# $\eta \to 3\pi$ decays and the $\rho\eta\pi$ coupling constant

Ll. Ametller and A. Bramon
*Departament de Física Teórica, Universitat Autónoma de Barcelona, Bellaterra, Barcelona, Spain*
(Received 18 March 1981)

An approach to the $\eta \to 3\pi$ decays based on $\eta$-$\pi^0$ mixing and resonant final-state interactions is presented. It successfully accounts for the experimental data and circumvents the difficulties of the traditional (current-algebra) approach. The value for the $G$-parity-violating coupling constant $|g_{\rho\eta\pi}| = 0.067 \pm 0.013$ is deduced from the data.

The understanding of the electromagnetic (EM) decays $\eta \to \pi^+\pi^-\pi^0$ and $\eta \to 3\pi^0$ has been a longstanding difficulty[1-3] for current algebra and partial conservation of axial-vector currents (PCAC). Recently,[4-7] the problem has been reconsidered from more modern (and more speculative) points of view and interest has been renewed in it. A definite and convincing explanation for all the aspects of the problem seems, however, to be still lacking. Our purpose consists in presenting an unconventional description of these $\eta$ decays which circumvents the traditional difficulties and accounts for the available data in a rather natural way.

The relevant experimental information concerning $\eta \to 3\pi$ decyas can be briefly summarized as follows. There is strong evidence that the $\eta \to \pi^+\pi^-\pi^0$ amplitude admits the linear parametrization[8,9]

$$A^{+-0}(s) = A[1 + 3a(s_0 - s)/2M_\eta(M_\eta - 2\mu - \mu_0)], \tag{1}$$

where $s \equiv (q^+ + q^-)^2 = (P - q^0)^2$, $s_0 = \frac{1}{3}(M_\eta{}^2 + 2\mu^2 + \mu_0{}^2) \simeq 0.12\ \text{GeV}^2$ corresponds to the center of the Dalitz plot and $P(M_\eta)$, $q^\pm(\mu)$, and $q(\mu_0)$ are the $\eta$, $\pi^\pm$, and $\pi^0$ four-momenta (masses). The slope parameter $a$ is found to be almost real and its value[8]

$$a \simeq \text{Re}a = -0.55 \pm 0.04 \tag{2}$$

gives an accurate description of the amplitude in the whole Dalitz region, i.e., for $4\mu^2 \leq s \leq (M_\eta - \mu_0)^2 \simeq 8.7\,\mu^2$. In particular, *assuming* that this simple linear behavior can be extrapolated to nonphysical values (soft-pion limits), one expects

$$A^{+-0}(s = 0.030\ \text{GeV}^2 \simeq \mu^2) = 0\,, \tag{3a}$$

$$A^{+-0}(s = M_\eta{}^2) = 3.0A\,. \tag{3b}$$

By contrast, the density of points in the Dalitz plot for the $\eta \to 3\pi^0$ decay and the corresponding amplitude $A^{000}$ have been observed[10] to be compatible with a constant. The partial widths of the $\eta \to \pi^+\pi^-\pi^0$ and $\eta \to 3\pi^0$ decays can be deduced from the experimental result[11] $\Gamma(\eta \to \gamma\gamma) = 324 \pm 46$ eV and the corresponding branching ratios.[9] They turn out to be

$$\Gamma(\eta \to \pi^+\pi^-\pi^0) = 200 \pm 29\ \text{eV}\,, \tag{4a}$$

$$\Gamma(\eta \to 3\pi^0) = 276 \pm 43\ \text{eV}\,. \tag{4b}$$

[The Particle Data Group[9] (PDG) average for Eq. (4b) is $\Gamma(\eta \to 3\pi^0) = 255 \pm 37$ GeV. Our value (4b) comes from Eq. (4a) and the mean value $\Gamma(\eta \to 3\pi^0)/\Gamma(\eta \to \pi^+\pi^-\pi^0) = 1.38 \pm 0.08$ deduced from all the experimental results which are insensitive to the controversial $\eta \to \pi^0\gamma\gamma$ decay mode.[9]]

