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Phase transition due to vortex condensation and the problem of quark confinement

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We study the phase transition of the Abelian Higgs model with many magnetic vortices, which turns out to give the Marshall-Ramond model of relativistic strings with the aid of the dual transformation technique. We show that the vortex condensation appears as a stable phase in the strong-coupling region. The relation between the dielectric permeability and the order parameter of the phase transition exhibits the antiscreening of electric charges in the vortex-condensed medium. We also demonstrate that the potential between two opposite charges is linearly rising, unless the distance between them exceeds a critical length where the two charges are liberated.

I. INTRODUCTION AND SUMMARY

Recently, Creutz and others[1] have extended the study of lattice gauge theories into weak-coupling regions, adopting Monte Carlo simulations, with an interesting indication that confinement and asymptotic freedom coexist in non-Abelian gauge theories.

In this article we investigate the problem of quark confinement in the Abelian Higgs model in the continuum. In particular we study the phase transition, which is caused by condensation of magnetic strings. In the strong-coupling phase quarks will be partially confined. We utilize the dual transformation technique so that we can study the strong-coupling phase rather than the weak-coupling region.

We first briefly sketch our ideas on quark confinement as follows.

(i) In order to confine quarks we must squeeze the electric flux between a quark and an antiquark. When the squeezing of the electric flux is achieved, the electric flux takes well-defined values at each space-time point, so that the conjugate magnetic fields should be undetermined from the uncertainty principle between electric and magnetic fields. This situation may be realized by preparing a medium where "magnetic objects" such as magnetic strings[2] or monopoles[3] are created and annihilated everywhere at any instant, which causes large fluctuation of magnetic fields.

(ii) In order to see what magnetic objects are important for confinement, we remember lattice gauge theories in which the confinement is trivially achieved, if the gauge coupling constant is large enough, both in Abelian and non-Abelian cases. Here the confinement does not depend upon whether it is Abelian or non-Abelian, but the essential point is the strong coupling. Therefore, we should take the magnetic vortex which exists both in Abelian and non-Abelian cases as the magnetic objects playing an important role for confinement, rather than magnetic monopoles which only appear in the latter case. There are two types of strings of magnetic vortices, closed and open. In the latter, we have to terminate magnetic strings with magnetic monopoles which do not appear in Abelian gauge theories but have to be introduced by hand. So, we restrict our consideration to closed strings of magnetic vortices.

(iii) Next, we consider in the strong-coupling region what phase is realized in the medium where magnetic vortex rings are created and annihilated everywhere at any instant. The phase is determined by the balance of action and entropy. We calculate the free energy F,

$$F = (\text{action}) - (\text{entropy}) , \tag{1.1}$$

where the action is given by $\alpha A/e^2$ with A the area of the world sheet of a vortex ring, e the coupling constant, and α a constant. The entropy is obtained by counting a number of states $\Omega(A)$ having the area A, as $\Omega(A) \propto e^{\beta A}$ where β is a constant depending on the space-time dimension. Therefore, the entropy βA dominates over the action in the strong-coupling region $e^2 > \alpha/\beta$, and the medium where vortices are condensed appears as the stable phase. This is a type of phase transition found by Kosterlitz and Thouless in the two-dimensional XY model.[4]

(iv) Introducing external classical charges into the medium, we show the squeezing of electric fluxes originating from the charges.

Our aim is to examine (i)–(iv) in the Abelian Higgs model without invoking lattice gauge theories. In this case a vortex is the magnetic string of Nielsen and Olesen.[2] We treat it perturbatively in the dual-transformed Abelian Higgs model which is equivalent[5] to the relativistic hydrodynamics of Kalb and Ramond and of Nambu.[6] The dual transformation[7-9] interchanges kinetic terms with mass terms, and hence it enables us to treat the self-energy of the vortex in a $1/e^2$ expansion. Throughout this paper we use a cutoff theory, i.e., we ignore small variations of string by introducing a hypercubic lattice with spacing a which is identified with the inverse of the momentum cutoff.

The phase transition in the Abelian Higgs model has been studied so far in lattice gauge theories.[10] In lattice theories magnetic monopoles are inherently included and the condensation of them causes the phase transition. It should be stressed that in this paper we study the condensation of magnetic closed strings rather than monopoles, since the conventional Abelian Higgs model does not involve monopole excitations.

We generalize the dual transformation in the Abelian Higgs model with one vorticity source[5] to the case with many vortices. We sum up various configurations of the world sheet of vortex rings, restricting ourselves only to the closed vortex with topological quantum number ± 1. We take the configuration sum by extending the method of Stone and Thomas.[11] They rewrite the configuration sum over all possible world lines of monopoles into a second-quantized scalar theory. In our case the configuration sum over all possible world sheets of closed strings is rewritten into a string theory of Marshall and Ramond.[12,13] This is the model of interacting closed strings, where the functional field $\Psi[C]$ for a closed string couples with the local field $W_{\mu\nu}(x)$ gauge invariantly in the sense of Kalb and Ramond.[6] As a result of the configuration sum, effects of the entropy appear as a negative mass squared in the action.

We calculate the effective potential for the string field $\Psi[C]$ in our dual-transformed model. We apply the Stueckelberg transformation[14] to the tensor field $W_{\mu\nu}$ which leads to $\partial^\mu D_{\mu\nu\prime\lambda\rho}(x)=0$ for the propagator of the field. This enables us to calculate the effective potential for the string field within the framework of local field theory, since the above condition leads to the automatic suppression of the diagrams including the propagator of the string field. Now we get self-energy for the string field and we study the phase transition due to condensation of magnetic vortices by comparing the self-energy with the entropy in (1.1). The order parameter $\psi_s(x)$ describing this phase transition is naturally given by a sum of the vacuum expectation value of $|\Psi[C]|^2$ over a set of curves $\{C\}$, i.e., ψ_s is the sum of the existence probability for closed strings over the set of curves passing through x in a fixed direction. We observe that the minimum of ψ_s [we denote it by $(\psi_s)_{\min}$] does not vanish and the vortex appears in the strong-coupling region $e^2 > e_c{}^2$, while $(\psi_s)_{\min}$ vanishes and the vortex does not appear in the weak-coupling region. When expressing the effective potential in terms of ψ_s (ψ_s includes $1/e^2$ in the definition), the gauge coupling e appears only in the entropy term. When e increases, the entropy term gradually dominates and $(\psi_s)_{\min}$ becomes larger. In order to obtain a physical picture of the medium with condensed vortex rings, we rewrite the system in terms of A_μ instead of $W_{\mu\nu}$ using the inverse dual transformation. We then can obtain an important relation between ψ_s and the dielectric permeability $\mathcal{E}$ of the medium:

$$\mathcal{E}=(1+\psi_s)^{-1}\,. \tag{1.2}$$

This relation immediately shows the antiscreening, as $\mathcal{E}$ takes the values less than unity in the phase with vortex condensation ($e^2 > e_c{}^2$) and it becomes smaller as e^2 increases, while $\mathcal{E}=1$ in the normal phase. We have now arrived at the effective Lagrangian similar to that of 't Hooft and of Kogut and Susskind[15] where $\mathcal{E}$ is introduced by hand.

Our mechanism is interpreted as follows in the dual-transformed model: A nonvanishing expectation value of the string field produces the mass of $W_{\mu\nu}$ fields which is added to the original mass term, just as the vacuum expectation value of the Higgs field gives a mass to gauge fields A_μ. This phenomenon is in turn interpreted as follows in the original Lagrangian: The vacuum expectation value of the string field decreases the dielectric permeability which is the coefficient of the kinetic term $-\frac{1}{4}(F_{\mu\nu})^2$. This corresponds to the phenomenon that the vacuum expectation value of the Higgs scalar decreases the coefficient of the kinetic term $-\frac{1}{2}(V_\mu)^2$ in the dual formulation. These correspondences reflect a feature characteristic of the dual transformation which interchanges the kinetic term with the mass term.

We next introduce two external classical particles with opposite charges into our medium and study the potential between them. Our dielectric permeability $\mathcal{E}$ does not vanish except for $e^2=\infty$ and differs from that assumed by 't Hooft and by Kogut and Susskind,[15] so that the Coulomb-type behavior of the electric flux still appears for finite e^2. In calculating the potential between two external charges in the strong-coupling region ($e^2 \gg e_c{}^2$) we assume A_μ to be massless in the region since we expect that the vacuum expectation value of the Higgs scalar takes a sufficiently small value as the Higgs field is repelled by the densely condensed vortex rings in our medium. For simplicity we take the potential $V(\psi_s)$ as

$$V(\psi_s) = -c_e\psi_s + d\psi_s^2 , \tag{1.3}$$

which retains the essential feature of our effective potential. In Eq. (1.3) the first term is the entropy effect and the second term is anticipated from the overlapping of strings and the requirement of renormalizability. The calculation of the effective potential suggests that d does not depend on e and only c_e does depend on e, $c_e \propto e^2$. Under these simplifications we evaluate the total energy for various solutions of Euler equations and look for the solution having the minimum energy. We take three typical solutions: (a) stringlike solution, (b) thick-tube-like solution, and (c) Coulomb-type solution. Comparison of these three cases indicates that the electric flux is squeezed and forms an electric string connecting two external charges if the distance l between the two charges is less than a critical length l_c. When the distance l becomes larger than l_c, the potential between the charges behaves like a Coulomb potential and the charges are liberated from each other. The liberation energy is, however, sufficiently large as compared with the energy of the string per unit length.

