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# Necessity of many-particle forces in relativistic Newtonian mechanics for massive and massless particles

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The Lorentz-invariance conditions for Newtonian equations of motion for three particles are assumed to be satisfied by sums of two-particle forces that satisfy the Lorentz-invariance conditions for two particles. Then it is shown that a particle can be accelerated only by forces from particles that do not accelerate, provided every particle has positive mass. There are exceptional cases when one or more of the particles has zero mass. Relativistic Newtonian mechanics for zero-mass particles is formulated two different ways. When the equations of motion specify the accelerations as functions of the positions and velocities, the result is the same as for positive-mass particles. When the time derivatives of the momenta are specified as functions of the positions and momenta, the result is that a particle can be accelerated only by forces from particles that do not accelerate continuously. However, there are forces that change the magnitude of the momentum without changing the velocity, for a particle with zero mass. They produce discontinuous acceleration when the velocity abruptly changes direction as the momentum reaches zero and changes sign. For a particle accelerated by a force from a massless particle that accelerates in this discontinuous way, there are two-particle forces with acceleration of both particles.

## INTRODUCTION

The conditions for Newtonian equations of motion to be Lorentz invariant are nonlinear equations for the functions that give the accelerations in terms of the positions and velocities at one time.[1-3] As soon as these conditions were formulated,[1,2] it was realized that they require many-particle forces for systems of more than two particles because the nonlinearity makes it impossible to use sums of two-particle forces.[2] However, a proof was not published.

Here we state this property precisely and give a proof. The point is to see if there are any exceptions, any two-particle forces that escape this implication of the nonlinearity, so that forces in systems of more than two particles can be sums of these two-particle forces. The obvious trivial exception is when a particle is accelerated by forces from particles that do not accelerate; then there are no nonlinear terms. Even exceptions of only a technical nature, which are not particularly meaningful physically, might be interesting because they would indicate what could and could not be proved in more sophisticated formulations. In Hamiltonian mechanics with constraints it was shown recently that for a system of more than two particles there are no forces that are sums of two-particle forces, and techniques were developed to calculate the many-particle forces.[4,5]

We consider two-particle forces, which satisfy the Lorentz-invariance conditions for systems of two particles, and we assume that sums of these two-particle forces satisfy the Lorentz-invariance conditions for systems of three particles. We show that then a particle can be accelerated only by forces from particles that do not accelerate, provided every particle has positive mass. There are exceptional cases when one or more of the particles has zero mass.

For particles with zero mass we formulate relativistic Newtonian mechanics two different ways. One way is to let the equations of motion specify the accelerations as functions of the positions and velocities. Then the result is the same as for particles with positive mass. The other way is to let the equations of motion specify the time derivatives of the momenta as functions of the positions and momenta. Then the result is that a particle can be accelerated only by forces from particles that do not accelerate continuously. However, the forces that cause no acceleration are not necessarily zero. They can change the magnitude of the momentum without changing the velocity, for a particle with zero mass, because the velocity is a vector of fixed length $c$, in the direction of the momentum, and the magnitude of the momentum is an additional variable independent of the velocity. If the momentum reaches zero and changes sign, the velocity abruptly reverses direction, so there is discontinuous acceleration. Thus, when a particle is accelerated by a force from a massless particle that accelerates in this discontinuous way, we find our assumptions allow two-particle forces with acceleration of both particles. These discontinuous accelerations can produce bound states of two massless particles.

## MASSIVE PARTICLES

Consider a classical-mechanical system of $N$ particles described by positions $\vec{x}^n$, velocities

$\vec{v}^n = d\vec{x}^n/dt$, and Newtonian equations of motion that give the accelerations as functions of the positions and velocities at one time,

$$d\vec{v}^n/dt = \vec{f}^n(\vec{x}^1, \vec{x}^2, \ldots, \vec{x}^N, \vec{v}^1, \vec{v}^2, \ldots, \vec{v}^N)\,, \tag{1}$$

for $n = 1, 2, \ldots, N$. We make these equations invariant for time translations by not letting $\vec{f}^n$ depend explicitly on time, for space translations by letting $\vec{f}^n$ depend on the positions only through the relative positions $\vec{x}^n - \vec{x}^m$, and for rotations by letting $\vec{f}^n$ be a vector function (that rotates as a vector when $\vec{x}^1, \ldots, \vec{x}^N, \vec{v}^1, \ldots, \vec{v}^N$ rotate). The Currie-Hill conditions for the equations of motion to be Lorentz invariant[1,2] are