The theoretical attempts to account for the preceding data have been traditionally performed in the context of current algebra and soft-pion techniques.[1] As Sutherland[2] first noticed, the application of these techniques to either charged pion in the $\eta \to \pi^+\pi^-\pi^0$ amplitude leads to the unambiguous prediction $A^{+-0}(s = \mu^2) = 0$, which is in excellent agreement with the extrapolated value quoted in Eq. (3a). Such an impressive success of this approach contrasts, however, with its inability to account for the remaining features of the data. Indeed, taking the soft limit for the neutral pion leads to $A^{+-0}(s = M_\eta{}^2) = 0$, in clear disagreement with Eq. (3b). For this and other theoretical reasons,[12] a tadpole term[13] was added to the initial, purely photonic interaction Lagrangian.[2] As a result, the successful prediction for $s \simeq \mu^2$ was preserved and the vanishing of the amplitude at $s = M_\eta{}^2$ could be avoided, but the value of $A^{+-0}(s = M_\eta{}^2)$ was still substantially lower[1,3] than the experimental one. The strategy of the recent work on the subject has mainly consisted in incorporating the appropriate enhancement mechanisms into the previous, traditional approach. Some interesting examples of these mechanisms are the introduction of large contributions coming from exotic $q^2\bar{q}^2$ scalar mesons[6] or from threshold effects in the pion-pion system,[7] the use of $SU(3) \times SU(3)$ chiral symmetry with large mass differences for the $u$ and $d$ current quarks,[4] and the possible relevance of the U(1) problem.[5]

In the present paper the $\eta \to 3\pi$ decays will be discussed from a different point of view. For convenience, we perform an isospin analysis of the $I=1$, $3\pi$ final states and distinguish between two types of contributions to the amplitudes $A^{+-0}$ and $A^{000}$. The first type contains the channels with an $I=1$ intermediate dipion system and contributes exclusively to $A^{+-0}$. Being dynamically dominated by the $\rho^{\pm}$-meson poles, this $I=1$ amplitude will be denoted $A_\rho(s)$ and it will be shown to supply the appropriate $s$ dependence to $A^{+-0}(s)$. The remaining, second type of amplitude, containing the channels with $I=0$ and 2 dipion states, contributes to both $A^{+-0}$ and $A^{000}$. Isospin and Bose-symmetry arguments allow us to write

$$A^{+-0}(s)=A+A_\rho(s)\,, \tag{5a}$$

$$A^{000}=3A\,, \tag{5b}$$

where use has been made of our main assumption, namely, that the non-$\rho$-dominated amplitude $A$ is approximately $s$ independent. The necessity of such an assumption is obvious from the observed[10] constancy of $A^{000}$, and its partial justification on dynamical grounds will be attempted later on.

The $\rho$ contribution proceeds through the decay chain $\eta\to\rho^{\pm}\pi^{\mp}\to\pi^+\pi^-\pi^0$ and is readily evaluated to be

$$A_\rho(s)=\lambda g_{\rho\pi\pi}{}^2\left[\frac{(P+q^-)\cdot(q^+-q)}{m_\rho{}^2-(q^++q)^2-im_\rho\Gamma_\rho}+\frac{(P+q^+)\cdot(q^--q)}{m_\rho{}^2-(q^-+q)^2-im_\rho\Gamma_\rho}\right], \tag{6}$$

where $\lambda g_{\rho\pi\pi}\equiv g_{\rho\eta\pi}$ is the $G$-parity-violating coupling constant associated with the first step and[9] $g_{\rho\pi\pi}{}^2/4\pi=12\,\Gamma(\rho\to\pi\pi)/m_\rho(1-4\mu^2/m_\rho^2)^{3/2}=3.0\pm0.1$ describes the final strong decay. The $\rho$ propagators appearing in Eq. (6) can be reasonably approximated by their common value at the center of the Dalitz plot (the error introduced is smaller than $\sim8\%$) and, then, one obtains

$$A^{+-0}(s)\simeq A+3\lambda g_{\rho\pi\pi}{}^2\frac{(s-s_0)}{m_\rho{}^2-s_0-im_\rho\Gamma_\rho(s_0)}\,. \tag{7}$$

Therefore, having imposed the constancy of $A^{000}=3A$, the $\rho$ contributions are able to reproduce the $s$ dependence of $A^{+-0}(s)$ in the whole physical region, where no potentially dangerous extrapolations are needed.

