Tests for gluonium or other non-$q\bar{q}$ admixtures in the $f(1270)$

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SU(3) and SU(6) suggest $\Gamma(f\to\pi\pi)=112$–115 MeV for a quark-antiquark state, while the observed value is 148 ± 17 MeV. This small discrepancy permits a non-$q\bar{q}$ admixture ("gg") in the f: $|f> = \cos\theta|q\bar{q}> + \sin\theta|gg>$. We examine the possibility that the orthogonal state $f^{\perp} = -\sin\theta|q\bar{q}> + \cos\theta|gg>$ decouples from $\pi\pi$, and find $\sin^2\theta = 0.15^{+0.05}_{-0.06}$. Consequences include a diminished predicted rate for $f\to\gamma\gamma$, in qualitative accord with experiment; a mass formula $f^{\perp} - A_2 = \cot^2\theta(A_2 - f)$ yielding $f^{\perp}$ in the range 1.45–1.87 GeV/c^2; and predictions for $f^{(\perp)}$ spectra as produced in $\gamma\gamma\to f^{(\perp)}$ or $J/\psi\to f^{(\perp)}+\gamma$ and detected in $f\to K\bar{K}$, $\rho\gamma$, or $\gamma\gamma$. The width of $f^{\perp}$ is estimated to be no more than 40 MeV under these assumptions.

I. INTRODUCTION

There is little serious doubt remaining that hadron spectroscopy is a consequence of the interactions of a color triplet of quarks by the exchange of an octet of vector gluons. This theory, quantum chromodynamics or QCD for short, also appears to predict the existence of bound states of gluons alone.[1,2] The expected properties of these objects are model-dependent, relying crucially on long-distance aspects of QCD at present not thoroughly understood. However, a very specific prediction has emerged within the context of the MIT bag model. This is that there should be a $J^{PC}=2^{++}$ gluonic bound state around 1.3 GeV, whose properties are significantly affected by mixing with the familiar $f(1270)$ resonance.[2,3]

The $f(1270)$ has been well understood for some time as a quark-antiquark ($q\bar{q}$) system composed primarily of nonstrange quarks: $f\simeq(u\bar{u}+d\bar{d})/\sqrt{2}$. If it is admixed with non-$q\bar{q}$ components, several observable consequences are expected. It is our purpose to study some of these effects.

We find that mixing of $q\bar{q}$ and gluonic ("gg") components in the f is not required, but also not ruled out, by present data. First, SU(3) and SU(6) analyses which we shall perform suggest $\Gamma(f\to\pi\pi)\simeq112$–$115$ MeV for a quark-antiquark ($q\bar{q}$) state, while the observed value is 148 ± 17 MeV. There thus is room for a gg component which interferes constructively in the $\pi\pi$ decay amplitude. Second, the observed rate for $f\to\gamma\gamma$ seems smaller than predicted for a pure $q\bar{q}$ state. Mixing can improve this situation, though not dramatically. Third, it is possible to take the small A_2-f mass difference seriously as an indication of mixing effects. If this is done, and if the orthogonal state $f^{\perp}$ is assumed to decouple from $\pi\pi$ (thereby explaining why it has not been seen until now), one finds that the $f^{\perp}$ mass should be between 1.45 and 1.87 GeV/c^2. The width of $f^{\perp}$ is estimated to be less than about 40 MeV.

As suggested in Ref. 2, the less prominent decay channels (other than $\pi\pi$) play a crucial role in finding effects of $f^{\perp}$. We find the reactions

$$J/\psi\to f^{(\perp)}+\gamma\,, \tag{1}$$

$$\gamma\gamma\to f^{(\perp)} \tag{2}$$

particularly useful for producing f's with "anomalous" behavior, and the decay channels $K\bar{K}$, $\rho\gamma$, and $\gamma\gamma$ particularly useful for detecting them. One particularly interesting process is found to be

$$J/\psi\to\gamma f^{(\perp)}\to\gamma K\bar{K}\,. \tag{3}$$

This paper is organized as follows. Mixing, SU(3), and SU(6) are discussed in Sec. II. The $\gamma\gamma$ and $\rho\gamma$ decays of f are treated in Sec. III. Mass formulas are contained in Sec. IV. Section V is devoted to an estimate of partial decay widths of $f^{\perp}$, while Sec. VI contains some examples of production and interference calculations. We summarize the present work and stress unanswered questions in Sec. VII.

II. MIXING, SU(3), AND SU(6)

Let us assume that the "ordinary" $f(1270)$ is an admixture of a $q\bar{q}$ and a "gg" component[4]

$$|f\rangle=\cos\theta|q\bar{q}\rangle+\sin\theta|gg\rangle\,. \tag{4}$$

Its orthogonal partner $f^{\perp}$ is

$$|f^{\perp}\rangle=-\sin\theta|q\bar{q}\rangle+\cos\theta|gg\rangle\,. \tag{5}$$

As in Ref. 2, we assume that the reason the f has not shown anomalous behavior previously is that it has been studied primarily in its major production and decay channel $\pi\pi\to f\to\pi\pi$, and that $f^{\perp}$ decouples from $\pi\pi$:

$$0=\langle\pi\pi|f^{\perp}\rangle=-\sin\theta\langle\pi\pi|q\bar{q}\rangle+\cos\theta\langle\pi\pi|gg\rangle\,. \tag{6}$$

