PHYSICAL REVIEW D VOLUME 24, NUMBER 5 1 SEPTEMBER 1981

Chiral dynamics and semileptonic τ decays

Gordon J. Aubrecht II,* Najib Chahrouri, and Konrad Slanec
Institut für Theoretische Physik, Universität Karlsruhe, 7500 Karlsruhe 1, Federal Republic of Germany
(Received 23 February 1981; revised manuscript received 17 June 1981)

We consider the semileptonic decays of the τ in a model of SU(3)$\times$SU(3) chiral dynamics using phenomenological Lagrangians. The effect of the resonant states is included to facilitate comparison with the data.

I. INTRODUCTION

As more data on τ decays become available,[1-6] it is of interest to consider the various semileptonic decays. The investigation of $\tau \rightarrow \nu\rho\pi$ by Geffen and Wilson[7] and the work of Fischer, Wess, and Wagner[8] on multipion decays of the τ based on the original ideas of Callen, Coleman, Wess, and Zumino,[9] has provided evidence that current algebra is also of interest for higher-energy phenomena. A further test of the applicability of these older ideas to the new realms of physics at high energy comes from consideration of the final states containing several types of particles. If these predictions are also borne out by experiment, we may place more reliance on the applicability of chiral groups to decays of the new particles. We consider here chiral SU(3) $\times$ SU(3), the simplest extension to more particles (π, K, and η). The various particles' decay constants differ from one another at the level of ~10%.[10,11] The effect of differing decay constants is, of course, of interest.[12] For our purposes, we shall consider the η to be identical to the η_8. The particle masses are introduced in the phase-space calculations because of the strong effect of the available phase space on the numerical values of the widths.

We shall use the method of phenomenological Lagrangians using nonlinear realizations of the chiral group, which relates amplitudes for the emission of various numbers of soft hadrons. The techniques adopted for the nonlinear realizations of chiral groups are all familiar, and shall not be recapitulated here (see, e.g., Refs. 8 and 9). We may summarize the result by stating that the amplitudes for multihadron final states depend only on the particle decay constant f_π (and in principle on f_K, f_η, determined years ago for low-energy phenomena[9,10]). In this sense, the predictions for τ decay are parameter-free (that is, the high-energy behavior is given in terms of the low-energy behavior). It is in this sense that the model provides a test of the applicability of the current algebra to higher-energy phenomena. In making the extension from low energy to high energy, it is of course necessary to introduce the possible resonant states. We explicitly consider the ρ, K^*, and ϕ states, as made up of two-particle pseudoscalar states in resonance, in our current matrix elements.

II. CHIRAL DYNAMICS AND MULTIHADRON PRODUCTION

We consider decays of the form

$$\tau \rightarrow \nu_\tau + m\pi + nK + p\eta \,.$$

Since the decay amplitudes may be written in terms of the matrix elements of the leptonic and hadronic currents, we write the matrix element for the above process

$$\mathfrak{M}(\tau \rightarrow \nu_\tau + m\pi + nK + p\eta) = \frac{G}{\sqrt{2}} \cos\theta_C \, \bar{u}(p_\tau)\gamma^\mu(1+\gamma_5)\bar{u}(p_\nu) J_\mu(m\pi, nK, p\eta) \tag{1}$$

in the strangeness-conserving case. In the case of strangeness-changing currents, the factor $\cos\theta_C$ is replaced by $\sin\theta_C$. In Eq. (1) the J_μ are the current matrix elements for the production of m pions, n kaons, and p η's (which are at this stage all massless). Because all masses are identical (and zero) all the decay constants are the same and all the currents J_μ are conserved. In view of Eq. (1), it is convenient to give the results of our calculations normalized to the leptonic decay rate

$$\Gamma(\tau \rightarrow \nu_\tau \mu \nu_\mu) = G^2 \cos^2\theta_C \, m_\tau^{\,5}/192\pi^3 \,.$$

The ratio $\Gamma/\Gamma(\tau \rightarrow \nu_\tau \mu\nu_\mu)$ shall be denoted γ (cf. Ref. 8). Hence, for strangeness-conserving current matrix elements

$$\gamma(m\pi, nK, p\eta) = 24(2\pi)^{6-(m+n+p)} m_\tau^{-6} \int \frac{d^3p_\nu}{2p_\nu^0} \left(p_\tau^\rho p_\nu^\sigma + p_\tau^\sigma p_\nu^\rho - p_\tau \cdot p_\nu g^{\rho\sigma}\right) \times \int \prod_{i,j,k} \frac{d^3p_i}{2p_i^0} \frac{d^3k_j}{2k_j^0} \frac{d^3\eta_k}{2\eta_k^0} \delta^4\left(p_\tau - p_\nu - \sum_i p_i - \sum_j k_j - \sum_k \eta_k\right) \times J_\rho^\dagger(m\pi, nK, p\eta) J_\sigma(m\pi, nK, p\eta) \,. \tag{2}$$

For strangeness-changing currents, the above result should be multiplied by $\tan^2\theta_C$.