$$\sum_{m=1}^{N}\sum_{l=1}^{3}(x^n{}_k - x^m{}_k)(f^m{}_l\,\partial f^n{}_j/\partial v^m{}_l + v^m{}_l\,\partial f^n{}_j/\partial x^m{}_l) - \sum_{m=1}^{N}\sum_{l=1}^{3} v^m{}_k v^m{}_l\,\partial f^n{}_j/\partial v^m{}_l + \sum_{m=1}^{N}\partial f^n{}_j/\partial v^m{}_k + 2v^n{}_k f^n{}_j + v^n{}_j f^n{}_k = 0 \tag{2}$$

for $j, k = 1, 2, 3$ and $n = 1, 2, \ldots, N$. We use units such that $c = 1$.

Suppose the force on each particle is a sum of Poincaré-covariant two-particle forces, so that

$$\vec{f}^n = \sum_{\substack{r=1\\ r\neq n}}^{N} \vec{f}^{nr}(\vec{x}^n - \vec{x}^r, \vec{v}^n, \vec{v}^r)\,, \tag{3}$$

where $\vec{f}^{nr}$ and $\vec{f}^{rn}$ are rotational-vector functions that satisfy the Lorentz-invariance conditions (2) for a system of two particles. We shall show that the Lorentz-invariance conditions (2) for three-particle systems imply that for each $n$ and $r$ either $\vec{f}^{nr}$ or $\vec{f}^r$ is zero; accelerations are caused only by forces from particles that do not accelerate.

By comparing the Lorentz-invariance conditions for two-particle and three-particle systems we find that

$$\sum_{l=1}^{3} f^{rs}{}_l\,\partial\vec{f}^{nr}/\partial v^r{}_l = 0 \tag{4}$$

for $r \neq n$, $s \neq r$, and $s \neq n$. The only common variable on which both $f^{rs}{}_l$ and $\partial\vec{f}^{nr}/\partial v^r{}_l$ depend is $\vec{v}^r$, so the only direction that can be a property of both these factors is the direction of $\vec{v}^r$. For the dot product (4) to be zero, the only possibilities are that one factor is zero or that one factor is collinear with $\vec{v}^r$ and the other factor is perpendicular to $\vec{v}^r$. Thus we see that either $\vec{f}^{rs}$ is zero, or

$$\sum_{l=1}^{3} f^{rs}{}_l v^r{}_l = 0\,, \tag{5}$$

or

$$f^{rs}{}_l \propto v^r{}_l\,, \tag{6}$$

or

$$\partial\vec{f}^{nr}/\partial v^r{}_l = 0 \tag{7}$$

for $l = 1, 2, 3$ in (6) and (7), with the proportionality factor in (6) independent of $l$. If $\vec{f}^{rs}$ is zero, then $\vec{f}^{rn}$ must be zero to allow for three-particle systems in which particles $n$ and $s$ cause identical forces. Altogether this means $\vec{f}^r$ is zero. We shall show that either (5) or (6) implies $\vec{f}^{rs}$ is zero. Therefore, $\vec{f}^r$ is zero in every case except the one described by (7). We shall show that (7) implies $\vec{f}^{nr}$ is zero. Thus in every case either $\vec{f}^{nr}$ or $\vec{f}^r$ is zero.

From (5) we get

$$-f^{rs}{}_k + (\vec{v}^r)^2 f^{rs}{}_k = 0 \tag{8}$$

by changing $n$ to $r$ in the Lorentz-invariance conditions for two particles, multiplying by $v^r{}_j$, and summing over $j = 1, 2, 3$. This implies $\vec{f}^{rs}$ is zero, if we assume the particle speeds $|\vec{v}^r|$ are less than $c$, which is 1 in the units we are using.

From (6), writing

$$\vec{f}^{rs} = a_{rs}\vec{v}^r, \tag{9}$$

and letting $\vec{e}^r$ be a vector perpendicular to $\vec{v}^r$ (for example $\vec{v}^r \times \vec{v}^s$), we get

$$a_{rs}e^r{}_k = 0 \tag{10}$$

by changing $n$ to $r$ in the Lorentz-invariance conditions for two particles, multiplying by $e^r{}_j$, and summing over $j = 1, 2, 3$. This implies $\vec{f}^{rs}$ is zero.