We also discuss the question of whether we can derive in the strong-coupling region an effective action similar to that of lattice gauge theories, by considering interactions of $W_{\mu\nu}$ with a closed string of magnetic flux. Although this question is partly answered in our study on some crude assumptions, a rigorous answer waits for future studies.

In Sec. II we give the dual transformation in the Abelian Higgs model with many vortices and derive the Marshall-Ramond model of string theories. In Sec. III the effective potential for the string field is calculated at the one-loop level and we study the phase transition due to vortex condensation. In Sec. IV we evaluate in the strong-coupling region the potential between two opposite external charges and obtain a linearly rising potential unless the distance between them exceeds a critical length. Section V is devoted to discussions on a relation between our model and the lattice gauge theories and on the renormalization-group equation in the presence of nonlocal vortex rings.

II. DUAL TRANSFORMATION OF THE ABELIAN HIGGS MODEL WITH MANY VORTICES

As a generalization of our earlier work,[5] in this section we present a dual transformation which relates the relativistic hydrodynamics of Kalb and Ramond and of Nambu coupled with string fields to the Abelian Higgs model with many vortices. We consider the Lagrangian of the Abelian Higgs model given by

$$\mathfrak{L} = -\tfrac{1}{4}(F_{\mu\nu})^2 + |(\partial_\mu + ieA_\mu)\phi|^2 - V(\phi) , \tag{2.1a}$$

$$V(\phi) = \mu^2\phi^\dagger\phi + \tfrac{1}{4}\lambda(\phi^\dagger\phi)^2 \quad (\mu^2 < 0) . \tag{2.1b}$$

The partition function of this model reads

$$Z \propto \int \mathfrak{D}A_\mu(x) \int \mathfrak{D}|\phi(x)|\,|\phi(x)| \times \int \mathfrak{D}\chi(x) \exp\left[i \int d^4x\, \mathfrak{L}(x)\right] , \tag{2.2}$$

where $|\phi|$ and χ are defined by $\phi = |\phi| e^{i\chi}$. The dual transformation with an antisymmetric tensor field $W_{\mu\nu}$ $(\tilde{W}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}W^{\lambda\rho})$ (Ref. 5)

$$\exp\left[i \int d^4x(-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu})\right] \propto \int \mathfrak{D}W_{\mu\nu}(x) \exp\left\{i \int d^4x \left[-\tfrac{1}{4}(m^2W_{\mu\nu}W^{\mu\nu} + 2m\tilde{W}_{\mu\nu}F^{\mu\nu})\right]\right\} . \tag{2.3}$$

reduces (2.2) to the following expression:

$$Z \propto Z^* = \int_0^{2\pi} \mathfrak{D}\chi(x) Z^*[\chi(x)] , \tag{2.4a}$$

$$Z^*[\chi(x)] \equiv \int \mathfrak{D}W_{\mu\nu}(x) \int \mathfrak{D}|\phi| \frac{1}{|\phi|^3} \exp\left[i \int d^4x\, \mathfrak{L}^*[\chi(x)]\right] , \tag{2.4b}$$

and

$$\mathfrak{L}^*[\chi(x)] = -\frac{m^2}{2e^2|\phi|^2}\frac{1}{2}(V_\mu)^2 - \tfrac{1}{4}m^2(W_{\mu\nu})^2 + \frac{1}{2}\frac{2\pi}{e}mW^{\mu\nu}\epsilon_{\mu\nu\lambda\rho}(\partial^\lambda\partial^\rho - \partial^\rho\partial^\lambda)\chi(x) + (\partial_\mu|\phi|)^2 - V(|\phi|) , \tag{2.4c}$$

where $V_\mu \equiv \partial^\nu \tilde{W}_{\nu\mu}$.[6]

The vorticity source defined by

$$\omega_{\mu\nu}(x) = \frac{1}{4\pi}\epsilon_{\mu\nu\lambda\rho}(\partial^\lambda\partial^\rho - \partial^\rho\partial^\lambda)\chi(x) \tag{2.5}$$

gives nonvanishing value only at the singular point of $\chi(x)$. Therefore, the integration over $\chi(x)$ in Eq. (2.4a) singles out all singular configurations of the $\chi(x)$ field. If the set of singularities of χ defines a world sheet $y^\mu(\tau,\sigma)$, then we have

$$\omega^{\mu\nu}(x)=n\iint d\tau\, d\sigma\left|\frac{\partial(y^\mu,y^\nu)}{\partial(\tau,\sigma)}\right|\delta^{(4)}(x-y(\tau,\sigma))\,, \tag{2.6}$$

where n is a topological quantum number.[16]

Now let us evaluate the $\chi(x)$ integral in (2.4). The difference of two singular configurations χ_s and χ_s' corresponding to the same $\omega^{\mu\nu}(x)$ is a regular one $\chi_r(x)$,

$$\chi_s(x)-\chi_s'(x)=\chi_r(x)\,, \tag{2.7}$$

so that (2.4a) is reduced to

$$Z^*=\left(\int_0^{2\pi}\mathfrak{D}\chi_r(x)\right)\sum_{\substack{\text{configuration}\\ \text{of }\omega^{\mu\nu}(x)}}Z^*[\omega(x)]\,. \tag{2.8}$$

We can omit an infinite factor $\int_0^{2\pi}\mathfrak{D}\chi_r(x)$ and this procedure is equivalent to gauge fixing. After substitution of Eqs. (2.5) and (2.6) into Eq. (2.8), the main part in (2.8) reads

$$\sum_{N=1}^{\infty}\frac{1}{N!}\sum_{S^{(1)}}\cdots\sum_{S^{(N)}}\sum_{n^{(1)}=-\infty}^{+\infty}\cdots\sum_{n^{(N)}=-\infty}^{+\infty}\exp\left\{i\frac{1}{2}\frac{2\pi}{e}m\sum_{i=1}^{N}\left[n^{(i)}\int_{S^{(i)}}d\sigma_{\mu\nu}^{(i)}(x)W^{\mu\nu}(x)\right]\right\}, \tag{2.9}$$

where we have considered the configuration made up of N connected world sheets $S^{(1)}$-$S^{(N)}$, and performed the summation over all possible configurations of $S^{(i)}$ and the summation over the topological quantum number $n^{(i)}$ for $S^{(i)}$.

In order to perform the configuration sum we extend the method of Stone and Thomas[11] in the case of pointlike excitations of monopoles to that of stringlike excitations of magnetic vortices. We define a measure to count the number of configurations of a world sheet by introducing a hypercubic lattice of spacing a, which is identified with the inverse of the cutoff Λ in momentum space. Throughout this paper we use a cutoff theory for two reasons.

(1) A complete theory of renormalization does not yet exist for a string coupled with an antisymmetric tensor field.

(2) A string in the real world has a finite width which plays a role of the cutoff.

