Corrections to the baryon magnetic moments in the quark model

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Recent high-precision baryon-magnetic-moment measurements show discrepancies with the broken quark model. These discrepancies can be explained by the presence of a correction term that transforms as the $U=1$, $U_3=Q=0$ component of an SU(3) decuplet, when the predictions are in nuclear magnetons. This property will be tested soon by ongoing experiments.

The treatment of the baryon magnetic moments by Coleman and Glashow[1] in terms of SU(3) symmetry was very successful compared to the data available at that time. Since then, the data have continually improved, showing increasingly clear deviations from the naïve model. The most recent data[2-4] are accurate to a few percent or better, and many attempts have been made to improve the theory to this level of accuracy. Many of these attempts have been to generalize the SU(3) treatment,[5] but most have been in terms of the quark model. For example, De Rújula, Georgi, and Glashow[6] argue that the mass-broken quark model[7] will be valid in quantum chromodynamics, and essentially identical results are obtained with bag models.[8] Fine-structure corrections, such as relativistic effects, configuration mixing, and other effects one would expect in a constituent model have been investigated,[9,10] but the 1% level of accuracy has not been reached. Still, the quark model is intimately related to SU(3) and SU(6). Many features of quark models can be interpreted as group-theoretical properties, and, conversely, the knowledge of empirical transformation properties of the data may be helpful to model builders. This was pointed out most recently by Tomozawa,[11] who stressed that a treatment based on the transformation property of the magnetic-moment operator would include the effects of interactions between the quarks. The purpose of this paper is to point out that a simple generalization of the SU(3) transformation property of the quark-model magnetic-moment operator leads to good agreement with the present data.

The magnetic-moment operator μ that acts on baryon states classified according to SU(3) may be expanded in terms of irreducible SU(3) tensor operators $T^{\lambda}_{U,U_3,Q}$, where λ can be $\underline{1}$, $\underline{8}$, $\underline{10}$, $\underline{10}^*$, or $\underline{27}$.[12] These are most conveniently classified by their U-spin properties. The simplest assumption is that μ is the $U=0$ component of an octet operator (in nuclear magnetons):

$$\mu^{\mathrm{CG}} = T^{8}_{000}\,. \tag{1}$$

This leads to the Coleman-Glashow relations.[1] The two parameters F and D may be reduced to one by choosing the SU(6) value of the F-to-D ratio.[13] The resulting predictions for the magnetic moments are equivalent to those of the symmetric additive quark model.[14] This model can be generalized by adding an SU(3) singlet T^{1}_{000} and the $U=1$ component of another octet operator $T^{8'}_{100}$:

$$\mu^{\mathrm{Q}} = T^{1}_{000} + T^{8}_{000} + T^{8'}_{100}\,. \tag{2}$$

By choosing the same SU(6) value of the F/D ratio for both octets, the five parameters can be reduced to the following three: S (from T^{1}_{000}), $F=-\sqrt{0.8}\,D$ (from T^{8}_{000}), and $F'=-\sqrt{0.8}\,D'$ (from $T^{8'}_{100}$). The Clebsch-Gordan coefficients corresponding to these parameters are easily calculated[15] and are shown in Table I. The magnetic-moment formulas resulting from these assumptions are equivalent to those of the mass-broken quark model,[7] in which the three parameters are the quark moments μ_u, μ_d, and μ_s. The connection between the two sets of parameters is

$$\begin{aligned}\mu_u &= S - \frac{1}{\sqrt{3}}F\,,\\ \mu_d &= S + \frac{1}{2\sqrt{3}}F - \tfrac{1}{2}F'\,,\\ \mu_s &= S + \frac{1}{2\sqrt{3}}F + \tfrac{1}{2}F\,.\end{aligned} \tag{3}$$

These three parameters may be reduced further to two by the natural assumption[16]

$$\mu_u = -2\mu_d \quad \text{or} \quad 3S = F'\,. \tag{4}$$

If the quarks have Dirac moments, (4) is equivalent to assuming that the up and down quarks have the same effective mass. The predictions of (2) and (4) agree roughly with the data, but it is clear that corrections need to be included.[10]

The generalization from (1) to (2) can be carried

TABLE I. Clebsch-Gordan coefficients and reduced matrix elements for the operators in Eqs. (2) and (6) with the SU(6) F/D ratios, and the predictions obtained from Eq. (6) under assumption (4). The data are from Refs. 2–4, 17, and 18.

