

Pointer basis of quantum apparatus: Into what mixture does the wave packet collapse?

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The form of the interaction Hamiltonian between the apparatus and its environment is sufficient to determine which observable of the measured quantum system can be considered "recorded" by the apparatus. The basis that contains this record—the *pointer basis* of the apparatus—consists of the eigenvectors of the operator which commutes with the apparatus-environment interaction Hamiltonian. Thus the environment can be said to perform a nondemolition measurement of an observable diagonal in the pointer basis.

I. WHAT IS MEASURED IN A QUANTUM MEASUREMENT?

Von Neumann¹ has shown that the unitary evolution alone suffices to establish a *nonseparable correlation* between the state vector $|A\rangle$ of the quantum apparatus \mathcal{A} and the state vector $|\psi\rangle$ of the quantum system \mathcal{S} which is to be measured:

$$|A_0\rangle \otimes |\psi\rangle = \left\{ \sum_s a_s |A_s\rangle \right\} \otimes \left\{ \sum_s c_s |s\rangle \right\} \\ \rightarrow \sum_s c_s |A_s\rangle \otimes |s\rangle. \quad (1.1)$$

Here $|A_s\rangle$ and $|s\rangle$ are basis vectors for the apparatus \mathcal{A} and system \mathcal{S} , respectively, while $|A_0\rangle$ is the initial state of the apparatus.

Equation (1.1) seems, at first sight, to solve the problem of measurement in quantum mechanics. States of the apparatus $\{|A_s\rangle\}$ are now correlated with the states of the quantum system $\{|s\rangle\}$. To the question "What has been measured on \mathcal{S} ?" one may be tempted to reply "The observable $\hat{S} = \sum_s e_s |s\rangle\langle s|$, of course." The apparatus \mathcal{A} , however, is itself presumably described by quantum mechanics. Therefore, nothing can prevent one from expressing the state of \mathcal{A} in terms of an alternative orthonormal basis $\{|A_r\rangle\}$ composed of superpositions of states $|A_s\rangle$:

$$|A_r\rangle = \sum_s \langle A_s | A_r \rangle |A_s\rangle. \quad (1.2)$$

In terms of this new apparatus basis the state of the combined $\mathcal{A}\mathcal{S}$ system can be readily rewritten:

$$\sum_s c_s |A_s\rangle \otimes |s\rangle = \sum_r |A_r\rangle \otimes \sum_s c_s \langle A_r | A_s \rangle |s\rangle \\ = \sum_r d_r |A_r\rangle \otimes |r\rangle. \quad (1.3)$$

This equation defines a set of Everett's *relative states* $\{|r\rangle\}$, i.e., normalized, but, in general, not mutually orthogonal states of the system \mathcal{S} relative to the arbitrarily chosen basis set $\{|A_r\rangle\}$ of the apparatus.² Does that imply that when the

measurement is completed the quantum system will in fact end up in one of the states $\{|r\rangle\}$ rather than in one of the states $|s\rangle$?

A particularly acute manifestation of this ambiguity in the choice of the preferred apparatus basis occurs when all the coefficients c_s in Eq. (1.1) happen to have the same magnitude. In that case, whenever the set $\{|A_r\rangle\}$ is orthonormal, the set of the relative states $\{|r\rangle\}$ is orthonormal as well. Then the apparatus by virtue of being correlated with the state of the system contains not only all the information about the observable $\hat{S} = \sum_s e_s |s\rangle\langle s|$; it must equally well contain all the information about many other observables $\hat{R} = \sum_r f_r |r\rangle\langle r|$, defined on the Hilbert space of the system \mathcal{S} . This is so despite the fact that \hat{R} and \hat{S} do not, in general, commute. Yet we know that quantum mechanics prevents one from measuring simultaneously two noncommuting observables with arbitrary accuracy. Moreover, everyday experience convinces us that the choice of "what has this apparatus measured" cannot be made arbitrarily, long after the apparatus-system interaction has taken place, as Eqs. (1.1)–(1.3) would seem to imply. The "real-world" apparatuses constructed to measure momentum do measure momentum and not the conjugate observable, position.

A question can then be raised: What does, in the real-world apparatuses, determine this apparently unique *pointer basis* $\{|A_p\rangle\}$, which records the corresponding relative states $\{|p\rangle\}$ of the system?

Interaction with the environment is the key feature that distinguishes the here-proposed model of the apparatus from the manifestly quantum systems. We argue that the apparatus cannot be observed in a superposition of the pointer-basis states because its state vector is being continuously collapsed. It is the "monitoring" of the apparatus by the environment which results in the apparent reduction of the wave packet. Correlations between states of the pointer basis and corresponding relative states of the system are

nevertheless preserved in the final mixed-state density matrix:

$$\sum_s \sum_{s'} c_s c_{s'}^* |A_s\rangle \langle A_{s'}| \otimes |s\rangle \langle s'| - \sum_p |b_p|^2 |A_p\rangle \langle A_p| \otimes |p\rangle \langle p|. \quad (1.4)$$

Hence, even though below we do not face the insoluble question of quantum theory of measurement: "What causes the collapse of the system-apparatus-environment combined wave function?" we do determine into what mixture the wave function appears to have collapsed.

In the following section we begin our discussion describing a simple example of the interaction between a pair of two-state systems—"bit-by-bit measurement"—which leads to correlations of the type described by Eqs. (1.1)–(1.3). We will argue that the nonuniqueness of the apparatus basis leads to apparent nonseparability paradoxes of the same nature as those encountered in the Einstein-Podolsky-Rosen (EPR) experiment.

In Sec. III we will use the von Neumann equation to show that the apparatus-environment interaction Hamiltonian must couple in a nonperturbative³⁻⁷ way into that apparatus observable that is diagonal in the pointer basis. This in turn determines relative states of the system which can be considered "recorded" by the apparatus. Section IV shows how the general results of Sec. III resolve issues raised for the bit-by-bit measurement problem in Sec. II. Section V contains a brief discussion of the most important new concept introduced in this paper—pointer basis—in the context of measurement theory and practice. Conclusions of the paper are stated in Sec. VI.