A more quantitative description of the $\eta\to3\pi$ data can be immediately achieved by fixing our two independent parameters $A$ and $\lambda=g_{\rho\eta\pi}/g_{\rho\pi\pi}$. Neglecting (see below) possible phase differences between $A$ and $A_\rho(s)$—as suggested by the experimental results (1) and (2)—and introducing the PDG-compilation values[9] for $M_\eta$, $\mu$, $\mu_0$, $m_\rho$, and $\Gamma_\rho$, one can deduce

$$|A|=0.20\pm0.02\,, \tag{8a}$$

$$\lambda=0.011\pm0.002 \tag{8b}$$

from the experimental values quoted in Eqs. (2) and (4a). Similarly, from Eq. (4b) one obtains

$$|A|=0.23\pm0.02 \tag{8c}$$

in reasonable agreement with Eq. (8a). Our result (8b) leads to $|g_{\rho\eta\pi}|=0.067\pm0.013$ representing a rather safe and accurate determination of the EM $\rho\eta\pi$ coupling.

In the rest of this paper we try to give some plausibility to our previous assumptions and to show that the values of $A$ and $\lambda$ quoted in Eqs. (8) could have been estimated from independent sources of experimental information. To this end, one has to introduce the well-known $\eta$-$\pi^0$ mixing[14] which is directly connected with $\lambda$. In this context the value of $\lambda$ is found to be[15] $\lambda=0.012$ or $\lambda=0.0107$ if a linear or quadratic mass formula for the pseudoscalar-meson masses[9] which includes EM effects[14] is used. From a combined study[16] of baryon and meson mass splittings, $\rho$-$\omega$ mixing, and $\eta\to3\pi$ decays one similarly obtains $\lambda=0.013\pm0.002$. Finally, from the difference in binding energies in light hypernuclei, one finds[15] $g(\Lambda\Lambda\pi)\simeq-0.027g(\Lambda\Sigma\pi)$, which implies $\lambda\simeq0.012$, after correcting for similar $\Lambda\Sigma^0$-mixing effects.[14] In summary, the available information on the parameter $\lambda$ is consistent with our value quoted in Eq. (8b).

Similarly, in the present approach, it seems natural to associate the amplitude $A$ with the contributions coming from scalar-meson intermediate states. Some aspects of these contributions have already been considered by other authors[6,7,17] when dealing with the $\eta\to3\pi$ problem and, with minor modifications, when describing the related $\eta'\to\eta\pi\pi$ strong decay.[18] The dominant contribution to $A^{+-0}$ comes from the lowest-lying $\epsilon(700)$ state which couples to $\pi^0\eta$ through the $\pi^0$ content of the physical $\eta$ originated, as in the $\rho\eta\pi$ case, by EM $\eta$-$\pi^0$ mixing and thus satisfying $\lambda\equiv g_{\rho\eta\pi}/g_{\rho\pi\pi}=g_{\epsilon\eta\pi}/g_{\epsilon\pi\pi}$. One has

$$A_\epsilon(s)=\lambda\,\frac{32\pi}{3}\,\frac{m_\epsilon\Gamma_\epsilon(1-4\mu^2/m_\epsilon{}^2)^{-1/2}}{m_\epsilon{}^2-s-im_\epsilon\Gamma_\epsilon(s)} \tag{9}$$

independently of the $q\bar q$ or $q^2\bar q^2$ quark content of $\epsilon(700)$. The assumed constancy of $A$ is now an approximate consequence of the small range of variation of $s$ around $s_0$, $\Delta s\simeq\pm2.6\mu^2\ll m_\epsilon{}^2$, which allows the transformations of Eq. (9) into

$$A\simeq\lambda\,\frac{32\pi}{3}\,m_\epsilon\Gamma_\epsilon/[m_\epsilon{}^2-s_0-im_\epsilon\Gamma_\epsilon(s_0)]\,. \tag{10}$$