The problem is thus to compute the current matrix elements J_μ (exactly conserved in the massless limit). Were we to introduce the masses at this stage, the currents would of course obey PCAC (partial conservation of axial-vector current). We shall calculate the current matrix elements from our nonlinear Lagrangian via the Noether theorem. We may write ($f=f_\pi$)

$$\mathcal{L} = f^2\,\mathrm{Tr}(\mathcal{P}_\mu \mathcal{P}^\mu)\,, \tag{3}$$

where

$$\mathcal{P}_\mu = \partial_\mu M - \frac{1}{3!}[M,[M,\partial_\mu M]] + \frac{1}{5!}[M,[M,[M,\partial_\mu M]]] + \cdots\,,$$

and M is the usual SU(3) matrix of pseudoscalars:

$$M = \sum_i (\varphi_i/f)(\lambda_i/2)\,.$$

Using the Noether theorem, we obtain the following vector and axial-vector currents:

$$\mathcal{V}^\mu = if^2\left\{[M,\partial^\mu M] - \frac{2}{3!}[M,[M,\partial^\mu M]]] + \cdots\right\}\,, \tag{4}$$

$$\mathcal{A}^\mu = f^2\{\partial^\mu M - \tfrac{2}{3}[M,[M,\partial^\mu M]] + \cdots\}\,. \tag{5}$$

These currents allow us to calculate the current matrix elements in Eq. (2). For a given process, we must use Eq. (3) as well as Eqs. (4) or (5) to obtain a parameter-independent result, since we must sum all possible tree diagrams contributing to that process. For example, the current matrix element for production of three kaons has a contribution from the current for production of a virtual kaon, followed by production of three kaons (Fig. 1). We are led to the following current matrix elements in our most general case (one, two, or three hadrons produced):

$$J_\mu(\pi^+) = -i\sqrt{2}fp_{+\mu}\,, \tag{6}$$

$$J_\mu(K^+) = -i\sqrt{2}\,fk_{+\mu}\,, \tag{7}$$

$$J_\mu(\pi^+\pi) = \sqrt{2}\,(p_+ - p)_\mu\,, \tag{8}$$

$$J_\mu(K^+\bar{K}) = -(k_+ - \bar{k})_\mu\,, \tag{9}$$

$$J_\mu(\pi K^+) = 1/\sqrt{2}\,(k_+ - p)_\mu\,, \tag{10}$$

$$J_\mu(\pi^+K) = (k - p_+)_\mu\,, \tag{11}$$

$$J_\mu(K^+\eta) = 3/\sqrt{6}\,(k_+ - \eta)_\mu\,. \tag{12}$$

In the following expressions, $\mathcal{P}_{\mu\nu}$ is given by $\mathcal{P}^A_{\mu\nu}(P) = g_{\mu\nu} - P_\mu P_\nu/(P^2 - m_A{}^2)$:

$$J_\mu(\pi_1^+\pi_2^+\pi^-) = -(i2\sqrt{2}/3f)(2p_- - p_{1+} - p_{2+})^\nu \mathcal{P}^\pi_{\mu\nu}(P)\,, \tag{13}$$

$$J_\mu(\pi_1\pi_2\pi^+) = (i2\sqrt{2}/3f)(2p_+ - p_1 - p_2)^\nu \mathcal{P}^\pi_{\mu\nu}(P)\,, \tag{14}$$

$$J_\mu(\pi^+K^+K^-) = (i\sqrt{2}/3f)(p_+ + k_+ - 2k_-)^\nu \mathcal{P}^\pi_{\mu\nu}(P)\,, \tag{15}$$

$$J_\mu(\pi^+K\bar{K}) = (i\sqrt{2}/3f)(p_+ + \bar{k} - 2k)^\nu \mathcal{P}^\pi_{\mu\nu}(P)\,, \tag{16}$$

