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Effect of the $\Sigma(1460)$ on the analysis of kaonic-hydrogen x-ray deexcitation

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The effect of a new Σ state on the $2p$-$1s$ transition in kaonic hydrogen is discussed. It is shown that the new state can only account for a small fraction of the effect one observes in the line energy shift. This negative result provides indirect support for the need of some more sophisticated treatment of the Coulomb corrections in K^-p scattering.

I. INTRODUCTION

Recently many discussions have been devoted to the problem of the x-ray deexcitation of kaonic hydrogen, where the observed $2p$-$1s$ transition indicates that the strong-interaction energy shift ϵ is smaller than about 60 eV and that the width Γ is smaller than about 230 eV.[1] These quantities can be related to the complex K^-p scattering length A_c by[2]

$$\epsilon+\tfrac{1}{2}i\Gamma=2\alpha^3\mu^2A_c\,, \qquad (1)$$

where α is the fine-structure constant and μ the reduced K^-p mass.

The experimental values of ϵ and Γ turn out to be smaller than what one would obtain by using as A_c in Eq. (1) the average A_p of A_0 and A_1, the $I=0$ and $I=1$ $\bar{K}N$ scattering lengths as determined by analyses of low-energy $\bar{K}N$ reactions. For example, the most recent analysis gives[3] $\frac{1}{2}(A_0+A_1)=-0.665+i0.64$ fm, which in turn gives $(\epsilon,\Gamma)=(-274,528)$ eV. It is difficult to estimate the uncertainty of this prediction. Even if the spread of the values of A_0 and A_1 found in different analyses is within 0.1 fm (with the exception of Re A_1, for which the oldest analyses gave values of the order of 0 fm and the most recent ones values of about 0.4 fm), it is probable that the statistical uncertainty is somewhat larger.[4] In connection with this matter, it is perhaps worthwhile recalling that the most recent analyses include a number of constraints, one of which is that the threshold amplitudes satisfy dispersion relations.[3]

II. CURRENT PROPOSALS OF EXPLANATION OF THE EFFECT

It has been suggested[5] that the effect may be explained with a suitable model for the $Y_0^*(1405)$, the resonance lying in the $\bar{K}N$ unphysical region. The relevant feature of this model is a rather rapid variation of Re A_0 around threshold. Using for A_1 the value provided by Chao's analysis,[6] this mechanism reduces ϵ to about 17 eV and Γ to 297 eV. The value of Γ is perhaps still a little bit high in comparison with the experimental result; moreover, it should be noted that a different choice of the $I=1$ scattering length, similar to, for instance, that of Martin's parametrization,[3] would give $(\epsilon,\Gamma)=(76,281)$ eV.

In Ref. 5 the hope was expressed that one could fit the $\bar{K}N$ low-energy scattering data with their model. However, it has been found[3,4,7] that by such a model no adequate description of the data is possible, if one requires that the $I=0$ amplitude has a pole near 1405 MeV.

If, instead, one requires that all $I=0$ K-matrix elements have a pole, then either this pole is considerably displaced from 1405 MeV or a background term must be introduced, with the effect of making the pole residues become very small. Probably these difficulties are related to the fact that this model is in contrast with the indication coming from the Coulomb-nuclear interference in low-energy K^-p interaction[8] that this is repulsive.

Another proposal which could explain the kaonic-hydrogen result has been presented by Deloff and Law.[9] These authors discuss the importance of a number of electromagnetic corrections to the naive assumption that A_p be equal to $\frac{1}{2}(A_0+A_1)$. The first of them consists in taking in consideration the mass difference of the K^-p and $\bar{K}^0n$ system. This leads to the following expression for the K^-p scattering length:

$$A_t=\frac{A_p+|k_0|A_0A_1}{1+|k_0|A_p}\,, \qquad (2)$$

k_0 being the modulus of the $\bar{K}^0$ c.m. momentum at K^-p threshold. The second consists in considering Coulomb corrections, which following Dalitz and Tuan,[10] leads to the following expression:

$$A_{DT}=\frac{A_t}{1-2\pi A_t\lambda/B}\,, \qquad (3)$$

where B is the kaonic-hydrogen Bohr radius and $\lambda=-(2\gamma+\ln 2R/B)/\pi$, γ being the Euler constant and the parameter R the range of $\bar{K}N$ force.