The unknown $\langle\pi\pi|gg\rangle$ then may be eliminated in the $f\to\pi\pi$ amplitude, yielding

$$\langle\pi\pi|f\rangle=\sec\theta\langle\pi\pi|q\bar q\rangle\,. \quad (7)$$

Equation (6) is a very powerful assumption, since although we know very little about the coupling of gg to $\pi\pi$, there are a number of constraints on the coupling of $q\bar q$ to $\pi\pi$. We can immediately estimate $\langle\pi\pi|q\bar q\rangle$ by comparison with other decays of $J^{PC}=2^{++}$ mesons to a pair of 0^- mesons [via SU(3)] or one 0^- and one 1^- meson [via SU(6)].

For a decay into final states with orbital angular momentum l, one would expect[5] the partial width Γ to be given by

$$\Gamma\sim\tilde\Gamma p^{*2l}(p^*/M)\,, \quad (8)$$

where $\tilde\Gamma$ denotes a partial width with kinematic factors taken out, p^* stands for the magnitude of the center-of-mass three-momentum, and M is the mass of the decaying particle. Equation (8) should be valid for small p^* but there may be correction factors[6] when a good deal of energy is released in the decay. We shall use Eq. (8) in making estimates based on SU(3).

Attempts to abstract the symmetries of the quark model in order to relate processes such as $2^{++}\to 0^-0^-$ and $2^{++}\to 1^-0^-$ culminated in the work of Melosh[7] and related phenomenological studies.[8,9] Most of these studies use PCAC (partial conservation of axial-vector current) to relate matrix elements of axial charges to amplitudes for pion emission in decays $A\to B\pi$. As a consequence, there arises in the amplitude a kinematic factor $p^{*0}\equiv(M_A{}^2-M_B{}^2)/2M_A$, the center-of-mass three-momentum for massless pion emission, and hence

$$\Gamma\sim\tilde\Gamma p^*(p^{*0})^2\,. \quad (9)$$

We shall use Eq. (9) for SU(6) estimates.

In Table I we list the processes of interest, their relevant kinematic variables p^* and p^{*0}, the observed partial widths,[10] and the predicted values of $\tilde\Gamma$. Some comments follow.

(1) We have adopted $\tilde\Gamma(f[q\bar q]\to\pi\pi)\equiv 1$ to set the scale in Table I. Accordingly [see Eq. (7)], $\tilde\Gamma(f(1270)\to\pi\pi)=\sec^2\theta$, as shown in the last column of the first line.

(2) In estimating $\Gamma(f\to K\bar K)$ we have assumed that the gg component of f couples to quarks in an SU(3)-invariant manner, e.g.,

$$\langle K^+K^-|gg\rangle=\langle\pi^+\pi^-|gg\rangle\,. \quad (10)$$

On the other hand, we take the $q\bar q$ component of f to be an "ideal" octet-singlet mixture: $q\bar q\equiv(u\bar u+d\bar d)/\sqrt2$, so that

$$\langle K^+K^-|q\bar q\rangle=\langle\pi^+\pi^-|q\bar q\rangle/2\,. \quad (11)$$

We thus find, using Eqs. (4)–(6), that

$$\langle K^+K^-|f\rangle=\tfrac12\cos\theta(1+2\tan^2\theta)\langle\pi^+\pi^-|q\bar q\rangle\,, \quad (12)$$

$$\langle K^+K^-|f^\perp\rangle=\tfrac12\sin\theta\,\langle\pi^+\pi^-|q\bar q\rangle\,. \quad (13)$$

The prediction in the last column of the second line then comes from Eq. (12) when we sum over charge states and recall

$$\Gamma(f\to\pi\pi)=\tfrac32\Gamma(f\to\pi^+\pi^-),\quad \Gamma(f\to K\bar K)=2\Gamma(f\to K^+K^-)\,.$$

TABLE I. Some two-body decays of $J^{PC}=2^{++}$ mesons.

Decay	p^* (MeV/c)	p^{*0} (MeV/c)	Experimental Γ (MeV) (Ref. 10)	Predicted $\tilde\Gamma$
$f\to\pi\pi$	621	636	148 ± 17	$\sec^2\theta$(def.)
$\to K\bar K$	397		5.0 ± 0.8	$\sec^2\theta(1+\sin^2\theta)^2/3$
$\to\eta\eta$	322		4	$\sec^2\theta(1+2\sin^2\theta)^2/27$ [a]
				$\sec^2\theta(1+\sin^2\theta)^2/12$ [b]
$A_2\to K\bar K$	434		4.9 ± 0.6	$\frac13$
$\to\eta\pi$	544		14.9 ± 1.4	$\frac29$ [a]
				$\frac13$ [b]
$\to\rho\pi$	414	430	71.4 ± 4.2	2
$K^{**}\to K\pi$	623	631	49.1 ± 5.2	$\frac12$
$\to K^*\pi$	424	440	27.0 ± 3.5	$\frac34$
$f'\to K\bar K$	572		≤67 ± 10 [c]	$\frac23$

[a] Value based on $\eta\equiv(u\bar u+d\bar d-2s\bar s)/\sqrt6$.
[b] Value based on $\eta\equiv(u\bar u+d\bar d-\sqrt2 s\bar s)/2$ (Ref. 11).
[c] Total width. Other decay modes (notably $K^*\bar K$+c.c.) possible.

