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Majorana neutrinos and low-energy tests of electroweak models

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Tests based on neutrinoless double-β decay and rare muon processes are proposed for the detection of a heavy Majorana neutrino with a mass of ~ 100 GeV. Existence of such a neutrino would distinguish between the standard and left-right-symmetric electroweak gauge models with U(1) in the latter identified with $(B-L)$. The sensitivity of these processes to the mass of the heavy Majorana neutrino is discussed.

I. INTRODUCTION

The successful $(V-A)$ (left-handed) charged-weak-current theory was derived in 1957 (Ref. 1) on the basis of γ_5 invariance, electron-muon universality, and a baryon-lepton symmetry principle couched in the permutation-invariance requirement: $p \leftrightarrow \nu$, $n \leftrightarrow e$, $\Lambda \leftrightarrow \mu$. The concept of γ_5 invariance, modeled on the Weyl equation for the massless neutrino,[2] was extended to the finite-mass charged leptons and the finite-mass baryons.[3] Further consequences of the aforementioned "baryon-lepton symmetry principle" were worked out in a paper by Gamba, Marshak, and Okubo[4] and led to the postulation of two global groups: "weak isospin" $\vec{I}_w$ and "weak hypercharge" Y_w, related through the "weak" Gell-Mann–Nishijima relation $Q = I_{3w} + Y_w/2$. It was observed that if one set $Y_w = B - L + T$ (with B and L the baryon and lepton numbers, respectively, and T a "triality" quantum number), then $T = 0$ for weak isodoublets and $T \neq 0$ for weak isosinglets. This baryon-lepton symmetry principle has undergone a series of reformulations, first when the SU(3) group (with its three quarks and single Cabibbo angle) was introduced,[5] then when two distinct neutrinos $(\nu_e \neq \nu_\mu)$ were identified[6] (leading to the hypothesis of the charmed quark[7]), and finally within the framework of the six-quark–six-lepton model.[8]

From 1961 to 1968, Glashow, Salam, and Weinberg[9] developed the standard gauge model, based on the electroweak group $SU(2)_L \times U(1)$ [with $SU(2)_L$ the left-handed weak-isospin group and U(1) a weak-hypercharge group]. This electroweak group was gauged with four massless vector bosons [three associated with $SU(2)_L$ and one associated with U(1)] in such a way that, after the symmetry was spontaneously broken by a doublet of Higgs bosons, the three W_L's of the weak interaction acquired masses determined by one parameter, $\sin^2\theta_W$ (θ_W is the Weinberg angle), and the photon retained its zero mass. The total neutral weak current was predicted to have the form $\rho(I_{3L} - \sin^2\theta_W Q)$ (with $\rho = 1$), a prediction that has been confirmed by all measurements up to the present. This minimal version of the standard gauge model of the electroweak interaction implicity assumed one zero-mass (left-handed) neutrino per generation[10] and accepted maximal parity violation (left-handed character) of the weak interaction as given.

Beginning in 1974, Mohapatra, Pati, Salam, and others[11] developed an alternative model of the electroweak interaction which restored parity to the status of a high-energy symmetry of weak interactions, namely the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$. This left-right-symmetric model implicitly assumed finite-mass neutrinos and two sets of W bosons, W_L and W_R, where, subsequent to the spontaneous breakdown, $m_{W_R} \gg m_{W_L}$. Recently, it has been shown by Marshak and Mohapatra[12] that unlike the case of the $SU(2)_L \times U(1)$ model, the vector U(1) generator in the $SU(2)_L \times SU(2)_R \times U(1)$ model can be identified with $(B-L)$ symmetry. This interpretation of U(1) has the important consequence that in the left-right-symmetric model, electric charge is given by $Q = I_{3L} + I_{3R} + \frac{1}{2}(B-L)$, where $\vec{I}_{L,R}$ are the generators of the $SU(2)_{L,R}$ groups. The above relation implies that $-\Delta I_{3R} = \frac{1}{2}\Delta(B-L)$ in the energy region where $SU(2)_L \times U(1)$ is a good symmetry. This is to say that the mass scale associated with spontaneous breakdown of parity, m_{W_R}, can also be related to the breakdown of local $(B-L)$ electroweak symmetry. We shall see how this simple physical idea can be exploited to predict the masses of the two Majorana neutrinos associated with each generation of quarks and leptons.[13]