$$J_\mu(\pi\bar{K}K^+) = -(i/f)(k_+ - \bar{k})^\nu \mathcal{P}^\pi_{\mu\nu}(P)\,, \tag{17}$$

$$J_\mu(\eta\bar{K}K^+) = (i/\sqrt{3}f)(2\eta - k_+ - \bar{k})^\nu \mathcal{P}^\pi_{\mu\nu}(P)\,, \tag{18}$$

$$J_\mu(K_1^+K_2^+K^-) = -(i2\sqrt{2}/3f)(2k_- - k_{1+} - k_{2+})^\nu \mathcal{P}^K_{\mu\nu}(P)\,, \tag{19}$$

$$J_\mu(K\bar{K}K^+) = -(i\sqrt{2}/3f)(2\bar{k} - k - k_+)^\nu \mathcal{P}^K_{\mu\nu}(P)\,, \tag{20}$$

$$J_\mu(\pi_1\pi_2K^+) = (i\sqrt{2}/6f)(2k_+ - p_1 - p_2)^\nu \mathcal{P}^K_{\mu\nu}(P)\,, \tag{21}$$

$$J_\mu(\pi^+\pi^-K^+) = (i\sqrt{2}/3f)(k_+ + p_+ - 2p_-)^\nu \mathcal{P}^K_{\mu\nu}(P)\,, \tag{22}$$

$$J_\mu(\pi\pi^+K) = -(i/f)(p_+ - p)^\nu \mathcal{P}^K_{\mu\nu}(P)\,, \tag{23}$$

$$J_\mu(\eta\pi^+K) = (i/\sqrt{3}f)(2k - p_+ - \eta)^\nu \mathcal{P}^K_{\mu\nu}(P)\,, \tag{24}$$

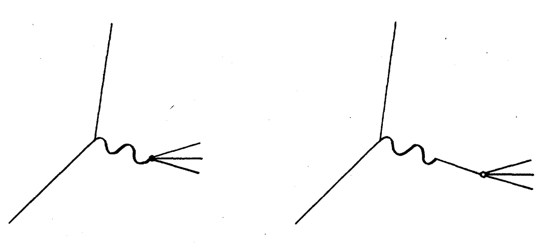
$$J_\mu(\eta\pi K^+) = (i/\sqrt{6}f)(2k_+ - p - \eta)^\nu \mathcal{P}^K_{\mu\nu}(P)\,, \tag{25}$$

$$J_\mu(\eta_1\eta_2K^+) = (i/\sqrt{2}f)(2k_+ - \eta_1 - \eta_2)^\nu \mathcal{P}^K_{\mu\nu}(P)\,. \tag{26}$$

With these formulas the widths γ of Eq. (2) are determined through production of three particles. Of course, our description in Eqs. (6)–(26) is incomplete in the sense that the known resonant behavior of the particles has been neglected. In other words, (6)–(26) are correct only in the low-energy limit. For higher energy, the resonant states must be explicitly included.

Possible resonant states are those for $\pi\pi$, πK, and $K\bar{K}$, that is, ρ, K^*, and ϕ, respectively (we restrict ourselves to two-particle resonant states). As in Ref. 8, we modify Eqs. (6)–(26) by including the form factor in such a way that the current conservation is preserved. The low-energy limit is then automatically recovered. We shall write the form factors $F_A(q)$, where

$$F_A(q) = (m_A{}^2 - \mathrm{i}m_A\Gamma_A)/(m_A{}^2 - \mathrm{i}m_A\Gamma_A - q^2)\,. \tag{27}$$



(a) (b)

FIG. 1. (a) Typical diagram, e.g., for $\tau \to \nu_\tau K K\bar{K}$ for production of three hadrons. (b) Contribution of a virtual intermediate particle, e.g., $\tau \to \nu_\tau K \to \nu_\tau KK\bar{K}$.

TABLE I. Reduced widths γ, hadron masses neglected.