Let us evaluate a configuration sum $K(C_1,C_2;A)$ defined by

$$K(C_1,C_2;A)=\sum_S\exp\left[i\frac{1}{2}\frac{2\pi}{e}m\int_S d\sigma^{\mu\nu}(x)W_{\mu\nu}(x)\right], \tag{2.10}$$

where the summation is taken over all configurations of the world sheet S connecting closed curves C_1 and C_2 with area A in Euclidean space-time. (We restrict the sheet S to be an orientable one without holes.) The following recursion equation holds for $K(C_1,C_2;A)$ (see Fig. 1):

$$K(C_1,C_2;A)=\sum_{\substack{\mu>0\\ \neq t}}\left[e^{i(2\pi/e)ma^2W_{\mu t}(x)}K(C_1,C_2-\delta_x^{\mu t}C;A-a^2)+e^{-i(2\pi/e)ma^2W_{\mu t}(x)}K(C_1,C_2+\delta_x^{\mu t}C;A-a^2)\right], \tag{2.11}$$

where $\delta_x^{\mu t}C$ denotes a keyboardlike variation of the curve C at the point x in the μ direction vertical to the tangential direction t of C. From Eq. (2.11) we have

$$\begin{aligned}\frac{1}{a^2}\times\frac{1}{a^2}&[K(C_1,C_2;A)-K(C_1,C_2;A-a^2)]\\
&=\sum_{\substack{\mu>0\\ \neq t}}\frac{1}{a^4}\left\{\left(1-i\frac{2\pi}{e}ma^2W_{\mu t}(x)\right)\left[K(C_1,C_2+\delta_x^{\mu t}C;A-a^2)-\left(1+i\frac{2\pi}{e}ma^2W_{\mu t}(x)\right)K(C_1,C_2;A-a^2)\right]\right.\\
&\qquad\left.-\left[K(C_1,C_2;A-a^2)-\left(1+i\frac{2\pi}{e}mW_{\mu t}(x)\right)K(C_1,C_2-\delta_x^{\mu t}C;A-a^2)\right]\right\}\\
&\quad-\frac{1}{a^4}[1-2(D-1)]K(C_1,C_2;A-a^2)\,,\end{aligned} \tag{2.12}$$

where D denotes space-time dimension, and a is supposed to be a small quantity. Equation (2.12) leads us to

$$\frac{\partial}{\partial \bar{A}} K(C_1, C_2; \bar{A}) = \left[\sum_{\mu \neq t} \left(\frac{\delta}{\delta \sigma^{\mu t}(x)} - i \frac{2\pi}{e} m W_{\mu t}(x) \right)^2 - M_0^2 \right] K(C_1, C_2; \bar{A}) \tag{2.13a}$$

$$= \left(\oint_{C_2} dx_t \right)^{-1} \left[\oint_{C_2} dx_t \sum_{\mu \neq t} \left(\frac{\delta}{\delta \sigma^{\mu t}(x)} - i \frac{2\pi}{e} m W_{\mu t}(x) \right)^2 - M_0^2 \right] K(C_1, C_2; \bar{A}) \tag{2.13b}$$

with $\bar{A} \equiv a^2 A$ and

$$M_0^2 \equiv \frac{1}{a^4}[1 - 2(D-1)]. \tag{2.14}$$

This diffusion equation (2.13) is a generalization of Foerster's equation[11] (13) for a free string extended to the case of a string interacting with an antisymmetric tensor field $W_{\mu\nu}$.

Let us notice that the negative value of M_0^2, representing entropy effects, does not immediately indicate the emergence of a phase transition. It occurs only if M_0^2 is not modified to a positive value by the self-energy, which will be calculated in the next section.

Now we solve the diffusion equation formally. Introducing an orthonormal set of eigenfunctionals $\{\Psi_n[C]\}$ and their eigenvalues $\{\omega_n^2\}$ defined by

$$\hat{H}_C \Psi_n[C] \equiv - \left[\left(\oint_C dx_t \right)^{-1} \oint_C dx_t \sum_{\mu \neq t} \left(\frac{\delta}{\delta \sigma^{\mu t}(x)} - i \frac{2\pi}{e} m W_{\mu t}(x) \right)^2 - M_0^2 \right] \Psi_n[C]$$
$$= \omega_n^2 \Psi_n[C], \tag{2.15}$$

we can write a solution for $K(C_1, C_2; \bar{A})$ as

$$K(C_1, C_2; \bar{A}) = \sum_n e^{-\omega_n^2 \bar{A}} \Psi_n[C_2] \Psi_n^*[C_1] \tag{2.16}$$

under the following normalization condition:

$$K(C_1, C_2; 0) = \sum_n \Psi_n[C_2] \Psi_n^*[C_1] = \delta(C_1, C_2). \tag{2.17}$$

$G(C_1, C_2)$ defined by[17]

$$G(C_1, C_2) = \int_0^\infty d\bar{A}\, K(C_1, C_2; \bar{A}) \tag{2.18}$$

represents a propagator of a closed string, since it satisfies

$$\hat{H}_{C_2} G(C_1, C_2) = \delta(C_1, C_2). \tag{2.19}$$

In the string theory, this propagator $G(C_1, C_2)$ can be obtained from the Lagrangian

$$\mathfrak{L}_S^*[C] \equiv - \left(\oint_C dx_t \right)^{-1} \oint_C dx_t \left[\sum_{\mu \neq t} \left| \left(\frac{\delta}{\delta \sigma^{\mu t}(x)} - i \frac{2\pi}{e} m W_{\mu t}(x) \right) \Psi[C] \right|^2 \right] - M_0^2 |\Psi[C]|^2, \tag{2.20}$$

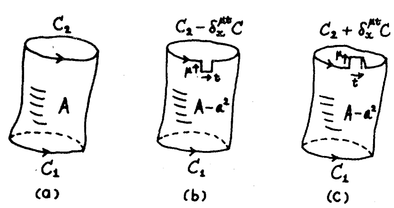

FIG. 1. (a), (b), and (c) express world sheets appearing in the definitions of $K(C_1, C_2; A)$, $K(C_1, C_2 - \delta_x^{\mu t} C_2; A)$, and $K(C_1, C_2 + \delta_x^{\mu t} C_2; A)$, respectively.

which was first considered by Marshall and Ramond.[12]

In order to rewrite the configuration sum into a partition function of (2.20), we must restrict the shape of world sheets to that of a torus. Indeed there are many other configurations of world sheets. It is, however, sufficient to take into account only the configurations of a torus in order to investigate the essence of the phase transition due to entropy effects. If we take into account more complicated configurations, the critical coupling of this phase transition will become small, since these configurations give larger entropy effects than a torus. Therefore, we neglect these complicated configurations in the following discussion. Restriction of the topological quantum number to $n=\pm 1$ comes from instability of strings with $|n|>1$. Under the above-mentioned restrictions, Eq. (2.9) reads

$$\exp\left\{\sum_{S=\text{torus}}\sum_{n=\pm 1}\exp\left[i\frac{1}{2}\frac{2\pi}{e}mn\oint_S d\sigma_{\mu\nu}(x)W^{\mu\nu}(x)\right]\right\}, \tag{2.21}$$

which is reduced to

$$\exp\left[\sum_C\int_0^\infty d\bar{A}\,\frac{1}{\bar{A}}K(C,C;\bar{A})\right]. \tag{2.22}$$

(See Fig. 2.)

Let us comment on the number of ways to trace a given torus in one direction. The factor $1/\bar{A}$ in Eq. (2.22) removes the overcounting in the choice of the starting curve C out of a set of curves representing the intermediate steps in tracing a given torus. (The number of curves in the set is equal to $\bar{A}/a^4$.) However, there is another ambiguity in selecting one set of curves representing one way for a closed curve to propagate on the given torus. This ambiguity corresponds to the number of ways to parametrize a world sheet of a string, and is attributed to the gauge degree of freedom in the string theory. With respect to this problem we must refer to the work of Kawai.[13] By substituting Eq. (2.16) into Eq. (2.22), we obtain

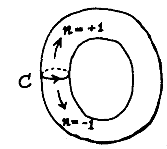

FIG. 2. Configuration of world sheets contributing to the partition function (2.25), where the two directions for tracing the torus correspond to $n=+1$ and $n=-1$.

$$\begin{aligned}
&\exp\left(\sum_n\int_0^\infty d\bar{A}\,\frac{1}{\bar{A}}\,e^{-\omega_n^2\bar{A}}\sum_C\Psi_n[C]\Psi_n^*[C]\right)\\
&\quad\propto\prod_n\frac{1}{\omega_n^2}=(\det\hat{H}_C)^{-1}\\
&\quad\propto\int\mathfrak{D}\Psi[C]\mathfrak{D}\Psi^*[C]\exp\left\{i\sum_C\left[-\left(\oint_C dx_t\right)^{-1}\oint_C dx_t\left|\left(\frac{\delta}{\delta\sigma^{\mu t}(x)}-i\frac{2\pi}{e}mW_{\mu t}(x)\right)\Psi[C]\right|^2-M_0^2|\Psi[C]|^2\right]\right\}\equiv Z_S^*\,.
\end{aligned}\tag{2.23}$$

Finally we obtain the dual form of the partition function of the Abelian Higgs model with many vortices as follows:

$$Z_{\text{Higgs}}\propto Z^*\,, \tag{2.24a}$$

$$Z^*\equiv\int\mathfrak{D}W_{\mu\nu}(x)\int\mathfrak{D}|\phi|\frac{1}{|\phi|^3}\int\mathfrak{D}\Psi[C]\mathfrak{D}\Psi^*[C]\exp\left\{i\left[\int d^4x\,\mathfrak{L}_P^*(x)+\sum_C\mathfrak{L}_S^*[C]\right]\right\}, \tag{2.24b}$$

where

$$\begin{aligned}\mathfrak{L}_P^*(x)=&-\frac{m^2}{2e^2|\phi|^2}\frac{1}{2}(V_\mu)^2-\tfrac{1}{4}m^2(W_{\mu\nu})^2\\&+(\partial_\mu|\phi|)^2-V(|\phi|)\,,\end{aligned} \tag{2.24c}$$

and $\mathfrak{L}_S^*[C]$ is defined by Eq. (2.20).