When the $SU(3)^c$ (color group) is adjoined to an electroweak group, one is led naturally to larger (grand unification and/or partial unification) groups. It is the purpose of this paper to stay at the electroweak level and to try to differentiate between the standard electroweak model and the left-right-symmetric electroweak model by focusing on the low-energy tests of the (finite-mass) neutrino predictions of these two models. In Sec. II, we summarize what each of the electroweak models has to tell us regarding the nature of the neutrino and its mass. We shall find that while the standard electroweak model can accommodate one light Majorana neutrino per generation, only the left-right-symmetric electroweak model predicts a heavy Majorana neutrino for each generation as well. The heavy Majorana neutrino is crucial for the two sections that follow: Sec. III on neutrinoless double-β decay and Sec. IV on the rare muon processes $\mu\to e\gamma$ decay and muon conversion into electrons or positrons. While the observation of the rare muon processes $\mu^-\to e^- +\gamma$ and $\mu^- +A(Z)\to e^- +A(Z)$ could occur via the mechanism of a heavy Dirac neutrino, neutrinoless double-β decay and the lepton-number violating process $\mu^- +A(Z)\to e^+ +A(Z-2)$ would only be detectable if a heavy Majorana neutrino exists. If evidence of the latter type is forthcoming, this would be a strong argument for the validity of the left-right-symmetric electroweak model. In Sec. V we shall offer some concluding remarks on the relative sensitivity of the various low-energy tests to the mass of the heavy Majorana neutrino.

II. NATURE OF THE NEUTRINO AND ITS MASS

Neutrino oscillations, if they occur, provide evidence on neutrino mass; the reverse is not true. The present experimental limits on the neutrino mass are given in Table I.

It appears from Table I that the neutrino may have a finite mass and, if so, one would have to understand a finite but small mass of the neutrino (i.e., much smaller than the mass of its associated lepton). We examine this problem within the context of the standard and left-right-symmetric electroweak models (the first generation is treated as typical).

Below we summarize the different possibilities regarding the nature of the neutrino and its mass.

A. Standard model: $SU(2)_L \times U(1)$

(i) Put ν_R in a singlet representation. Using the usual Higgs doublet $\phi(\frac{1}{2},1)$ with $\langle\phi\rangle = \binom{0}{\lambda}$, the neutrino acquires a Dirac mass given by the first term of the following expression:

$$h_1(\bar{\nu}_L\bar{e}_L)\begin{pmatrix}\lambda\\0\end{pmatrix}\nu_R + \text{H.c.} + h_2(\bar{\nu}_L\bar{e}_L)\begin{pmatrix}0\\\lambda\end{pmatrix}e_R + \text{H.c.} \tag{1}$$

But the mass of ν_e here is arbitrary and there does not appear to be a natural way to make it small, i.e., much smaller than m_e.

(ii) We start with the usual lepton doublet

$$\psi_L = \begin{pmatrix}\nu_L\\e_L\end{pmatrix}$$

but in addition to the Higgs doublet ϕ, we postulate a Higgs triplet $\Delta(1,2)$ with the vacuum expectation value

$$\langle\Delta\rangle = \begin{pmatrix}0 & 0\\ v & 0\end{pmatrix}. \tag{2}$$

Then the neutrino can acquire a Majorana mass

$$h_3\psi_L^T C i\tau_2\langle\Delta\rangle\psi_L + \text{H.c.} \tag{3a}$$

so that

$$m_{\nu_e} = h_3 v. \tag{3b}$$

Δ also contributes to vector-boson masses and one obtains

$$\rho = \frac{m_W^2}{m_Z^2\cos^2\theta_W}\, 1 - \frac{4v^2}{\lambda^2}. \tag{4}$$

Further, from the second term of Eq. (1)

$$m_e = h_2\lambda. \tag{5}$$

Since ρ is very nearly 1, $v^2/\lambda^2 \ll 1$ and thus m_{ν_e} could be quite small. In fact, since one expects $h_3/h_2 \leq 1$, one obtains

$$\frac{m_{\nu_e}}{m_e} \leq \tfrac{1}{2}(1-\rho)^{1/2}. \tag{6}$$

Using the present experimental limit on $(1-\rho)$, say 0.02, we have

TABLE I. Limits on neutrino masses.

Neutrino type	Mass	Type of evidence
ν_e	30 ± 10 eV	Shape of H^3 β spectrum [Ref. 14(a)]
	~1 eV	Neutrino oscillations [Ref. 14(b)]
ν_μ	<0.57 MeV	Muon range in $\pi^+\to\mu^+ + \nu_\mu$ [Ref. 14(c)]
ν_τ	<250 MeV	τ-decay spectrum

$m_{\nu_e} < 35$ keV.