From (7) we see that

$$\partial\vec{f}^{nr}/\partial x^r{}_l = 0 \tag{11}$$

for $l = 1, 2, 3$ because in the Lorentz-invariance conditions for two particles $\vec{v}^r$ occurs only in

$$\sum_{l=1}^{3} v^r{}_l\,\partial f^{nr}{}_j/\partial x^r{}_l\,.$$

This means $\vec{f}^{nr}$ is a function of only $\vec{v}^n$. Then, to be a rotational vector, $\vec{f}^{nr}$ must be proportional to $\vec{v}^n$, so $\vec{f}^{nr}$ satisfies (6) with $rs$ changed to $nr$, which implies $\vec{f}^{nr}$ is zero, as we have already seen.

## MASSLESS PARTICLES DESCRIBED BY VELOCITIES

Almost everything done in the last section holds equally well if one or more of the particles has zero mass and speed $c$. We still may consider

Newtonian equations of motion of the form (1). If particle $n$ has speed $c$, the variables $\vec{v}^n$ are restricted to

$$(\vec{v}^n)^2 = 1 \tag{12}$$

(since $c$ is 1 in the units we are using), and the function $\vec{f}^n$ must satisfy

$$\vec{v}^n \cdot \vec{f}^n = 0 \tag{13}$$

to keep (12) from changing in time. The Lorentz-invariance conditions (2) are the same, because the restrictions (12) and (13) are consistent with Lorentz invariance.

Almost all the results of the last section are still valid because they are consequences of the Lorentz-invariance conditions (2). The only exception is that when $(\vec{v}^r)^2$ is 1 we learn nothing from Eq. (8), so we no longer have a proof that (5) implies $\vec{f}^{rs}$ is zero. However, to make the dot product (4) zero we need not only (5) but also

$$\partial \vec{f}^{nr}/\partial v^r{}_l \propto v^r{}_l \tag{14}$$

for $l = 1, 2, 3$ with a proportionality factor independent of $l$. For the case we need to consider we also have

$$(\vec{v}^r)^2 = 1\,, \tag{15}$$

$$\vec{v}^r \cdot \vec{f}^r = 0\,. \tag{16}$$

We shall show that all this implies $\vec{f}^{nr}$ is zero. Thus it is still true that either $\vec{f}^{nr}$ or $\vec{f}^r$ is zero in every case.

After using (14)–(16) we have

$$(x^n{}_k - x^r{}_k)\sum_{l=1}^{3} v^r{}_l \partial f^{nr}{}_j/\partial x^r{}_l - v^n{}_k \sum_{l=1}^{3} v^n{}_l \partial f^{nr}{}_j/\partial v^n{}_l + \partial f^{nr}{}_j/\partial v^n{}_k + 2v^n{}_k f^{nr}{}_j + v^n{}_j f^{nr}{}_k = 0 \tag{17}$$

remaining as the Lorentz-invariance condition (2) for $\vec{f}^{nr}$ in the system of two particles $n$ and $r$. Differentiating this with respect to $v^r{}_i$ and using (14) we see that

$$\partial f^{nr}{}_j/\partial x^r{}_i \propto v^r{}_i\,, \tag{18}$$

for $i = 1, 2, 3$ with a proportionality factor independent of $i$. Then, differentiating (17) with respect to $x^r{}_i$, multiplying by the $i$th component of a vector perpendicular to $\vec{v}^r$, and summing over $i = 1, 2, 3$, we see that

$$\sum_{l=1}^{3} v^r{}_l \partial f^{nr}{}_j/\partial x^r{}_l = 0\,. \tag{19}$$

Therefore, $\partial f^{nr}{}_j/\partial x^r{}_l$ is zero for $l = 1, 2, 3$. Then $\vec{f}^{nr}$ is a function of only $\vec{v}^n$ and $\vec{v}^r$, which means it is of the form

$$\vec{f}^{nr} = A\vec{v}^n + B\vec{v}^r + C\vec{v}^n \times \vec{v}^r\,, \tag{20}$$