	T^{1}_{000}	T^{8}_{000}	$T^{8'}_{100}$	T^{10+}_{100}	Prediction	Experimental data
p	S	$-\frac{\sqrt{3}}{2}F$	$+\frac{1}{6}F'$	$-\delta$	input	2.793
n	S	$+\frac{1}{\sqrt{3}}F$	$-\frac{2}{3}F'$	$+\delta$	input	−1.913
Λ	S	$+\frac{1}{2\sqrt{3}}F$	$+\frac{1}{2}F'$		input	-0.614 ± 0.005
Σ^+	S	$-\frac{\sqrt{3}}{2}F$	$-\frac{1}{6}F'$	$+\delta$	2.40	2.33 ± 0.13
Σ^0	S	$-\frac{1}{2\sqrt{3}}F$	$-\frac{1}{2}F'$		0.79	
Σ^-	S	$+\frac{1}{2\sqrt{3}}F$	$-\frac{5}{6}F'$	$-\delta$	−0.82	-1.41 ± 0.25
Ξ^0	S	$+\frac{1}{\sqrt{3}}F$	$+\frac{2}{3}F'$	$-\delta$	−1.25	-1.250 ± 0.014
Ξ^-	S	$+\frac{1}{2\sqrt{3}}F$	$+\frac{5}{6}F'$	$+\delta$	−0.68	-0.75 ± 0.06
$\Sigma\Lambda$		$-\frac{1}{2}F$	$+\frac{1}{2\sqrt{3}}F'$		1.52	$1.82^{+0.25}_{-0.18}$

one step further, by including, for example, an SU(3)-decuplet contribution. Assuming time-reversal invariance, only the combination

$$T^{\underline{10}^+}_{100}=T^{\underline{10}}_{100}+T^{\underline{10}^*}_{100} \tag{5}$$

contributes,[12] so

$$\mu=T^{\underline{1}}_{000}+T^{\underline{8}}_{000}+T^{\underline{8}'}_{100}+T^{\underline{10}^+}_{100}\,. \tag{6}$$

This introduces a single new parameter δ:

$$\delta=\langle\underline{8}\,\|\,T^{\underline{10}^+}\,\|\,\underline{8}\rangle/\sqrt{15}\,. \tag{7}$$

The formulas obtained by calculating the Clebsch-Gordan coefficients[15] are shown in Table I. They can be obtained more easily by using the fact that $T^{\underline{10}^+}_{100}$ is both a $U=1$, $U_3=0$ operator and an $I=1$, $I_3=0$ operator.

The parameters F, F', and δ may be calculated from the experimental p, n, and Λ moments under assumption (4), once the scale is fixed. Assuming that (6) gives the moments in nuclear magnetons yields

$$F=-2.89\ \mu_N\,,$$
$$F'=0.266\ \mu_N\,, \tag{8}$$
$$\delta=-0.153\ \mu_N\,.$$

The predictions resulting from these values are shown in Table I, along with the experimental data.[2–4,17,18] The agreement is surprisingly good, especially for Ξ^0. Aside from the input, this is the only moment measured to 1% accuracy, and the prediction lies within the error bars. The only disagreements are the 2σ deviation from the Σ^- value and the 1.6σ deviation from the Σ-Λ transition moment. These are also the moments with the largest experimental errors. Along with Σ^+ and Ξ^-, they are presently being measured to higher precision.[4]

Under the assumption that the quarks have Dirac moments $\mu_q=e_q m_p/m_q$ (in nuclear magnetons), the parameters (8) correspond to $m_u=m_d=355$ and $m_s=509$ MeV. The ratio $\xi=m_u/m_s=0.70$ lies between the value $\xi=0.62$ determined by De Rújula, Georgi, and Glashow[6] from the baryon masses and the value $\xi=0.77$ estimated by Tomozawa[11] from meson masses. However, these comparisons should not be taken too seriously, since many effects have been ignored. For example, the gyromagnetic ratios of the quarks could vary from $g=2$ due to gluon or other corrections.

A decuplet operator cannot contribute to the magnetic moments of the decuplet baryons, so the quark-model prediction[16] $\mu(\Omega^-)=3\mu(\Lambda)$ is unchanged.

The close agreement seen in Table I is remarkable for several reasons. First, the predictions of Eq. (6) are in nuclear magnetons. Although this is natural in a nonrelativistic quark model, it would be surprising in a relativistic one.[9,19] Also, the transformation properties of the electromagnetic current suggest that the predictions of SU(3) should be in intrinsic magnetons rather than nuclear magnetons.[20] If Eq. (6) is taken to be in intrinsic magnetons, the agreement with the data is very poor. Second, supposing that the predictions should indeed be in nuclear magnetons, the success of Eq. (6) implies that all of the complicated corrections one might expect to find (e.g., two- or three-body quark and gluon interactions) must all add up to the same effect as a decuplet operator. Effects transforming as a 27-plet would seem to cancel out or be highly suppressed. Finally, it is puzzling that such good agreement could come from such a simple assumption. Whether or not this agreement is just coincidence may soon be cleared up by more accurate data.

Note added. Yossef Dothan has found that decuplet corrections improve his five-parameter linear-symmetric-model fits [University of Minnesota report, 1981 (unpublished)].

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