PHYSICAL REVIEW D VOLUME 24, NUMBER 5 1 SEPTEMBER 1981

Asymmetry parameter as a function of transverse momentum in the production of dileptons with polarized beam and target

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(Received 14 April 1980)

We calculate in quantum-chromodynamic perturbation theory the asymmetry parameter A in the production of massive dileptons in collisions of polarized hadrons as a function of transverse momentum, using various models of spin-dependent parton distribution functions. We find A to be negative, large, and model-dependent for p-$\bar{p}$ collisions, whereas for p-p collisions A is not strongly model dependent and varies from -5 to 7% for various values of transverse momentum.

I. INTRODUCTION

In this paper we study the Drell-Yan[1] production of massive lepton pairs, modified by quantum chromodynamics (QCD), in longitudinally polarized hadron collisions for nonzero transverse momentum of the dilepton. We are avoiding zero transverse momentum of the dilepton to avoid the mass singularity. We calculate the asymmetry parameter,

$$A=\frac{m^2(d^2\sigma/dm^2dq_T{}^2)(H_1(+)H_2(+)\to\mu^+\mu^-X)-m^2(d^2\sigma/dm^2dq_T{}^2)(H_1(+)H_2(-)\to\mu^+\mu^-X)}{m^2(d^2\sigma/dm^2dq_T{}^2)(H_1(+)H_2(+)\to\mu^+\mu^-X)+m^2(d^2\sigma/dm^2dq_T{}^2)(H_1(+)H_2(-)\to\mu^+\mu^-X)},$$

as a function of $r_T=q_T/\sqrt{s}$, where m^2 is the invariant mass squared, q_T is the transverse momentum of the lepton pair, and $\sqrt{s}$ is the center-of-mass energy of the colliding particles.

The QCD subprocesses giving rise to nonzero dilepton transverse momentum are (i) q-$\bar{q}$ annihilation giving rise to a gluon and a virtual photon eventually decaying into a lepton pair, Fig. 1(a), and (ii) q-g Compton-type interaction giving rise to a virtual photon eventually decaying into a lepton pair, Fig. 1(b). We calculate the asymmetry parameter A assuming four different models of spin-dependent parton distribution functions in a polarized nucleon for p-$\bar{p}$ collisions and p-p collisions and plot A against r_T. It must be emphasized that these results are valid only for those values of $q_T{}^2$ having $m^2\simeq q_T{}^2>1$ (GeV/c)^2. Below this the nonperturbative effects play a significant role.

The arrangement for the rest of the paper is as follows. In Sec. II we define the variables to be used in the paper, derive the differential cross section for QCD subprocesses for definite helicity states of the interacting partons giving rise to the dimuon transverse momentum, and then derive the expression for the asymmetry parameter A. In Sec. III we discuss the various models for spin-dependent parton distribution functions in longitudinal polarized nucleons (antinucleons). Section IV deals with results and discussions.

II. DERIVATION OF THE QCD SUBPROCESS CROSS SECTION $m^2d^2\hat{\sigma}/dm^2dq_T{}^2$ AND THE EXPRESSION FOR THE ASYMMETRY PARAMETER A

First we give the definitions of the various variables that we use in the rest of the paper. We consider the following process:

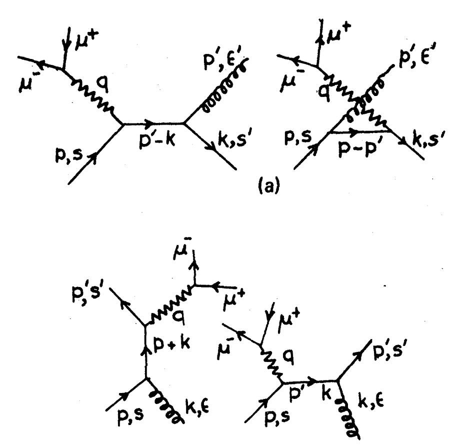

FIG. 1. (a) The diagram contributing to the subprocess $q\bar{q}\to g\mu^+\mu^-$. (b) The diagram contributing to the subprocess $qg\to q\mu^+\mu^-$.

$$H_1(P_1)+H_2(P_2)\to \mu^+(K_1)\mu^-(K_2)X . \quad (1)$$

H_1 and H_2 denote the colliding hadrons with momenta P_1 and P_2, respectively, and K_1, K_2 are the momenta of the muon pair. We work in the hadron center-of-mass frame. The QCD subprocesses producing the lepton pair are the following:

$$\text{(i) } q(p,s)+\bar q(k,s')\to g(p',\epsilon')+\mu^+(K_1,s_1) + \mu^-(K_2,s_2), \quad (2)$$

where q, $\bar q$ are the annihilating quark and antiquark with momenta, p, k and spin s, s', respectively, and g is the gluon produced with momenta p' and polarization ϵ'. This process is depicted in Fig. 1(a).

$$\text{(ii) } q(p,s)+g(k,\epsilon)\to q(p',s')+\mu^+(K_1,s_1) + \mu^-(K_2,s_2)$$

and

$$\bar q(p,s)+g(k,\epsilon)\to \bar q(p',s')+\mu^+(K_1,s_1) + \mu^-(K_2,s_2) . \quad (3)$$

Figure 1(b) depicts this process in which a quark or an antiquark of momentum p and spin s interact with a gluon of momentum k and polarization ϵ giving rise to a quark or antiquark of momentum p' and spin s' and a pair of leptons.

