Raising the sideways scale

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(Received 19 May 1981)

Extended hypercolor theories have been plagued with inherent flavor-changing effects occurring above experimental levels. The problem may be alleviated in a scenario in which hypercolor interactions are *not* asymptotically free. Then the scale of broken gauged family symmetry may be higher than before. The masses of the pseudo-Goldstone bosons of hypercolor may also be increased.

One of the hopes of a fundamental theory of quarks and leptons would be to relate their masses in some natural way to the mass scale of weak interactions. Theories with elementary scalar fields and adjustable Yukawa couplings shed no light on this question. But there is another class of theories which incorporate new strong interactions[1] (hypercolor) to provide the mass scale for the weak interactions. By extending the theory in some way it is possible to have naturally small quark and lepton masses appear in an effective Lagrangian below the hypercolor scale. Such an extension can involve new gauge interactions, with their effects suppressed either by their large mass relative to hypercolor[2,3] or by their weak couplings.[4] Extended hypercolor theories involving a new heavy mass scale will be labeled hypercolor sideways (HS) theories.

But besides the fermion-mass terms in the effective Lagrangian, one also has to consider the four-fermion terms. In a realistic theory incorporating several generations, these effective terms can in general be flavor changing ($\Delta G = 0, 1$, or 2).[3,5] But there are already strong experimental restrictions on such terms. The most stringent restriction[5] applies to the coefficients of the $\Delta S = 2$ operators, which must be less than $\frac{1}{2}(10^6\ \mathrm{GeV})^{-2}$.

In the context of HS theories, explicit flavor-changing currents in fact must be present to avoid light Goldstone bosons. The coefficient of a $\Delta S = 2$ operator expected from a simple sideways exchange is $\sim 1/2M^2$ where M absorbs the inverse coupling, $M \equiv g_S^{-1} M_S$. It is also reasonable to take M_S as the mass of the sideways gauge boson which feeds a mass to the down quark. The standard expression is

$$m_d \simeq \frac{\langle 0|\bar{D}D|0\rangle}{2M^2} \ . \tag{1}$$

The hypercolor condensate $\langle 0|\bar{D}D|0\rangle \equiv m_H^3$ is estimated by simply scaling up quantum chromodynamics. It is fairly safe to assume that m_H lies between 200 and 400 GeV. This yields $M \simeq (2-6)\times 10^4$ GeV. This discrepancy with the experimental $\Delta S = 2$ situation is a challenge for this type of theory, as emphasized in Ref. 5.

Note that this is a discrepancy in the real part of the coefficient of the $\Delta S = 2$ operator. Experimentally, the imaginary part is suppressed by a further factor of $\sim 10^{-3}$. Thus the above discrepancy will be far more severe unless the imaginary part is naturally suppressed by some other mechanism in these theories. For now, such a suppression is assumed. (It occurs in the model of Ref. 6.)

In HS theories, a single exchange of a neutral (color-singlet or -octet) pseudo-Goldstone boson (PGB) can also produce $\Delta G = 2$ operators. To sufficiently suppress these,[5] the mass of the PGB must be $\gtrsim 6\times 10^3 \theta$ GeV, where θ is a "Cabibbo-suppression" factor. This seriously conflicts with standard estimates of PGB masses. One can ensure that θ is sufficiently small by imposing "monophagy."[7] This requires that the *only* fermion-mass-generating terms take the form

$$\sum_{s,t} (\Gamma^u_{st} u^\dagger_{Rs} u_{Lt} U^\dagger_L U_R + \Gamma^d_{st} d^\dagger_{Rs} d_{Lt} D^\dagger_L D_R + \Gamma^e_{st} e^\dagger_{Rs} e_{Lt} E^\dagger_L E_R) + \mathrm{H.c.}$$