At the end of this section we remark that Eguchi and Nambu have proposed a new approach to the free string theory in which the area of the world sheet plays the role of the evolution parameter.[18] Our treatment of the string is very similar to theirs.

III. EFFECTIVE POTENTIAL FOR STRING FIELD AND PHASE TRANSITION DUE TO VORTEX CONDENSATION

In this section we evaluate the effective potential for the string field $\Psi[C]$ at the one-loop level, and discuss the phase transition due to the condensation of vortex rings.

The diagrams we are going to evaluate are given in Fig. 3. Let us compare these diagrams with those in scalar quantum electrodynamics discussed by Coleman and Weinberg.[19] In scalar quantum electrodynamics, if we take the Landau gauge the diagrams having a vertex $\varphi^\dagger\overleftrightarrow{\partial}_\mu\varphi A^\mu$ do not contribute to the effective potential. In our string theory,

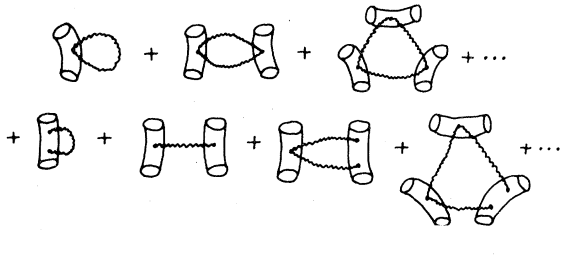

FIG. 3. Diagrams contributing to the effective potential for $\Psi[C]$ at the one-loop level, where a wavy line and a tube represent propagators of $W_{\mu\nu}$ and $\Psi[C]$, respectively.

a similar phenomenon occurs: After the Stueckelberg transformation[14] is performed, the propagator of the $W_{\mu\nu}$ field, denoted by $D_{\mu\nu,\lambda\rho}(x)$ satisfies

$$\partial^\mu D_{\mu\nu,\lambda\rho}(x)=0\,, \tag{3.1}$$

and we can show that the diagrams having a vertex of the type

$$\Psi^*[C]\frac{\overleftrightarrow{\delta}}{\delta\sigma^{\mu t}(x)}\Psi[C]W^{\mu t}(x)$$

do not contribute to the effective potential.

Proof of this statement is as follows: Let us insert an identity

$$1=\int \mathcal{D}B_\mu(x)\exp\left[\frac{i}{2\alpha}\int d^4x\left(\partial^\mu W_{\mu\nu}+\frac{1}{m}[\partial^\mu(\partial_\mu B_\nu-\partial_\nu B_\mu)+\alpha m^2 B_\nu]\right)^2\right], \tag{3.2}$$

into the integrand of the partition function (2.24b), and perform the gauge transformation of Kalb and Ramond[6]:

$$W_{\mu\nu}\to W_{\mu\nu}-\frac{1}{m}(\partial_\mu B_\nu-\partial_\nu B_\mu) \tag{3.3a}$$

and

$$\Psi[C]\to\exp\left[-i\frac{2\pi}{e}\oint_C dx^\mu B_\mu(x)\right]\Psi[C]. \tag{3.3b}$$

[In Eq. (3.2) $B_\mu(x)$ is an auxiliary field and α is a parameter.] Then Z^* [Eq. (2.4)] is rewritten as follows:

$$Z^*\propto\int\mathcal{D}W_{\mu\nu}(x)\int\mathcal{D}B_\mu(x)\int\mathcal{D}|\phi|\,\frac{1}{|\phi|^3}\int\mathcal{D}\Psi[C]\mathcal{D}\Psi^*[C]\exp\left\{i\left[\int d^4x\,\tilde{\mathcal{L}}_P^*(x)+\sum_C\mathcal{L}_S^*[C]\right]\right\}, \tag{3.4}$$

where

$$\tilde{\mathcal{L}}_P^*\equiv-\frac{1}{4}\frac{m^2}{e^2|\phi|^2}(V_\mu)^2-\tfrac{1}{4}m^2(W_{\mu\nu})^2+\frac{1}{2\alpha}(\partial^\mu W_{\mu\nu})^2-\tfrac{1}{4}(\partial_\mu B_\nu-\partial_\nu B_\mu)^2+\tfrac{1}{2}m^2\alpha(B_\mu)^2+(\partial_\mu|\phi|)^2-V(|\phi|)\,. \tag{3.5}$$

The essential point is that the auxiliary field B_μ decouples from the other fields and can be neglected in the following discussion. Here we fix m^2 to be $2e^2\langle\phi\rangle^2$, where $\langle\phi\rangle=(2\mu^2/\lambda)^{1/2}$. This corresponds to the expansion around the value $(2\mu^2/\lambda)^{1/2}$. Then the momentum representation of the free propagator of $W_{\mu\nu}$ reads

$$\hat{D}_{\mu\nu,\lambda\rho}(k)=\frac{i}{k^2-m^2}\left[g_{\mu[\lambda}g_{\nu\rho]}-(1-\alpha)\frac{g_{\mu[\lambda}k_\nu k_{\rho]}-g_{\nu[\lambda}k_\mu k_{\rho]}}{k^2-\alpha m^2}\right], \tag{3.6}$$

for an arbitrary α, and it takes a simple form for $\alpha=0$, namely,

$$\hat{D}_{\mu\nu,\lambda\rho}(k)\big|_{\alpha=0}=\frac{2i}{k^2-m^2}[I_{\mu\nu,\lambda\rho}-L_{\mu\nu,\lambda\rho}(k)]\,, \tag{3.7}$$

where

$$I_{\mu\nu,\lambda\rho}\equiv\tfrac{1}{2}g_{\mu[\lambda}g_{\nu\rho]}\,, \tag{3.8a}$$

$$L_{\mu\nu,\lambda\rho}(k)\equiv\frac{1}{2k^2}(g_{\mu[\lambda}k_\nu k_{\rho]}-g_{\nu[\lambda}k_\mu k_{\rho]})\,. \tag{3.8b}$$

Hereafter, we set $\alpha=0$, so that Eq. (3.1) is satisfied. In the graphs having a vertex of the type

$$\Psi^*[C]\frac{\overleftrightarrow{\delta}}{\delta\sigma^{\mu t}(x)}\Psi[C]W^{\mu t}(x)\,,$$

$\Psi^*[C]$ or $\Psi[C]$ belongs to an external line and $W^{\mu t}(x)$ is replaced by the propagator of $W_{\mu\nu}$. Therefore, it is sufficient to consider the following expression:

$$\sum_C \left(\oint_C dx_t \right)^{-1} \oint_C dx_t \sum_\mu \langle \Psi[C] \rangle^* \frac{\overleftrightarrow{\delta}}{\delta\sigma_{\mu t}(x)} \Psi[C] D_{\mu t,\nu s}(x-y) \,,$$

where $\langle \Psi[C] \rangle$ denotes the vacuum expectation value of $\Psi[C]$. Integration by parts leads us to

$$\sum_C \left(\oint_C dx_t \right)^{-1} \oint_C dx_t \sum_\mu \left[2 \left(\frac{\delta}{\delta\sigma_{\mu t}(x)} \langle \Psi[C] \rangle^* \right) \Psi[C] D_{\mu t,\nu s}(x-y) + \langle \Psi[C] \rangle^* \Psi[C] \frac{1}{a} \partial^\mu D_{\mu t,\nu s}(x-y) \right], \tag{3.9}$$

where we assume that the curve C is sufficiently large so that we can neglect the influence of the functional derivative on the length of C, $\oint_C dx_t$. On the assumption of

$$\frac{\delta}{\delta\sigma_{\mu t}(x)} \langle \Psi[C] \rangle = 0 \,, \tag{3.10}$$

Eq. (3.9) vanishes on account of Eq. (3.1). This assumption is similar to the assumption $\partial_\mu \langle \phi(x) \rangle = 0$ in scalar quantum electrodynamics.[19] Eventually it is sufficient to consider the graphs with an internal loop of $W_{\mu\nu}$, for calculating the effective potential for $\langle \Psi[C] \rangle$.