We define

$$\begin{aligned}
&s=(P_1+P_2)^2\simeq 2P_1\cdot P_2 ,\\
&q=K_1+K_2 ,\\
&q^2=m^2=\text{invariant mass squared of the dilepton pair},\\
&\hat s=(p+k)^2=(x_1P_1+x_2P_2)^2\simeq x_1x_2 s ,\\
&\hat t=(p-p')^2 ,\\
&\hat u=(q-p)^2 ,\quad \hat s+\hat t+\hat u=m^2 ,\\
&\tau=m^2/s ,\\
&r_T=q_T/\sqrt{s} ,\quad q_T=\left(\frac{\hat u\hat t}{\hat s}\right)^{1/2} = \text{transverse momentum of the dimuon.}
\end{aligned} \quad (4)$$

$f^i_{H(+)}(x,\pm)$, $f^i_{H(-)}(x,\pm)$ denote the probabilities of finding a quark i (or antiquark $\bar i$) of helicity $\pm$ with fraction x of the parent hadron's momentum inside a hadron H of helicity + and −, respectively, and $g_{H(+)}(x,\pm)$, $g_{H(-)}(x,\pm)$ are the respective probabilities for gluons. We define

$$\begin{aligned}
&\Delta f^i_{H(\pm)}(x)=f^i_{H(\pm)}(x,+)-f^i_{H(\pm)}(x,-) ,\\
&\Delta g_{H(\pm)}(x)=g_{H(\pm)}(x,+)-g_{H(\pm)}(x,-) ,\\
&f^i_H(x)=f^i_{H(+)}(x,+)+f^i_{H(+)}(x,-)\\
&\qquad =f^i_{H(-)}(x,-)+f^i_{H(-)}(x,+) ,\\
&g_{H(x)}=g_{H(+)}(x,+)+g_{H(+)}(x,-)\\
&\qquad =g_{H(-)}(x,-)+g_{H(-)}(x,+) .
\end{aligned} \quad (5)$$

Parity invariance demands that

$$f^i_{H(+)}(x,+)=f^i_{H(-)}(x,-) ,\quad f^i_{H(+)}(x,-)=f^i_{H(-)}(x,+) ,$$
$$g_{H(+)}(x,+)=g_{H(-)}(x,-) ,\quad g_{H(+)}(x,-)=g_{H(-)}(x,+) ,$$

and thus

$$\begin{aligned}
&\Delta f^i_{H(+)}(x)=-\Delta f^i_{H(-)}(x)=\Delta f^i_H(x) ,\\
&\Delta g_{H(+)}(x)=-\Delta g_{H(-)}(x)=\Delta g_H(x) .
\end{aligned} \quad (6)$$

The process $q(p,s)+\bar q(k,s')\to g(p',\epsilon')+\mu^+(K_1,s_1)+\mu^-(K_2,s_2)$. This process is depicted in Fig. 1(a). By straightforward calculation we arrive at the differential cross section

$$m^2\frac{d^2\hat\sigma}{dm^2d\hat t}(q(\eta_1)\bar q(\eta_2)\to\mu^+\mu^-X) =\frac{4}{9}\left(\frac{2\alpha^2\alpha_s e_i^2}{3\hat s^2}\right)(1-\eta_1\eta_2)\left(-2+\frac{\hat s^2+m^4}{\hat t\hat u}\right), \quad (7)$$

where $\alpha=e^2/4\pi$ is the QED coupling constant and $\alpha_s=g^2/4\pi$ is the QCD running coupling constant, η_1 and η_2 are the helicities of the quark and the antiquark, respectively. Using the identity $q_T^2=\hat u\hat t/\hat s$ we deduce

$$m^2\frac{d^2\hat\sigma}{dm^2dq_T^2}(q(\eta_1)\bar q(\eta_2)\to\mu^+\mu^-X) =\left(\frac{16}{27}\right)\frac{(1-\eta_1\eta_2)\alpha^2\alpha_s e_i^2}{\hat s[(\hat s-m^2)^2-4\hat s q_T^2]^{1/2}}\left(\frac{\hat s^2+m^4}{\hat s q_T^2}-2\right) . \quad (8)$$

Thus we see that the only quark and antiquark of opposite helicity couple:

$$m^2\frac{d^2\hat\sigma}{dm^2dq_T^2}(q(\pm)\bar q(\mp)\to\mu^+\mu^-X) =\frac{32\alpha^2\alpha_s e_i^2}{27\hat s[(s-m^2)^2-4\hat s q_T^2]^{1/2}}\left(\frac{\hat s^2+m^4}{\hat s q_T^2}-2\right) . \quad (9)$$

The process $q(p,s)+g(k,\epsilon)\to q(p',s')+\mu^+(K_1,s_1)+\mu^-(K_2,s_2)$. This process is depicted in Fig. 1(b). By straightforward calculation we arrive at the following expression:

$$m^2\frac{d^2\hat\sigma}{dm^2d\hat u}(q(\eta)g(\Lambda)\to\mu^+\mu^-X)=\frac{\alpha^2\alpha_s e_i^2}{9\hat s^2}\left[-\frac{(\hat u^2+\hat s^2+2m^2\hat t)}{\hat s\hat u}+\eta\Lambda\left(\frac{\hat u-2m^2}{\hat s}-\frac{\hat s-2m^2}{\hat u}\right)\right], \quad (10)$$

where η and Λ are the helicities of the quark and the gluon, respectively. Using the identity $q_T^2=\hat u\hat t/\hat s$ we get

$$m^2\frac{d^2\hat\sigma}{dm^2dq_T^2}(q(\eta)g(\Lambda)\to\mu^+\mu^-X)=\frac{\alpha^2\alpha_s e_i^2}{9\hat s[(\hat s-m^2)^2-4\hat s q_T^2]^{1/2}}\left\{\frac{\hat s+3m^2}{\hat s}+\frac{(\hat s-m^2)}{q_T^2}\left[1-\frac{2m^2(\hat s-m^2)}{\hat s^2}\right]\right.$$
$$\left.+\eta\Lambda\left[-\frac{(\hat s+3m^2)}{\hat s}+\frac{(\hat s-m^2)}{q_T^2}\left(1-\frac{2m^2}{\hat s}\right)\right]\right\}. \qquad (11)$$