Produced hadrons	ρ (%)	K^* (%)	ϕ (%)	Continuum (%)	Total	Experiment
π^+				100	0.63	0.56 ± 0.12[a]
K^+				100	0.03	
$\pi^+\pi$	100				1.26	1.22 ± 0.21[b]
$K^+\bar{K}$				100	0.13	
π^+K		100			0.14	0.13 ± 0.05[c]
πK^+		100			7.1×10^{-2}	
$K^+\eta$				100	1.2×10^{-2}	
$\pi^+(\pi^+\pi^-, \pi\pi)$	100				0.24	0.31 ± 0.13[d]
$\pi^+(K^+K^-, K\bar{K})$		11.8	87.7	0.5	0.38	
$\pi\bar{K}K^+$		93.2		6.8	0.11	
$K^+\bar{K}\eta$				100	1.7×10^{-2}	
$K^+K^+K^-$			100	0.04	0.58	
$K\bar{K}K^+$			98.1	1.9	2.2×10^{-2}	
$\pi\pi K^+$		73.0		27	3.0×10^{-4}	
$\pi^+\pi^-K^+$	35	64.3		0.6	4.4×10^{-3}	
$\pi\pi^+K$	100				7.2×10^{-3}	
$\pi^+K\eta$		98.4		1.6	1.8×10^{-2}	
$\pi K^+\eta$		98.8		1.2	2.5×10^{-3}	
$K^+\eta\eta$				100	1.6×10^{-3}	

[a]Reference 3; this number is given as 0.68 ± 0.17 in Ref. 5.
[b]References 2, 5, and 6.
[c]Reference 5; the number quoted is only the resonant part.
[d]Reference 1.

It is also necessary to ensure that only the $J = \frac{1}{2}$ part of $K\pi$ be allowed to form K^*. Similarly, only the $I = 1$ part of $\pi\pi$ can form ρ, and only the $J = 0$ piece of $K\bar{K}$ can form ϕ. In practice, this means that terms such as $(k - p_+)_\mu$ must be replaced by

$$\tfrac{1}{3}(k - p_+)_\mu + \tfrac{2}{3}(k - p_+)_\mu F_{K^*}(k + p_+)$$

when one correctly introduces the resonance in the three-particle currents. With these requirements, we obtain the following current matrix elements:

$$J_\mu(\pi^+\pi) = \sqrt{2}\,(p_+ - p)_\mu F_\rho(p + p_+)\,, \tag{8'}$$

$$J_\mu(\pi K^+) = (1/\sqrt{2})(k_+ - p)_\mu F_{K^*}(p + k_+)\,, \tag{10'}$$

$$J_\mu(\pi^+ K) = (k - p_+)_\mu F_{K^*}(p_+ + k)\,, \tag{11'}$$

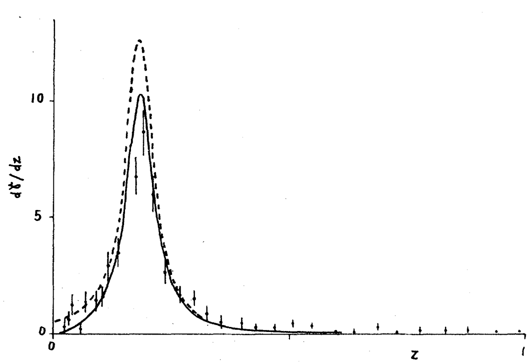


FIG. 2. Differential distribution of the reduced width for $\tau \to \nu_\tau \pi^+\pi$. The data are from Refs. 4–6. The masses of the decay hadrons are taken to be zero for the dashed curve; actual hadron masses are used for the solid curve.

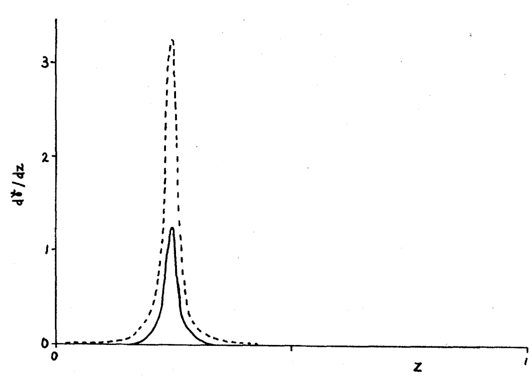


FIG. 3. Differential distribution of the reduced width for $\tau \to \nu_\tau \pi^+ K$. The masses of the decay hadrons are taken to be zero for the dashed curve; actual hadron masses are used for the solid curve.