(3) We have omitted from Table I processes considered in other fits[5] for which partial widths are only poorly known, such as $K^{**} \to K\eta$, $A_2 \to \pi\eta'$, etc. These do not provide useful constraints. We do include $f \to \eta\eta$ since this was mentioned in Refs. 2 as a mode of possible interest.

(4) In addition to ignoring the strange-quark content of the f, we ignore the nonstrange-quark content of the f'. Moreover, we ignore any "gg" admixture in the $f' \approx s\bar{s}$. The observed branching ratio for $f' \to \pi\pi$ is very small,[12] so this is a reasonable assumption, but it should be relaxed in a more detailed treatment.

(5) We neglect electromagnetic impurities[13] in the f^0 and A_2^0, assuming them to contain equal numbers of u (or $\bar{u}$) and d (or $\bar{d}$).

(6) The predictions for $A_2 \to \pi\eta$ depend on the octet-singlet mixing assumed for the η.[11] We shall omit $\Gamma(A_2 \to \pi\eta)$ from our fits and calculate it at the end.

(7) The branching ratio for $f' \to K\bar{K}$ is not well known; it could be somewhat less than 100%. We omit $\Gamma(f' \to K\bar{K})$ from the fits and calculate it at the end.

The results of an SU(3) fit, an SU(6) fit, and a combined fit are shown in Figs. 1–3 and Table II. Nonzero values of θ are favored by all three fits.

The SU(3) fit is impelled toward $\theta \neq 0$ by the predicted values of $\Gamma(f \to \pi\pi)$ and $\Gamma(f \to K\bar{K})$. It is prevented from having large values of θ primarily by the predictions

$$\frac{\tilde{\Gamma}(f \to K\bar{K})}{\tilde{\Gamma}(f \to \pi\pi)} = \frac{1}{3}(1+\sin^2\theta)^2 \tag{14}$$

or

$$\frac{\tilde{\Gamma}(f \to K\bar{K})}{\tilde{\Gamma}(A_2 \to K\bar{K})} = \cos^2\theta(1+2\tan^2\theta)^2 \tag{15}$$

and the observed value of $\tilde{\Gamma}(f \to K\bar{K})$.[14]

A previous SU(3) fit[5] to the decays of 2^{++} mesons did not require any effect such as the one we are discussing. What has changed since that fit was performed?

First, the effect in question is not an overwhelming one; the SU(3) fit favors $\sin^2\theta = 0.14^{+0.05}_{-0.06}$ on the basis of $\Delta\chi^2 = 1$, so it can be viewed as just a little over 2 standard deviations. The predicted value of $\Gamma(f \to \pi\pi)$ is not changed very much in the

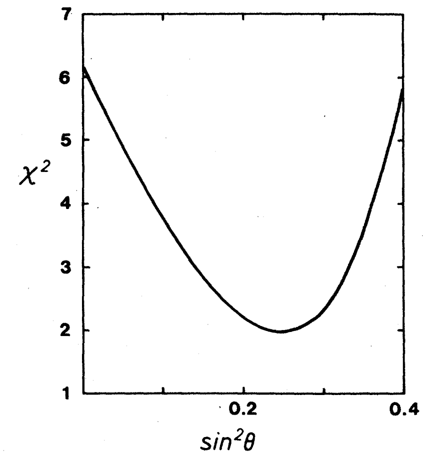

FIG. 2. χ^2 vs $\sin^2\theta$ for a fit to decays of tensor mesons to a pion and a pseudoscalar or vector meson, based on SU(6).

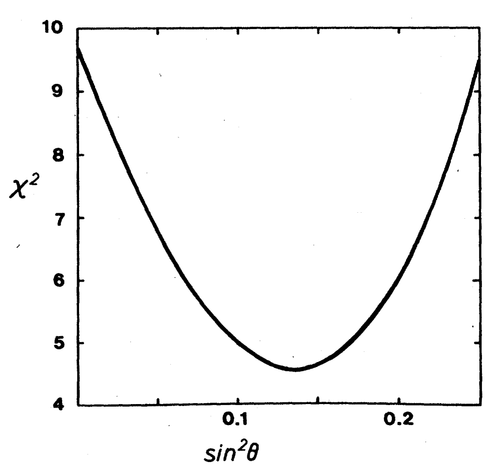

FIG. 1. χ^2 vs $\sin^2\theta$ for a fit to decays of tensor mesons to a pair of pseudoscalar mesons, based on SU(3).

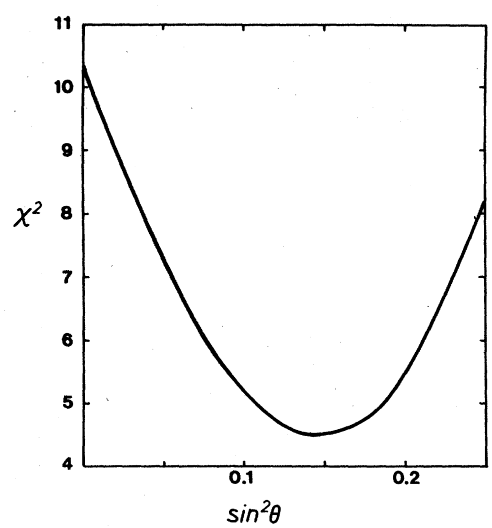

FIG. 3. χ^2 vs $\sin^2\theta$ for a fit to decays of tensor mesons to a pair of pseudoscalar mesons or a pion and a vector meson, based on SU(3) and SU(6).