This is not a very useful limit and is, for example, much larger than the present limit on m_{ν_e} from H^3 (i.e., 30 eV). To sum up, in this case the charged weak current is still pure $(V-A)$ but the neutrino is a Majorana particle and has acquired a finite mass through the violation of lepton-number conservation [cf. Eq. (3a)] and thus can give rise to neutrinoless double-β decay. But as we shall see, its contribution to this decay is too small to be detectable.

B. Left-right-symmetric model: $SU_L(2) \times SU_R(2) \times U_{B-L}(1)$ $(g_L = g_R = g)$

Lepton doublets before symmetry breaking are

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \begin{pmatrix} N_R \\ e_R \end{pmatrix}. \tag{7}$$

The Higgs structure is

$$\Phi(\tfrac{1}{2}, \tfrac{1}{2}, 0), \quad \langle\Phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix},$$
$$\Delta_L(1,0,2), \quad \langle\Delta_L\rangle = 0; \quad \Delta_R(0,1,2), \quad \langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v & 0 \end{pmatrix}, \tag{8}$$

where $\kappa' \ll \kappa \ll v$. Neutrinos must have Majorana masses and this case has been considered in detail by Mohapatra and Senjanovic.[13] The physical Majorana neutrinos ν_e and N_e have masses

$$m_{\nu_e} = \frac{h_1{}^2\kappa^2}{h_3 v}, \quad m_{N_e} = h_3 v, \tag{9a}$$

and the mixing angle between ν_L and N_R is

$$|\gamma| \simeq \frac{h_1\kappa}{h_3 v} \ll 1. \tag{9b}$$

Here h_1, h_2, and h_3 are the respective Yukawa coupling constants of the Φ, $\bar{\Phi} \equiv \tau_2\Phi^*\tau_2$, Δ_R Higgs particles with the leptons. On the other hand,

$$m_{W_R} = gv, \quad m_e = h_2\kappa, \tag{10}$$

so that (for $h_1 \sim h_2$)

$$m_{\nu_e} \sim a\frac{m_e{}^2}{m_{W_e}}, \tag{11a}$$

$$m_{N_e} = \frac{1}{a} m_{W_R}, \tag{11b}$$

$$|\gamma| \lesssim a\frac{m_e}{m_{W_R}}, \tag{11c}$$

where $a = g/h_3$. It is interesting that $m_{\nu_e} m_{N_e} \simeq m_e{}^2$ (independent of a). Also it should be noted that present experimental limits[13] give $m_{W_R} \gtrsim 300$ GeV so that $m_{\nu_e} \lesssim 1$ eV and $m_{N_e} \gtrsim 100$ GeV.

We thus arrive at the interesting conclusion that reasonable extensions[15] of the standard electroweak model can accommodate one light Dirac or Majorana neutrino per generation of arbitrary mass; for the purposes of future calculations we use the experimental upper limit of 30 eV. On the other hand, the left-right-symmetric electroweak model predicts two Majorana neutrinos per generation, a light one having a mass less than about 1 eV (predominantly coupled to the lighter W_L boson) and a massive one with greater than about 100 GeV (predominantly coupled to the heavier W_R boson).

III. NEUTRINOLESS DOUBLE-β DECAY

Neutrinoless double-β decay becomes an interesting test of the models, as we shall see below. As is well known, neutrinoless double-β decay $[(\beta\beta)_0]$ can occur if the "intermediate" electron neutrino is a Majorana particle.

The $(\beta\beta)_0$ decay has been analyzed in terms of the amplitude η, the lepton-number-nonconserving parameter, where η appears in the leptonic current as follows:

$$i\bar{e}\gamma_\lambda[1+\gamma_5) + \eta(1-\gamma_5)]\nu_e.$$

Even though this decay occurs mainly through the finite mass of the electron neutrino, the contribution of the latter is expressed in terms of an "equivalent" η by Halprin, Minkowski, Primakoff, and Rosen.[16] This has the advantage that the uncertainty due to nuclear matrix elements is eliminated. The important point of the analysis is that for m_{ν_e} negligible, the Coulomb potential appears, while for the neutrino mass large, the potential is a δ function. Thus the matrix elements in the latter case are essentially determined by the modulus square of the wave function at the origin, $|\psi(0)|^2$, for the two-nucleon system or the quark system, depending upon whether one assumes that the basic $(\beta\beta)_0$ process involves nucleons or quarks (see below). The present limit on η, consistent with $(\beta\beta)_0$ decay rates of $^{48}Ca \to {}^{48}Ti$, $^{76}Ge \to {}^{76}Se$, and $^{82}Se \to {}^{82}Kr$ is[17]