where, to make $\vec{f}^{nr}$ a rotational vector, $A, B, C$ are functions of only $(\vec{v}^n)^2$ and $\vec{v}^n \cdot \vec{v}^r$, since $(\vec{v}^r)^2$ is 1. The parity-conserving part involving $A, B$ and the parity-violating part involving $C$ must satisfy (17) separately. From the $A, B$ part in (17) we get a term $\delta_{jk}A$ and terms proportional to $v^n{}_j v^n{}_k$, $v^n{}_j v^r{}_k$, $v^r{}_j v^n{}_k$, and $v^r{}_j v^r{}_k$. Each of these must be zero. Therefore, $A$ is zero. The remaining terms are $Bv^n{}_j v^r{}_k$ and terms proportional to $v^r{}_j v^n{}_k$ and $v^r{}_j v^r{}_k$. Therefore, $B$ is zero. From the $C$ part of (20) in (17) we get a term

$$C\sum_{l=1}^{3} \epsilon_{jkl} v^r{}_l$$

and terms proportional to $(\vec{v}^n \times \vec{v}^r)_j v^n{}_k$, $(\vec{v}^n \times \vec{v}^r)_j v^r{}_k$, and $v^n{}_j (\vec{v}^n \times \vec{v}^r)_k$. Each of these must be zero. Therefore, $C$ is zero. Thus altogether $\vec{f}^{nr}$ is zero.

## MASSLESS PARTICLES DESCRIBED BY MOMENTA

We may also use relativistic momenta $\vec{u}^n$, for the particles $n = 1, 2, \ldots, N$, so that the velocities are

$$\vec{v}^n = [(\vec{u}^n)^2 + m_n{}^2]^{-1/2}\vec{u}^n\,,$$

where $m_n$ is the mass of particle $n$, and $(u^n{}_0, \vec{u}^n)$ transforms as a four-vector with

$$u^n{}_0 = [(\vec{u}^n)^2 + m_n{}^2]^{1/2}\,.$$

Then we use equations of motion

$$d\vec{u}^n/dt = \vec{F}^n(\vec{x}^1, \vec{x}^2, \ldots, \vec{x}^N, \vec{u}^1, \vec{u}^2, \ldots, \vec{u}^N)\,. \tag{21}$$

As before, we make these equations invariant for time translations by not letting $\vec{F}^n$ depend explicitly on time, for space translations by letting $\vec{F}^n$ depend on the positions only through the relative positions $\vec{x}^n - \vec{x}^m$, and for rotations by letting $\vec{F}^n$ be a vector function (that rotates as a vector when $\vec{x}^1, \ldots, \vec{x}^N$, $\vec{u}^1, \ldots, \vec{u}^N$ rotate). The conditions for Lorentz invariance[6] are now

$$\sum_{m=1}^{N}\sum_{l=1}^{3}(x^n{}_k - x^m{}_k)\left\{F^m{}_l \partial F^n{}_j/\partial u^m{}_l + [(\vec{u}^m)^2 + m_m{}^2]^{-1/2} u^m{}_l \partial F^n{}_j/\partial x^m{}_l\right\} + \sum_{m=1}^{N}[(\vec{u}^m)^2 + m_m{}^2]^{1/2}\partial F^n{}_j/\partial u^m{}_k + [(\vec{u}^n)^2 + m_n{}^2]^{-1/2} u^n{}_k F^n{}_j - \delta_{jk}[(\vec{u}^n)^2 + m_n{}^2]^{-1/2}(\vec{u}^n \cdot \vec{F}^n) = 0 \tag{22}$$

for $j, k = 1, 2, 3$ and $n = 1, 2, \ldots, N$. If the mass of every particle is positive, this description in terms of momenta is completely equivalent to the previous description in terms of velocities. However, we are interested in the case where the mass of one or more of the particles is zero. Then the two descriptions are not equivalent. For a particle with zero mass, the velocity is just the unit vector in the direction of the momentum, and the magnitude of the momentum is an additional variable independent of the velocity. Since the $\vec{F}^n$ are functions of more variables than the $\vec{f}^n$ are, we can expect to find more solutions of the Lorentz-invariance conditions for the $\vec{F}^n$ than for the $\vec{f}^n$. On the other hand, the Lorentz-invariance conditions (22) for the $\vec{F}^n$ are derived from the assumption that the momenta transform as four-vectors, and that is not implied by transformations of velocities.