TABLE II. Predictions for decay widths of $J^{PC}=2^{++}$ mesons.

	Predicted Γ (MeV)					
	SU(3)		SU(6)		Combined	
Decay	$\sin^2\theta=0$	$\sin^2\theta=0.14$	$\sin^2\theta=0$	$\sin^2\theta=0.25$	$\sin^2\theta=0$	$\sin^2\theta=0.15$
$f\to\pi\pi$	112	121	115	149	112	125
$\to K\bar{K}$	4.0	5.6			3.8	5.5
$\to\eta\eta$ [a]	0.16	0.27			0.15	0.27
	0.35	0.49			0.34	0.48
$A_2\to K\bar{K}$	6.0	5.6			5.8	5.3
$\to\eta\pi$ [a]	11.2	10.5	16.0	15.6	13.5	13.0
	16.8	15.7	24.1	23.4	20.3	19.5
$\to\rho\pi$			70.0	67.9	70.3	69.7
$K^{**}\to K\pi$	50.4	46.8	56.7	55.0	52.9	50.5
$\to K^*\pi$			28.2	27.3	28.3	28.0
$f'\to K\bar{K}$	42	39			40	37
χ^2/DF [b]	9.6/3	4.6/2	6.2/3	2.0/2	10.3/4	4.5/3

[a] First set of values based on $\eta\equiv(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$; second set based on $\eta\equiv(u\bar{u}+d\bar{d}-\sqrt{2}s\bar{s})/2$.
[b] Values of $\Gamma(f\to\eta\eta)$, $\Gamma(A_2\to\eta\pi)$, and $\Gamma(f'\to K\bar{K})$ omitted from fitting procedure and calculated at end (see text).

best SU(3) fit from its value at $\theta=0$. Second, the values $\Gamma(A_2\to K\bar{K})=6.1\pm1.0$ MeV, $\Gamma(K^{**}\to K\pi)=63\pm7.5$ MeV were used in the fit of Ref. 5. The values in Table I are lower, with smaller experimental errors. Third, the fit of Ref. 5 made use of a value $\Gamma(A_2\to\pi\eta)=17\pm2$ MeV, for an octet η. We do not use $\Gamma(A_2\to\pi\eta)$ in the fitting procedure, for reasons mentioned above.

We thus regard the SU(3) fit as suggestive but by no means conclusive evidence that the $f(1270)$ may have a non-$q\bar{q}$ admixture.

The SU(6) fit is impelled toward $\theta\neq\theta$ by the predicted value of $\Gamma(f\to\pi\pi)$ alone. The predictions of the remaining rates are almost independent of θ, indicating a substantial amount of consistency among the SU(6) predictions for $\Gamma(A_2\to\rho\pi)$, $\Gamma(K^{**}\to K\pi)$, and $\Gamma(K^{**}\to K^*\pi)$. It has been noticed before[15] that an SU(6) fit with $f=q\bar{q}$ predicts a value of $\Gamma(f\to\pi\pi)$ somewhat lower than the experimental one. On the basis of the SU(6) fit, we would estimate $\sin^2\theta=0.25^{+0.08}_{-0.11}$.

There is some ambiguity in how to perform a combined SU(3) and SU(6) fit, since the predictions of the two schemes (even for identical values of θ) differ somewhat—presumably by virtue of the approximate nature of the kinematic corrections (8) and (9). We have performed a fit by using separate reduced matrix elements for SU(3) and SU(6) predictions, and averaging the χ^2 contributions from $f\to\pi\pi$ and $K^{**}\to K\pi$, where both models make predictions. The values quoted in Table II are the averages of these predictions. The result is

$$\sin^2\theta=0.15^{+0.05}_{-0.06}, \tag{16}$$

rather similar to that from SU(3) alone. This is only a 2.5-standard-deviation effect. Nevertheless, we could imagine improvements both in data and in the accuracy of the theoretical description that would make us wish to take the effect seriously. The remainder of this paper is devoted to the consequences of assuming that $\sin^2\theta$ is, indeed, nonvanishing, and lies within the bounds of Eq. (16).