$$|\eta| \lesssim 5\times10^{-4}. \tag{12}$$

This value of η corresponds to $m_{\nu_e} \leqslant 1$ keV for the light neutrino and implies a half-life for $(\beta\beta)_0$ decay of ^{82}Se, for example

$$T_{1/2}^{(\beta\beta)_0} \simeq \frac{1.1\times10^{14\pm2}}{\eta^2}\ \text{yr} \gtrsim 4\times10^{20\pm2}\ \text{yr}, \tag{13}$$

where $10^{\pm2}$ reflects the uncertainty in the nuclear part of the matrix elements.[18]

The observation of $(\beta\beta)_0$ decay requires a Majorana neutrino and would thus exclude the case A(i) of Sec. II. Moreover, if one sets $\eta = 0$, as is the situation for the case A(ii), and takes, for example, m_{ν_e} to be 30 eV (its present upper limit), the half-life for $(\beta\beta)_0$ decay would be about 10^3 larger than the half-life (13). Thus in this case $(\beta\beta)_0$ decay would be hard to detect.

In a model where there are two Majorana neutrinos, as in Sec. II B where m_{ν_e} is very small ($\lesssim 1$ eV), $(\beta\beta)_0$ decay would get most of its contribution from the heavy neutrino N_e which has predominantly right-handed couplings. This case we now discuss.

The basic process at the "quark level" is shown in Fig. 1. If u and d quarks are, respectively, replaced by n and p, then the analysis of Halprin *et al.*[16] for $A \simeq 100$ $(\beta\beta)_0$ nuclei gives for the (right-handed) heavy neutrino N_e the equivalent[19]

$$\eta \simeq \left(\frac{m_{W_L}{}^2}{m_{W_R}{}^2}\right)^2 \frac{1}{m_{N_e}} (3.5)\, 10^{-1}\ \text{GeV}, \tag{14}$$

i.e.,

$$\eta \leqslant (3.5) 10^{-5}$$

for $m_{N_e} \geqslant 100$ GeV and $m_{W_L}{}^2/m_{W_R}{}^2 \lesssim \frac{1}{10}$. For the value of η in Eq. (14), $(\beta\beta)_0$ would require a half-life measurement of order $8\times 10^{22\pm 2}$ yr.

To sum up, a measurement of a half-life for $(\beta\beta)_0$ decay in the range 10^{20} to 10^{24} yr would help to distinguish the various cases discussed in Sec. II and to place a limit on m_{N_e} for the left-right-symmetric electroweak model if the limit on $(m_{W_L}{}^2/m_{W_R}{}^2)$ is known from other considerations. Possible candidates for $(\beta\beta)_0$ decay are ^{48}Ca, ^{76}Ge, ^{82}Se, ^{128}Te, ^{130}Te, ^{136}Xe, ^{150}Nd.[20,21]

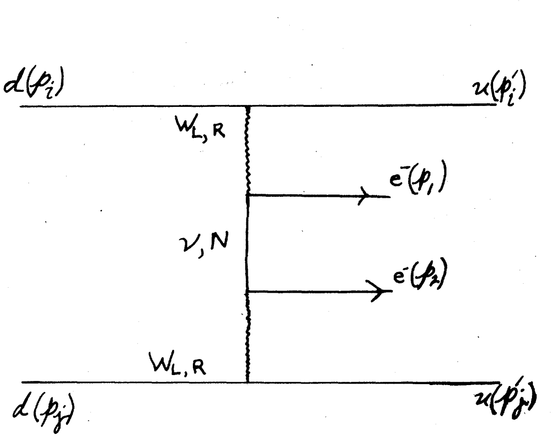

FIG. 1. Neutrinoless double-β decay at quark level.

IV. RARE MUON PROCESSES

Another low-energy test of electroweak models is to study rare muon processes, particularly those involving intergenerational mixing of the neutrinos. These processes can be divided into two basic classes, according to whether they violate muon number only or both muon and lepton numbers:

$$\text{A: (i)}\quad \mu \to e + \gamma, \tag{15}$$

$$\text{(ii)}\quad \mu^- + A(Z) \to e^- + A(Z);$$

$$\text{B:}\ \mu^- + A(Z) \to e^+ + A(Z-2). \tag{16}$$

The former (A) can occur independently of whether the neutrinos in the "intermediate state" are Dirac or Majorana while the latter (B) occurs only if the neutrinos are Majorana. In each case there is a mixing between electron- and muon-type neutrinos. Thus the occurrence of B would signal the Majorana nature of the neutrinos and if conditions are favorable, a measurement of

$$\frac{\Gamma(\mu^- + A(Z) \to e^- + A(Z))}{\Gamma(\mu^- + A(Z) \to e^+ + A(Z-2))}$$

would not depend on the intergeneration mixing angle. Let us consider the three processes A(i), A(ii), and B in turn.

