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Neutron-antineutron oscillations in an applied magnetic field

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The phenomenology of neutron-antineutron (n-$\bar{n}$) oscillations in the presence of an applied external magnetic field is developed. Conditions are derived for optimizing the growth of $\bar{n}$ probability in a neutron beam. For the case of a space-varying field (precession around a field axis), no "long-time" or "high-field" solutions exist. For the case of a time-varying field, long-time solutions exist but with a quadratic growth coefficient about $\frac{1}{3}$ that of the degaussed (zero-field) solution. No high-field solutions were found to be better than the zero-field solutions. However, it may be experimentally advantageous to apply a driving field instead of degaussing to the levels required for the zero-field case.

Since Mohapatra and Marshak[1] have discussed neutron-antineutron (n-$\bar{n}$) oscillation phenomenology based on gauge models with spontaneously broken local $B-L$ symmetry leading to large $\Delta B=2$, $\Delta L=0$ nucleon transition amplitudes, some experimental proposals[2–4] have been put forward to search for neutron-antineutron mixing in a free neutron beam. In order to make the experiments succeed, they all have to degauss the earth's magnetic field to the order of 10^{-3} G along the beam paths of meters or tens of meters. It seems to be an expensive and a difficult process to shield the earth's magnetic field.

The purpose of this paper is to extend the phenomenology of n-$\bar{n}$ oscillations in the presence of an applied external magnetic field. The field may be space varying (precessing around a certain axis) or time oscillating. Our aim is to see whether the application of such a field can enhance the probability of oscillations or make their detection feasible without the earth's magnetic field being degaussed.

In the presence of a static applied magnetic field, the probability of oscillation is given by

$$P_{\bar{n}}(t)=\left(\frac{\omega_m}{\omega_0}\right)^2\sin^2\omega_0 t\ , \tag{1}$$

where

$$\omega_0=(\omega_B{}^2+\omega_m{}^2)^{1/2}\ ,$$

$$\omega_m=\delta m/\hbar\ ,$$

$$\omega_B=\mu B_0/\hbar$$

(μ is the neutron magnetic moment; B_0, the static field; and δm, the mass difference between $n+\bar{n}$ and $n-\bar{n}$ mass eigenstates). The maximum probability given by Eq. (1) will be unmeasurable unless the magnetic field is sufficiently small, in which case Eq. (1) can be approximated by

$$P_{\bar{n}}(t)\simeq(\omega_m t)^2\ . \tag{2}$$

We shall refer to Eq. (2) as the "zero-field" solution. For a typical experiment with thermal neutrons, $t\sim10^{-2}$ sec and it would be necessary to degauss the earth's magnetic field by a factor of 10^3 in order for Eq. (2) to be valid.

We shall examine the behavior of $P_{\bar{n}}$ under conditions of applied time- (or space-) varying magnetic field at large scale times ($\omega_B t$ is the scale time). We denote these solutions as "high-field" or "long-time" solutions. We obtain $P_{\bar{n}}$ from solutions to the equation

$$\frac{d}{dt}\begin{pmatrix}N\\ \bar{N}\end{pmatrix}=-\frac{i}{\hbar}\begin{bmatrix}M_0+\mu\vec{\sigma}\cdot\vec{B} & \delta m\\ \delta m & M_0-\mu\vec{\sigma}\cdot\vec{B}\end{bmatrix}\begin{pmatrix}N\\ \bar{N}\end{pmatrix}\ , \tag{3}$$

where N ($\bar{N}$) are the state functions for neutron (antineutron); B is the applied (varying) field, and M_0 is the neutron (or antineutron) mass.

The M_0 can be eliminated by defining

$$\begin{pmatrix}n\\ \bar{n}\end{pmatrix}=e^{+iM_0t/\hbar}\begin{pmatrix}N\\ \bar{N}\end{pmatrix}\ .$$

Then n ($\bar{n}$) satisfies

$$\frac{dn}{dt}=-i\omega_B n-i\omega_m\bar{n}\ , \tag{4a}$$

$$\frac{d\bar{n}}{dt}=i\omega_B\bar{n}-i\omega_m n\ , \tag{4b}$$

where

$$\omega_B=\frac{\mu}{\hbar}\vec{\sigma}\cdot\vec{B};\quad \omega_m=\frac{\delta m}{\hbar}\ .$$