$$J_\mu(\pi_1^+\pi_2^+\pi^-) = -(i2\sqrt{2}/3f)(p_- - p_{1+})^\nu F_\rho(p_- + p_{1+})\mathcal{P}^\pi_{\mu\nu}(P) - (i2\sqrt{2}/3f)(p_- - p_{2+})^\nu F_\rho(p_- + p_{2+})\mathcal{P}^\pi_{\mu\nu}(P)\,, \tag{13'}$$

$$J_\mu(\pi_1\pi_2\pi^+) = (i2\sqrt{2}/3f)(p_+ - p_1)^\nu F_\rho(p_+ + p_1)\mathcal{P}^\pi_{\mu\nu}(P) + (i2\sqrt{2}/3f)(p_+ - p_2)^\nu F_\rho(p_+ + p_2)\mathcal{P}^\pi_{\mu\nu}(P)\,, \tag{14'}$$

$$J_\mu(\pi^+K^+K^-) = (i/3\sqrt{2}f)(k_+ - k_-)^\nu F_\phi(k_+ + k_-)\mathcal{P}^\pi_{\mu\nu}(P) + (i2\sqrt{2}/9f)(p_+ - k_-)^\nu F_{K^*}(p_+ + k_-)\mathcal{P}^\pi_{\mu\nu}(P) + (i/9\sqrt{2}f)(3k_+ + 2p_+ - 5k_-)^\nu \mathcal{P}^\pi_{\mu\nu}(P)\,, \tag{15'}$$

$$J_\mu(\pi^+K\bar{K}) = (i2\sqrt{2}/9f)(p_+ - k)^\nu F_{K^*}(p_+ + k)\mathcal{P}^\pi_{\mu\nu}(P) + (i/3\sqrt{2}f)(\bar{k} - k)^\nu F_\phi(k + \bar{k})\mathcal{P}^\pi_{\mu\nu}(P) + (i/9\sqrt{2}f)(3\bar{k} + 2p_+ - 5k)^\nu \mathcal{P}^\pi_{\mu\nu}(P)\,, \tag{16'}$$

$$J_\mu(\pi\bar{K}K^+) = (i/3f)(p - k_+)^\nu F_{K^*}(p + k_+)\mathcal{P}^\pi_{\mu\nu}(P) + (i/3f)(\bar{k} - p)^\nu F_{K^*}(p + \bar{k})\mathcal{P}^\pi_{\mu\nu}(P) + (i2/3f)(\bar{k} - k_+)^\nu \mathcal{P}^\pi_{\mu\nu}(P)\,, \tag{17'}$$

$$J_\mu(K_1^+K_2^+K^-) = (i\sqrt{2}/3f)(k_{1+} - k_-)^\nu F_\phi(k_{1+} + k_-)\mathcal{P}^K_{\mu\nu}(P) + (i\sqrt{2}/3f)(k_{2+} - k_-)^\nu F_\phi(k_{2+} + k_-)\mathcal{P}^K_{\mu\nu}(P) + (i\sqrt{2}/3f)(k_{1+} - k_-)^\nu \mathcal{P}^K_{\mu\nu}(P) + (i\sqrt{2}/3f)(k_{2+} - k_-)^\nu \mathcal{P}^K_{\mu\nu}(P)\,, \tag{19'}$$

$$J_\mu(K\bar{K}K^+) = (i\sqrt{2}/6f)(k - \bar{k})^\nu F_\phi(k + \bar{k})\mathcal{P}^K_{\mu\nu}(P) + (i\sqrt{2}/6f)(2k_+ + k - 3\bar{k})^\nu \mathcal{P}^K_{\mu\nu}(P)\,, \tag{20'}$$

$$J_\mu(\pi_1\pi_2K^+) = (i\sqrt{2}/36f)(k_+ - p_1)^\nu F_{K^*}(k_+ + p_1)\mathcal{P}^K_{\mu\nu}(P) + (i\sqrt{2}/36f)(k_+ - p_2)^\nu F_{K^*}(k_+ + p_2)\mathcal{P}^K_{\mu\nu}(P) + (i\sqrt{2}/18f)(2k_+ - p_1 - p_2)^\nu \mathcal{P}^K_{\mu\nu}(P)\,, \tag{21'}$$

$$J_\mu(\pi^+\pi^-K^+) = (i\sqrt{2}/3f)(p_+ - p_-)^\nu F_\rho(p_+ + p_-)\mathcal{P}^K_{\mu\nu}(P) + (i2\sqrt{2}/9f)(k_+ - p_-)^\nu F_{K^*}(k_+ + p_-)\mathcal{P}^K_{\mu\nu}(P) + (i\sqrt{2}/9f)(k_+ - p_-)^\nu \mathcal{P}^K_{\mu\nu}(P)\,, \tag{22'}$$

$$J_\mu(\pi\pi^+K) = (i/f)(p - p_+)^\nu F_\rho(p + p_+)\mathcal{P}^K_{\mu\nu}(P)\,, \tag{23'}$$

