)$ is the Coulomb Breit interaction:

$$V(r) = -\frac{\alpha}{r} + \frac{1}{2}\left[\vec{\alpha}^{(1)}\cdot\alpha^{(2)} + \frac{(\vec{r}\cdot\vec{\alpha}^{(1)})(\vec{r}\cdot\vec{\alpha}^{(2)})}{r^2}\right]\frac{\alpha}{r}. \tag{2}$$

Equation (1) leads to the following equations for the positive-parity ground-state ($j = l = 0$) radial wave function[5]:

$$\frac{df}{dr} + \left(\epsilon + \frac{\alpha'}{r}\right)g = 0 ,$$
$$-\left(\frac{d}{dr} + \frac{2}{r}\right)g + \left(\epsilon + \frac{\alpha'}{r}\right)f = 0 , \tag{3}$$

where

$$\epsilon = \frac{E}{2}, \tag{4}$$

$$\alpha' = \frac{\alpha}{2}. \tag{5}$$

System (3) is formally identical with the radial equation for one massless Dirac particle in an external Coulomb field, with the value of Dirac's quantum number κ taken to be 1.[6] For $\alpha' < 1$ we can therefore solve Eq. (3) by analogy with the solution to the corresponding Dirac equation.[7]

We find that the partial-wave S matrix is given by ($\alpha' < 1$)

$$S = -e^{-i\pi\gamma}\frac{1}{\gamma + i\alpha'}\frac{\Gamma(\gamma - i\alpha')}{\Gamma(\gamma + i\alpha')}, \tag{6}$$

where

$$\gamma^2 = 1 - \alpha'^2 . \tag{7}$$

In order to solve Eq. (3) for $\alpha' > 1$, we analytically continue the solution valid for $\alpha' < 1$, i.e., we take

$$\gamma = -i(\alpha'^2 - 1)^{1/2} = -i\lambda . \tag{8}$$

The physical meaning of our solution is then the following. For $\alpha' < 1$, (3) describes elastic scattering of the two particles, and one has $|S| = 1$. However, for $\alpha' > 1$, pair creation becomes possible. The probability for elastic scattering $|S|^2$ is less than 1, while from (6) and (8) the probability for pair creation W_P is given by ($\alpha' > 1$):

$$W_P = 1 - |S|^2 = 1 - e^{-2\pi\lambda} \frac{\sinh\pi(\alpha' - \lambda)}{\sinh\pi(\alpha' + \lambda)}. \qquad (9)$$

As a check of our method of solution, we have computed the probability for pair creation of massive Klein-Gordon particles in an external Coulomb field. Our result coincides with that found by using a Feynman propagator technique.[8] We wish to emphasize that an attractive feature of the analytic continuation method is that it avoids the introduction of any cutoff parameter at the quark level. On the other hand, this method can only be applied because of the singular nature of the point Coulomb field for $\alpha' > 1$ (Ref. 9) and could not be used as such to compute, for instance, pair creation is the field of a finite-size nucleus.

Strictly speaking, one gets from (9) that the probability W_P is equal to 1 only when α is infinite. However, a numerical calculation shows that $W_P = 0.999\,923$ already for $\alpha = 2.500$, so that, in practice, values of the coupling constant not far from the critical one are sufficient in order that the probability for free particles decaying into some multiparticle condensate be reasonably close to 1.

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