The relevant interaction for these graphs reads

$$-\sum_C \left(\oint_C dx_t \right)^{-1} \oint_C dx_t \sum_\mu \left(\frac{2\pi}{e} \right)^2 m^2 [W_{\mu t}(x)]^2 |\Psi[C]|^2 = -\int d^4x \, \tfrac{1}{4} m^2 \hat{\psi}_s \sum_{\mu,\nu} [W_{\mu\nu}(x)]^2 \,, \tag{3.11}$$

with

$$\hat{\psi}_s \equiv 4 \left(\frac{2\pi}{e} \right)^2 \sum_{C_{x,t}} \left(a^3 \oint_C dx_t \right)^{-1} |\Psi[C]|^2 \,, \tag{3.12}$$

where we have used a formula

$$\sum_C \oint_C dx_t F(C,x) = \int d^4x \sum_t \sum_{C_{x,t}} a^{-3} F(C,x) \,, \tag{3.13}$$

and $C_{x,t}$ denotes a closed curve passing through a point x in a fixed direction t. Now the effective potential $V(\psi_s)$ (ψ_s denotes the vacuum expectation value of $\hat{\psi}_s$) can be obtained up to one-loop level as follows:

$$V(\psi_s) = \left(\frac{e}{2\pi} \right)^2 M_0^2 \psi_s + i \sum_{n=1}^{\infty} \frac{1}{2n} (-i \tfrac{1}{2} m^2 \psi_s)^n \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\hat{D}_{\mu\nu,\lambda\rho}(k)]^n \tag{3.14a}$$

$$= -\left(\frac{e}{2\pi} \right)^2 5\Lambda^4 \psi_s + i \int_{0<|k|<\Lambda} \frac{d^4k}{(2\pi)^4} (-\tfrac{3}{2}) \ln \left[\frac{k^2 - m^2(1+\psi_s)}{k^2 - m^2} \right] \tag{3.14b}$$

$$= -\left(\frac{e}{2\pi} \right)^2 5\Lambda^4 \psi_s + \frac{3}{2(4\pi)^2} \left\{ \tfrac{1}{2} \Lambda^4 \ln \left(1 + \frac{m^2}{\Lambda^2 + m^2} \psi_s \right) + \tfrac{1}{2} \Lambda^2 m^2 \psi_s + \tfrac{1}{2} m^4 (1+\psi_s)^2 \ln \left[\frac{1+\psi_s}{\Lambda^2 m^2 + (1+\psi_s)} \right] \right\}, \tag{3.14c}$$

where the momentum cutoff Λ is identified with $1/a$. For $\Lambda^2 \gg m^2$, it is approximately equal to

$$V(\psi_s) \approx \frac{1}{(4\pi)^2} \left[(-20e^2\Lambda^4 + \tfrac{3}{2} \Lambda^2 m^2) \psi_s - \tfrac{3}{4} m^4 \psi_s^2 \ln \left(\frac{\Lambda^2}{m^2} \right) + \tfrac{3}{4} m^4 (1+\psi_s)^2 \ln(1+\psi_s) \right]. \tag{3.15}$$

Let us notice that ψ_s is originally proportional to $1/e^2$ so that the expansion of the effective potential with respect to ψ_s is considered as an expansion in $1/e^2$. Then the dual-transformed model of the Abelian Higgs model is suitable for the strong-coupling expansion. The physical meaning of ψ_s is the sum of existence probability of closed string C over a certain set of curves $C_{x,t}$. Therefore, when the minimum of the potential $V(\psi_s)$ indicates $\psi_s \neq 0$, the physically stable vacuum is filled with magnetic closed strings. The phase transition from the normal vacuum $\psi_s = 0$ to the vacuum with strings condensed $\psi_s \neq 0$ can be characterized by a mass parameter of ψ_s in Eq. (3.15):

$$M^2 \equiv \frac{1}{(4\pi)^2} \Lambda^4 \left(-20e^2 + \frac{3}{2} \frac{m^2}{\Lambda^2} \right), \tag{3.16}$$

where the first term on the right-hand side of this equation comes from the configuration sum of closed strings (derived in Sec. II) and indicates entropy effects, and the second term represents the self-energy of the string due to radiative corrections caused by the $W_{\mu\nu}$ field.

Then there exists a critical coupling e_c^2 defined by

$$e_c^2 = \frac{3}{40} \frac{m^2}{\Lambda^2}. \tag{3.17}$$

For $e^2 < e_c^2$, the normal vacuum appears $[(\psi_s)_{\min} = 0]$, while the vacuum filled with strings $[(\psi_s)_{\min} \neq 0]$ appears for $e^2 > e_c^2$). The third term proportional to $(1+\psi_s)^2 \ln(1+\psi_s)$ in the potential (3.15) prevents $(\psi_s)_{\min}$ from going to infinity for $e^2 > e_c^2$, if the second term proportional to $\psi_s^2 \ln(\Lambda^2/m^2)$ gives a finite contribution after renormalization. An interaction term $Km^4\psi_s^2$, necessary for this renormalization, may be introduced for the following reasons: The self-energy of a string with a higher topological quantum number ($n > 2$) is roughly proportional to n^2 and deviates from the simple sum of energies of n strings. This deviation may be corrected by introducing an interaction like $Km^4\psi_s^2$. This interaction is also introduced in order to correct the following overcounting problem. When two tube-like world sheets share the same contour, two different graphs from the viewpoint of propagation of closed strings correspond to one field configuration.

It is also noted that $(\psi_s)_{\min}$ becomes large as e increases, because e appears only in the entropy term. The immediate consequence of $(\psi_s)_{\min} \neq 0$ is the increase of mass for $W_{\mu\nu}$. From Eqs. (3.5) and (3.11), the mass for $W_{\mu\nu}$ denoted by m_W reads

$$(m_W)^2 = m^2[1 + (\psi_s)_{\min}]. \tag{3.18}$$

This mechanism is analogous to the Higgs mechanism. In the Higgs mechanism the vacuum expectation value of a scalar field generates the mass of the vector field, while in our model, the vacuum expectation value of a string field increases the mass of the tensor field $W_{\mu\nu}$.

Now let us relate our model to the effective-Lagrangian approach of 't Hooft and of Kogut and Susskind.[15] In their model, electric confinement occurs, if the dielectric permeability $\epsilon(\varphi)$ is assumed to vanish sufficiently fast as

$$\varepsilon(\varphi) \propto (\varphi - \varphi_{\min})^{2\alpha} \quad (\alpha \geqq 1), \tag{3.19}$$

near a minimum point $\varphi_{\min}$ of the potential. We derive in the following a very important relation between the dielectric permeability ε and the quantity ψ_s representing the degree of vortex condensation:

$$\varepsilon(\psi_s) = (1 + \psi_s)^{-1}. \tag{3.20}$$

For this purpose, we apply the inverse dual transformation to Z^* [Eqs. (2.24a)–(2.24c)]. After reviving the integration over A_μ and performing the integration over $W_{\mu\nu}$ as

$$\begin{aligned}\int \mathfrak{D}W_{\mu\nu}(x) \exp\left\{i \int d^4x \left[-\frac{1}{4}\left(m^2 \frac{1}{\varepsilon(\psi_s)} W_{\mu\nu}W^{\mu\nu} + 2m\tilde{W}_{\mu\nu}(F^{\mu\nu} - \mathcal{J}_s^{\mu\nu})\right)\right]\right\} \\ \propto \prod_x \varepsilon(\hat{\psi}_s)^3 \exp\left\{i \int d^4x \left[-\tfrac{1}{4}\varepsilon(\hat{\psi}_s)(F^{\mu\nu} - \mathcal{J}_s^{\mu\nu})^2\right]\right\},\end{aligned} \tag{3.21}$$

we obtain the following expression for Z:

$$Z^* \propto Z', \tag{3.22a}$$

$$Z' = \int \mathfrak{D}A_\mu \int \mathfrak{D}|\phi|\,|\phi| \int \mathfrak{D}\Psi[C]\mathfrak{D}\Psi^*[C] \prod_x \varepsilon(\hat{\psi}_s)^3 \exp\left\{i\left[\int d^4x\, \mathcal{L}_P(x) + \sum_C \mathcal{L}_S[C]\right]\right\}, \tag{3.22b}$$

where

$$\mathcal{L}_P(x) = -\tfrac{1}{4}\varepsilon(\hat{\psi}_s)(F^{\mu\nu} - \mathcal{J}_s^{\mu\nu})^2 + e^2|\phi|^2(A_\mu)^2 + (\partial_\mu|\phi|)^2 - V(|\phi|), \tag{3.22c}$$

$$\mathcal{L}_S[C] = -\left(\oint_C dx_t\right)^{-1} \oint_C dx_t \left[\sum_{\mu \neq t} \left|\frac{\delta}{\delta\sigma^{\mu t}(x)}\Psi[C]\right|^2\right] - M_0^2|\Psi[C]|^2, \tag{3.22d}$$

and

$$J_s^{\mu\nu}(x) \equiv -2i\left(\frac{2\pi}{e}\right) \sum_{C_{x,\nu}} \left(a^3 \oint_C dx_t\right)^{-1} \Psi^*[C] \frac{\overleftrightarrow{\delta}}{\delta\sigma^{\mu t}(x)} \Psi[C]. \tag{3.22e}$$

Equations (3.21) and (3.22b) show that $\varepsilon(\psi_s)$ defined by (3.20) plays the role of the dielectric permeability. $\varepsilon(\psi_s)$ takes the value unity for $e^2 < e_c^2$, while it becomes smaller than unity as e^2 increases beyond e_c^2.