The relative phase between the amplitudes $A_\rho(s)$ and $A$, i.e., the phase of $a$, can be deduced from

Eqs. (1), (7), and (10) giving

$$a \simeq -\frac{3g_\rho^2\pi\pi}{16\pi}\,\frac{M_\eta(M_\eta - 3\mu)}{m_\epsilon\Gamma_\epsilon}\,\frac{m_\epsilon^2 - s_0 - im_\epsilon\Gamma_\epsilon(s_0)}{m_\rho^2 - s_0 - im_\rho\Gamma_\rho(s_0)}\,, \tag{11}$$

which leads to the required small phase due to the smallness of $m_\epsilon\Gamma_\epsilon(s_0) - m_\rho\Gamma_\rho(s_0)$ at $s \simeq s_0$. Finally, for the reasonable input $m_\epsilon \sim m_\rho$ and $\Gamma_\epsilon \simeq 360$ MeV, Eq. (11) reproduces the experimental result (2) for $a$, and Eqs. (8b) and (9) give $A \simeq 0.20$ in good agreement with Eqs. (8a) and (8c).

In conclusion, a satisfactory description of the $\eta \to \pi^+\pi^-\pi^0$ and $\eta \to 3\pi^0$ decays can be achieved in a simple framework incorporating $\eta$-$\pi^0$ EM mixing and resonant final-state interactions. Among these, a definite $\rho$ contribution, which plays an essential role in order to account for the $\pi^0$-energy dependence in the $\eta \to \pi^+\pi^-\pi^0$ Dalitz plot, can be extracted from the data. It leads to the accurate value for the modulus of the EM $\rho\eta\pi$ coupling constant $|g_{\rho\eta\pi}| = \lambda g_{\rho\pi\pi} = 0.067 \pm 0.013$ for which no other estimates are known.


[1] A. Raby, Phys. Rev. D 13, 2594 (1975). This paper reviews the history behind the decay $\eta \to 3\pi$.

[2] D. G. Sutherland, Phys. Lett. 23, 384 (1966).

[3] S. Weinberg, Phys. Rev. D 11, 3583 (1975).

[4] P. Langacker and H. Pagels, Phys. Rev. D 19, 2070 (1979); 10, 2904 (1974).

[5] K. A. Milton, W. F. Palmer, and S. S. Pinsky, Phys. Rev. D 22, 1124; 22, 1647 (1980).

[6] R. Aaron and H. Goldberg, Phys. Rev. Lett. 45, 1752 (1980).

[7] C. Roiesnel and T. N. Truong, Palaiseau Report No. A423.1080, 1980 (unpublished).

[8] A summary and list of references can be found in J. G. Layter *et al.*, Phys. Rev. D 7, 2565 (1973).

[9] Particle Data Group, Rev. Mod. Phys. 52, S1 (1980).

[10] C. Baglin *et al.*, Phys. Lett. 29B, 445 (1969).

[11] A. Browman *et al.*, Phys. Rev. Lett. 32, 1067 (1974).

[12] K. Wilson, Phys. Rev. 179, 1499 (1969).

[13] S. Coleman and S. L. Glashow, Phys. Rev. 134, B67 (1964).

[14] P. Carruthers, *Introduction to Unitary Symmetry* (Wiley, New York, 1966).

[15] M. M. Nagels *et al.*, Nucl. Phys. B147, 189 (1979).

[16] P. Langacker, Phys. Lett. 90B, 447 (1980).

[17] Y. T. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. 161, 1612 (1967).

[18] C. A. Singh and J. Pasupathy, Phys. Rev. Lett. 35, 1193 (1975); A. Bramon and E. Massó, Phys. Lett. 93B, 65 (1980); G. W. Intemann and G. K. Greenhut, Phys. Rev. D 22, 1669 (1980).