To write an expression for the asymmetry parameter A, we need the convoluted cross section

$$m^2\frac{d^2\sigma^{\text{annih}}}{dm^2dq_T^2}(H_1(h_1)H_2(h_2)\to\mu^+\mu^-X),$$

owing to the quark-antiquark annihilation subprocess, and

$$m^2\frac{d^2\sigma^{\text{Comp}}}{dm^2dq_T^2}(H_1(h_1)H_2(h_2)\to\mu^+\mu^-X)$$

owing to the quark- (or antiquark-) gluon Compton process. Using Eqs. (8) and (11) we get the following expression for these cross sections:

$$m^2\frac{d^2\sigma^{\text{annih}}}{dm^2dq_T^2}(H_1(h_1)H_2(h_2)\to\mu^+\mu^-X)$$
$$=\frac{16\alpha^2\alpha_s}{27}\int_0^1dx_1\int_0^1dx_2\theta((\hat s-m^2)^2-4\hat s q_T^2)$$
$$\times 2B\sum_{i=u,d,s}e_i^2\{[f^i_{H_1(h_1)}(x_1,+)f^{\bar i}_{H_2(h_2)}(x_2,-)+f^i_{H_1(h_1)}(x_1,-)f^{\bar i}_{H_2(h_2)}(x_2,+)]+[x_1\leftrightarrow x_2,H_1\leftrightarrow H_2]\}, \qquad (12)$$

$$m^2\frac{d^2\sigma^{\text{Comp}}}{dm^2dq_T^2}(H_1(h_1)H_2(h_2)\to\mu^+\mu^-X)$$
$$=\frac{\alpha^2\alpha_s}{9}\int_0^1dx_1\int_0^1dx_2\theta((\hat s-m^2)^2-4\hat s q_T^2)\sum_{i=u,d,s}e_i^2\{Cg_{H_1(h_1)}(x_1)[f^i_{H_2(h_2)}(x_2)+f^{\bar i}_{H_2(h_2)}(x_2)]$$
$$+D\Delta g_{H_1(h_1)}(x_1)[\Delta f^i_{H_2(h_2)}(x_2)+\Delta f^{\bar i}_{H_2(h_2)}(x_2)]$$
$$+[x_1\leftrightarrow x_2,H_1\leftrightarrow H_2]\}, \qquad (13)$$

where h_1 and h_2 are the helicities of hadrons H_1 and H_2, respectively, and

$$B=\left(\frac{\hat s^2+m^4}{\hat s q_T^2}-2\right)\Big/\{\hat s[(\hat s-m^2)^2-4\hat s q_T^2]^{1/2}\}, \qquad (14)$$

$$C=\left\{\frac{\hat s-3m^2}{\hat s}\ \frac{\hat s-m^2}{q_T^2}\left[1-\frac{2m^2(\hat s-m^2)}{\hat s^2}\right]\right\}\Big/\{\hat s[(\hat s-m^2)^2-4\hat s q_T^2]^{1/2}\}, \qquad (15)$$

$$D=\left[\frac{-\hat s-3m^2}{\hat s}+\frac{\hat s-m^2}{q_T^2}\left(1-\frac{2m^2}{\hat s}\right)\right]\{\hat s[(\hat s-m^2)^2-4\hat s q_T^2]^{1/2}\}. \qquad (16)$$

Using Eqs. (12) and (13), and (5) and (6), we arrive at the following expression for the asymmetry parameter A:

$$A=\left[\frac{\mathfrak{D}^{\text{annih}}}{(\Sigma^{\text{annih}}+\Sigma^{\text{Comp}})}+\frac{\mathfrak{D}^{\text{Comp}}}{(\Sigma^{\text{annih}}+\Sigma^{\text{Comp}})}\right], \qquad (17)$$

$$\mathfrak{D}^{\text{annih}}=m^2\frac{d^2\sigma^{\text{annih}}}{dm^2dq_T^2}(H_1(+)H_2(+)\to\mu^+\mu^-X)-m^2\frac{d^2\sigma^{\text{annih}}}{dm^2dq_T^2}(H_1(+)H_2(-)\to\mu^+\mu^-X)$$
$$=-\frac{16\alpha^2\alpha_s}{27}\int_0^1dx_1\int_0^1dx_2\theta((\hat s-m^2)^2-4\hat s q_T^2)2B\sum_{i=u,d,s}e_i^2[\Delta f^i_{H_1(+)}(x_1)\Delta f^{\bar i}_{H_2(+)}(x_2)+(i\leftrightarrow\bar i)], \qquad (18)$$

$$\mathfrak{D}^{\text{Comp}} = m^2 \frac{d^2\sigma^{\text{Comp}}}{dm^2 dq_T{}^2}(H_1(+)H_2(+) \to \mu^+\mu^- X) - m^2 \frac{d^2\sigma^{\text{Comp}}}{dm^2 dq_T{}^2}(H_1(+)H_2(-) \to \mu^+\mu^- X)$$
$$= \frac{\alpha^2\alpha_s}{9}\int_0^1 dx_1 \int_0^1 dx_2 \theta((\hat{s}-m^2)^2 - 4\hat{s}q_T{}^2) 2D \sum_{i=u,d,s} e_i{}^2 \{\Delta g_{H_1(+)}(x_1)[\Delta f^i_{H_2(+)}(x_2) + \Delta f^{\bar{i}}_{H_2(+)}(x_2)] + [x_1 \leftrightarrow x_2, H_1 \leftrightarrow H_2]\}, \tag{19}$$