II. A BIT-BY-BIT MEASUREMENT

To explore the physical consequences of the nonuniqueness of the basis chosen to represent an apparatus correlated with a quantum system, it is best to examine a simple thought experiment. Here we shall consider a two-state quantum apparatus used to "measure" (the word "premeasurement" would be more exact but less convenient) the other two-state system. (Two states allow one to store one bit of information and hence a "bit-by-bit measurement.")

Three Stern-Gerlach magnets can be arranged to first split, and then recombine, the spin- $\frac{1}{2}$ beam. This is the usual reversible Stern-Gerlach setup (RSG).^{8,9} We shall supplement it by a bistable atom acting as a (quantum) apparatus. Such an atom, inserted along one of the two possible trajectories of the spin- $\frac{1}{2}$ particle provides a possible, if impractical, realization of the bit-

by-bit measurement (see Fig. 1). This same example has been recently employed by Scully, Shea, and McCullen,¹⁰ who used a bistable atom as a microscopic model of "Wigner's friend."¹¹

During the passage through the first of the Stern-Gerlach magnets, momentum and hence position of the spin become correlated with the eigenstate of the spin component along the z axis. In particular, if a spin enters RSG in an initially pure eigenstate $|\odot\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$ of the spin in the direction x , then the splitting of the beam can be represented by

$$|\odot\rangle \otimes |\phi(\vec{r}, t)\rangle = [|\uparrow\rangle \otimes |\phi_{\uparrow}(\vec{r}, t)\rangle + |\downarrow\rangle \otimes |\phi_{\downarrow}(\vec{r}, t)\rangle] / \sqrt{2}. \quad (2.1)$$

Here $|\phi(\vec{r}, t)\rangle$, $|\phi_{\uparrow, \downarrow}(\vec{r}, t)\rangle$ describe the time-dependent position of the wave packet. In the third Stern-Gerlach magnet, $|\phi_{\uparrow}\rangle$ becomes identical with $|\phi_{\downarrow}\rangle$, or, in other words, the two beams correlated with spin states $|\uparrow\rangle$ and $|\downarrow\rangle$ finally recombine. If there were no measurements made on the position of the spin or on the spin itself inside RSG, the spin would leave in the pure state $|\odot\rangle$.

Insert now a bistable atom to serve as an apparatus along the trajectory of the spin $|\uparrow\rangle$. Suppose it is initially in one of its two states $|\neq\rangle$. Suppose, moreover, that the interaction Hamiltonian between atom and spin is given by

$$H_{SA} = g v (\vec{r} - \vec{r}_A) \otimes (|\uparrow\rangle \langle \uparrow| + |\downarrow\rangle \langle \downarrow|) \otimes (|\neq\rangle \langle \neq| + |\neq\rangle \langle \neq|), \quad (2.2)$$

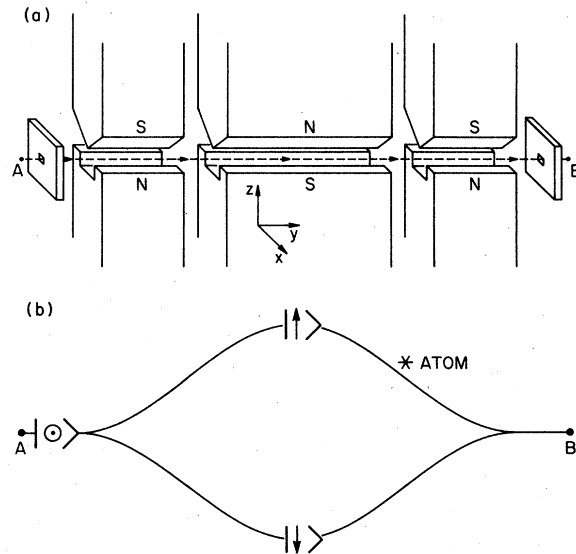


FIG. 1. (a) Reversible Stern-Gerlach setup. (b) Schematic representation of the trajectory of the spin carrier in the bit-by-bit measurement. Possible location of the bistable atom given by an asterisk.

where g is a coupling constant, v is a short-range interaction potential, e.g., $v(|\vec{r} - \vec{r}_A|) \propto \delta(\vec{r} - \vec{r}_A)$, and \vec{r}, \vec{r}_A are positions of the spin and the atom, respectively. With this interaction Hamiltonian, it is not difficult to demonstrate that the final state of the atom contains a record of the path

$$|\Psi\rangle = |\otimes\rangle \otimes |\pm\rangle \otimes |\phi\rangle - \{|\uparrow\rangle \otimes |\phi_{\uparrow}\rangle + |\downarrow\rangle \otimes |\phi_{\downarrow}\rangle\} \otimes |\pm\rangle / \sqrt{2}. \quad (2.3)$$

When the spin-atom interaction begins, one must use the Schrödinger equation to calculate the effect of the passing spin on the atom

$$i\hbar|\dot{\Psi}\rangle = (H_S + H_A + H_{SA})|\Psi\rangle. \quad (2.4)$$

This yields¹⁰

$$|\dot{\Psi}\rangle = \{|\uparrow\rangle \otimes [a(t)|\pm\rangle + b(t)|\mp\rangle] \otimes |\phi_{\uparrow}\rangle + |\downarrow\rangle \otimes |\pm\rangle \otimes |\phi_{\downarrow}\rangle\} / \sqrt{2}, \quad (2.5)$$

where the time-dependent coefficients a and b satisfy coupled equations

$$\begin{aligned} i\hbar\dot{a} &= \epsilon a + gb\gamma(t), \\ i\hbar\dot{b} &= -\epsilon b + ga\gamma(t). \end{aligned} \quad (2.6)$$

Here 2ϵ is the difference between the energy of the states $|\pm\rangle$ and $|\mp\rangle$, while

$$\gamma(t) = \int d^3r \langle \phi_{\uparrow}(\vec{r}, t) | v(\vec{r} - \vec{r}_A) | \phi_{\downarrow}(\vec{r}, t) \rangle. \quad (2.7)$$