Generally one must consider the spin components of the n ($\bar{n}$) states, in which case n ($\bar{n}$) are two-component vectors (spin up, spin down) and

$$P_{\bar{n}}=\sum_{j=1}^{2}\bar{n}_j{}^*\bar{n}_j\ .$$

If the magnetic field has only one component, say B_Z, there is no mixing of spin states and n ($\bar{n}$) is one-dimensional. In this case

$$P_{\bar{n}} = \bar{n}^* \bar{n} \ .$$

Since $\omega_B >> \omega_m$, we shall solve Eq. (4) perturbatively. First, we denote the unitary solutions ($\delta m = 0$, no state mixing) as v_i (for n) and as u_i (for $\bar{n}$). The subscript (i) denotes the fact that, in the case of spin mixing, there will be two such solutions. In the case of a one-dimensional field, the subscript can be ignored. Expanding in terms of the unitary solutions

$$\bar{n} = \sum_i \alpha_i u_i \ ,$$

we obtain for the expansion coefficient α_i, using the orthogonality of the unitary solutions

$$\frac{d}{dt}\alpha_i = -i\omega_m u_i^\dagger n \ .$$

$P_{\bar{n}}$ becomes

$$P_{\bar{n}} = \sum_i \alpha_i^2 = (\omega_m^2 t^2) \sum_i \left[\frac{1}{t}\int_0^t u_i^\dagger n \, dt'\right]^2 \tag{5}$$

or

$$\frac{P_{\bar{n}}}{\omega_m^2 t^2} = \sum_i \left|\frac{1}{t}\int_0^t u_i^\dagger n \, dt'\right|^2 \ .$$

We observe that at "small times," u_i and n can be taken outside the integral of Eq. (5) giving

$$\frac{P_{\bar{n}}}{\omega_m^2 t^2} = 1 \ (\text{"small" time})$$

which is the zero-field limit.

We now apply the general procedure above to two different cases:

Case I. Longitudinal driving field with no spin mixing,

$$B = B_0(1 - r \sin\omega_d t) \ . \tag{6}$$

The unitary solutions (v,u) are, respectively,

$$v = e^{-if(t)} = n(t) \ , \quad u = e^{+if(t)} \ , \tag{7}$$

where

$$f(t) = \omega_B t - \frac{2r\omega_B}{\omega_d}\sin^2\frac{\omega_d t}{2} \ , \quad \omega_B = \mu B_0/\hbar \ .$$

This gives

$$\frac{P_{\bar{n}}}{\omega_m^2 t^2} = \left[\frac{1}{t}\int_0^t e^{-2if(t')}\,dt'\right]^2 \ . \tag{8}$$

The integral of Eq. (8) will go to zero as $t \to \infty$ unless $f(t)$ is periodic (say with period τ) so that $f(t+\tau) = f(t) + \pi$, in which case the integral becomes the periodic average

$$\frac{1}{t}\int_0^t e^{-2if(t')}\,dt' = \frac{1}{\tau}\int_0^\tau e^{-2if(t')}\,dt' \ . \tag{9}$$

The periodicity requirement is satisfied whenever

$$\omega_d = \frac{2\omega_B}{n}, \quad n = 1, 2, 3, \ldots \ . \tag{10}$$

The integral of Eq. (8) is then a function of the parameter r of Eq. (6).

Figure 1 shows a plot of $P_{\bar{n}}/\omega_m^2 t^2$ as a function of r (0 to 7) for $n=1$ and $n=2$. The optimal choice corresponds to $n=1$, $r=1.84$ for which the function $P_{\bar{n}}/\omega_m^2 t^2 = 0.34$.

Equation (4) can, of course, be solved numerically to obtain $P_{\bar{n}}(t)$ as illustrated in Fig. 2 for the optimal case ($n=1$, $r=1.84$). Here we have used ω_B

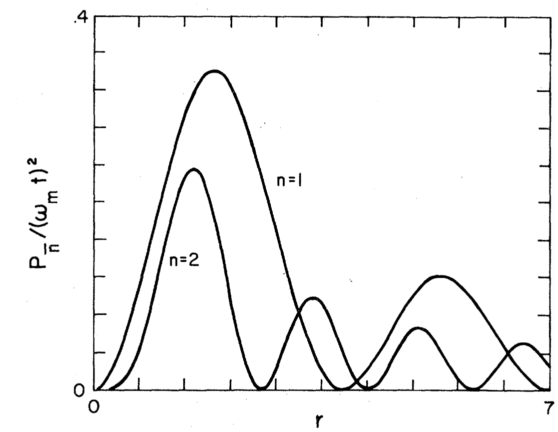

FIG. 1. Plot of $P_{\bar{n}}/\omega_m^2 t^2$ as a function of the ratio of driving to the static magnetic field r for n (which is twice the ratio of Larmor frequency to driving frequency) equals 1 and 2.