At the end of this section we compare two representations, Z' [Eq. (3.22)] and Z [Eq. (2.24)] from the viewpoint of the dual transformation. The nonvanishing expectation value of the Higgs field $\langle\phi\rangle$ makes the gauge field A_μ massive in the original representation Z' (Higgs mechanism), while it is a multiplicative factor of the kinetic term in the dual representation Z^* and behaves as a viscosity for the velocity vector V_μ in relativistic hydrodynamics. As for the vacuum expectation value of the string field $\Psi[C]$, especially ψ_s, it increases the mass of $W_{\mu\nu}$ in Z^* (analogous to the Higgs mechanism). On the other hand, in Z', ψ_s changes the value of the dielectric permeability $\varepsilon(\psi_s)$ which is a multiplicative factor of the kinetic term $-\frac{1}{4}(F_{\mu\nu})^2$ and is considered as a viscosity for the electric field.

IV. PARTIAL CONFINEMENT POTENTIAL FOR EXTERNAL CLASSICAL CHARGES IN THE STRONG-COUPLING REGION

In this section we evaluate the potential between two classical particles with opposite charges introduced into the vacuum with vortices condensed. For this purpose we simplify our Lagrangian in the strong-coupling region $e^2 \gg e_c^2$ [see Eqs. (3.22) and (3.15)] into the following one:

$$\mathcal{L} = -\tfrac{1}{4}\varepsilon(\psi_s)(F_{\mu\nu})^2 + J_\mu^{\text{ext}} A^\mu - V(\psi_s)\,, \tag{4.1}$$

where $\varepsilon(\psi_s) = (1+\psi_s)^{-1}$, J_μ^{ext} denotes the external electric current, and $V(\psi_s)$ is the effective potential for the string field ψ_s.

The effective potential (3.15) evaluated in the last section possesses the following properties:

(1) The gauge coupling e appears only in the linear term for ψ_s, and the sign of this term determines whether the vortices are condensed or not.

(2) Even in the strong-coupling region $e^2 > e_c^2$, $(\psi_s)_{\min}$ must be finite.

Keeping these properties, we take the following simple form for $V(\psi_s)$ instead of (3.15):

$$V(\psi_s) = -c_e \psi_s + d\psi_s^2\,, \tag{4.2a}$$

where

$$\begin{aligned} c_e &\equiv \frac{1}{(4\pi)^2}(20e^2\Lambda^4 - \tfrac{3}{2}\Lambda^2 m^2) \\ &= c(e^2 - e_c^2) \\ &\xrightarrow[e^2 \gg e_c^2]{} ce^2 \end{aligned} \tag{4.2b}$$

and the positive constant d does not depend on the coupling constant e.

We also comment on the absence of $e^2|\phi|^2(A_\mu)^2$ in (4.1) for $e^2 \gg e_c^2$. In this strong-coupling region, the density of vortices is so large that $e^2|\phi|^2$ takes the value sufficiently inside the vortex. Therefore, the mass of the A_μ field can be neglected in comparison with m.

Introducing the electromagnetic induction tensor $D_{\mu\nu}$ by

$$D_{\mu\nu} \equiv \varepsilon(\psi_s) F_{\mu\nu}\,, \tag{4.3}$$

we obtain the following Euler equations for our model [Eq. (4.1)]:

$$\partial^\mu D_{\mu\nu} + J_\nu^{\text{ext}} = 0\,, \tag{4.4}$$

$$\frac{dV}{d\psi_s} - \tfrac{1}{4}(D_{\mu\nu})^2 = 0\,. \tag{4.5}$$

We put $+Q$ charge at $\vec{x} = -\vec{l}/2$ and $-Q$ charge at $\vec{x} = +\vec{l}/2$, so that we have

$$J_0^{\text{ext}} = Q\left[\delta^{(3)}\left(\vec{x} + \frac{\vec{l}}{2}\right) - \delta^{(3)}\left(\vec{x} - \frac{\vec{l}}{2}\right)\right], \tag{4.6a}$$

$$D^{0i} \equiv (\vec{D})^i \quad (i = 1,2,3)\,, \tag{4.6b}$$

$$\vec{J}^{\text{ext}} = \vec{0} \text{ and } D^{ij} = 0 \quad (i,j = 1,2,3)\,. \tag{4.6c}$$

Then Eqs. (4.4) and (4.5) are reduced to

$$\vec{\nabla} \cdot \vec{D} = J_0^{\text{ext}}\,, \tag{4.7}$$

$$\frac{dV}{d\psi_s} + \tfrac{1}{2}(\vec{D})^2 = 0\,. \tag{4.8}$$

Equation (4.8) shows that the introduction of the electric flux $\vec{D}$ changes the vacuum structure: ψ_s in the presence of $\vec{D}$ is obtained as the minimum of the following modified potential:

$$V_{\vec{D}}(\psi_s) \equiv V(\psi_s) + \tfrac{1}{2}(\vec{D})^2 \psi_s\,. \tag{4.9}$$

Here $\frac{1}{2}(\vec{D})^2\psi_s$ is considered as an additional mass term for ψ_s. Therefore, when $(\vec{D})^2$ is sufficiently large,

$$\tfrac{1}{2}(\vec{D})^2 > c_e\,, \tag{4.10}$$

the vacuum becomes normal $[(\psi_s)_{\min} = 0]$ (see Fig. 4). This mechanism is desirable for electric confinement, because the electric flux originating from an external charge tends to flow in a thin tunnel of the normal vacuum dug in the condensed vacuum. There are many solutions satisfying the two equations (4.7) and (4.8). In order to find the physically realized solution, we must compare the total energy

$$H = \int d^3x [\tfrac{1}{2}(1+\psi_s)(\vec{D})^2 + V(\psi_s)] \tag{4.11}$$

and select the minimum-energy solution. In the following, let us study three typical solutions: (A) stringlike, (B) thick-tube-like, and (c) Coulomb-type solutions.

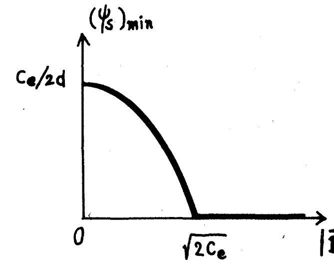

FIG. 4. Relation between $(\psi_s)_{\min}$ and $|\vec{D}|$ with the potential of (4.2a) in the strong-coupling region.

A. Stringlike solution

This solution is axially symmetric. We use cylindrical coordinates (r, θ, z) and put the external charges $+Q$ and $-Q$ at $z=-l/2$ and $z=+l/2$, respectively. The nonvanishing component D_z is assumed as follows:

$$D_z = \begin{cases} \dfrac{Q}{\pi w^2} & \text{for } 0 \leqq r \leqq w\,,\ -\dfrac{l}{2} \leqq z \leqq \dfrac{l}{2}\,, \\ 0\,, & \text{otherwise}\,. \end{cases} \tag{4.12}$$

We further assume that $|D_z|$ for $0 \leqq r \leqq w$ and $-l/2 \leqq z \leqq l/2$ satisfies the condition (4.10). On this assumption, the solution of Eq. (4.8) becomes very simple:

$$\psi_s = \begin{cases} 0 \text{ for } 0 \leqq r \leqq w\,,\ -\dfrac{l}{2} \leqq z \leqq \dfrac{l}{2}\,, \\ \dfrac{c_e}{2d}\,, \quad \text{otherwise}. \end{cases} \tag{4.13}$$

Then the total energy measured from the vacuum value reads

$$H_A(w) = \left[\frac{1}{2}\left(\frac{Q}{\pi w^2}\right)^2 + \frac{c_e^{\,2}}{4d}\right]\pi w^2 l\,, \tag{4.14}$$

which takes the minimum value

$$H_A = \frac{Q^2}{\pi w_A^{\,2}}\, l \tag{4.15}$$

at

$$w = w_A \equiv \sqrt{2}\left(\frac{d}{c_e}\right)^{1/4} w_0\,. \tag{4.16}$$

Here w_0 is defined by

$$\frac{1}{2}\left(\frac{Q}{\pi w_0^{\,2}}\right)^2 = c_e \text{ or } w_0 = 4\left(\frac{Q^2}{2\pi^2 c_e}\right)^{1/2}.$$

The condition (4.10) is satisifed when the coupling constant e is sufficiently large and an inequality $c_e > 4d$ holds.