For simplicity we shall assume $\epsilon = 0$, and, for all times, $\langle \phi_{\uparrow}(\vec{r}, t) | v(\vec{r} - \vec{r}_A) | \phi_{\downarrow}(\vec{r}, t) \rangle = 0$. Now the solution of Eq. (2.6) is straightforward:

$$\begin{aligned} a(t) &= \cos A(t), \\ b(t) &= -i \sin A(t). \end{aligned} \quad (2.8)$$

Here A is the action, measured in units of \hbar :

$$\hbar A(t) = g \int^t \gamma(t) dt. \quad (2.9)$$

After the spin has emerged from the reversible Stern-Gerlach setup, A becomes a constant:

$$A = \lim_{t \rightarrow \infty} A(t).$$

Consequently, the final wave function is given by¹²

$$|\Psi\rangle = \{|\uparrow\rangle \otimes [\cos A|\pm\rangle - i \sin A|\mp\rangle] + |\downarrow\rangle \otimes |\pm\rangle\} \otimes |\phi\rangle / \sqrt{2}. \quad (2.10)$$

Clearly, the final state is still pure and no irreversible measurement, no collapse of the wave function $|\Psi\rangle$ could have occurred. However, it is straightforward to demonstrate that the illusion of a collapse may arise when one considers the spin and the atom as two independent systems. This is best seen if $A = \pi/2$, i.e.,

$$|\Psi\rangle = \{|\uparrow\rangle \otimes |\pm\rangle - i|\downarrow\rangle \otimes |\mp\rangle\} \otimes |\phi\rangle / \sqrt{2}. \quad (2.11)$$

of the spin, and hence the record of the value of the spin itself.

The evolution of the wave function $|\Psi\rangle$ of the combined spin-atom system as it proceeds from the initial, pure state can be written as

Now whenever the atom is found in the state $|\pm\rangle$, the spin is in the state $|\uparrow\rangle$, and vice versa. The state of the spin has become nonseparably correlated with the state of the atom. This final state with perfect correlation, Eq. (2.11), will be used throughout the rest of this paper.

At first sight it might appear that the problem of measurement in quantum mechanics has already been solved. After all, the atom apparatus contains a record of whether the spin (system) has assumed the $|\uparrow\rangle$ or $|\downarrow\rangle$ state while traversing the reversible Stern-Gerlach setup. This can be easily checked by measuring the state of the atom and verifying that the spin can be found in the appropriate, correlated direction. This is represented by projections $|\pm\rangle\langle\pm|$ and $|\mp\rangle\langle\mp|$:

$$\begin{aligned} |\pm\rangle\langle\pm| \Psi\rangle &= |\uparrow\rangle \otimes |\pm\rangle \otimes |\phi\rangle / \sqrt{2}, \\ |\mp\rangle\langle\mp| \Psi\rangle &= -i|\downarrow\rangle \otimes |\mp\rangle \otimes |\phi\rangle / \sqrt{2}. \end{aligned} \quad (2.12)$$

There can be no doubt that quantum mechanics predicts 100% correlation between the state of the atom and the state of the spin.

The question "What has been recorded by the atom apparatus?" becomes nevertheless apparent when rather than using $|\pm\rangle\langle\pm|$ and $|\mp\rangle\langle\mp|$ one employs projection operators corresponding to the alternative basis:

$$\begin{aligned} |+\rangle &= (|\pm\rangle + |\mp\rangle) / \sqrt{2}, \\ |-\rangle &= (|\pm\rangle - |\mp\rangle) / \sqrt{2}. \end{aligned} \quad (2.13)$$

Clearly

$$\begin{aligned} |+\rangle\langle+| \Psi\rangle &= -i\{|\uparrow\rangle \otimes (|\uparrow\rangle + i|\downarrow\rangle) \otimes |\phi\rangle / \sqrt{2} \\ &= -i\{|\uparrow\rangle \otimes |-\rangle\} \otimes |\phi\rangle / \sqrt{2}, \end{aligned} \quad (2.14a)$$

and, by the same token,

$$|-\rangle\langle-| \Psi\rangle = i\{|\downarrow\rangle \otimes |-\rangle\} \otimes |\phi\rangle / \sqrt{2}. \quad (2.14b)$$

Here

$$\begin{aligned} |\rightarrow\rangle &= (|\uparrow\rangle + i|\downarrow\rangle)/\sqrt{2} \\ \text{and} \\ |\leftarrow\rangle &= (|\uparrow\rangle - i|\downarrow\rangle)/\sqrt{2} \end{aligned} \quad (2.15)$$

define a new basis for the spin spanned by its eigenstates along the y axis. Thus, the wave function $|\Psi\rangle$ given by Eq. (2.11) can be equally well written as

$$|\Psi\rangle = -i\{|\rightarrow\rangle \otimes |\rightarrow\rangle - |\rightarrow\rangle \otimes |\leftarrow\rangle\} \otimes |\phi\rangle/\sqrt{2}. \quad (2.16)$$

Hence, there is also a 100% correlation between the state of the atom and the spin in a basis completely different from the one used to correlate the value of the spin with its position, i.e., $|\uparrow\rangle$, $|\downarrow\rangle$. We have used a Stern-Gerlach magnet with a field gradient in the direction z to measure spin in the direction y . Moreover, we can choose what we shall measure on the atom long after the spin has ceased to interact with it.¹³

Counterintuitive predictions of quantum mechanics for this bit-by-bit measurement correspond closely to the nonseparability of the "Einstein-Podolsky-Rosen paradox."¹⁴ For what is measured on the state of the atom influences the state of the spin. Moreover, even though the spin has been split inside RSG into the two well-separated beams, one definitely carrying $|\uparrow\rangle$ and the other $|\downarrow\rangle$, once these beams are recombined the atom can supply definite and correct information about the spin's alignment (i.e., parallel or antiparallel) with respect to other directions.

Wigner's friend—to use the analogy of Ref. 10—not only ends up in a superposition after taking a look at the single spin passing through RSG, he can also be led to admit that he has seen—inside the RSG magnets with magnetic field gradients in the z direction—spin aligned along the y axis.