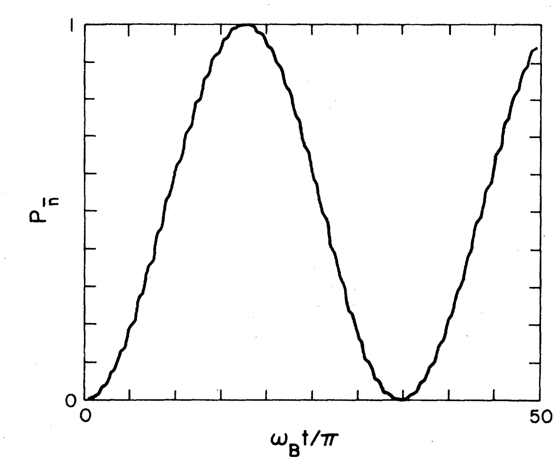

FIG. 2. Plot of $P_{\bar{n}}(t)$ for the optimal case $n=1$, $r=1.84$.

$=20\omega_m$ and have plotted $P_{\bar{n}}$ as a function of a scaled time $(\omega_B t/\pi)$. The curve of Fig. 2 is approximately representable as

$$P_{\bar{n}} \sim \sin^2(\sqrt{0.34\omega_m t}) \quad . \tag{11}$$

An important consequence of the asymptotic $(t \to \infty)$ solutions is that they are independent of the magnitude of the applied field and therefore do not require degaussing.

Case II. Rotating-field configuration. In this case a rotating field around the Z axis so ω_B can be written as

$$\omega_B = \begin{pmatrix} \omega_Z & \omega_T e^{-i\omega_r t} \\ \omega_T e^{i\omega_r t} & -\omega_Z \end{pmatrix} \quad , \tag{12}$$

where $\omega_Z = \mu B_Z/\hbar$, $\omega_T = \mu B_\perp/\hbar$, and ω_r is the rotation frequency. The unitary solutions[5] to (4b) can be written as

$$u_\pm = \frac{1}{N_\pm} \begin{pmatrix} e^{-i\omega_r t/2} & 0 \\ 0 & e^{i\omega_r t/2} \end{pmatrix} \begin{bmatrix} \omega_T \\ -\left(\omega_Z + \dfrac{\omega_r}{2}\right) \mp \tilde{\omega} \end{bmatrix} e^{\pm i\tilde{\omega} t} \quad ,$$

where

$$\tilde{\omega} = \left[\left(\omega_Z + \frac{\omega_r}{2}\right)^2 + \omega_T{}^2\right]^{1/2} \quad ,$$
$$N_\pm = \left[2\tilde{\omega}\left\{\tilde{\omega} \mp \left(\omega_Z + \frac{\omega_r}{2}\right)\right\}\right]^{1/2} \quad . \tag{13}$$

This gives

$$\frac{P_{\bar{n}}}{\omega_m{}^2 t^2} = \left|\frac{1}{t}\int_0^t u_+^\dagger n\right|^2 + \left|\frac{1}{2}\int_0^t u_-^\dagger n\right|^2 \quad . \tag{14}$$

If a unitary solution for n is used in Eq. (14) the integrals become pure time-dependent phase factors whose "periodic" averages are always zero; thus there are no high-field solutions for this case.

Conclusions. We have considered the effect on n-$\bar{n}$ oscillations of applying a driving magnetic field. It has been shown that applying a sinusoidally varying field in the direction of the static field will produce a quadratic growth (in time) coefficient which is, optimally, about a factor of 3 less than is obtained by degaussing the earth's magnetic field; however, the solution thus obtained is *independent* of field strength and may provide a useful alternative to methods presently being considered which require degaussing of the earth's magnetic field by about a factor of 1000. The necessary condition is obtained simply by picking a driving frequency (ω_d) equal to $2\mu B_0/\hbar$ where B_0 is *any* static magnetic field. We further show that no useful solutions are obtained for the case of a space-rotating configuration of field.

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[1]R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. 44, 1316 (1980); and Phys. Lett. 94B, 183 (1980); cf. also S. L. Glashow, Harvard University Reports No. HUTP-79/A029 and No. HUTP-79/A059 (unpublished); cf. also L. N. Chang and N. P. Chang, Phys. Rev. Lett. 45, 1540 (1980); Phys. Lett. 92B, 103 (1980); T. K. Kuo and S. T. Love, Phys. Rev. Lett. 45, 93 (1980).

[2]G. R. Young, H. O. Cohn, T. A. Gabriel, R. A. Lillie, P. D. Miller, and R. R. Spence, Oak Ridge National Laboratory; M. S. Goodman and R. Wilson, Harvard University; and W. M. Bugg, T. G. Condo, T. Handler, and E. L. Hart, University of Tennessee (unpublished).

[3]Experiment proposal to be performed at ILL Grenoble by M. Baldo-Ceolin, University of Padova (unpublished).

[4]Experiment proposal to be performed at KEK National Laboratory, Japan by H. Yoshiki (unpublished).

[5]I. I. Rabi, Phys. Rev. 51, 652 (1937).