Suppose, as before, the force on each particle is a sum of Poincaré-covariant two-particle forces, so that

$$\vec{F}^n = \sum_{\substack{r=1 \\ r\neq n}}^{N} \vec{F}^{nr}(\vec{x}^n - \vec{x}^r, \vec{u}^n, \vec{u}^r) , \tag{23}$$

where $\vec{F}^{nr}$ and $\vec{F}^{rn}$ are rotational-vector functions that satisfy the Lorentz-invariance conditions (22) for a system of two particles. We shall show that the Lorentz-invariance conditions (22) for three-particle systems imply that for each $n$ and $r$ either $\vec{F}^{nr}$ causes no acceleration of particle $n$ or $\vec{F}^r$ causes no continuous acceleration of particle $r$. As before, accelerations are caused only by forces from particles that do not accelerate continuously. However, the forces that cause no acceleration are not necessarily zero. We find some that change the magnitude of the momentum without changing the velocity, for a particle with zero mass. In one case this produces a discontinuous acceleration when the velocity abruptly reverses direction as the momentum reaches zero and changes sign. Then there is acceleration of both particles.

By comparing the Lorentz-invariance conditions for two-particle and three-particle systems as before, we find that

$$\sum_{l=1}^{3} F^{rs}{}_l \partial \vec{F}^{nr}/\partial u^r{}_l = 0 \tag{24}$$

for $r\neq n$, $s\neq r$, and $s\neq n$. By the same reasoning we used to get from Eq. (4) to Eqs. (5)–(7), we deduce that either $\vec{F}^{rs}$ is zero, or

$$\sum_{l=1}^{3} F^{rs}{}_l u^r{}_l = 0 , \tag{25}$$

or

$$F^{rs}{}_l \propto u^r{}_l , \tag{26}$$

or

$$\partial \vec{F}^{nr}/\partial u^r{}_l = 0 \tag{27}$$

for $l=1,2,3$ in (26) and (27), with the proportionality factor in (26) independent of $l$. We shall show that (25) implies $\vec{F}^{rs}$ is zero and (26) implies either $\vec{F}^{rs}$ is zero or $m_r$ is zero and $\vec{F}^{rs}$ causes no continuous acceleration of particle $r$. By considering three-particle systems in which particles $n$ and $s$ cause identical forces, we conclude that if $\vec{F}^{rs}$ is zero, or if $\vec{F}^{rs}$ causes no continuous acceleration of particle $r$, then $\vec{F}^{rn}$ causes no continuous acceleration of particle $r$. Altogether this means $\vec{F}^r$ causes no continuous acceleration of particle $r$. We shall show that (27) implies $\vec{F}^{nr}$ is zero or $m_n$ is zero and $\vec{F}^{nr}$ causes no acceleration of particle $n$. Thus in every case either $\vec{F}^{nr}$ causes no acceleration of particle $n$ or $\vec{F}^r$ causes no continuous acceleration of particle $r$. However, in some cases both $\vec{F}^{nr}$ and $\vec{F}^r$ are nonzero, and in one case both cause acceleration, when particle $r$ has zero mass and a change in the sign of its momentum causes a discontinuous acceleration as the velocity abruptly reverses direction.

From (25) we get

$$[(\vec{u}^r)^2 + m_r{}^2]^{1/2} \sum_{j=1}^{3} u^r{}_j \partial F^{rs}{}_j/\partial u^r{}_k = 0 \tag{28}$$

by changing $n$ to $r$ in the Lorentz-invariance conditions for two particles, multiplying by $u^r{}_j$, and summing over $j=1,2,3$. Then

$$F^{rs}{}_k = \partial \sum_{j=1}^{3} u^r{}_j F^{rs}{}_j/\partial u^r{}_k = 0 \tag{29}$$

for $k=1,2,3$.