This conclusion appears preposterous. For quantum systems it has been nevertheless convincingly verified in experiments stimulated by the EPR paradox. Furthermore, if one denies (in disagreement with original proposals of von Neumann,¹ London and Bauer,¹⁵ and Wigner¹¹) any special role to consciousness, there is seemingly nothing that could keep one from describing an arbitrary system, no matter how large, by a state vector and Schrödinger equation. After all, there is nothing in the laws of physics that would make quantum mechanics applicable to a few-body system but render it invalid for a truly many-body system, even if it contains 10^{25} or more atoms as long as it remains isolated.

III. POINTER BASIS OF THE APPARATUS

It is usually taken for granted that the apparatus measured—perfectly or imperfectly—a particular observable \hat{P} . For, human or nonhuman observers

who consult the pointer of the ideal apparatus learn that the system is in one of the eigenstates of \hat{P} and not in some arbitrarily chosen "relative state." However, as we have seen so far, quantum mechanics alone, when applied to an isolated, composite object consisting of an apparatus and a system, cannot in principle determine which observable has been measured. Below we shall argue that the possibility of a natural choice of "what has been measured" arises when one recognizes the following: (a) The apparatus \mathcal{Q} interacts with its environment \mathcal{E} via some specific interaction Hamiltonian $\mathcal{H}_{\mathcal{Q}\mathcal{E}}$. (b) The observer \mathcal{O} consults only the pointer of the apparatus and not the state of the environment.

The apparatus-environment interaction can then be regarded as an additional measurement establishing nonseparable correlations between the apparatus and the environment. As a result, information about the environment (usually regarded as "noise") obliterates the information about the just premeasured quantum system \mathcal{S} . However, when the Hamiltonian $\mathcal{H}_{\mathcal{Q}\mathcal{E}}$ commutes with the observable $\hat{\Pi}$ of the apparatus, then this particular observable will not be perturbed. Only the basis consisting of the eigenstates of $\hat{\Pi}$, the pointer basis, will contain nothing but the information about the quantum system itself. Moreover, the combined $\mathcal{Q}\mathcal{S}$ system is now represented by a mixture diagonal in a particular product basis, consisting of the eigenvectors of the pointer basis of the apparatus and the corresponding relative states of the system.

One can anticipate the main result of this section by saying that the pointer basis of the apparatus \mathcal{Q} is chosen by the form of the apparatus-environment interaction: It is this basis which contains a reliable record of the state of the system \mathcal{S} . This in turn determines uniquely those relative states of the system which are correlated with the apparatus. Moreover, apparatus-environment correlations do not allow one to observe the $\mathcal{Q}\mathcal{S}$ combination in a superposition. Instead, it becomes a mixture diagonal in the basis constructed from the pointer-basis eigenstates $|A_p\rangle$ and the corresponding relative states of the system. Let us stress that details of the state of the environment itself are not necessary to determine the pointer basis. The form of the apparatus-environment interaction $\mathcal{H}_{\mathcal{Q}\mathcal{E}}$ suffices for that purpose.

To verify the existence and further clarify the role of the pointer basis $\{|A_p\rangle\}$, consider evolution of the density matrix $\rho_{\mathcal{S}\mathcal{Q}\mathcal{E}}$ of the three combined systems: the to-be-measured quantum system \mathcal{S} , the apparatus \mathcal{Q} , and the environment \mathcal{E} :

$$-i\hbar\dot{\rho}_{\mathcal{S}\mathcal{Q}\mathcal{E}} = [\rho_{\mathcal{S}\mathcal{Q}\mathcal{E}}, \mathcal{H}_{\mathcal{S}} + \mathcal{H}_{\mathcal{Q}} + \mathcal{H}_{\mathcal{E}} + \mathcal{H}_{\mathcal{S}\mathcal{Q}} + \mathcal{H}_{\mathcal{Q}\mathcal{E}} + \mathcal{H}_{\mathcal{S}\mathcal{E}}]. \quad (3.1)$$

The correspondence between the components of the total Hamiltonian and the elements of the interacting combination $\mathcal{S}\mathcal{Q}\mathcal{E}$ is evident from the notation. We purposely disregard the interaction between the environment \mathcal{E} and the "rest of the world." This interaction is of no importance for the choice of the pointer basis, at least as long as it does not alter the form of $\mathcal{H}_{\mathcal{Q}\mathcal{E}}$. To write Eq. (3.1) we have moreover assumed (1) that all the interactions are pairwise, i.e., that $\mathcal{H}_{\mathcal{S}\mathcal{Q}\mathcal{E}}=0$, and (2) that the environment \mathcal{E} can be regarded as a quantum system. Assumption (1) is customary and clear, even though it may prevent one from even an approximate treatment of the gravitational interaction beyond its Newtonian pairwise form. Assumption (2) deserves further scrutiny. One should clarify what is meant by the "environment," i.e., which degrees of freedom of the Universe should be taken into account in determining "what mixture does the wave packet collapse into."

We define the environment \mathcal{E} as consisting of all those degrees of freedom which contribute significantly to the evolution of the state of the apparatus. For a given experimental setup one could devise a criterion based on the average strength of the interaction, or, alternatively, on the amount of action exchanged between the environment and the apparatus. The objective of such a criterion is to exclude all those degrees of freedom whose total contribution to the environment-apparatus interaction can be, for all practical purposes, disregarded. Eventually, one can distinguish between the immediate environment \mathcal{E} —and include it in the density matrix $\rho_{\mathcal{S}\mathcal{Q}\mathcal{E}}$ of Eq. (3.1)—and "the rest of the world," or a "remote environment \mathcal{R} " which need not be taken into account. Interaction between \mathcal{E} and \mathcal{R} can be in general quite strong. As we have already decided we shall not take it into account in Eq. (3.1). This additional term would complicate our arguments, and our final conclusion does not depend on whether we set it equal to zero—"decouple" the environment from the rest of the Universe—or take it into account and consider $\rho_{\mathcal{S}\mathcal{Q}\mathcal{E}\mathcal{R}}$ rather than $\rho_{\mathcal{S}\mathcal{Q}\mathcal{E}}$. Having agreed that the environment may be in principle regarded as isolated, we can introduce an appropriate basis system $|\epsilon\rangle$ spanning its Hilbert space.