A(i). $\mu \to e + \gamma$ decay[2]

Let us define the mass eigenstates of the neutrinos as

$$\begin{aligned} \nu_1 &= \cos\theta\, \nu_e + \sin\theta\, \nu_\mu, \\ \nu_2 &= -\sin\theta\, \nu_e + \cos\theta\, \nu_\mu, \end{aligned} \tag{17}$$

where the ν's refer to the light neutrinos. If there are heavy neutrinos (as in the left-right-symmetric model), then we can also write

$$\begin{aligned} N_1 &= \cos\theta'\, N_e + \sin\theta'\, N_\mu, \\ N_2 &= -\sin\theta'\, N_e + \cos\theta'\, N_\mu. \end{aligned} \tag{18}$$

The branching ratios for $\mu \to e + \gamma$ in the two cases, respectively, are given by[23]

$$B_L = \frac{\Gamma(\mu \to e + \gamma)}{\Gamma_\mu^{\text{total}}} = \frac{3\alpha}{32\pi}\left(\sin\theta\cos\theta\,\frac{m_{\nu_2}{}^2 - m_{\nu_1}{}^2}{m_{W_L}{}^2}\right)^2, \tag{19}$$

$$B_R = \frac{3\alpha}{32\pi}\left(\frac{m_{W_L}{}^2}{m_{W_R}{}^2}\right)^4 \left(\sin\theta'\cos\theta'\,\frac{m_{N_2}{}^2 - m_{N_1}{}^2}{m_{W_L}{}^2}\right)^2. \tag{20}$$

Equation (19) holds for the standard $SU(2)_L \times U(1)$ model; for $m_{W_L} \sim 80$ GeV and the limiting values $m_{\nu_e} \lesssim 30$ eV, $m_{\nu_\mu} \sim 0.6$ MeV, $B_L < 4.5\times 10^{-26}$, which is too small to be measurable. Equations (19) and (20) hold for the left-right-symmetric model where the ν's are predominantly left-handed and light while the N's are predominantly right-handed and heavy, but have masses smaller than but of the same order as m_{W_R}. In this case, Eq. (20), for $m_{W_L}{}^2/m_{W_R}{}^2 \sim \frac{1}{10}$, $m_{N_2}{}^2 - m_{N_1}{}^2 \sim 10^4$ GeV^2 (all crude estimates),

$$B_R \approx 4\times10^{-8}\,(\sin\theta'\cos\theta')^2\,. \tag{21}$$

No real prediction for θ' exists but we could argue from the cosmological limits on the ν_e, ν_μ masses and Eqs. (11a) and (11b) that $\theta' \lesssim (m_e/m_\mu)^{1/2}$ (Ref. 24). With this value of θ', Eq. (21) yields the not uninteresting branching ratio

$$B_R \lesssim 2\times 10^{-10}\,. \tag{22}$$

We thus find that the heavy neutrino predicted by the left-right-symmetric model may bring within the realm of detection a rare weak process that is hopeless to measure if only the light neutrino exists (as predicted by the standard model).

The above analysis can be carried over directly to the case of the rare process $\tau \to \mu(e)+\gamma$ if μ is replaced by τ and e by μ or e.

A(ii). $\mu^- + A(Z) \to e^- + A(Z)$

We shall represent this process by

$$\mu^- + p \to e^- + p\,.$$

The dominant diagrams for this process are shown in Fig. 2(a). These diagrams, evaluated on the assumption that the external momenta can be neglected compared to the internal momenta, give[25]

$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{8\pi}\epsilon\,\sin\theta'\cos\theta'[\bar{e}\gamma_\mu(1-\gamma_5)\mu] \times[\bar{p}(C_V\gamma_\mu - C_A\gamma_\mu\gamma_5)p] \tag{23a}$$

with

$$C_V = 4 - a(1-4\sin^2\theta_W)\,,\quad C_A = 4 - a(1-2\sin^2\theta_W)\,. \tag{23b}$$