$$J_\mu(\eta\pi^+K) = (i4/3\sqrt{3}f)(k - p_+)^\nu F_{K^*}(k + p_+)\mathcal{P}^K_{\mu\nu}(P) + (i/3\sqrt{3}f)(2k + p_- - 3\eta)^\nu \mathcal{P}^K_{\mu\nu}(P)\,, \tag{24'}$$

$$J_\mu(\eta\pi K^+) = (i2/3\sqrt{6}f)(k_+ - p)^\nu F_{K^*}(p + k_+)\mathcal{P}^K_{\mu\nu}(P) + (i/3\sqrt{6}f)(4k_+ - 3\eta - p)^\nu \mathcal{P}^K_{\mu\nu}(P)\,. \tag{25'}$$

We are now in a position to calculate the widths of Eq. (2), as well as the distribution $d\gamma/dP^2$, where, as above, $P = p_\tau - p_{\nu_\tau}$. Instead of using the variable P^2, we consider the distributions with respect to the dimensionless variable $z = P^2/m_\tau^2$, which runs from zero to one in the massless case and from $(m_1 + m_2 + m_3)^2/m_\tau^2$ for the case in which three massive particles are produced. In Table I we give the reduced widths as calculated for massless hadrons, and in Figs. 2–4 the distributions $d\gamma/dz$ for the case of two-particle final states containing resonances and in Figs. 5–8 a selection from the three-particle final-state distributions. Since we are producing hadrons whose masses are

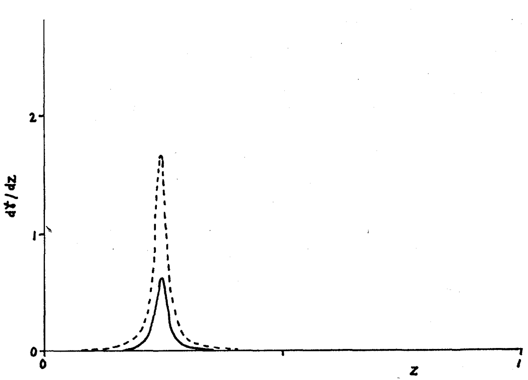


FIG. 4. Differential distribution of the reduced width for $\tau \to \nu_\tau \pi K^+$. The masses of the decay hadrons are taken to be zero for the dashed curve; actual hadron masses are used for the solid curve.

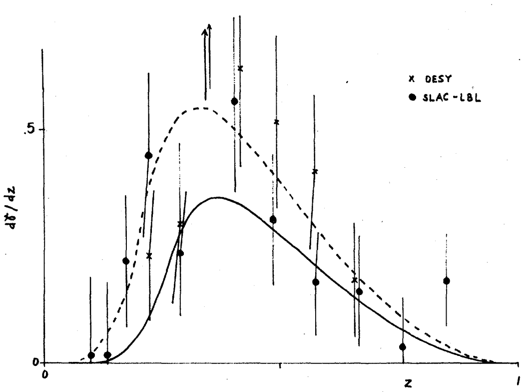


FIG. 5. Differential distribution of the reduced width for $\nu \to \nu_\tau \pi^+\pi^+\pi^-$. The data are from Ref. 1 and J. A. Jaros *et al.*, Phys. Rev. Lett. 40, 1120 (1978). The masses of the decay hadrons are taken to be zero for the dashed line; actual hadron masses are used for the solid line. The data are normalized to the total area given in Table I for comparison. Note that this means that only the shape of the distribution is important.