$$\Sigma^{\text{annih}} = m^2 \frac{d^2\sigma^{\text{annih}}}{dm^2 dq_T{}^2}(H_1(+)H_2(+) \to \mu^+\mu^- X) + m^2 \frac{d^2\sigma^{\text{annih}}}{dm^2 dq_T{}^2}(H_1(+)H_2(-) \to \mu^+\mu^- X)$$
$$= \frac{16\alpha^2\alpha_s}{27}\int_0^1 dx_1 \int_0^1 dx_2 \theta((\hat{s}-m^2)^2 - 4\hat{s}q_T{}^2) 2B \sum_{i=u,d,s} e_i{}^2 [f^i_{H_1}(x_1) f^{\bar{i}}_{H_2}(x_2) + (i \leftrightarrow \bar{i})], \tag{20}$$

$$\Sigma^{\text{Comp}} = m^2 \frac{d^2\sigma^{\text{Comp}}}{dm^2 dq_T{}^2}(H_1(+)H_2(+) \to \mu^+\mu^- X) + m^2 \frac{d^2\sigma^{\text{Comp}}}{dm^2 dq_T{}^2}(H_1(+)H_2(-) \to \mu^+\mu^- X)$$
$$= \frac{\alpha^2\alpha_s}{9}\int_0^1 dx_1 \int_0^1 dx_2 \theta((\hat{s}-m^2)^2 - 4\hat{s}q_T{}^2) 2C \sum_{i=u,d,s} e_i{}^2 \{g_{H_1}(x_1)[f^i_{H_2}(x_2) + f^{\bar{i}}_{H_2}(x_2)] + [x_1 \leftrightarrow x_2, H_1 \leftrightarrow H_2]\}. \tag{21}$$

III. SPIN DEPENDENCE OF PARTON DISTRIBUTION FUNCTIONS

We take the following four different models for the spin-dependent parton distribution functions in longitudinally polarized nucleons (antinucleons).

A. SU(6) model

We assume the sea and the gluon to be unpolarized and take the spin dependence of quark distribution $f(x, \pm)$ based on a nonrelativistic SU(6) wave function of the nucleon, i.e.,

$$\begin{aligned}
f^u_{p(+)}(x,+) &= \tfrac{5}{6} f^{uv}_p(x) + \tfrac{1}{2} f^{us}_p(x) = f^{\bar{u}}_{\bar{p}(+)}(x,+), \\
f^u_{p(+)}(x,-) &= \tfrac{1}{6} f^{uv}_p(x) + \tfrac{1}{2} f^{us}_p(x) = f^{\bar{u}}_{\bar{p}(+)}(x,-), \\
f^d_{p(+)}(x,+) &= \tfrac{1}{3} f^{dv}_p(x) + \tfrac{1}{2} f^{ds}_p(x) = f^{\bar{d}}_{\bar{p}(+)}(x,+), \\
f^d_{p(+)}(x,-) &= \tfrac{2}{3} f^{dv}_p(x) + \tfrac{1}{2} f^{ds}_p(x) = f^{\bar{d}}_{\bar{p}(+)}(x,-),
\end{aligned} \tag{22}$$

where the second superscripts v and s denote valence and sea contributions. Thus

$$\begin{aligned}
\Delta f^u_p(x) &= \Delta f^{\bar{u}}_{\bar{p}}(x) = \tfrac{2}{3} f^{uv}_p(x), \\
\Delta f^d_p(x) &= \Delta f^{\bar{d}}_{\bar{p}}(x) = -\tfrac{1}{3} f^{dv}_p(x), \\
\Delta f^{\bar{d}}_{\bar{p}}(x) &= \Delta f^{\bar{u}}_p(x) = \Delta f^d_{\bar{p}}(x) = \Delta f^u_{\bar{p}}(x) \\
&= \Delta f^s(x) = \Delta f^{\bar{s}}(x) = 0, \\
\Delta g_p(x) &= \Delta g_{\bar{p}}(x) = 0.
\end{aligned} \tag{23}$$

B. Sehgal model

In this model we use the Sehgal[2] parametrization for $\Delta f^u_p(x)$ and $\Delta f^d_p(x)$. Since for a nucleon

$$\langle J_z \rangle = \langle S_z \rangle_{\text{quarks}} + \langle S_z \rangle_{\text{gluon}} + \langle L_z \rangle, \tag{24}$$

we take

$$\langle L_z \rangle = 0,$$

$$\langle S_z \rangle_{\text{quarks}} = \tfrac{1}{2}\int_0^1 dx[\Delta f^u(x) + \Delta f^{\bar{u}}(x) + \Delta f^d(x) + \Delta f^{\bar{d}}(x) + \Delta f^s(x) + \Delta f^{\bar{s}}(x)], \tag{25}$$

$$\left(\frac{G_A}{G_V}\right)_{n\to p} = \int_0^1 dx[\Delta f^u(x) + \Delta f^{\bar{u}}(x) - \Delta f^d(x) - \Delta f^{\bar{d}}(x)], \tag{26}$$

$$\left(\frac{G_A}{G_V}\right)_{\Xi^-\to\Xi^0} = \int_0^1 dx[\Delta f^d(x) + \Delta f^{\bar{d}}(x) - \Delta f^s(x) - \Delta f^{\bar{s}}(x)]. \tag{27}$$