From (27) we see that

$$\partial \vec{F}^{nr}/\partial x^r{}_l = 0 \tag{30}$$

for $l=1,2,3$ because in the Lorentz-invariance conditions for two particles $\vec{u}^r$ occurs only in

$$[(\vec{u}^r)^2 + m_r{}^2]^{-1/2} \sum_{l=1}^{3} u^r{}_l \partial F^{nr}{}_j/\partial x^r{}_l .$$

This means $\vec{F}^{nr}$ is a function of only $\vec{u}^n$. To be a vector for rotations, it must be of the form

$$\vec{F}^{nr} = A_{nr}\vec{u}^n , \tag{31}$$

where $A_{nr}$ is a function of $(\vec{u}^n)^2$. Substituting this in the Lorentz-invariance conditions, we find they imply that either $A_{nr}$ is zero or $m_n$ is zero and $A_{nr}|\vec{u}^n|$ is a constant. The latter means $d\vec{u}^n/dt$ is constant, either plus or minus, in the direction of $\vec{u}^n$. Then the magnitude of the momentum changes at a constant rate, but there is no change in the velocity, not even a reversal of direction, because the momentum cannot change direction or even change sign. Thus either $\vec{F}^{nr}$ is zero or $m_n$ is zero and $\vec{F}^{nr}$ causes no acceleration of particle $n$.

From (26), writing

$$\vec{F}^{rs} = A_{rs}\vec{u}^r , \tag{32}$$

and letting $\vec{e}^r$ be a vector perpendicular to $\vec{u}^r$ (for example $\vec{u}^r \times \vec{u}^s$), we get

$$A_{rs} m_r^2 [(\vec{u}^r)^2 + m_r^2]^{-1/2} e^r_k = 0 \quad (33)$$

by changing $n$ to $r$ in the Lorentz-invariance conditions for two particles, multiplying by $e^r_j$, and summing over $j = 1, 2, 3$. This implies either $\vec{F}^{rs}$ is zero or $m_r$ is zero. If $m_r$ is zero, it is clear from (26) that $\vec{F}^{rs}$ causes no continuous acceleration of particle $r$.

Together with (26) we need

$$\sum_{l=1}^{3} u^r_l \partial \vec{F}^{nr} / \partial u^r_l = 0 \quad (34)$$

to make the dot product (24) zero. As an example, suppose both $\vec{F}^{nr}$ and $\vec{F}^{rn}$ satisfy both conditions (26) and (34), so one formula can be used for all the two-particle forces in systems of two or more massless particles. This means $\vec{F}^{nr}$ satisfies (34) and is of the form (31) to satisfy (26) with $rs$ changed to $nr$. Let us assume the equations of motion (21) are invariant for space reflection as well as rotation. Then $A_{nr}$ in (31) is a function of $(\vec{x}^n - \vec{x}^r)^2$, $(\vec{x}^n - \vec{x}^r) \cdot \vec{u}^n$, $(\vec{x}^n - \vec{x}^r) \cdot \vec{u}^r$, $(\vec{u}^n)^2$, $(\vec{u}^r)^2$, and $\vec{u}^n \cdot \vec{u}^r$. The solution of (34) and the Lorentz-invariance conditions is that $A_{nr}|\vec{u}^n|$ is a function of the two variables

$$(\vec{x}^n - \vec{x}^r) \cdot \vec{u}^r / (|\vec{u}^n||\vec{u}^r| - \vec{u}^n \cdot \vec{u}^r), \quad (35)$$

$$(\vec{x}^n - \vec{x}^r)^2 + \frac{2[(\vec{x}^n - \vec{x}^r) \cdot \vec{u}^n][(\vec{x}^n - \vec{x}^r) \cdot \vec{u}^r]}{(|\vec{u}^n||\vec{u}^r| - \vec{u}^n \cdot \vec{u}^r)}. \quad (36)$$

This force can change the sign of the momentum, so there is discontinuous acceleration when the velocity abruptly reverses direction. In fact this can produce bound states of two massless particles.

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[1]D. G. Currie, Phys. Rev. 142, 817 (1966).
[2]R. N. Hill, J. Math. Phys. 8, 201 (1967).
[3]D. G. Currie and T. F. Jordan, in *Lectures in Theoretical Physics*, edited by A. O. Barut and W. E. Brittin (Gordon and Breach, New York, 1968), Vol. XA, p. 91.
[4]H. Sazdjian, report, 1980 (unpublished).
[5]F. Rohrlich, Phys. Rev. D 23, 1305 (1981).
[6]D. G. Currie and T. F. Jordan, Phys. Rev. 167, 1178 (1968).