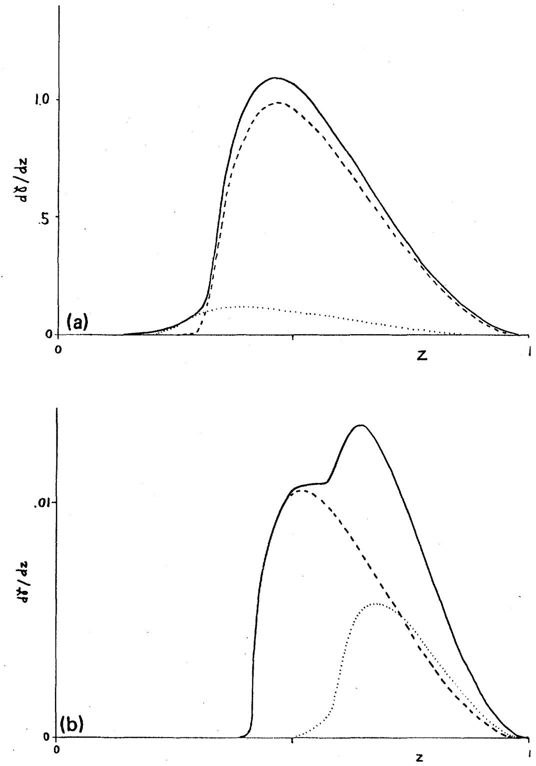


FIG. 6. Differential distribution of the reduced width for $\tau \rightarrow \nu_\tau \pi^+ K^+ K^-$. The contributions of the ϕ (dashed curve) and K^* (dotted curve) are shown; that of the continuum is not explicitly shown. The masses of the decay hadrons are taken to be zero. (b) Same as Fig. (a) except that the actual hadron masses are used.

not negligible in determining available phase space, we redo all calculations to take the restricted phase space into account. These results are presented in Table II, and are also shown in Figs.

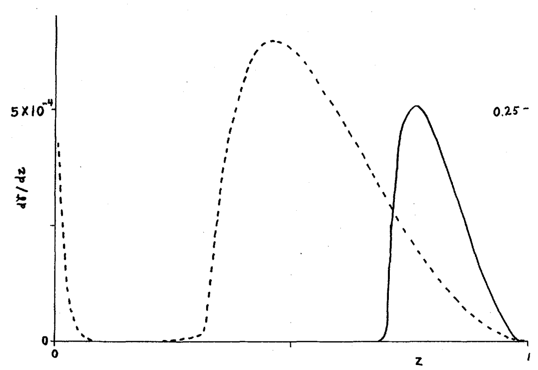


FIG. 7. Differential distribution of the reduced width for $\tau \rightarrow \nu_\tau K^+ K^+ K^-$. The masses of the decay hadrons are taken to be zero for the dashed line (scale on right); actual hadron masses are used for the solid line (scale on left).

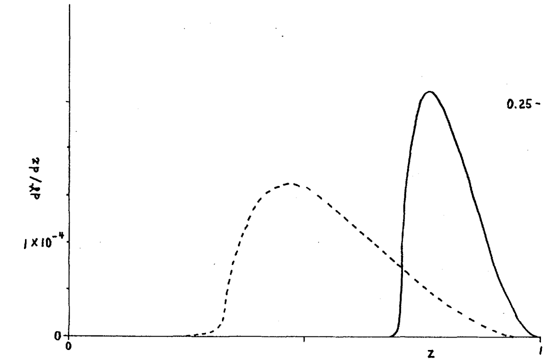


FIG. 8. Differential distribution of the reduced width for $\tau \rightarrow \nu_\tau K^+ K \bar{K}$. The masses of the decay hadrons are taken to be zero for the dashed line (scale on right); the actual hadron masses are used for the solid line (scale on left).

2–8.

We note that the widths involving particles other than the pion are rather small, and earlier calculations[8] showed that the widths for $\tau \rightarrow \nu_\tau \pi^+ \pi^+ \pi^- \pi$ and $\tau \rightarrow \nu_\tau \pi\pi\pi\pi^+$ were rather small. We shall therefore not present results for the other 28 four-particle decay widths here. The size of the correction resulting from the breaking of the symmetry, so that $f_\pi \neq f_K \neq f_\eta$, is currently being investigated.[12]

III. SUMMARY AND CONCLUSIONS

We see from Tables I and II that the resonances contribute the bulk of the decay width. It would thus be of interest to consider a gauge theory of SU(3) $\times$SU(3) so that the resonances can be included in the theory rather than being introduced phenomenologically as in this work. A gauge theory of SU(2)$\times$ SU(2) has recently been considered by Fischer, Klüver, and Wagner.[13] They have found that the effect can be rather large in the massless case.

The effect of the phase-space restriction from the hadronic masses is particularly clear in the cases KKK, $KK\eta$, and $K\eta\eta$, the heaviest mesons. However, the effect of the mass is not negligible even in the case of the two-pion final state. We illustrate this point using the $\pi^+\pi$ decay width. Were we to naively apply the narrow-resonance approximation and neglect the fact that the ν_τ carries off energy, we would obtain, taking $\Gamma_\rho = 150$ MeV, $\gamma(\tau \rightarrow \nu_\tau \pi^+ \pi) = 1.49$. If we then consider the effect of the distribution in z, still leaving the pions massless, we obtain as in Ref. 8 a factor $(1-z)^2$ which modifies the result to 1.36, for $z = m_\rho^2/m_\tau^2$. A full calculation using the proper upper and lower limits yields a value 1.26 for the same width, a reduction of about 20%. Now, how-

TABLE II. Reduced widths γ, actual hadron masses.