The solution (A) obtained here represents an electric string with the transverse width w_A and the potential between the charges is linear rising. It is worthwhile to notice that w_A is a width where two forces are balanced: One force comes from the string potential and tends to narrow the normal region and the other force comes from the electric field and tends to widen the normal region. This phenomenon may be called the dual Meissner effect,[20] because the usual Meissner effect is obtained by interchanging the roles of $\vec{D}$ and $\vec{B}$ and the roles of ψ_s and $|\phi|$.

B. Thick-tube-like solution

To obtain a thick-tube-like solution, we choose the nonvanishing components of $\vec{D}$ as follows:

$$D_z = \frac{Q}{\pi w^2} \quad \text{in the region I}\left(0 \leqq r \leqq w\,, -\frac{l}{2} + \delta \leqq z \leqq \frac{l}{2} - \frac{\delta}{2}\right), \tag{4.17a}$$

$$D_r = \frac{Q}{2\pi\delta}\,\frac{w^2 - r^2}{w^2 r} \quad \text{in the region II}\left(0 \leqq r \leqq w, -\frac{l}{2} \leqq z \leqq -\frac{l}{2} + \delta\,, \text{ and } \frac{l}{2} - \delta \leqq z \leqq \frac{l}{2}\right), \tag{4.17b}$$

where we assume

$$w^2 \gg w_0^{\,2} \tag{4.18}$$

in order that D_z does not satisfy the condition (4.10). We further assume δ to be very small and of the same order as the string width w_A of the solution (A). The r dependence of D_r is determined so as to satisfy the conservation law of the electric flux.

In the region I, where the vacuum is filled with vortices because of (4.18), the energy is given by

$$H_{\mathrm{I}} = \int d^3x \left[\frac{1}{2}\left(1 + \frac{c_e}{2d}\right)\vec{D}^2 - \frac{1}{16d}(\vec{D}^2)^2\right] \tag{4.19a}$$

$$= \frac{Q^2(l - 2\delta)}{\pi w^2}\left[\frac{1}{2}\left(1 + \frac{c_e}{2d}\right) - \frac{Q^2}{16\pi^2 d}\,\frac{1}{w^2}\right]. \tag{4.19b}$$

The region II is divided into a normal region $(0 \leqq r \leqq r_c)$ and a region filled with vortices $(r_c \leqq r \leqq w)$. From Eqs. (4.10) and (4.17b), the critical radius r_c is known to be the root of the following equation:

$$r_c^{\,2} + 4\sqrt{2}\left(\frac{d}{c_e}\right)^{1/4} \frac{w^2}{w_0}\, r_c - w^2 = 0\,. \tag{4.20}$$

If we restrict ourselves to the case

$$w^2 \gg \left(\frac{c_e}{d}\right)^{1/2} w_0^{\,2}\,, \tag{4.21}$$

we obtain a simple solution

$$r_c^{\,2} = \frac{1}{8}\left(\frac{c_e}{d}\right)^{1/2} w_0^{\,2}\,. \tag{4.22}$$

The energy in the region II is evaluated as follows:

$$H_{\rm II}=\frac{Q^2}{2\pi\delta}\left(\tfrac{1}{2}\ln\frac{w^2}{\delta^2}-\frac{11}{4}+\frac{\delta^2}{w^2}-\frac{\delta^4}{4w^4}\right)+\frac{Q^2}{2\pi\delta}\frac{c_e}{2d}\left[\tfrac{1}{2}\ln\frac{w^2}{r_c^2}-\frac{3}{4}+\frac{1}{2}\left(\frac{r_c^2}{w^2}\right)\times\left(2\ln\frac{w^2}{r_c^2}+\frac{1}{3}+\frac{5}{2}\frac{r_c^2}{w^2}-\frac{r_c^4}{w^4}+\frac{1}{6}\frac{r_c^6}{w^6}\right)\right]. \tag{4.23}$$

This is approximately equal to

$$H_{\rm II}\approx\frac{Q^2}{2\pi\delta}\frac{c_e}{4d}\left(\ln\frac{w^2}{r_c^2}\right), \tag{4.24}$$

in the strong-coupling limit $c_e \gg d$ under the condition (4.21)

$$w_A{}^2=\delta^2\ll r_c{}^2\ll w^2\,. \tag{4.25}$$

In the same limit (4.25), $H_{\rm I}$ reads

$$H_{\rm I}\approx\frac{Q^2}{2\pi\delta}\frac{c_e}{2d}\left(\frac{\delta l}{w^2}\right). \tag{4.26}$$

Now the total energy $H_B=H_{\rm I}+H_{\rm II}$ of the solution (B) takes the minimum value

$$H_B\approx\frac{Q^2}{2\pi\delta}\frac{c_e}{4d}\left[\ln\left(\frac{2\delta l}{r_c^2}\right)+1\right], \tag{4.27}$$

at $w_B{}^2=2\delta l$. Consistency of this solution with (4.21) and (4.25) holds if

$$l^2\gg\frac{r_c{}^4}{4d^2}=\frac{1}{29}\left(\frac{c_e}{d}\right)^{3/2}w_0{}^2\,. \tag{4.28}$$

This solution (B) expresses a logarithmic potential.

C. Coulomb-type solution

The dielectric permeability (3.19) assumed by 't Hooft and by Kogut and Susskind excludes the existence of Coulomb-type solutions.[14] The dielectric permeability ε derived in our model [Eq. (3.20)] takes the value less than the normal value unity in the region where $\frac{1}{2}\vec{D}^2<c_e$, but does not vanish except for an extreme case $e^2=\infty$. Therefore, our model allows the existence of a Coulomb-type solution.

The electric flux in the Coulomb-type solution is given by

$$\vec{D}=\frac{Q}{4\pi}\left(\vec{\nabla}\frac{1}{|\vec{x}-\vec{l}/2|}-\vec{\nabla}\frac{1}{|\vec{x}+\vec{l}/2|}\right). \tag{4.29}$$

The normal region appears in the neighborhood of external sources:

$$\left|\vec{x}\pm\frac{\vec{l}}{2}\right|^2<\rho_c{}^2\equiv\frac{w_0{}^2}{4}, \tag{4.30}$$

where we assume that $l^2\gg\rho_c{}^2$. The total energy (4.11) consists of two parts H_v and H_n, where H_v comes from the condensed region

$$H_v=\int_{|\vec{x}\pm\vec{l}/2|>\rho_c}d^3x\left[\frac{1}{2}\left(1+\frac{c_e}{2d}\right)\vec{D}^2-\frac{1}{16d}(\vec{D}^2)^2\right], \tag{4.31}$$

and H_n comes from the normal region

$$H_n=\int_{\delta<|\vec{x}\pm\vec{l}/2|<\rho_c}d^3x\left(\tfrac{1}{2}\vec{D}^2+\frac{c_e}{4d}\right). \tag{4.32}$$

In Eq. (4.22), we also use the cutoff δ identified with ω_A, because the region $|\vec{x}\pm\vec{l}/2|<\delta$ is extremely close to the external charges and the energy in this region is considered to be common in all the solutions (A)–(C).

The most dominant terms with respect to ρ_c/l and δ/ρ_c of H_v and H_n are evaluated as follows:

$$H_v\approx\frac{Q^2}{4\pi}\left(1+\frac{c_e}{2d}\right)\left(\frac{1}{\rho_c}-\frac{1}{l}\right)-\frac{1}{16d}\frac{Q^4}{(4\pi)^3}\frac{2}{5}\frac{1}{\rho_c{}^5}, \tag{4.33}$$

$$H_n\approx\frac{Q^2}{4\pi}\left(\frac{1}{\delta}-\frac{1}{\rho_c}\right)+\frac{c_e{}^2}{2d}\frac{4\pi}{3}(\rho_c{}^3-\delta^3)\,. \tag{4.34}$$

Then we have

$$H_C=H_v+H_n\approx\frac{Q^2}{4\pi\rho_c}\frac{c_e}{2d}\left(\frac{16}{15}-\frac{\rho_c}{l}\right), \tag{4.35}$$

which describes the Coulomb potential between the external charges $+Q$ and $-Q$. The approximations used in deriving Eq. (4.35) are valid under the conditions $l^2\gg\rho_c{}^2$ and $c_e\gg d$, which guarantees $\rho_c{}^2\gg\delta^2=w_A{}^2$.

Now we compare three solutions (A)–(C) and select the minimum-energy solution. The distance $l_{\rm AC}$, where the energies of two solutions (A) and (C) coincides, is obtained on the assumption $l_{\rm AC}\gg\rho_c$ as follows:

$$l_{\rm AC}=\frac{w_A{}^2}{\rho_c}\frac{c_e}{d}\frac{2}{15}=\frac{8}{15}\left(\frac{c_e}{d}\right)^{1/2}w_0\,. \tag{4.36}$$

This is consistent with the assumption $l_{\rm AC}\gg\rho_c$ in the strong-coupling limit. The stringlike solution is realized for $l<l_{\rm AC}$, while the Coulomb-type solution for $l>l_{\rm AC}$.