These corrections are not sufficient to account for the effect since they lead, respectively, to $(\epsilon,\Gamma)=(-425,625)$ eV and $(-407,528)$ eV when the parameters of Ref. 3 are used and to similar values when those of the previous Martin analysis[11] are used.[9] The effect does not affect the calculation of Ref. 5 because their very small value of A_0 allows A_{DT}, A_t, and A_p to be very close to each other. Thus Deloff and Law proposed a more complete treatment of the Coulomb correction, with an approach similar to that used by Sauer in pp scattering.[12] The correction they propose is

$$A_c = \frac{A_t - R\Delta(1-R/B)(1+A_t/R) - R(R+2A_t)/B}{1-2\pi\lambda A_t/B + 2R/B + \Delta(1+A_t/R)[1+2R(1+\pi\lambda)/B]}\,, \tag{4}$$

where Δ is a complex parameter, whose theoretical determination is model dependent, and such that for certain values it may reproduce the kaonic-hydrogen result. In Ref. 7 it has been pointed out that the modification (4) is not compelling; in particular McGinley finds by two simple model calculations that the correction due to Δ is small; this does not exclude that other models might give different results. He also developed a more sophisticated formalism, which unfortunately is not as easy to apply as those of Refs. 9 and 10. The difficulty with all these formalisms is that they require the knowledge of the detailed structure of the strong potential within the range of the forces. In Ref. 7 the hope is expressed that small individual corrections in the K-matrix elements might lead to a sizable effect on the value of the K^-p scattering length.

In view of this situation it is interesting to see whether at least part of the effect may be traced back to some different source, so that the electromagnetic corrections could be applied to a value of the scattering length which disagrees less with the kaonic-hydrogen result.

III. THE EFFECT OF THE Σ(1460) ON THE K^-p SCATTERING LENGTH

A possibility in the direction just discussed is provided by the recently reported discovery[13] of an enhancement in the K^-n mass spectrum at 1.46 GeV, seen in the reaction $\pi^-p \to K^-K^0\pi^+n$ at 3.95 GeV/c. If one were to interpret this enhancement as a resonance, this would have a mass $M_R = 1464 \pm 8$ MeV and a width $\Gamma_R = 44 \pm 24$ MeV. In the past a Σ state in the same energy region had been observed in π^+p interactions at 1.7 GeV/c (Ref. 14); however, since this state dominantly decays in $\pi\Lambda$, whereas that seen by the CERN–Collège de France–Madrid–Stockholm collaboration is mainly coupled to $\pi\Sigma$ and $\bar{K}N$, the two are not likely to be the same state.

The existing analyses of $\bar{K}N$ low-energy scattering do not include such a state; therefore we want to investigate its effect on the problem of the kaonic hydrogen, and we are stimulated to this by the consideration that a resonance always produces an increase of the real part in the energy region just below it, so that one can hope that this effect on ReA_p is sufficient to explain at least part of the effect on ϵ.

Obviously the correct procedure would be to reanalyze together low-energy $\bar{K}N$ data and the result of Ref. 1, including the new state in the parametrization. However, in this paper we shall limit ourselves to try to make a rough estimate of the Σ(1460) effect and assume that, as a first approximation, one can account for it by adding its contribution to the value calculated from the K-matrix parametrization. This assumption is supported by the well-known fact that K-matrix solutions are far from being unique[3,4] and that to a large extent the actual solution is determined by data taken at energies where the effect of the new state should be negligible.

We checked this point calculating how the new state affects the real part of the $I=1$ amplitude in the energy region above the resonance. The procedure we followed is the same by which the effect on the scattering length was calculated and we are going to describe it in the next section. We found that the shifts on the mentioned real part in the laboratory was −3, −1.5, −1, and −0.26 fm at 260, 290, 320, and 500 MeV/c, respectively.

IV. NUMERICAL RESULTS

The size of the correction ΔA_1 to the value of the $I=1$ scattering length provided by the K matrix was evaluated using a classical form of dispersion relation, that for the function $[F_-(\omega) - F_+(\omega)]$ (Ref. 15) where ω is the kaon laboratory energy and $F_\pm(\omega)$ are the $K^\pm n$ scattering amplitudes. We find that the variation of the $I=1$ laboratory amplitude at threshold is equal to

$$\delta = \frac{m}{4\pi^2}\int_{\text{res}} \frac{\text{Im} f_{\text{res}}(\omega')d\omega'}{\omega' - m^2} \tag{5}$$

and ΔA_1 can be calculated from δ, making use of the usual relation

$$\Delta A_1 = \frac{M}{M+m}\,\delta\,. \tag{6}$$

In Eqs. (5) and (6) m and M are the kaon and nucleon mass, respectively.