Produced hadrons	ρ (%)	K^* (%)	ϕ (%)	Continuum (%)	Total
π^+				100	0.62
K^+				100	0.03
$\pi^+\pi$	100				0.99
$K^+\bar{K}$				100	1.2×10^{-2}
π^+K		100			5.2×10^{-2}
πK^+		100			2.6×10^{-3}
$K^+\eta$				100	9.2×10^{-4}
$\pi^+(\pi^+\pi^-,\pi\pi)$	100				0.14
$\pi^+(K^+K^-,K\bar{K})$		28.9	70.4	0.7	4.5×10^{-3}
$\pi\bar{K}K^+$		99.6		0.4	3.1×10^{-3}
$K^+\bar{K}\eta$				100	1.6×10^{-6}
$K^+K^+K^-$			100	0.02	1.2×10^{-4}
$K\bar{K}K^+$			99.3	0.7	1.6×10^{-5}
$\pi\pi K^+$		77		23	4.8×10^{-5}
$\pi^+\pi^-K^+$	40.8	58.6		0.7	5.2×10^{-4}
$\pi\pi^+K$	100				7.6×10^{-4}
$\pi^+K\eta$		84.4		15.6	1.1×10^{-4}
$\pi K\eta$		93.2		6.8	4.4×10^{-5}
$K^+\eta\eta$				100	4.0×10^{-8}

ever, if we take the pion masses into account to reduce the available phase space, the latter two results are modified to 1.12 and 0.99, respectively, about a 25% effect. Thus the effect of the restricted phase space is more important than that of properly introducing the distribution in the lepton energy even for the low-mass pion. Experimentally, the ratio is 1.2 ± 0.3,[5,6] so all the calculations are consistent with experiment. It would be desirable to have the ratio larger for the "realistic" case in which the masses are included. This would be expected on the basis of the results of Ref. 13 in the case of a gauge theory of τ decay, at the price of introduction of a new parameter. One would then have to investigate effects on three-pion decay to ensure that the procedure makes sense.

The effect of the true particle masses in restricting the available decay phase space does not appear to be negligible even for particles whose masses are as low as those of the pions. For the more massive particles, the widths as predicted are reduced by factors as large as 10^4. This reduction may be enough that the total τ decay width is too small, a situation presumably ameliorated by the introduction of the vector bosons in the gauge theory of τ decay.

ACKNOWLEDGMENTS

One of us (G.A.) would like to thank the Alexander von Humboldt Foundation for financial support, for the opportunity to work in Germany, and for much help during his stay. He wishes also to express his appreciation for the hospitality extended to him by Professor J. Wess and his colleagues in the Institute. We have greatly benefited from conversations with R. Fischer, A. Klüver, R. Wagner, and J. Wess.

---

*On leave from Department of Physics, The Ohio State University, Columbus, OH 43210 (permanent and present address).

[1]Pluto Collaboration, G. Alexander *et al.*, Phys. Lett. 73B, 99 (1978); W. Wagner *et al.*, Z. Phys. C 3, 193 (1980).

[2]B. H. Wiik, Report No. DESY 78/23, 1978 (unpublished).

[3]S. Flügge, Report No. DESY 78/42, 1978 (unpublished).

[4]G. S. Abrams *et al.*, Phys. Rev. Lett. 43, 1555 (1979).

[5]C. A. Blockner, Report No. LBL-10801, 1980 (unpublished).

[6]J. Dorfan, Report No. SLAC-PUB-2590, 1980 (unpublished).

[7]D. A. Geffen and W. J. Wilson, Phys. Rev. D 18, 2488 (1978).

[8]R. Fischer, J. Wess, and F. Wagner, Z. Phys. C 3, 313 (1980).

[9]C. Callen, S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177B, 2247 (1969).

[10]M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev.

175, 2195 (1968).
[11]D. McKay, W. Palmer, and R. Sarraga, Phys. Rev. D 8, 2532 (1973).
[12]G. Aubrecht, N. Chahrouri, and K. Slanec (unpublished).
[13]R. Fischer, A. Klüver, and F. Wagner, Report No. SLAC-PUB-2608, 1981 (unpublished).