Three more assumptions of an essentially technical nature can now be introduced:

(i) $\mathcal{H}_{\mathcal{S}\mathcal{E}}=0$ —the quantum system itself remains isolated from the environment. If this assumption is violated after the premeasurement has occurred, then the apparatus will contain the information about the state in which the quantum system *was*, but not necessarily *is*, any more.

(ii) $\mathcal{H}_{\mathcal{S}\mathcal{Q}}$ acts only for a very short period of time. During that time interval $\mathcal{H}_{\mathcal{S}\mathcal{Q}} \gg \mathcal{H}_{\mathcal{Q}\mathcal{E}}$ and a correla-

tion of the form of Eq. (1.1) is established. Afterwards the interaction between the system and the apparatus is effectively nil, i.e., $\mathcal{H}_{\mathcal{Q}\mathcal{E}} \gg \mathcal{H}_{\mathcal{S}\mathcal{Q}}$, and $\mathcal{H}_{\mathcal{S}\mathcal{Q}}$ can be set equal to zero.

(iii) All the vectors of the pointer basis correspond to the same degenerate energy eigenstate, i.e., $\mathcal{H}_{\mathcal{Q}}|A_p\rangle = E|A_p\rangle$, where E is not a function of p . (However, the $|A_p\rangle$ are not degenerate any further.) This is equivalent to a physical requirement that the measurement should not lead to the exchange of energy between the system and the apparatus.¹⁶ All of these assumptions are stronger than is absolutely necessary. Adopting them in this form makes the following discussion much simpler and allows us to concentrate on the main idea rather than on the detailed and cumbersome review of the subcases.

The density matrix of the $\mathcal{S}\mathcal{Q}\mathcal{E}$ combination evolves then—immediately after the correlation between the system \mathcal{S} and the apparatus \mathcal{Q} has been established—according to the equation

$$\begin{aligned} -i\hbar\dot{\rho}_{\mathcal{S}\mathcal{Q}\mathcal{E}} &= [\rho_{\mathcal{S}\mathcal{Q}\mathcal{E}}, \mathcal{H}_{\mathcal{S}} + \mathcal{H}_{\mathcal{Q}} + \mathcal{H}_{\mathcal{E}} + \mathcal{H}_{\mathcal{Q}\mathcal{E}}] \\ &= [\rho_{\mathcal{S}\mathcal{Q}\mathcal{E}}, \mathcal{H}_{\mathcal{S}} + \mathcal{H}_{\mathcal{Q}} + \mathcal{H}_{\mathcal{E}}] + [\rho_{\mathcal{S}\mathcal{Q}\mathcal{E}}, \mathcal{H}_{\mathcal{Q}\mathcal{E}}]. \end{aligned} \quad (3.2)$$

The effect of the first commutator bracket can now be disregarded. This follows from the fact that the time evolution of the apparatus states $|A_p(t)\rangle = \exp[-(i/\hbar)Et]|A_p(0)\rangle$ leaves the diagonal terms of the density matrix invariant. Therefore, evolution of the apparatus \mathcal{Q} due to $\mathcal{H}_{\mathcal{Q}}$ does not obliterate the information about the system \mathcal{S} .

The second commutator $[\rho_{\mathcal{S}\mathcal{Q}\mathcal{E}}, \mathcal{H}_{\mathcal{Q}\mathcal{E}}]$ introduces correlations between the apparatus and its environment. It will leave diagonal terms of the diagonal terms of the density matrix invariant only if it will commute with the very projection operators $|A_p\rangle\langle A_p|$ which are to appear on the diagonal. Consequently, if $|A_p\rangle$ is to remain correlated with the relative state of the quantum system \mathcal{S} , $\mathcal{H}_{\mathcal{Q}\mathcal{E}}$ must satisfy

$$[\mathcal{H}_{\mathcal{Q}\mathcal{E}}, \sum_p \pi_p |A_p\rangle\langle A_p|] = 0 \quad (3.3)$$

for any arbitrary choice of coefficients π_p . Defining the *pointer observable*,

$$\hat{\Pi} = \sum_p \pi_p |A_p\rangle\langle A_p|, \quad (3.4)$$

where we now require π_p to be strictly real, one can reexpress the above condition by stating that the pointer basis $\{|A_p\rangle\}$ is a complete set of eigenfunctions of the operator $\hat{\Pi}$ that commutes with the Hamiltonian $\mathcal{H}_{\mathcal{Q}\mathcal{E}}$:

$$[\hat{\Pi}, \mathcal{H}_{\mathcal{Q}\mathcal{E}}] = 0. \quad (3.5)$$

The above condition can be interpreted by analogy with quantum nondemolition measurements, con-

sidered recently by Braginsky and his group,⁵ the Caltech group,⁶ and Unruh.⁷ There the to-be-measured quantum system is the Weber bar used also as the detector of a gravitational wave. Here it is the apparatus itself. There the measurement is performed by a complicated setup designed not to perturb, say, the eigenstate of the phonon number operator or some other suitably chosen observable. Here the "measurement" is performed by the environment itself, and the apparatus-environment interaction Hamiltonian chooses the pointer observable $\hat{\Pi}$ as the one which will be measured by the environment in a nonperturbative fashion.

The interaction Hamiltonian can then depend on only one apparatus observable—on $\hat{\Pi}$.³⁻⁷ In particular, any interaction Hamiltonian of the form

$$\mathcal{H}_{\alpha\mathcal{S}} = \sum_p \sum_{\epsilon' \neq \epsilon} |A_p\rangle \langle A_p| \otimes (g_{\epsilon\epsilon'}^{(p)} |\epsilon\rangle \langle \epsilon'| + g_{\epsilon'\epsilon}^{(p)*} |\epsilon'\rangle \langle \epsilon|) \quad (3.6)$$

does satisfy condition (3.5). Moreover, both $|\epsilon\rangle$ and $g_{\epsilon\epsilon'}^{(p)}$ may explicitly depend on time due to the interaction with the remote environment \mathcal{R} . As long as the interaction remains diagonal in the pointer basis, it will not disturb correlations of the apparatus with the states of the system relative to the pointer basis.