Since it is possible to choose fields to diagonalize Γ^u, Γ^d, and Γ^e, it can be shown[7] that the couplings of the neutral PGB's are flavor conserving, up to order[8] m_f^2/F_H^2. But monophagy may not be as easy to realize as first suggested. It is *not* sufficient to require that sideways gauge bosons only couple each type (u,d,e) of fermion to the same type of hyperfermion. One must identify symmetries to forbid additional terms. The unbroken gauge symmetries below the sideways scale is $G_H \times \mathrm{SU}(3) \times \mathrm{SU}_L(2) \times \mathrm{U}_Y(1)$. This symmetry does not forbid the undesired terms

$$\sum_{s,t} (\Gamma'^d_{st} d^\dagger_{Rs} d_{Lt} E^\dagger_L E_R + \Gamma'^e_{st} e^\dagger_{Rs} e_{Lt} D^\dagger_L D_R) + \mathrm{H.c.}$$

One can imagine some additional global symmetry respected by the sideways breaking to prevent these terms. But this symmetry would eventually be broken by hypercolor, resulting in a light axion. Avoiding this leads to the statement of Ref. 3 that there must be some kind of (hyper)quark-(hyper)lepton unification above the sideways scale. As an example, the easy way to accomplish this is via a Pati-Salam SU(4). [It is assumed that SU(4) breaks to SU(3) color at the scale of the sideways breakdown.[6]] Then SU(4) and sideways effects combined will give rise to the above terms. There is also no *a priori* reason why Γ^d and Γ'^d should be simultaneously diagonalizable. Thus with monophagy lost one returns to the problem of suppressing $\Delta G = 2$ effects from a single-PGB exchange.

But there is a scenario in the context of HS theories which may circumvent these problems. Hypercolor in these theories is usually taken to be asymptotically free. The breakdown of hyperflavor symmetry well below the sideways scale can then be attributed to the rise of the hypercolor running coupling constant $g_H(k)$ at lower momentum scales. But the latter property is not unique to asymptotically free theories. The $\beta(g_H)$ function may instead be as depicted in Fig. 1, with hypercolor being "born" in the region with $\beta(g_H)$ negative. $g_H(k)$ again continues to rise at smaller momentum scales. But the high-momentum properties of the theory are now governed by the fixed point g_H^*. This has implications on ordinary-fermion-mass generation.

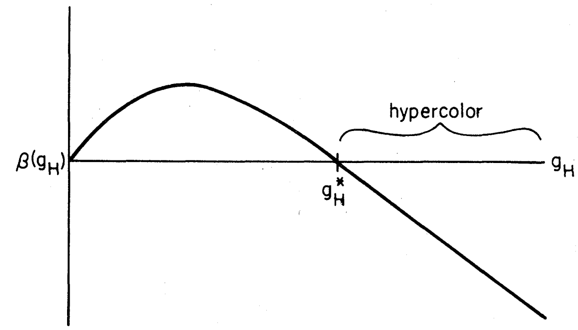

FIG. 1. An example of $\beta(g_H)$ for the new scenario.

A characteristic fermion mass can be defined by

$$\gamma_5 m_f \equiv \frac{-i}{2} \int \frac{d^4p}{(2\pi)^4} \frac{g_S{}^2}{p^2 + M_S{}^2} \int d^4x\, e^{ipx} \{\gamma_5, \langle 0 | T[H(x)\overline{H}(0)] | 0 \rangle\} \; . \tag{2}$$

In the limit $|p^2| \to \infty$ the operator-product expansion gives

$$\frac{1}{\gamma_5} \int d^4x\, e^{ipx} \{\gamma_5, \langle 0 | T[H(x)\overline{H}(0)] | 0 \rangle\} \simeq U(p, m_H, g_H) \langle 0 | \overline{H}H(0) | 0 \rangle = m_H{}^3 U(p, m_H, g_H) \; . \tag{3}$$

m_H is being treated as the renormalization point. Standard renormalization-group analysis[9] of the Wilson coefficient function U gives

$$\lim_{\kappa \to \infty} U(\kappa p, m_H, g_H) = \frac{c}{(\kappa p)^4} \left(\frac{\kappa p}{m_H} \right)^{[\gamma_{\overline{H}H}(g_H^*) + \gamma(g_H^*)]} \; . \tag{4}$$