Thus,

$$\langle S_z \rangle_{\text{quarks}} = \tfrac{1}{2}\left[\left(\frac{G_A}{G_V}\right)_{n\to p} + 2\left(\frac{G_A}{G_V}\right)_{\Xi^-\to\Xi^0}\right] + \tfrac{5}{2}\int_0^1 dx[\Delta f^s(x) + \Delta^{\bar{s}}(x)]. \tag{28}$$

Neglecting the strange-quark contribution, one gets

$$\langle s_z \rangle_{\text{quarks}} = \tfrac{1}{2}(3F - D), \tag{29}$$

where

$$\left(\frac{G_A}{G_V}\right)_{n\to p} = F + D,$$

$$\left(\frac{G_A}{G_V}\right)_{\Xi^-\to\Xi^0} = F - D.$$

Using the experimental values of $D + F = 1.25 \pm 0.01$ and $F/D = 0.5 \pm 0.03$, one gets $\langle s_z \rangle_{\text{quark}} = 0.03 \pm 0.03$. The average value of F/D over all X is taken here, whereas it may change[3,4] with x.

Thus,

$$\langle s_z \rangle_{\text{gluon}} = 0.2 \pm 0.05. \tag{30}$$

Assuming the sea to be unpolarized, we get

$$0.3 = \langle s_z \rangle_{\text{quarks}} = \tfrac{1}{2}\int_0^1 dx\,[\Delta f^u(x) + \Delta f^d(x)] \tag{31}$$

and the Bjorken sum rule is

$$1.23 = \int_0^1 dx[\Delta f^u(x) - \Delta f^d(x)]\,. \tag{32}$$

Thus, Eqs. (31) and (32) give

$$\begin{aligned} \Delta f^u(x) &= 0.456 f^{uv}(x)\,, \\ \Delta f^d(x) &= -0.315 f^{dv}(x)\,. \end{aligned} \tag{33}$$

We parametrize the spin dependence of the gluon distribution as $\Delta g(x) = \beta x g(x)$ such that (30) is satisfied,

$$\begin{aligned} 0.2 = \langle s_z \rangle_{\text{gluon}} &= \int_0^1 x\beta g(x)dx = \beta\,\frac{(m+1)}{2}\int_0^1 (1-x)^m dx \\ &= \beta/2\,. \end{aligned}$$

Thus, $\beta = 0.4$.

Hence,

$$\begin{aligned} \Delta g(x) &= 0.4 x g(x)\,, \\ g(x) &= \frac{(m+1)}{2}(1-x)^m/x\,. \end{aligned} \tag{34}$$

We call this the Sehgal model. Such a polarization of the gluon without a corresponding polarization of the quark sea is not consistent, as the quark sea is supposed to arise from the gluons. However, this calculation is done only with the aim of seeing the effect of gluon polarization. The effect of sea polarization is discussed in the Babcock-Monsay-Sivers model.

C. Carlitz-Kaur model

In this model, valence quarks interact with the sea and hence their spins are diluted. Let $\sin^2\theta$ be the probability that a valence quark's spin changes in interaction with the sea. Suppose $H(x)$ is the probability of spin-flip interaction between the valence and the sea and $N(x)$ is the density of the sea relative to the valence. Then from statistical consideration

$$\sin^2\theta = \frac{1}{2}\,\frac{H(x)N(x)}{(H(x)N(x)+1)}\,. \tag{35}$$

Carlitz and Kaur,[5] assuming the sea to be unpolarized, deduce the expression

$$H(x)N(x) = H_0(1-x)^2 x^{-1/2}\,. \tag{36}$$

The value $H_0 = 0.052$ is set by the Bjorken sum rule, Eq. (32). A measure of the spin dilution induced by these interactions is given by

$$\begin{aligned} \cos 2\theta &= (H(x)N(x)+1)^{-1} \\ &= (1 + H_0(1-x)^2 x^{-1/2})^{-1}\,. \end{aligned} \tag{37}$$

The spin-dependent structure functions are simply given as the product of a function describing the asymmetries of valence-quark spins as given by the broken-SU(6) model[4,6] in the absence of the interaction with the spin dilution factor $\cos 2\theta$. The structure functions for scattering off longitudinally polarized nucleons thus have the form

$$\begin{aligned} 2g_1^p(x) &= \cos 2\theta\left[\tfrac{4}{9} f_p^{uv}(x) - \tfrac{3}{9} f_p^{dv}(x)\right], \\ 2g_1^n(x) &= \cos 2\theta\left[\tfrac{1}{9} f_p^{uv}(x) - \tfrac{2}{9} f_p^{dv}(x)\right], \end{aligned} \tag{38}$$

since

$$\begin{aligned} 2g_1^p(x) &= \tfrac{4}{9}\,\Delta f_p^u(x) + \tfrac{1}{9}\,\Delta f_p^d(x)\,, \\ 2g_1^n(x) &= \tfrac{4}{9}\,\Delta f_p^d(x) + \tfrac{1}{9}\,\Delta f_p^u(x)\,. \end{aligned} \tag{39}$$

Equations (38) and (39) give

$$\begin{aligned} \Delta f_p^u(x) &= \cos 2\theta\left[f_p^{uv}(x) - \tfrac{2}{3} f_p^{dv}(x)\right], \\ \Delta f_p^d(x) &= \cos 2\theta\left[-\tfrac{1}{3} f_p^{dv}(x)\right]. \end{aligned} \tag{40}$$