A Breit-Wigner parametrization of the resonance is used, and it is assumed that the resonance is purely elastic. (Indeed from the $\pi\Sigma/\bar{K}N$ ratio one would find an elasticity of the order of 0.6 ± 0.2.[13]) We evaluated the integral (5) in the narrow-width approximation. One could consider these approximations too crude; however, it has to be borne in mind that the resonance parameters present rather large uncertainties, so that even if one could perform a more accurate calculation, this actually would be rather superfluous for our purpose of estimating the order of magnitude of the effect. This attitude will be confirmed by our results.

From Eq. (5) one obtains

$$\delta = \frac{m}{2} \Gamma_R \left(\frac{M_R}{M}\right)^2 \frac{1}{K_R{}^3}, \tag{7}$$

where K_R is the kaon laboratory momentum at the resonance position. In this calculation the $\Sigma(1460)$ is assumed to be s wave; this is in agreement with the type of fit by which its parameters were determined[13] and is supported by the isotropy of its decay in K^-n. Thus we find $\delta = 0.448$ fm. We also studied what the effect would be of a p-wave resonance located at the same energy and with the same width. In our approximations this would modify the value of δ by a factor of 3. The changes induced by such a state on the value of the K^-p scattering length are summarized in Table I, whereas those on (ϵ, Γ) are presented in Table II. The value of the scattering lengths A_0 and A_1, from which we start, are those of Ref. 3 and the results are displayed for both assumptions about the resonating wave.

From this calculation it emerges that one can succeed in changing $\mathrm{Re}A_p$ substantially and in the right direction, but unfortunately this achievement is unstable with respect to the electromagnetic corrections and, when these are considered, again ϵ becomes substantial. Probably in this type of calculation the only way of avoiding this feature is if, as in Ref. 5, one of the scattering lengths is very small, but we have already mentioned the difficulties related to such a possibility. From the results of these tables, using the values of ϵ corresponding to A_{DT}, the overall effect of the new state is to reduce the value of ϵ by about 5%. The effect is somewhat larger (~9%) for the p-wave case, where one may also notice that in the naive approach of neglecting electromagnetic corrections one can get a value of ϵ rather close to the experimental results.

TABLE I. Values of the K^-p scattering length calculated assuming that (a) the $\Sigma(1460)$ is an s-wave resonance and (b) the $\Sigma(1460)$ is a p-wave resonance. Units are fm.

	A_p	A_t	A_{DT}
(a)	$-0.518+0.64i$	$-0.978+0.776i$	$-0.940+0.658i$
(b)	$-0.224+0.64i$	$-0.882+0.736i$	$-0.858+0.636i$

TABLE II. Values of (ϵ, Γ) for the value of the K^-p scattering length indicated in the corresponding entry of Table I. Units are eV.

(a)	(−213, 528)	(−403, 640)	(−388, 542)
(b)	(−92, 528)	(−363, 607)	(−354, 524)

Another point which emerges from our calculations is that there is no way of reducing Γ.

One could wonder whether, at least concerning ϵ, the situation can be improved by taking into account the effect of the uncertainties on M_R and Γ_R. We investigated this point fixing our attention on the changes of these quantities which would lead to a larger value of $\mathrm{Re}A_1$, i.e., those corresponding to a larger width and to a position of the $\Gamma(1460)$ closer to the K^-p elastic threshold, since these are the changes which may reduce the size of ϵ to that required by the kaonic-hydrogen result.

The results of this calculation are presented in Table III. It was found that when only one of the uncertainties was taken into account, very similar values of A_p were obtained; therefore, in order to illustrate the effect we only present here the results obtained by changing Γ_R. Moreover, we present those obtained in correspondence to a variation of both M_R and Γ_R. All above calculations were performed at the level of one standard deviation variation.