The terms proportional to a arise from the Z_R-exchange diagram. The parameters a and ϵ are given by

$$\frac{\cos^2\theta_W}{\cos 2\theta_W}\frac{m_{Z_L}{}^2}{m_{Z_R}{}^2} = a\left(\frac{m_{W_L}{}^2}{m_{W_R}{}^2}\right), \tag{24a}$$

$$\epsilon = \frac{m_2{}^2 - m_1{}^2}{\sin^2\theta_W m_{W_L}{}^2}\left(\frac{m_{W_L}{}^2}{m_{W_R}{}^2}\right)^2\left(\ln\frac{m_{W_R}{}^2}{M^2} - 2\right), \tag{24b}$$

where

$$M = \frac{m_1 + m_2}{2}\,. \tag{24c}$$

In evaluating the loop integral, we have assumed that the masses m_1, m_2 of the heavy neutrinos N_1 and N_2 are such that

$$\frac{m_1{}^2, m_2{}^2}{m_{W_R}{}^2} \lesssim \frac{1}{10},\quad \frac{m_2 - m_1}{m_2 + m_1} \lesssim \frac{1}{3}\,. \tag{25}$$

The parameter a depends on some details of the L-R-symmetry breaking. If the symmetry is broken by the Higgs multiplets $(\frac{1}{2},\frac{1}{2},0)$, $(1,0,2)$, and $(0,1,2)$ (giving Majorana neutrinos as in Sec. II), then

$$a \approx \frac{1}{2\cos^2\theta_W} \simeq \frac{2}{3}\,. \tag{26}$$

The formulas (23) and (24) give the branching ratio of the capture rates for the processes $\mu^- + p \to e^- + p$ with respect to $\mu^- + p \to \nu + n$ as

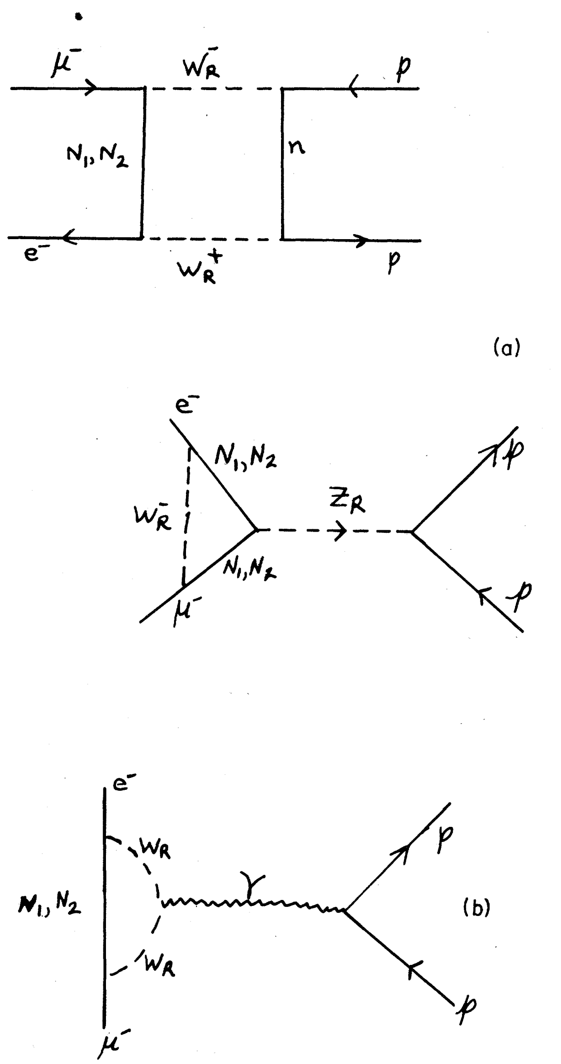

FIG. 2. Feynman diagrams for the process $\mu^- + p \to e^- + p$.

$$B=\frac{[\Lambda]_{\mu^-\to e^-}}{[\Lambda]_{\mu^-\to\nu}}$$
$$\simeq 10^{-6}\epsilon^2\sin^2\theta'\cos^2\theta' . \quad (27a)$$

Taking $M^2/m_{W_R}{}^2\simeq\frac{1}{10}$, we get

$$\epsilon\lesssim 1.2\times10^{-2} \quad (27b)$$

for

$$\frac{m_2{}^2-m_1{}^2}{m_{W_L}{}^2}\leqslant 1 \text{ and } \frac{m_{W_L}{}^2}{m_{W_R}{}^2}\leqslant\frac{1}{10} .$$