III. ELECTROMAGNETIC DECAYS OF f

Another process which is sensitive to admixtures of non-$q\bar{q}$ components in the f is the decay $f\to\gamma\gamma$. Let us assume that this proceeds entirely via the $q\bar{q}$ component, since only the quarks and not the gluons are charged. Then we expect

$$\Gamma(f\to\gamma\gamma)=\cos^2\theta\,\Gamma(f[q\bar{q}]\to\gamma\gamma), \tag{17}$$

$$\Gamma(f^\perp\to\gamma\gamma)=\sin^2\theta\,\Gamma(f[q\bar{q}]\to\gamma\gamma). \tag{18}$$

Both f and $f^\perp$ can be produced in the reactions

$$ee\to eef \tag{19}$$

proceeding via two-photon exchange. Estimates of $\Gamma(f[q\bar{q}]\to\gamma\gamma)$ vary considerably[16] but tend to lie above 5 keV. On the basis of SU(6) and vector dominance we have recently estimated[9]

$$\Gamma(f[q\bar{q}]\to\gamma\gamma)=7.7\pm2\text{ keV}. \tag{20}$$

The experimental values, based on

$$ee\to eef, \tag{21}$$

appear to be somewhat lower:

$$\Gamma(f\to\gamma\gamma)=\begin{cases}2.3\pm 0.5\pm 0.35\text{ keV (PLUTO) (Ref. 16)} & (22a)\\ 3.2\pm 0.2\pm 0.6\text{ keV (TASSO) (Ref. 17)} & (22b)\\ 3.5\pm 0.6\text{ or }\leq 4.7\text{ keV (95\% C.L.)} & (22c)\\ \text{(Mark II) (Ref. 18)} & \end{cases}$$

In (22a) and (22b) the first set of errors is statistical; the second set is systematic.

The $f^\perp$ is assumed to decouple from $\pi\pi$. If so, the experiments in Eq. (22) are seeing mainly the state in Eq. (17), and comparison with the prediction (17) suggests $\cos^2\theta < 1$. This is in qualitative accord with our estimate (10) though it is too early to tell whether it is consistent with the range $0.80 \leq \cos^2\theta \leq 0.91$ implied by (16). In any event, the effect of mixing is not expected to be large here. Incorporating this mixing, we now predict

$$4.6 \leq \Gamma(f\to\gamma\gamma) \leq 8.8\text{ keV}. \qquad (23)$$

If $f\to\rho\gamma$ also proceeds exclusively via $q\bar{q}$, we expect

$$\Gamma(f\to\rho\gamma)=\cos^2\theta\,\Gamma(f[q\bar{q}]\to\rho\gamma), \qquad (24)$$

$$\Gamma(f^\perp\to\rho\gamma)=\sin^2\theta\,\Gamma(f[q\bar{q}]\to\rho\gamma). \qquad (25)$$

We estimated

$$\Gamma(f[q\bar{q}]\to\rho\gamma)=1.35\pm 0.2\text{ MeV} \qquad (26)$$

in Ref. 9. The f (and not the $f^\perp$) would be produced in the reaction $\pi^- p\to fn$ via pion exchange. Hence, in

$$\pi^- p\to fn\to\rho\gamma n, \qquad (27)$$

one would expect to see a slightly diminished (but by no means extinct) signal. With Eqs. (16), (24), and (26), we expect

$$0.9 \leq \Gamma(f\to\rho\gamma) \leq 1.4\text{ MeV} \qquad (28)$$

and hence a branching ratio of at least $\frac{1}{2}$% for $f\to\rho\gamma$.

IV. MASS FORMULAS

Another circumstance suggests that the f may contain a non-$q\bar{q}$ admixture. This is the small A_2-f mass difference. (We thus neglect failures of ideal mixing, or isospin admixtures, discussed further in Ref. 13.) The ρ and ω, both nonstrange $q\bar{q}$ states, are degenerate to within several MeV. By contrast, the f lies 44 ± 7 MeV/c^2 below the A_2.[10] We can use this difference, together with our bounds on θ, to obtain crude limits on the $f^\perp$ mass in a two-state mixing model.

We denote the mass of the gg state before mixing as G; the unmixed $q\bar{q}$ state is assumed to be degenerate with the A_2 and we so denote its mass. The mixing parameter may be denoted δ. Then

$$\begin{pmatrix}A_2 & \delta\\ \delta & G\end{pmatrix}\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}=f\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}, \qquad (29)$$

$$\begin{pmatrix}A_2 & \delta\\ \delta & G\end{pmatrix}\begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}=f\begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}. \qquad (30)$$

Eliminating δ from the equations implied by the first row of (29) and (30), we find

$$f^\perp - A_2 = \cot^2\theta(A_2 - f). \qquad (31)$$

With the help of (16) and the aforementioned A_2-f difference, we estimate the mass range shown in Fig. 4, or

$$1.45\text{ GeV}/c^2 \leq f^\perp \leq 1.87\text{ GeV}/c^2. \qquad (32)$$

If the $f^\perp$ lies in this mass range, its mixing with the f'(1515) must also be important. We have neglected this effect in the present simplified model.[19]

The mass of the unmixed state G is of interest for comparison with QCD or bag-model calculations. Since the trace of a matrix is the sum of its eigenvalues, $A_2 + G = f + f^\perp$, or

$$f^\perp - G = A_2 - f = 44\pm 7\text{ MeV}/c^2. \qquad (33)$$

The unmixed G state thus lies between 1.4 and 1.8 GeV/c^2.

The mixing parameter δ is given by

$$\delta = (f - A_2)\cot\theta = -74\text{ to }-162\text{ MeV}/c^2. \qquad (34)$$

This seems a reasonable magnitude to arise if the mixing proceeds via a shared intermediate $\pi\pi$ state, but we have not attempted a quantitative estimate.