The $\langle J_z \rangle$ sum rule indicates that 11.6% of the proton's helicity is due to gluons. We parametrize the spin dependence of the gluon distribution as before, i.e.,

$$\Delta g(x) = Cxg(x) = \frac{C(m+1)}{2}(1-x)^m\,, \tag{41}$$

and fix C such that the gluon carries 11.6% of the proton's helicity. Thus,

$$\Delta g(x) = 0.058(m+1)(1-x)^m\,. \tag{42}$$

D. Babcock-Monsay-Sivers model

This is the model of spin-dependent distribution functions in nucleons given by Babcock, Monsay, and Sivers.[7] In this model they assume the sea to be polarized and parametrize the sea distribution, based on perturbation-theory diagrams in QCD and the generation of the sea,[8,9] as

$$\begin{aligned} f_{p(+)}^s(x,+) &= cf_p^s(x)[2 + (1-x)^2]\,, \\ f_{p(+)}^s(x,-) &= cf_p^s(x)[1 + 2(1-x)^2]\,, \end{aligned} \tag{43}$$

where c is to be fixed by the amount of momentum carried by the sea. Thus,

$$\Delta f_p^s(x) = cf_p^s(x)x(2-x)\,. \tag{44}$$

They parametrize the gluon distribution, on similar considerations, as

$$\begin{aligned} g_{p(+)}(x,+) &= kg(x)[2 + (1-x)^2]\,, \\ g_{p(+)}(x,-) &= kg(x)[1 + 2(1-x)^2]\,. \end{aligned} \tag{45}$$

Thus

$$\Delta g(x) = kg(x)x(2-x)\,, \tag{46}$$

where k is to be fixed by the fact that 50% of the nucleon's momentum is carried by the gluon. For $g(x)$ of the form $(1-x)^m/x$, k becomes

$$k=\frac{(m+1)(m+3)}{12(m+2)}\,,\qquad g(x)=(1-x)^m/x\,. \tag{47}$$

$\Delta f_p^u(x)$ and $\Delta f_p^d(x)$ are fixed by the Bjorken sum rule (32) and the $\langle J_z\rangle$ sum rule (24). If one assumes the spin-averaged parton distribution as given by[10]

$$\begin{aligned} f_p^{uv}(x)&=1.79(1-x)^3(1+2.3x)/\sqrt{x}\,,\\ f_p^{dv}(x)&=1.07(1-x)^{3.1}/\sqrt{x}\,,\\ f_p^{s}(x)&=0.15(1-x)^7/x\,,\\ f_p^{\bar d}(x)&=f_p^{\bar u}(x)=f_p^{\bar s}(x)=f_p^{s}(x)\,, \end{aligned} \tag{48}$$

then

$$\begin{aligned} \Delta f_p^{s}(x)&=\Delta f_p^{\bar d}(x)=\Delta f_p^{\bar u}(x)=0.028(1-x)^7(2-x)\,,\\ \Delta f_p^{u}(x)&=0.456 f_p^{uv}(x)\,,\\ \Delta f_p^{d}(x)&=-0.315 f_p^{dv}(x)\,. \end{aligned} \tag{49}$$

IV. RESULTS AND DISCUSSION

We compute the asymmetry parameter A as a function of r_T for the longitudinally polarized proton-antiproton collision and proton-proton collision for the four various models of spin-dependent parton and gluon distribution functions given in Sec. III. For the spin-averaged parton distribution we take the ones given by Peierls, Trueman, and Wang[10] and the spin-averaged gluon distribution is taken from the form $k(1-x)^m/x$, where k is fixed by the fact that gluons carry 50% of the nucleon's momentum. We do our calculations for $m=5$ and $m=8$ and take $\sqrt{s}=27.4$ GeV for values of $\tau=0.02$, 0.06, 0.1, and 0.4, r_T from 0 to 0.5. It should be noted that perturbative QCD is valid when $r_T>0.1$ for $\tau=0.1$ and $r_T>0.2$ for $\tau=0.4$.[11]

For p-$\bar p$ collisions, the gluon contribution is understandably very much less than the valence quark-antiquark contribution. We make the following remarks regarding the p-$\bar p$ case.

(i) For all spin-dependent parton distributions, A is negative for all values of r_T.

(ii) For the distribution obtained from SU(6) $|A|$ increases as r_T increases [Figs. 2(a) and 2(b)], the value of A ranging between -30% to -44%. The Drell-Yan model[12] (with $r_T=0$) also gives -44% when SU(6) is used. This limit can be understood simply because the u quarks dominate A and $\Delta f^u(x)=\frac{2}{3}f^{uv}(x)$. Thus,

$$A\simeq\frac{\frac{4}{9}\frac{2}{3}f_p^{uv}(x)\frac{2}{3}f_{\bar p}^{\bar u v}(x)}{\frac{4}{9}f_p^{uv}(x)f_{\bar p}^{\bar u v}(x)}=\frac{4}{9}\,.$$

(iii) For the Sehgal model the parameter A is

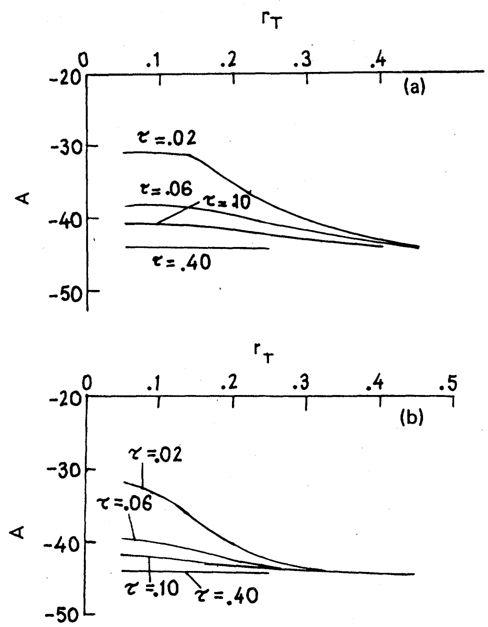

FIG. 2. (a) Graphs of A (expressed in percentage), for $p\bar p$ collision, plotted against r_T for $\tau=0.02$, 0.06, 0.1, 0.4 for the SU(6) model of spin-dependent parton distribution, the gluon distribution being $3(1-x)^5/x$. (b) Same, except that the gluon distribution is $4.5(1-x)^8/m$.