$\gamma_{\overline{H}H}$ and γ are the conventionally defined[9] anomalous dimensions for the operator $\overline{H}H(0)$ and the inverse of the hyperfermion propagator, respectively. Using this asymptotic form in Eq. (2) is justified if $\gamma^* \equiv \gamma_{\overline{H}H}(g_H^*) + \gamma(g_H^*)$ is $0 < \gamma^* < 2$, in which case momenta of order $M >> m_H$ dominate a convergent integral. If one sets

$$\frac{c}{16\pi} \csc \left(\frac{\gamma^* \pi}{2} \right) = 1 \; ,$$

then the result is

$$m_f = \frac{m_H{}^3}{2M^2} \left(\frac{g_S M}{m_H} \right)^{\gamma^*} \; . \tag{5}$$

$\gamma^* > 0$ is precisely what is desired in order to raise the sideways scale M for given m_f and m_H. $\gamma^* > 0.7$ yields $M > 10^6$ GeV for $m_H = 300$ GeV, $m_f = 10$ MeV, and $\alpha_S = 1$. The point is that coefficients of effective flavor-changing operators induced by a sideways exchange are characterized by $1/M^2$. Box diagrams have also been considered. So from the previous discussion, $M \gtrsim 10^6$ GeV sufficiently suppresses all flavor-changing processes induced by sideways exchange.

Note that the above analysis has been made somewhat easier by assuming that $g_H(M) \simeq g_H^*$. Then anomalous dimensions are constant over the important range of integration. But it is clear that this is not a required assumption for the effect noted.

An important question for this scenario is whether the hierarchy between the sideways and hypercolor mass scales remains natural. From the definition of $\beta(g_H)$ one has

$$\frac{M}{m_H} = \exp \left[- \int_{g_H(M)}^{g_H(m_H)} \frac{dg_H}{\beta(g_H)} \right] \; , \tag{6}$$

where $g_H(m_H)$ is the running coupling necessary for the spontaneous breakdown of hyperflavor symmetry. $\beta(q_H)$ is completely unknown in this region and it does not appear unnatural to have $M/m_H \sim 10^4$. But it is interesting that arbitrarily large hierarchies are not to be expected.

But what effect does this scenario have on other predictions made by these theories? One change will be in the masses of pseudo-Goldstone bosons. A single exchange of a gauge boson, whether of a typical sideways mass or of $SU(3)\times SU_L(2)\times U_Y(1)$, will induce terms in a phenomenological Lagrangian for PGB's.[10] These terms, when expanded out, are the PGB mass terms as well as multilinear PGB couplings. The coefficients of these terms are proportional to the difference in vacuum energy for two different vacuum orientations, for which the gauge generator is broken and unbroken, respectively. This is represented as

$$C_Z=\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\frac{g_Z{}^2}{p^2+M_Z{}^2}\int d^4x\, e^{ipx}\langle 0|T[J_b^\mu(x)J_{b\mu}(0)-J_u^\mu(x)J_{u\mu}(0)]|0\rangle \quad . \tag{7}$$

Here, J_b^μ and J_u^μ can be any pair of broken and unbroken currents. If $M_Z=M_S'$ is typical of the sideways scale then one can proceed as before. The result for $0<\gamma_{JJ}^*<2$ is

$$C_S\simeq\frac{m_H{}^6}{2M'^2}\left(\frac{g_S'M'}{m_H}\right)^{\gamma_{JJ}^*} \quad . \tag{8}$$

$\gamma_{JJ}^*\equiv\gamma_{JJ}(g_H^*)$ is the common anomalous dimension for the $J^\mu J_\mu(0)$ operators. $M'\equiv M_S'g_S'^{-1}$ and g_S' need not be identical to M and g_S. By comparing Eqs. (5) and (8) one can see that, roughly speaking, C_S will be larger in the new scenario if $\gamma_{JJ}^*>\gamma^*$. This is plausible since if the only set of diagrams contributing to Eq. (7) had the form of Fig. 2, then one would have $\gamma_{JJ}^*=2\gamma^*$.