As in the previous calculation it is impossible to obtain a reduction in Γ; concerning ϵ, only the p-wave calculation performed changing both Γ_R and M_R can reduce it appreciably (to about −190 eV).

TABLE III. Effect of the uncertainty on the width [(a1), (b1)] and on width and position [(a2), (b2)] of the $\Sigma(1460)$; here (a) and (b) have the same meaning as in Table I. Units are fm.

	A_p	A_t	A_{DT}
(a1)	$-0.438+0.64i$	$-0.939+0.617i$	$-0.896+0.529i$
(a2)	$-0.333+0.64i$	$-0.915+0.798i$	$-0.893+0.686i$
(b1)	$0.015+0.64i$	$-0.808+0.832i$	$-0.804+0.727i$
(b2)	$-0.666+0.64i$	$-0.416+0.743i$	$-0.444+0.691i$

V. DISCUSSION OF THE RESULTS AND CONCLUSIONS

It has to be remarked that, as a consequence of our making different assumptions on the wave where the Σ(1460) would appear as well as on its mass and width, our initial values of ReA_p span a rather large interval. This includes values which uncorrected would lead to an energy shift of the $2p$-$1s$ line in agreement with the experimental result. However, we have seen that this agreement is lost when electromagnetic corrections are included. This suggests that if one wants to achieve the result of reproducing (ϵ, Γ), one must be able to modify the value of the imaginary part of the K^-p scattering length. This quantity has, however, remained quite stable all through the years[16] and we have already mentioned the difficulties related to an approach similar to that of Ref. 5, i.e., one which reduces ϵ and Γ at the level of A_p.

Thus, from our calculations we think that not only can one conclude that the Σ(1460) is insufficient to reduce both ϵ and Γ, but also that this cannot be done at the level of A_p.

Such a conclusion provides support for the need of using some sophisticated mechanism of Coulomb correction; in particular, even within the difficulties pointed out in Ref. 7, the approach of Deloff and Law[9] is able to reduce the scattering length to the size required by kaonic-hydrogen x-ray deexcitation, without affecting ImA_p.

It has to be pointed out that this has an immediate effect on the low-energy analyses of $\bar{K}N$ scattering, since it implies the need of a revision of the K^-p amplitude determinations, due to the fact that current analyses include Coulomb correction in the form proposed in Ref. 10. Thus, if the result of Ref. 1 is confirmed in the new experiment which is being performed at CERN, a new analysis is urged, which includes both the new Σ state and some mechanism of Coulomb correction similar to that proposed by Deloff and Law.

Having in mind this perspective, together with the difficulty of a too large value of Δ (Ref. 7) we studied further how well one can reproduce the kaonic-hydrogen result by means of Eq. (4). As starting values of A_t we used those of Tables I and III.

On the basis of the result of Ref. 9 we varied R in the interval 0.3–0.5 fm and Δ in the complex-plane rectangle $(0.3-3)+i(-0.1, 0.1)$. Our results look very similar when we start either from the (a) or from the (b) value of A_t given in Table I.

In particular we find that the condition that Γ be in agreement with the experimental limit implies Re$\Delta \gtrsim 0.85$, whereas the corresponding condition for ϵ implies Re$\Delta \lesssim 1.8$. In the interval we studied no stringent limits were found for ImΔ. If one reduces the range of nuclear forces the allowed interval for ReΔ increases and vice versa. For example, for $R = 0.3$ fm one finds that the condition imposed by Γ becomes Re$\Delta \gtrsim 0.55$ and that imposed by ϵ becomes Re$\Delta \lesssim 2.1$, whereas for $R = 0.5$ fm one finds that the two limits are Re$\Delta \gtrsim 1.05$ and Re$\Delta \lesssim 1.6$, respectively.

Nothing really new is learned when the A_t input values are taken from Table III, except that for the choice corresponding to (b2) one finds that the allowed range of values for ReΔ is shifted by about 0.3 towards smaller values.

From this calculation it is clear that the kaonic-hydrogen result alone is not able to fix the value of the parameters appearing in Eq. (4), so that a value such as that reported in Ref. 9 is simply a representative example of a rather wide set of parameter values which may reproduce the x-ray data. It is hoped that this flexibility of Eq. (4) may allow the simultaneous fit of $\bar{K}N$ low-energy data and of the kaonic-hydrogen result for some value of Δ which can be adequately justified by model calculations.

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