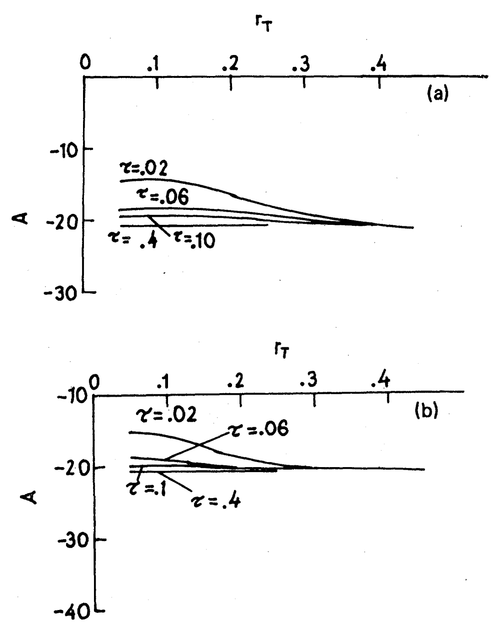

FIG. 3. (a) Graphs of A (expressed in percentage), for $p\bar p$ collision, plotted against r_T for $\tau=0.02$, 0.06, 0.1, 0.4 for the Sehgal model, the gluon distribution being $3(1-x)^5/x$. (b) Same, except that the gluon distribution is $4.5(1-x)^8/x$.

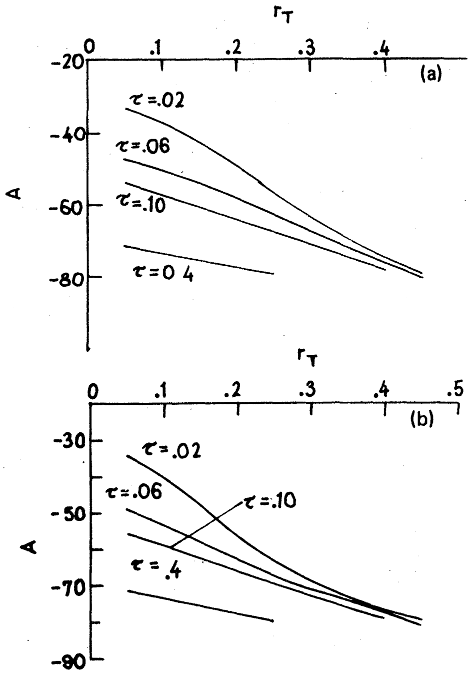

FIG. 4. (a) Graphs of A (expressed in percentage), for $p\bar{p}$ collision, plotted against r_T for $\tau = 0.02$, 0.06, 0.1, 0.4 for the Carlitz-Kaur model, the gluon distribution being $3(1-x)^5/x$. (b) Same, except that the gluon distribution is $4.5(1-x)^8/x$.

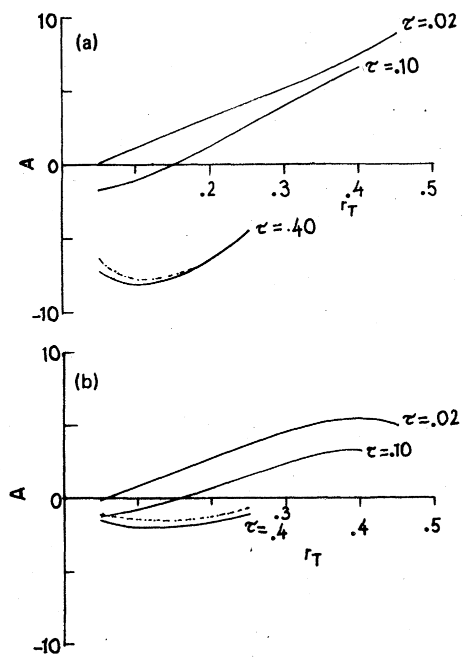

FIG. 5. (a) Graphs of A (expressed in percentage), for pp collision, plotted against r_T for $\tau = 0.02$, 0.1, 0.4 for the Sehgal model, the gluon distribution being $3(1-x)^5/x$. Dashed lines are obtained using q^2-dependent distribution functions. (b) Same, except that the gluon distribution is $4.5(1-x)^8/x$.

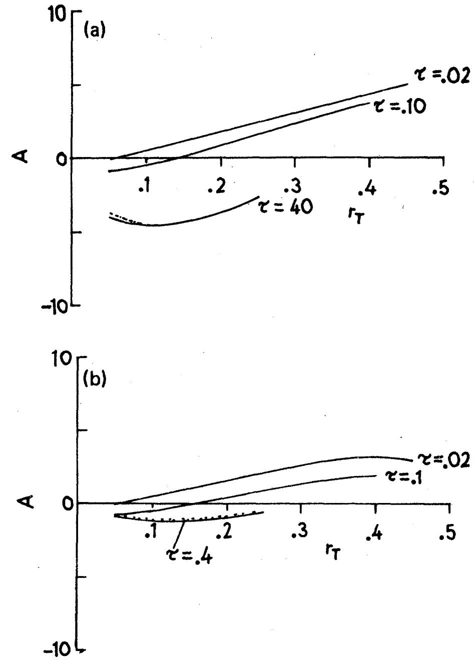

FIG. 6. (a) Graphs of A (expressed in percentage), for pp collision, plotted against r_T for $\tau = 0.02$, 0.1, 0.4 for the Carlitz-Kaur distribution, the gluon distribution being $3(1-x)^5/x$. Dashed lines are obtained using q^2 dependent distribution functions. (b) Same, except that the gluon distribution is $4.5(1-x)^8/x$.

more or less similar to the SU(6) model but values are less. The results are shown in Figs. 3(a) and 3(b).