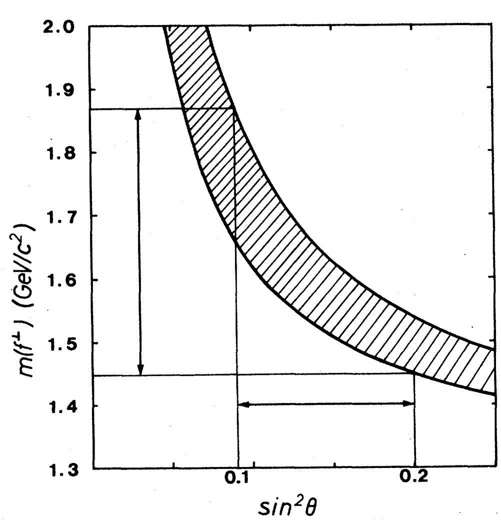

FIG. 4. Mass of $f^\perp$ vs $\sin^2\theta$. Shaded band corresponds to range of f and A_2 masses. Vertical lines correspond to range of $\sin^2\theta$ favored in combined fit; horizontal lines denote the corresponding range of $m(f^1)$.

V. PARTIAL-WIDTH ESTIMATES

The known decay modes of $f(1270)$ include $\pi\pi$, $2\pi^+$, $2\pi^-$, $K\bar{K}$, and $\gamma\gamma$. There are probably also $\pi^+\pi^-$, $2\pi^0$, $4\pi^0$, and $\eta\eta$ modes. All but the $\pi\pi$ mode should be shared by the $f^\perp$. We have estimated already

$$\langle\gamma\gamma|f\rangle = \cos\theta\,\langle\gamma\gamma|q\bar{q}\rangle, \tag{35}$$

$$\langle\gamma\gamma|f^\perp\rangle = -\sin\theta\,\langle\gamma\gamma|q\bar{q}\rangle, \tag{36}$$

so

$$\frac{\tilde{\Gamma}(f^\perp\to\gamma\gamma)}{\tilde{\Gamma}(f\to\gamma\gamma)} = \tan^2\theta = 0.10 \text{ to } 0.25. \tag{37}$$

On the basis of (12) and (13) we expect

$$\frac{\tilde{\Gamma}(f^\perp\to K\bar{K})}{\tilde{\Gamma}(f\to K\bar{K})} = \frac{\tan^2\theta}{(1+2\tan^2\theta)^2} = 0.07 \text{ to } 0.11. \tag{38}$$

A similar calculation leads to $\langle\eta\eta|f\rangle = \langle K^+K^-|f\rangle$ and $\langle\eta\eta|f^\perp\rangle = \langle K^+K^-|f^\perp\rangle$ [for $\eta = (u\bar{u}+d\bar{d}-\sqrt{2}\,s\bar{s})/2$], or $\tilde{\Gamma}(f\to\eta\eta) = \frac{1}{4}\tilde{\Gamma}(f\to K\bar{K})$, $\tilde{\Gamma}(f^\perp\to\eta\eta) = \frac{1}{4}\tilde{\Gamma}(f^\perp\to K\bar{K})$ (when we take account of the identity of the η's). The small values for the $f^\perp\to K\bar{K}$ and $f^\perp\to\eta\eta$ amplitudes reflect destructive interference between the $q\bar{q}$ and gg contributions, a vestige of the assumption that $f^\perp$ decouples from $\pi\pi$. The cancellation is sensitive to any SU(3) breaking effects that may occur in the $\langle 0^-0^-|gg\rangle$ coupling. For example, gluons may have different amplitudes for producing strange quarks and nonstrange ones. Nonetheless, we find it hard to escape the conclusion that $\Gamma(f^\perp\to 0^-0^-)$ should be quite small, of the order of several MeV. We present the results of three sample calculations, for $\sin^2\theta = 0.1$, 0.15, and 0.2, in Table III. We have taken the SU(3) reduced matrix element associated with the combined fit quoted in Table II, and have assumed $M(A_2) - M(f) = 44$ MeV/c^2, $M(A_2) = 1317$ MeV/c^2. Small values of θ lead to high predicted masses for $f^\perp$ (and hence more favorable barrier factors for decays) but the decay matrix elements are correspondingly smaller. For small θ and large $f^\perp$ masses we begin to mistrust the simple barrier factor (8), which probably overestimates the decay width when p^* is large.

TABLE III. Predictions for $\Gamma(f^\perp\to 0^-0^-)$ in MeV.

	$\sin^2\theta=0.1$	$\sin^2\theta=0.15$	$\sin^2\theta=0.20$
0^-0^- state			
$K\bar{K}$	4.6	3.6	3.2
$\eta\eta$ [a]	0.8	0.6	0.5
	1.5	1.1	0.9
$M(f)$ (GeV/c^2)	1.71	1.57	1.49

[a] First set of values based on $\eta = (u\bar{u}+d\bar{d}-\sqrt{2}\,s\bar{s})/2$; second set based on $\eta = (u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$.

The multipion decays of f and $f^\perp$ are not easily discussed within known symmetry schemes. On the basis of branching ratios for $\pi\pi$ and $K\bar{K}$ quoted in Ref. 10, we estimate that $B(f\to 4\pi) < 16\%$, and hence

$$\Gamma(f\to 4\pi) < 30 \text{ MeV}. \tag{39}$$

Certainly this same figure should be a reasonable upper limit for $f^\perp\to 4\pi$ decays, since $f^\perp$ is mainly gg and—if gg represents a gluonic bound state—the decay rates of such states to hadrons are generally assumed to be smaller than those of $q\bar{q}$.[2,20] Thus, we estimate

$$4 \le \Gamma(f^\perp) \le 40 \text{ MeV}. \tag{40}$$

Such a narrow resonance should be fairly prominent in inelastic (non-$\pi\pi$) channels.