A lower bound on M', $M'\gtrsim 300$ TeV, was obtained[11] in the case that the gauge boson was of the Pati-Salam SU(4) type (as in the model of Ref. 6). This translated into an upper bound on the mass of the neutral, color-singlet PGB, which receives a mass

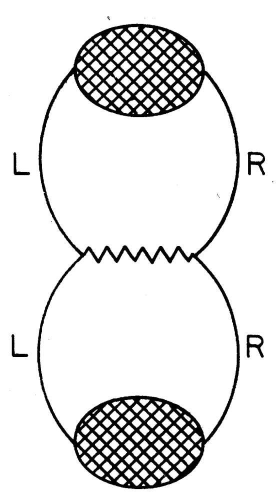

FIG. 2. A class of diagrams contributing to Eq. (7).

only from this source. This is the PGB most dangerous for flavor-changing processes. The result was $m_{p^0}\lesssim 2$ GeV.[10,11] Now in the new scenario, this can be used to place a *lower* bound on *any* PGB mass receiving a contribution from this source. Thus

$$m_{\rm PGB}>(2\ {\rm GeV})\left(\frac{3\times10^5\ {\rm GeV}}{M'}\right)\left(\frac{g_S'M'}{m_H}\right)^{\gamma_{JJ}^*/2} \quad . \tag{9}$$

Such a bound can arise in any scheme in which effective four-hyperfermion terms are produced from a symmetry breakdown at a scale M' well above the hypercolor scale. If $\alpha_S'=1$ and $M'=10^6$ (10^7) GeV, then the lower bound is 65 (20), 425 (210), 2800 (2200) GeV for $\gamma_{JJ}^*=1$, 1.4, 1.8.

The coupling of a PGB to fermions should be the same, m_f/F_H, as before. This is because Eq. (2) gives the coefficient of the PGB-fermion term in the phenomenological Lagrangian.[10] The surprising possibility is that the flavor-changing effects due to PGB exchange are suppressed simply by large PGB masses.

A question which remains to be investigated in this scenario is the behavior of the running coupling constants of $SU(3)\times SU_L(2)\times U_Y(1)$ above the hypercolor scale. Because hypercolor is not asymtotically free, it does not appear sufficient to calculate the β functions to lowest order in the hypercolor interactions (i.e., one loop with N, E, U, or D). It is possible that the β functions feel the effect of hyperhadrons above the mass scale of hyperhadrons. For example, the large number of colored hyperhadrons may make the SU(3) running coupling grow above the hypercolor scale and become strong at the sideways scale. This would make it more plausible that SU(3) emerges from the breakdown of a strongly interacting gauge theory at the sideways scale.

A model which may realize this scenario is presented in Ref. 6. Here the gauged symmetry breaks from $SO(6)\times SU(4)\times SU_L(2)\times SU_R(2)$ to $SO(3)\times SU(3)\times SU_L(2)\times U_Y(1)$ at the sideways scale. Both SO(6) and SU(4) are asymptotically free, whereas the SO(3) (hypercolor) is not. Other choices for hypercolor were not permitted.[6]

In summary, this scenario may effectively remove two features of HS theories which previously characterized these theories: low-mass PGB's and flavor-changing effects. The failure to observe Higgs scalars *and* PGB's at expected energies could be taken as a signal for this scenario.

I have appreciated discussions with S. Dimopoulos, H. Georgi, J. Preskill, L. Susskind, and S. Weinberg. I thank the Natural Sciences and Engineering Research Council of Canada for their financial support. I am grateful to the University of Texas, Austin, for their help and support. Research was supported in part by the National Science Foundation under Grant No. PHY77-22864.

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