Of course, the environment-apparatus interaction that allows for the existence of the pointer basis does not suffice to ensure successful functioning of the apparatus. Premeasurement, which correlates the quantum state of the apparatus with the state of the system, plays an absolutely essential role. Moreover, if after the measurement we expect the measured system to collapse into one of the mutually orthogonal eigenstates of the measured observable \hat{P} , then immediately after the premeasurement the combined \mathcal{QS} wave function should be of the form $\sum_p b_p |A_p\rangle \otimes |p\rangle$, where $\{|p\rangle\}$ constitutes that orthonormal basis composed of the eigenstates of the observable \hat{P} . When the states relative to the orthonormal pointer basis are not mutually orthogonal, the measurement will be only imperfect.

In this section we have established the main result of this paper: For the quantum systems known as apparatuses, there exists some basis $\{|A_p\rangle\}$ not perturbed by the interaction with the environ-

ment. This so-called pointer basis retains the information about the outcome of the premeasurement despite the imperfect isolation of the apparatus from its surroundings. In the next section we return to the example of the bit-by-bit measurement to see how the interaction with the environment can select the unique pointer basis.

IV. POINTER BASIS IN A BIT-BY-BIT MEASUREMENT

Consider the state of the bistable atom-spin system—familiar from Sec. II—after a perfect correlation between the two has been established:

$$|\Psi\rangle = \{-i|\uparrow\rangle \otimes |=\rangle + |\downarrow\rangle \otimes |=\rangle\} / \sqrt{2}. \quad (4.1)$$

(Here and below we shall omit the cumbersome and, for our purposes, irrelevant spatio-temporal component of the total wave function, $|\phi(\vec{r}, t)\rangle$.) The model environment consists of an additional bistable atom, basis states of which are $|\epsilon\rangle = |=\rangle$ and $|\epsilon'\rangle = |=\rangle$. Note that the states of the environment atom are denoted by normal "brackets" rather than the usual "bra-kets." This corresponds to an implicit assumption that the environment atom can be distinguished from the apparatus atom. The apparatus-environment interaction Hamiltonian

$$\begin{aligned} \mathcal{H}_{\alpha\mathcal{S}} &= g(|\uparrow\rangle \langle \uparrow| + |\downarrow\rangle \langle \downarrow|) \\ &\otimes (|=\rangle \langle =| - |=\rangle \langle =|) \\ &\otimes (|=\rangle \langle =| + |=\rangle \langle =|), \end{aligned} \quad (4.2)$$

is nonperturbative with respect to the basis $|=\rangle$, $|=\rangle$ of the apparatus, but does influence the state of the environment as well as the state of the system in any basis other than $|=\rangle$, $|=\rangle$. $\mathcal{H}_{\alpha\mathcal{S}}$ obviously commutes with the "spin operator" $\sigma_x = \hat{\Pi} = |=\rangle \langle =| - |=\rangle \langle =|$, as $\mathcal{H}_{\alpha\mathcal{S}} = g\hat{\Pi}(|=\rangle \langle =| + |=\rangle \langle =|)$.

Let us suppose, for example, that the initial state of the environment was given by $|=\rangle$, so that the state of the combined $\mathcal{S} + \alpha + \mathcal{E}$ system immediately after the measurement has occurred can be written as a direct product

$$\begin{aligned} |\Phi\rangle &= |\Psi\rangle \otimes |=\rangle \\ &= \{-i|\uparrow\rangle \otimes |=\rangle + |\downarrow\rangle \otimes |=\rangle\} \otimes |=\rangle / \sqrt{2}. \end{aligned} \quad (4.3)$$

Now it is not difficult to show that this initial state under the influence of the interaction Hamiltonian $\mathcal{H}_{\alpha\mathcal{S}}$ will evolve into

$$|\Phi\rangle = \{-i|\uparrow\rangle \otimes |=\rangle \otimes [\cos A(t)|=\rangle - i \sin A(t)|=\rangle] + |\downarrow\rangle \otimes |=\rangle \otimes [\cos A(t)|=\rangle + i \sin A(t)|=\rangle]\} / \sqrt{2}. \quad (4.4)$$

If we choose the interaction between the apparatus and environment to terminate when $A = \pi/4$, then the states of the apparatus become correlated with the two orthogonal states of the environment atom

$|\perp\rangle$ and $|\top\rangle$ as follows:

$$|\Phi\rangle = \{-i|\uparrow\rangle \otimes |=\rangle \otimes |\perp\rangle + |\downarrow\rangle \otimes |=\rangle \otimes |\top\rangle\} / \sqrt{2}. \quad (4.5)$$

Here

$$\begin{aligned} |\perp\rangle &= (|\equiv\rangle + i|\neq\rangle)/\sqrt{2}, \\ |\top\rangle &= (|\equiv\rangle - i|\neq\rangle)/\sqrt{2}. \end{aligned} \quad (4.6)$$

Now, if we act on the apparatus by means of the projection operators $|\equiv\rangle\langle\equiv|$ or $|\neq\rangle\langle\neq|$, we find, as in Sec. II, spins in the corresponding states $|\uparrow\rangle$ or $|\downarrow\rangle$, respectively. This is readily verified by checking that

$$\begin{aligned} |\equiv\rangle\langle\equiv|\Phi\rangle &= |\uparrow\rangle \otimes |\equiv\rangle \otimes |\top\rangle/\sqrt{2}, \\ |\neq\rangle\langle\neq|\Phi\rangle &= -i|\downarrow\rangle \otimes |\neq\rangle \otimes |\perp\rangle/\sqrt{2}. \end{aligned} \quad (4.7)$$

The environment has become correlated with the state of the apparatus without perturbing the eigenstates of the observable $\hat{\Pi}$ of the apparatus. Therefore, we can still trust the memory of the apparatus concerning the value of the spin in the direction z .