VI. PRODUCTION AND INTERFERENCE

We now turn to the prospect of observing effects of $f^\perp$ directly. We must produce and detect this resonance without making use of the $\pi\pi$ channel, from which it is assumed to decouple.

One production mechanism which may excite the gg component directly is the decay (1): $J/\psi\to f+\gamma$. Since $\tan^2\theta<1$, we expect this mechanism to produce $f^\perp$ more abundantly than f. The $ee\to eef$ reaction, as we mentioned, probably excites the $q\bar{q}$ component and thus also produces both f and $f^\perp$.

Suppose the $f^\perp$ and f interfere with relative amplitude r in a process $A\to f^{(\perp)}\to B$. Then the amplitude may be written as the sum of two interfering Breit-Wigner contributions:

$$A(A\to B) \sim \frac{1}{\epsilon+i} + \frac{r}{\epsilon-\delta+i\gamma}, \tag{41}$$

where

$$\epsilon \equiv [E-M(f)]/[\Gamma(f)/2], \tag{42}$$

$$\delta \equiv [M(f^\perp)-M(f)]/[\Gamma(f)/2], \tag{43}$$

$$\gamma \equiv \Gamma(f^\perp)/\Gamma(f). \tag{44}$$

Since[10] $\Gamma(f) = 178\pm 20$ MeV, we expect on the basis of (40) that $0.03 \le \gamma \le 0.25$.

A. The process $\gamma\gamma\to f^{(\perp)}\to K\bar{K}$

We have already estimated the required couplings in Eqs. (12), (13), (35), and (36). The result is

$$r = -\sin^2\theta/(1+\sin^2\theta) = -0.08 \text{ to } -0.17. \tag{45}$$

The additional contribution of the A_2 resonance is very important in this process.[21] It interferes constructively in K^+K^- and destructively in $K^0\bar{K}^0$.

The effect of $f^{\perp}$ is quite noticeable in both final states. Explicit calculations, also including the f' contribution, are in progress.[22]

B. The process $J/\psi \to f^{(\perp)}\gamma$, $f^{(\perp)} \to K\bar{K}$

Here we assume[23]

$$\frac{\langle \gamma f^{\perp} | J/\psi \rangle}{\langle \gamma f | J/\psi \rangle} = \frac{\langle f^{\perp} | gg \rangle}{\langle f | gg \rangle} = \cot\theta \,. \tag{46}$$

In conjunction with (35) and (36) we then have

$$r = (1 + 2\tan^2\theta)^{-1} = 0.67 \text{ to } 0.83 \,. \tag{47}$$

For this range of values we find the shapes of the intensity curves to be characterized primarily by the width of the narrow resonance. If $\Gamma(f^{\perp})$ approaches the upper bound of ~40 MeV, constructive interference between f and $f^{\perp}$ can lead to a modest enhancement with respect to the narrow resonance alone. A narrower $f^{\perp}$ would almost totally dominate the process. If no spectacular signal is seen, one can probably rule out values of $\Gamma(f^{\perp})$ toward the lower end of the range (40).

C. The process $J/\psi \to f^{(\perp)}\gamma$, $f^{(\perp)} \to \gamma\rho$ or $\gamma\gamma$

Here (46), (35), and (36) lead to $r = -1$. Since $\gamma \ll 1$, the consequences of destructive interference tend to be minimal. Again, the intensity curves resulting from (41) are not much different from their shapes in the presence of the narrow resonance alone.

D. Hadronic production of f

There are many ways of producing f in hadronic reactions without pion exchange. Examples are:

$$\pi^- p \to fn \text{ (high energy and/or large } |t|, \; A_2 \text{ exchange)}, \tag{48}$$

$$K^- p \to f\Lambda \;\; (K, K^*, K^{**} \text{ exchange}), \tag{49}$$

$$pp \to f + \ldots \text{ (possible coupling to } \bar{p} \text{ or production in central region)}. \tag{50}$$

No specific prediction is possible of the relative production of f and $f^{\perp}$ without further knowledge of the production mechanism. We suggest detecting the $f^{(\perp)}$, again, via its $K\bar{K}$ mode. (*Note added.* All hadronic experiments which produce the f' should also produce $f^{\perp}$. The very small f' signal in Ref. 12, for example, could be due instead to a combination of f' and $f^{\perp}$. The author thanks H. J. Lipkin for mentioning this point.) The estimates in Table III and Eq. (40) imply that $B(f^{\perp} \to K\bar{K})$ probably is at least as large as $B(f \to K\bar{K})$, so that except for very special (small) values of r the narrow resonance again should not be affected much by interference with the f.