(iv) The Carlitz-Kaur model has a much larger value of $\Delta f^u(x)$ for large x. This leads to a much larger asymmetry for large x as shown in Figs. 4(a) and 4(b). Even though the model is not very reliable in this limit, it will be of interest to test whether the asymmetry is as large as 80% for $r_T = 0.45$. This would clearly distinguish it from the other models.

(v) The Babcock-Monsay-Siver distribution, when the sea is also polarized, does not lead to any significant difference from the Sehgal distribution. This is understandable because the contribution due to the sea is insignificant.

We now turn to the p-p case where the quark-gluon contribution is as important as the quark-antiquark contribution. This happens because the quark-antiquark contribution is small as the antiquark distribution function in a proton is suppressed. We give the following comments.

(i) For SU(6) A is zero because neither the antiquarks nor the gluons are assumed to be polarized.

(ii) For the Sehgal distribution the asymmetry is shown in Figs. 5(a) and 5(b). This comes entirely from quark-gluon scattering, as the anti-

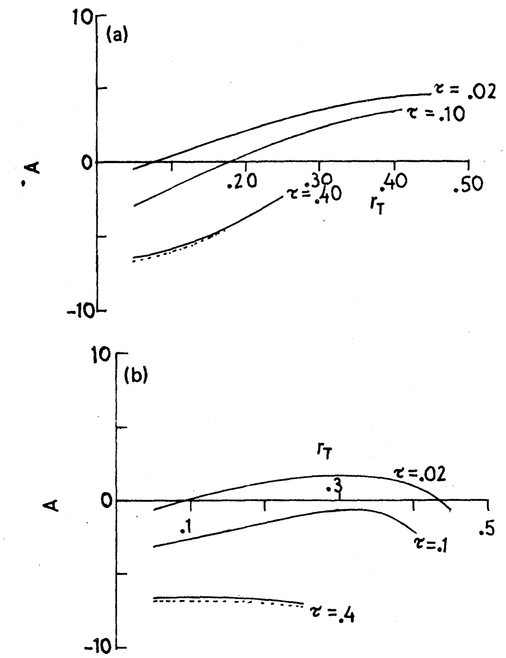

FIG. 7. (a) Graphs of A (expressed in percentage), for pp collision, plotted against r_T for $\tau = 0.02$, 0.1, 0.4 for the Babcock-Monsay-Siver distribution, the gluon distribution being $3(1-x)^5/x$. Dashed lines are obtained using q^2-dependent distribution functions. (b) Same, except that the gluon distribution is $4.5(1-x)^8/x$.

quarks are assumed to be unpolarized. The asymmetry has a strong dependence on the gluon distribution. This can be seen as we change the gluon distribution from $3(1-x)^5/x$ to $4.5(1-x)^8/x$. The same remark holds for the Carlitz-Kaur distribution [Figs. 6(a) and 6(b)].

(iii) The antiquarks are polarized in the Siver model and hence both the quark-antiquark and the quark-gluon processes contribute to the asymmetry. These are shown in Fig. 7(a) and 7(b).

We have calculated the asymmetry parameter A in the scaling limit. However, we can also include the scale-violating effects by treating the parton functions as a function of q^2. We use the Gluck-Reya[13] distributions for the q^2 dependence which is

$$f_p^{uv}(x,q^2) = f_p^{uv}(x,q_0^2)\left[\frac{\ln[q^2/(3\times10^{-6})]}{\ln[q_0^2/(3\times10^{-6})]}\right]^{0.951-x\ln(q^2/0.012)},$$

$$f_p^{dv}(x,q^2) = f_p^{dv}(x,q_0^2)\left[\frac{\ln[q^2/(7\times10^{-7})]}{\ln[q_0^2/(7\times10^{-7})]}\right]^{0.791-x\ln(q^2/0.0015)},$$

$$f_p^{s}(x,q^2) = f_p^{s}(x,q_0^2)\left[\frac{\ln[q^2/(1\times10^{-3})]}{\ln[q_0^2/(1\times10^{-3})]}\right]^{0.67-x\ln(q^2/0.583)},$$

$$g(x,q^2) = g(x,q_0^2)\left[\frac{\ln[q^2/(4\times10^{-5})]}{\ln[q_0^2/(4\times10^{-5})]}\right]^{0.288-x\ln(q^2/0.031)},$$

where $q_0^2 = 3$ GeV^2, and q^2 is in GeV^2. In the case of p-$\bar{p}$ collisions there is no substantial scaling violation. However, there is a slight effect in the p-p case with $\tau = 0.4$ and low values of r_T [Figs. 5(a), 5(b), 6(a), 6(b), 7(a), 7(b)]. Since the maximum value of $m_{\mu\mu}$ considered is approximately 16 GeV (for $\sqrt{s} = 27.4$ GeV and $\tau = 0.4$), the effect of the neutral vector boson Z^0 will be negligible.[14,15]

We conclude by observing that A for p-$\bar{p}$ collision shows striking variation with r_T and varies strongly with the model chosen for the spin-dependent valence-quark distribution functions in nucleons. Thus A as a function of r_T will serve as a good test for the correct spin-dependent distribution function of valence quarks in nucleons and hence is worth measuring.

After the completion of this work, we received a copy of a similar work by Keisho Hidaka.[16] He calculates the asymmetry only for p-p collision, whereas we calculate it both for p-p collision and p-$\bar{p}$. Our results are in good agreement with his work.

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