The solution (B) is solved only in the region (4.28), where its energy is sufficiently larger than the asymptotic value of the Coulomb-type solution (C):

$$H_B>\frac{Q^2}{2\pi w_0}\frac{1}{4\sqrt{2}}\left(\frac{c_e}{d}\right)^{5/4}\gg H_C(l=+\infty)\simeq\frac{Q^2}{2\pi w_0}\frac{8}{15}\left(\frac{c_e}{d}\right). \tag{4.37}$$

The distance $l_{\rm AB}$, where the energies of the solutions (A) and (B) coincide, is estimated as follows:

$$l_{\rm AB}=\frac{1}{4\sqrt{2}}\left(\frac{c_e}{d}\right)^{3/4}w_0\,. \tag{4.38}$$

This length is sufficiently larger than l_{AC} characterizing the transition from linear to Coulomb potential, so that we exclude the solution (B).

Now, we obtain a partial confinement potential in the strong-coupling limit $c_e \gg d$: In the region $l \lesssim l_{AC}$, two charges are connected by an electric string whose energy per unit length is equal to

$$\frac{1}{2\pi\alpha'} = \frac{Q^2}{\pi w_A{}^2} = Q\sqrt{2d}\left(\frac{c_e}{d}\right), \tag{4.39}$$

where we have used tentatively the notation α' expressing the slope of the Regge trajectory of meson states. When $l > l_{AC}$, the potential energy of the system becomes Coulombic. The asymptotic value $H_C(l=\infty)$, which is the energy necessary to liberate one external charge from the other, has the following relation with α':

$$H_C(l=\infty) = \frac{4Q}{15\pi}\left(\frac{c_e}{d}\right)^{3/4}\frac{1}{\sqrt{\alpha'}} \gg \frac{1}{\sqrt{\alpha'}}\,. \tag{4.40}$$

Finally, we give some remarks. Although we have neglected it in this paper, there may exist a kinetic term for ψ_s. This, however, does not change the essential feature of the potential between external charges. It only moderates the transition between the normal region and the condensed region.

The aim of this paper is only to show the appearance of a linear potential between external charges as a result of the condensation of the magnetic vortices in the strong-coupling region, and we do not intend to obtain qualitative results. The qualitative study of our model is left to future investigations.

V. DISCUSSION

In this section we discuss several points not studied in detail in this paper. The first point is on the renormalization problem or on the renormalization-group equations. We have used a momentum cutoff Λ for regularizing the ultraviolet (UV) divergences. In addition to UV divergences coming from the high-momentum region of the $W_{\mu\nu}$ propagator, there also exist UV divergences appearing from small variations of world sheets of strings. Therefore, the renormalization should include those of local field theory and those of non-local field theory.

In order to consider this problem, it will be helpful to examine renormalization in soliton theories. The renormalizability of soliton theories has already been studied by several authors.[21] If we can renormalize our model and formulate renormalization-group equations with respect to the gauge coupling constant and the mass of the vortex ring in a similar manner as was done in the two-dimensional XY model,[22] we will be able to clarify the properties of the phase transition due to vortex condensation.

The next point is whether we can derive in the strong-coupling region an effective action similar to that of lattice gauge theories by considering interactions of $W_{\mu\nu}$ with a closed magnetic string or not. Characteristic features of lattice gauge theories, confinement, strong-coupling expansion, and others, come from the following cosine-type action[23]:

$$\begin{aligned} S_{\text{lattice}} &= \frac{1}{2e^2}\sum_{x,\mu\nu}\cos[ea^2F_{\mu\nu}(x)] \\ &= \text{const} + \frac{1}{2e^2}\sum_{x,\mu\nu}\{-\tfrac{1}{2}[ea^2F_{\mu\nu}(x)]^2 + \cdots\}, \end{aligned} \tag{5.1}$$

which is periodic with respect to a shift, $eaA_\mu(x) \to eaA_\mu(x) + 2\pi$ (a = lattice constant).[24]

So far we have considered only the vacuum expectation value of $\Psi[C]$, namely, ψ_s. Even in this case the interaction of $(F_{\mu\nu})^2$ with ψ_s has introduced terms with higher power of $(F_{\mu\nu})^2$ into the action through Euler equations [see, for example, Eq. (4.3)]. Taking into account quantum flucutation of $\Psi[C]$, we evaluate the effective potential for $W_{\mu\nu}$ at the one-loop level.

A set of graphs to be considered is given in Fig. 5, where the torus represents a closed path of a string. Contribution of these graphs to the partition function is given by

$$\frac{\det\hat{H}_c^0(M^2)}{\det\hat{H}_c(M^2)} = \exp\left[-\mathrm{Tr}\ln\frac{\hat{H}_c(M^2)}{\hat{H}_c^0(M^2)}\right], \tag{5.2}$$

where $\hat{H}_c(M^2)$ denotes the operator defined by Eq. (2.15) and $\hat{H}_c^0(M^2)$ is the corresponding operator in a free string theory ($1/e=0$). On the following assumptions,

$$\langle W_{\mu\nu}\rangle = 0 \text{ and } \langle W_{\mu\nu}W^{\mu\lambda}\rangle = g_\nu^\lambda W^2\,, \tag{5.3}$$

$\hat{H}_c(M^2)$ can be replaced by $\hat{H}_c^0(M^2 + (2\pi/e)^2m^2W^2)$. Although our operator $\hat{H}_c^0$ is identical to that of Marshall and Ramond,[12] that is,

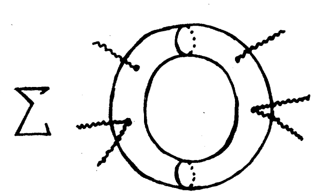

FIG. 5. Diagrams contributing to the effective potential for $W_{\mu\nu}$ at the one-loop level, where the torus represents a track of closed-string propagation.

$$\hat{H}_c^0(M^2)=\int_0^{2\pi}d\sigma\frac{1}{[-(x'(\sigma)^2]^{1/2}}\left[\frac{1}{(2\pi)^2}\left(g_{\mu\nu}-\frac{x'_\mu(\sigma)x'_\nu(\sigma)}{[x'(\sigma)]^2}\right)\frac{\delta^2}{\delta x_\mu(\sigma)\delta x_\nu(\sigma)}-M^2[x'(\sigma)]^2\right]\bigg/\int_0^{2\pi}d\sigma\{-[x'(\sigma)]^2\}^{1/2}, \quad (5.4)$$

we replace $\hat{H}_c^0$ by the following one $\hat{H}_c^{0\prime}$ for the sake of simplicity:

$$\hat{H}_c^{0\prime}(M^2)=\int_0^{2\pi}\frac{d\sigma}{2\pi}\left\{\frac{\delta^2}{\delta x_\mu(\sigma)\delta x^\mu(\sigma)}-(2\pi)^2M^2[x'(\sigma)]^2\right\}, \quad (5.5)$$

which leads to the string propagator of Kaku and Kikkawa.[25]

The eigenvalues of the new operator $\hat{H}_c^{0\prime}$ reads

$$\omega_{p_E,\{m_{n\mu}\}}{}^2=p_E{}^2+\sum_{n=0}^{\infty}\sum_{\mu=0}^{3}n(2m_{n\mu}+1)2\pi M, \quad (5.6)$$

where ω^2 are labeled by Euclidean momenta p_E and by the number of excitations $m_{n\mu}$ for the harmonic oscillator with n nodes in the μ direction. Then, Eq. (5.2) is reduced to

$$\int\frac{d^4p_E}{(2\pi)^4}\sum_{\{m_{n\mu}\}}\ln\left\{\frac{p_E{}^2+2\pi[M^2+(2\pi/e)^2m^2W^2]^{1/2}\sum_{n\mu}n(2m_{n\mu}+1)}{p_E{}^2+2\pi M\sum_{n\mu}n(2m_{n\mu}+1)}\right\}. \quad (5.7)$$

As is known from the dual transformation (3.21), $\langle m\tilde{W}_{\mu\nu}\rangle$ is identical to $\langle\varepsilon(\hat{\psi}_s)F_{\mu\nu}\rangle=\langle D_{\mu\nu}\rangle$.[5,7] Therefore, (5.7) expresses a complicated function of $(F_{\mu\nu})^2$. Although this expression is as yet far from the action (5.1), our present discussion is suggestive for future study to connect lattice gauge theories with the continuum gauge theories.

At the end we would like to suggest to the solid-state experimentalists to search for materials that may sustain the condensation of closed magnetic vortices of Abrikosov[2] and then attempt to squeeze external electric fluxes under suitable circumstances. Since the Landau-Ginzburg theory of superconductivity[26] is identical to the Abelian Higgs model studied in this paper, such an experiment will lead to valuable results.

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