VII. SUMMARY AND DISCUSSION

We have shown by examining SU(3) and SU(6) predictions for $\Gamma(f \to \pi\pi)$ that this quantity appears experimentally to be slightly larger than one might predict if f were a pure $q\bar{q}$ state. This allows us to pursue the possibility that f contains a non-$q\bar{q}$ component which also couples to $\pi\pi$. The orthogonal state $f^{\perp}$ is assumed to decouple from $\pi\pi$. This allows us to place bounds on mixing angles. The partial widths for $f \to \gamma\gamma$ and $f \to \rho\gamma$ are predicted to be diminished somewhat from their values expected if f were a pure $q\bar{q}$ state. For $f \to \gamma\gamma$, this is in qualitative accord with experiment. If the small A_2-f mass difference is taken seriously, the mass of $f^{\perp}$ is estimated in a simple two-state mixing model to lie between 1.45 and 1.87 GeV/c^2. The width of $f^{\perp}$ is estimated in this simple model to be less than 40 MeV. Other predictions, of visible effects of $f^{\perp}$ in $\gamma\gamma \to f^{(\perp)} \to K\bar{K}$ and in $J/\psi \to f^{(\perp)}\gamma$ followed by $f^{(\perp)} \to K\bar{K}$, $\rho\gamma$, or $\gamma\gamma$, remain to be tested.

Very little of what we have done requires the non-$q\bar{q}$ component of the f to be due to a gluonic bound state. [We *have* assumed this component to couple to 0^-0^- pairs as if it were an SU(3) singlet.] There should also be $qq\bar{q}\bar{q}$ mesons with $J^{PC} = 2^{++}$. One specific bag-model calculation[24] places the lightest of these mesons at 1650 MeV/c^2, presumably able to account for at least some of the mixing effects noted here.

In the two-state mixing model, the main component of $f^{\perp}$ should be gg, since the mixing angle θ is substantially below 45°. The proximity of $f^{\perp}$ to f' clearly requires further study of the mixing problem. The f' seems to be broader than one would infer from an SU(3) analysis (if its width is due mainly to the $K\bar{K}$ final state), as one can see from Table I. Perhaps it contains a component of $f^{\perp}$.

One interesting possibility is that the f' does not mix at all with gg, explaining its apparent absence in $J/\psi \to \gamma f'$.[25,26] There is a small peak of four events in the K^+K^- mass spectrum between 1553 and 1572 MeV/c^2 obtained from $J/\psi \to \gamma K^+K^-$ in the experiment of Ref. 25. Because of its mass, the authors hesitate to identify this peak with f'. (The f' mass quoted in Ref. 10 is 1516 ± 12 MeV/c^2.) On the other hand, for $\sin^2\theta = 0.15$, the simple two-state mixing model presented here does predict the $f^{\perp}$ mass to lie somewhat above the f' (see Table III). Perhaps the $f^{\perp}$ is responsible for these four events. If so, the signal is actually somewhat too small to accommodate within the present simplified model. To see this, we note that Ref. 25 quotes a number which can be interpreted for our purposes as

$$\frac{B(J/\psi \to \gamma f^{\perp})B(f^{\perp} \to K\bar{K})}{B(J/\psi \to \gamma f)} < 0.3 . \qquad (51)$$

For $\sin^2\theta = 0.15$, we expect

$$\frac{B(J/\psi \to \gamma f^{\perp})}{B(J/\psi \to \gamma f)} = \cot^2\theta \simeq 5.7 . \qquad (52)$$

These two relations imply that $B(f^{\perp} \to K\bar{K}) \lesssim 0.053$, which would be hard to accommodate with the prediction $(f^{\perp} \to K\bar{K}) \simeq 3.6$ MeV of Table III and the observed fairly narrow width of the observed signal. Clearly a study of $J/\psi \to \gamma K^+K^-$ with improved statistics would be highly worthwhile.

It is possible that our description of the $f(1270)$ could be essentially correct without there being *any* low-mass 2^+ state composed mainly of gg. In that case we would expect gg admixtures in many of the radial excitations of the f as well, with a total of more states than would be given by a pure $q\bar{q}$ picture. This is a very real possibility if the level density of gg states is substantially less than that of $q\bar{q}$ states, as would occur if the gg interaction is stronger than the $q\bar{q}$ interaction.[27]

We have assumed that the proposed $f^{\perp}$ decouples from $\pi\pi$, since otherwise it already would have been seen in high-statistics $\pi\pi$ scattering experiments.[28] Some reexamination of these experiments may be in order. There also exist extensive studies of $\pi\pi \to K\bar{K}$, both in the K^+K^- and $K_S^0K_S^0$ channels.[29] We are not aware of claims for narrow 2^+ resonances in any of these works at present.

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[19]With T. DeGrand and M. Nikolic, we have begun to investigate $f^{\perp}$-f' mixing (e.g., to suppress the $K\bar{K}$ decay of one state), and to relax the condition $\Gamma(f'\not\to\pi\pi)$ to the extent permitted by the data of Ref. 12. In the three-state $(f,f',f^{\perp})$ problem the f' also generally will acquire a gluon component.

[20]In the absence of mixing, for instance, we would estimate from Eqs. (6), (7), and (16) that for a pure gg state $f[gg]$, $\Gamma(f[gg]\to\pi\pi)=\sin^2\theta\,\Gamma(f\to\pi\pi)=10$–$35$ MeV. (We thank P. Fishbane for drawing this to our attention.) The 4π decay of the gg state should be correspondingly less important.

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