However, if we try to use projection operators $|+\rangle\langle+|$ or $|-\rangle\langle-|$ [see Eq. (2.14)] in an attempt to establish whether the spin was $|+\rangle$ or $|-\rangle$, we find it impossible to accomplish. To argue this we first calculate

$$\begin{aligned} |+\rangle\langle+|\Phi\rangle &= \{-i|\uparrow\rangle \otimes |\top\rangle + |\downarrow\rangle \otimes |\perp\rangle\} \otimes |+\rangle/2, \\ |-\rangle\langle-|\Phi\rangle &= \{-i|\uparrow\rangle \otimes |\top\rangle + |\downarrow\rangle \otimes |\perp\rangle\} \otimes |-\rangle/2. \end{aligned} \quad (4.8)$$

Clearly, $|\Phi\rangle$ cannot be written as a sum of two terms in the form

$$|\Phi\rangle = \alpha_+ |S_+\rangle \otimes |+\rangle \otimes |\epsilon_+\rangle + \alpha_- |S_-\rangle \otimes |-\rangle \otimes |\epsilon_-\rangle. \quad (4.9)$$

Because of the correlations with the environment, knowing the state of the apparatus in the $|+\rangle$, $|-\rangle$ basis does not suffice any more to determine the state of the system. Part of the information about the state of the spin has been "transferred" from the apparatus to the environment. And both the environment and the apparatus are correlated with $|\uparrow\rangle$ or $|\downarrow\rangle$ states of the spin.

We can therefore conclude that when the environment atom is present and interacts with the apparatus via $\mathcal{K}_{\alpha\mathcal{S}}$ given by Eq. (4.2), and the amount of exchanged action is 0.125 in the units of \hbar , then the apparatus-spin system will retain perfect correlation in only one product basis $\{|\uparrow\rangle \otimes |\neq\rangle, |\downarrow\rangle \otimes |\equiv\rangle\}$ of the direct-product space. Hence, $\{|\neq\rangle, |\equiv\rangle\}$ is the pointer basis of the apparatus, which will eventually appear on the diagonal of the density matrix obtained by tracing out "environmental degrees of freedom," i.e., the state of the environment atom. Measurements made by the apparatus on a spin eigenfunction along any other direction are to some degree obliterated by the interaction with the environment. In particular, no information about the

orientation of the spin in the direction of the y axis can be derived from the state of the apparatus alone.

While calculations leading to the above conclusions are certainly correct, one might object that the assumptions of a particular initial state for the apparatus and a specific amount of action exchanged between apparatus and environment are unrealistically restrictive. For example, if the apparatus-environment interaction were not terminated after $A=0.125\hbar$, but continued forever, then every once in a while the state of the system-apparatus combination would become independent of the state of the environment. This would occur whenever $A = \pm(n/4)\hbar$, where n is an integer. Thus, we would again, once in a while, face the paradox of not knowing what has been measured by the apparatus. This objection is certainly serious. One can show, however, as we shall do in a following paper, that when the size of the environment increases, the state of the apparatus-system combination becomes pure more and more rarely, and a close analogy between "memory recovery time" and the Poincaré recurrence time can be made.

V. DISCUSSION

Von Neumann, facing the necessity for the reduction of the wave packet, rejected the idea that an additional quantum apparatus \mathcal{Q}' , coupled to the original \mathcal{Q} , can be of any help in resolving difficulties of the measurement problem. For, he reasoned, the state of the combined $\mathcal{S}\mathcal{Q}\mathcal{Q}'$ system, after all the correlations have been established, would finally evolve into

$$\begin{aligned} |A'_0\rangle \otimes |A_0\rangle \otimes |\psi\rangle &\xrightarrow{\mathcal{K}_{\alpha\mathcal{S}}} |A'_0\rangle \otimes \left\{ \sum_p b_p |A_p\rangle \otimes |p\rangle \right\} \\ &\xrightarrow{\mathcal{K}_{\alpha'\mathcal{Q}}} \sum_p b_p |A'_p\rangle \otimes |A_p\rangle \otimes |p\rangle. \end{aligned} \quad (5.1)$$

Thus \mathcal{Q}' stands in the same relation to \mathcal{Q} as \mathcal{Q} with respect to \mathcal{S} . The final $\mathcal{S}\mathcal{Q}\mathcal{Q}'$ wave function is still pure. No reduction of the wave packet has been accomplished.

The reasoning of von Neumann presented above is no doubt correct. Yet, the goal of this paper is to show that when the environment \mathcal{E} , playing the role of the additional apparatus, is taken properly into account the question "What mixture does the wave packet collapse into?" acquires a definite answer. It may be surprising that one can say so much about the collapse without having to specify where or how it takes place. The aim of this section is to argue that the very question discussed in this paper, as well as many other physically interesting questions concerning the pro-

cess of measurement, can be answered without having to decide whether, where, when, or how the ultimate collapse occurs.

We have to agree with von Neumann that adding more and more apparatuses only delays the moment when the reduction of the wave packet would have to occur. This infinite regress can be terminated only if there are some entities in the Universe which can put an end to the unitary evolution, be it macroscopic objects of Copenhagen interpretation¹⁷ or conscious beings preferred by von Neumann himself,¹ as well as by other prominent physicists.^{14,15} Alternatively, one could resign himself to the unitary evolution predicted by the quantum theory for the Universe. Then it is the individual consciousness alone which appears to evolve in a nonunitary fashion choosing a single path in this labyrinth, "many world" Universe. Both of these opposing views of the collapse problem have been criticized and defended. What is important for us is that both of them agree on one absolutely crucial point: To describe the world "as we know it" there must be two distinct types of evolution—the reversible, deterministic one, which has been confirmed for the microworld, as well as the irreversible, random one which must provide for the choices experienced by the consciousness. The two interpretations differ only as to where the boundary between the unitary and nonunitary domains should be drawn. The many world interpretation allows nothing but the individual consciousness in the "random" domain. The Copenhagen interpretation extends that domain to include "macroscopic objects." Both interpretations do agree that the boundary must be there; "in the empty courtyard many a game cannot be a game until a line has been drawn—... no matter where—to separate one side from the other."¹⁸