From (27), we obtain

$$B\lesssim 1.3\times10^{-10}\sin^2\theta'\cos^2\theta' . \quad (28)$$

If we choose the "canonical" value of the mixing angle $\theta'\sim(m_e/m_\mu)^{1/2}$ we find

$$B\lesssim 6\times10^{-13} . \quad (29)$$

A Virginia Polytechnic Institute experiment is now under way at TRIUMF to measure this conversion process at the 10^{-12} level.[26]

Before we leave this process, we wish to point out that the diagram involving one-photon exchange shown in Fig. 2(b) involves the effective $\mu\to e\gamma$ vertex which has the form (neglecting m_e compared to m_μ)

$$\mathfrak{M}_\lambda=H\bar{u}_\mu(p')\sigma_{\lambda\nu}q_\nu(1+\gamma_5)u_e(p) \quad (30a)$$

$(q=p-p')$, where it has been estimated[25] that

$$H\simeq e\frac{G_F}{32\pi^2\sqrt{2}}m_\mu\frac{m_2{}^2-m_1{}^2}{m_{W_R}{}^2}\cos\theta'\sin\theta' . \quad (30b)$$

Thus the one-photon-exchange contribution to the process $\mu^-+p\to e^-+p$ is suppressed because of the absence of $\sin^2\theta_W$ in the denominator and the factor $\ln m_{W_R}{}^2/M^2$ as compared to Eq. (23) with ϵ given in Eq. (24b).

B. $\mu^-+A(Z)\to e^++A(Z-2)$

This process has been studied previously by Kamal and Ng[27] in a four-lepton-doublets (left-handed) sequential standard model.

We treat the nucleus as an "elementary particle" and assume that it has spin 0. We have here only the box diagram shown in Fig. 3, where we assume that the $A(Z-1)$ nucleus in the intermediate state and $A(Z-2)$ also have spin 0. The spin-0 assumption has the consequence that the factors l_β and l_α (where l denotes an internal momentum) appear at the $A(Z)+W_R^-\to A(Z-1)$ and $A(Z-1)+W_R^-\to A(Z-2)$ vertices. These factors have the important effect of suppressing the $m_{W_R}{}^2$ factor in the W_R propagator when the loop integration is performed. It is also easy to see that from the left of the diagram of Fig. 3, one would obtain a term of the form

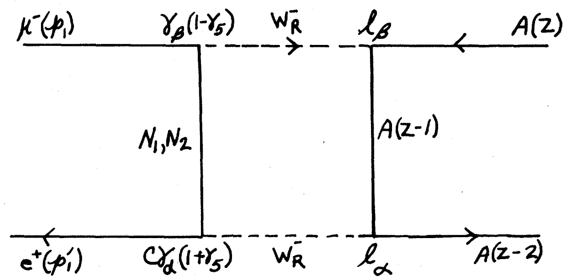

FIG. 3. Feynman diagram for the process $\mu^-+A(Z)\to e^++A(Z-2)$.

$$\sin\theta'\cos\theta' v^T(p_1')C\gamma_\alpha(1+\gamma_5)$$
$$\times\left(\frac{i\not{l}-m_1}{l^2+m_1{}^2}-\frac{i\not{l}-m_2}{l^2+m_2{}^2}\right)\gamma_\beta(1-\gamma_5)u(p_1) . \quad (31)$$

Thus the terms $i\not{l}\equiv i\gamma_\mu l_\mu$ do not contribute. With the above considerations, one finds

$$H_{\text{eff}}=\cos\theta'\sin\theta'\frac{G_F}{\sqrt{2}}\frac{\alpha}{8\pi}\frac{1}{\sin^2\theta_W}\frac{m_{W_L}{}^2}{m_{W_R}{}^2}I$$
$$\times v^T(p_1')C(1-\gamma_5)u(p_1) , \quad (32a)$$

where

$$I=\int_0^\infty\frac{x\,dx}{(x+1)^2}\left(\frac{-m_1}{x+y_1}-\frac{-m_2}{x+y_2}\right),$$
$$y_{1,2}=\frac{m_{1,2}{}^2}{m_{W_R}{}^2} . \quad (32b)$$

For the inequalities (25), $I\simeq(m_2-m_1)$. Equation (32) then gives for the capture rate with respect to that for $\mu^-+p\to\nu+n$ in the spin-singlet state of the (μ^--p) atom as follows:

$$B=\tfrac{1}{32}\sin^2\theta'\cos^2\theta'\left(\frac{\alpha}{8\pi}\right)^2\frac{1}{\sin^4\theta_W}\left(\frac{m_{W_L}{}^2}{m_{W_R}{}^2}\right)^2\left(\frac{m_2-m_1}{m_A}\right)^2 , \quad (33)$$

where m_A is the target mass. Thus

$$B\lesssim 4\times10^{-10}\sin^2\theta'\cos^2\theta'\left(\frac{m_2-m_1}{m_A}\right)^2 . \quad (34)$$

But $[(m_2-m_1)/m_A]^2$ could easily be greater than ten giving a B of the same order as (or even greater than) for $\mu^-+A(Z)\to e^-+A(Z)$ [cf. Eq. (28)].