One may divide all the questions that can be posed about quantum measurement into those which have answers depending on *where* the line is drawn, as opposed to the ones whose answers depend only on *whether* the line is drawn. We claim that the question "Which mixture does the wave packet appear to have collapsed into?" belongs to the second category. Clearly, there is just one pointer basis $\{|A_p\rangle\}$ in which the additional measurement

$$\begin{aligned} |A'_0\rangle \otimes |A_0\rangle \otimes |\psi\rangle &\xrightarrow{\mathcal{C}_{\mathcal{A}\mathcal{S}}} |A'_0\rangle \otimes \sum_p b_p |A_p\rangle \otimes |p\rangle \\ &\xrightarrow{\mathcal{C}_{\mathcal{A}'\mathcal{G}}} \sum_p \tilde{b}_p |A'_p\rangle \otimes |A_p\rangle \otimes |p\rangle \end{aligned} \quad (5.2)$$

allows the equality

$$|b_p|^2 = |\tilde{b}_p|^2. \quad (5.3)$$

In an arbitrary basis $\{|A_p\rangle\}$ different from the pointer basis, this equality will not hold; an additional measurement by \mathcal{G}' will destroy the perfect correlation between \mathcal{A} and \mathcal{S} established in the first step:

$$\begin{aligned} |A'_0\rangle \otimes \sum_s c_s |A_s\rangle \otimes |s\rangle \\ \xrightarrow{\mathcal{C}_{\mathcal{A}'\mathcal{G}}} \sum_s d_s |A'_s\rangle \otimes |A_s\rangle \otimes |s\rangle \end{aligned} \quad (5.4)$$

with

$$|c_s|^2 \neq |d_s|^2. \quad (5.5)$$

What has been measured on \mathcal{S} by \mathcal{G} is no longer up to an observer to decide. Already at the level of the above equations the pointer basis $\{|A_p\rangle\}$ determines the corresponding relative basis $\{|p\rangle\}$ as the only choice. And the collapse of the wave function has not yet occurred; both Eqs. (5.2) and (5.4) represent, beyond doubt, pure states. In this sense the above considerations answer a question about the appearance of the collapse without having to specify where one draws the "line."

In real-world apparatuses the role of the "additional apparatus \mathcal{G}' " or, equivalently, the role of the "environment \mathcal{S} " is usually played by part of the physical setup of the apparatus itself. For, what we have called "the apparatus \mathcal{A} " is just a small part of the complete setup, which can be fully described by a state vector in nondegenerate Hilbert space spanned by a set of basis vectors $\{|A_p\rangle\}$. In contrast to this simplified model, setups of real-world apparatuses are much more complicated and demand extensive product spaces to allow for a complete description. Out of this vast product Hilbert space we have singled out just one subspace, claiming it describes the "pointer," and hence epitomizes the apparatus itself. The rest of the apparatus setup described by the remaining parts of the product space—by far larger than the subspace used to represent the apparatus proper—describes then a natural immediate environment \mathcal{E} . As long as the coupling between the apparatus \mathcal{A} and its "built-in" environment allows for the existence of the pointer basis, the apparatus will be able to record the corresponding relative states of the to-be-measured quantum system \mathcal{S} . Therefore, part of the apparatus setup, the built-in environment, can be said to act as an interface between the apparatus proper \mathcal{A} and the rest of the world.

The function of this interface is not to isolate \mathcal{A} , as one might have at first guessed. On the contrary, it proves advantageous to couple \mathcal{A} via a well-defined and carefully controlled $\mathcal{C}_{\mathcal{A}\mathcal{S}}$ which

leaves the pointer basis of the apparatus undisturbed. Consequently, the time evolution of the combined $\mathcal{S}\mathcal{Q}\mathcal{E}$ object preserves correlations between the pointer-basis eigenvectors $|A_p\rangle$ and the relative states $|p\rangle$ of the measured system \mathcal{S} . The difficulty of isolating large quantum systems, stressed, by Zeh,¹⁹ among others, and more recently by Wigner,^{20,21} emerges as the crucial motivation: It proves easier to construct a controlled coupling than to isolate.

Let us moreover note that in the context of "many world" interpretation, Deutsch²² has recently postulated existence of a preferred basis, which he calls the "interpretation basis." It is determined by the requirement that, at the instant of completion of any interaction, a measurement has indeed taken place.

Finally, it is worth adding that situations where a quantum system acquires a preferred basis because of its coupling to another system have already been discussed in the context of quantum theory of measurement. Simonius²³ has noticed that quantum systems, interacting with their natural environment interpreted as a "background of probes, like photons or particles," will reveal "classical" features, i.e., localization of macroscopic bodies, localization of atoms within molecules, and stability of metastable compounds. Moreover, in the "Zeno Paradox"²⁴⁻²⁶ metastable states of quantum systems are stabilized by the appropriate coupling with other quantum systems, playing the role of external observers.

Each of the developments described in Refs. 19-

26 is intimately related to the existence of the here-discussed pointer basis. We hope to give a more complete discussion of this relation in future publications.

VI. CONCLUSIONS

We have shown that the interaction between the quantum apparatus \mathcal{Q} and its environment \mathcal{E} may single out a preferred pointer basis of the apparatus. This will happen always when the interaction Hamiltonian $\mathcal{H}_{\mathcal{Q}\mathcal{E}}$ commutes with an apparatus observable $\hat{\Pi}$. Correlations between the set of eigenfunctions of $\hat{\Pi}$ and the corresponding relative states of the system will then remain unperturbed despite the evolution of the apparatus generated by $\mathcal{H}_{\mathcal{Q}\mathcal{E}}$. The choice of $\hat{\Pi}$ determines what states of the quantum system \mathcal{S} can be recorded. Thus, in a certain sense it is the environment of the apparatus which participates in deciding what the apparatus measures: The pointer observable $\hat{\Pi}$ of the apparatus, the one on which the environment performs the "nondemolition measurement," remains the only one endowed with the maximum information about the state of the quantum system \mathcal{S} .

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