The above estimate of B for the process $\mu^-\to e^+$ is peculiar to the spin-0 assumption. For instance, for a spin-$\frac{1}{2}$ target and spin-$\frac{1}{2}$ particles in the intermediate and final states, we find

$$B \simeq \tfrac{1}{4}\sin^2\theta'\cos^2\theta'\left(\frac{\alpha}{4\pi}\right)^2 \frac{1}{4\sin^4\theta_W}\left(\frac{m_{W_L}{}^2}{m_{W_R}{}^2}\right)^2 \times\left(\frac{m_2-m_1}{m_{W_R}}\right)^2\left(\frac{m_A}{m_{W_R}}\right)^2\left(\ln\frac{m_{W_R}{}^2}{M^2}-1\right)^2, \tag{35}$$

which would give a much smaller B.

Comparison of Eqs. (28) and (34) shows that the ratio of B's is independent of the mixing angle as expected. Obviously, this statement is true if the $\mu^- \to e^-$ and $\mu^- \to e^+$ conversions are studied in different nuclei. Detection of both conversion processes with comparable branching ratios would be clear-cut evidence for the existence of heavy Majorana neutrinos and for the left-right-symmetric electroweak model.

V. CONCLUDING REMARKS

It is clear from the foregoing that detection in the next few years of any of the processes considered—neutrinoless double-β decay, $\mu \to e+\gamma$, or conversion of muon into electron or positron (through nuclear interaction)—would provide strong evidence for a heavy neutrino in the mass range ~100 GeV (which is predicted by the left-right-symmetric but not by the standard electroweak model). Detection of $(\beta\beta)_0$ or $\mu^- + A(Z) \to e^+ + A(Z-2)$ would settle the Majorana character of the heavy neutrino. Measurements of several of those processes would, in principle, permit a determination of the parameters entering the theoretical estimates following from the left-right electroweak model: m_N, m_{W_R}, and θ' (the mixing angle between the first and second lepton generations).

The estimates of the transition probabilities for the low-energy processes treated in Secs. III and IV were based on values of m_N (~100 GeV), m_{W_R} (~300 GeV), and θ' $[(m_e/m_\mu)^{1/2} \approx \frac{1}{15}]$, all compatible with present experimental limits. The resulting numbers brought all of these rare processes within range of the next round of difficult (albeit not impossible) experiments. But suppose despite these efforts, none of the aforementioned processes is detected. Would this rule out the left-right-symmetric electroweak model? The answer, of course, is "no" but this statement should be qualified.

The basic ingredients of the $SU_L(2)\times SU_R(2)\times U_{B-L}(1)$ electroweak group are that parity conservation is restored at sufficiently high energy and that the breaking of parity is related to the breaking of local $(B-L)$ symmetry. The immediate consequences are the prediction of two Majorana neutrinos per generations [one light (ν) and one heavy (N)] and relations between neutrino masses and m_{W_R} [inverse between m_ν' and m_{W_R}—see Eq. (11a)—and direct between m_N and m_{W_R}—see Eq. (11b)]. What is not fixed is the actual value of m_{W_R}. Since $m_N \sim g m_{W_R}$, we can summarize the dependence of the various low-energy processes on m_{W_R} as follows:

$(\beta\beta)_0$: $T_{1/2} \sim m_{W_R}{}^{10}$,

$\mu \to e+\gamma$: $B \sim m_{W_R}{}^{-4}$,

$\mu^- + A(Z) \to e^- + A(Z)$: $B \sim m_{W_R}{}^{-4}$,

$\mu^- + A(Z) \to e^+ + A(Z-2)$: $B \sim m_{W_R}{}^{-2}$ (spin-0 nucleus).

We cannot judge the refinements possible in the above experiments but it is evident that the reduction due to increased m_{W_R} is by all odds minimized for the last process.

In any case, a substantially larger value of m_{W_R}, in the range 10^3–10^4 GeV, will still permit the detection of neutron oscillations.[12,28]

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