Hiding of the conserved (anti)baryonic charge into black holes

A. D. Dolgov
Institute of Theoretical and Experimental Physics, B. Cheremushkinskaya ul. 25, 117259 Moscow, Union of Soviet Socialist Republic
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It is shown that the total number of baryons evaporated by a black hole can differ from that of antibaryons, even if baryonic charge is microscopically conserved. The baryonic asymmetry of the Universe which can be generated by black-hole evaporation in a specific mechanism first proposed by Zeldovich is estimated.

It was stated recently[1] that no baryonic excess in outer space can be generated by black-hole evaporation[2] if baryonic charge is microscopically conserved. The authors of Ref. 1 considered thermally emitted particles propagating through the gravitational field of the black hole, the C- and CP-violating processes of mutual particle transition in the gravitational field being permitted: $\mathcal{L}_{\rm int}=\varphi_i^* V_{ij}\varphi_j$ (here φ_i stands for a field operator. Note that this interaction conserves the total number of φ particles of all types. Using the CPT theorem and the generalization[1,3] of the detailed-balance condition to the case of violation of time reversibility, one can rigorously show that the net baryonic flux from a black hole vanishes in this case. This is the result of Ref. 1. (If there is no invariance with respect to time reversal, the detailed-balance condition is no longer fulfilled. However, unitarity of the S matrix enforces the equality of the total sum of probabilities of all direct and inverse processes in thermal equilibrium. So balance is achieved by summing up all possible cycles and the corresponding relation among transitions can be called the cyclic balance condition.[3])

However, if the total particle number in the course of propagating through the black-hole field is not conserved, the arguments of Ref. 1, as they are, are not applicable, because in this case the equations of motion become nonlinear and so the result can be invalid. It is shown in what follows that the mechanism proposed several years ago by Zeldovich[4] does indeed produce nonvanishing baryonic flux into outer space and the hiding of an equal number of antibaryons inside the black hole. (The idea of generation of baryon asymmetry of the Universe by black-hole evaporation was first formulated by Hawking.[2])

Assume that there exists a heavy meson A which has (among others) the decay channels

$$A\to H\bar{L} \quad \text{and} \quad A\to \bar{H}L\,,$$

where L is a light baryon and H is a heavy one. Because of C and CP violation the decay probabilities can be different:

$$[\Gamma(A\to L\bar{H})-\Gamma(A\to H\bar{L})]/\Gamma_{\rm tot}=\Delta>0\,. \tag{1}$$

Of course some other decay channels and rescattering processes should be possible to provide this inequality.

Because of the larger mass of H, as compared to that of L, the probability of back capture of H (and $\bar{H}$) by a black hole is larger. So the process of the A-meson evaporation and its subsequent decay in the gravitational field of the black hole leads to a baryonic excess in the outer world even if the baryonic number is strictly conserved. Particle scattering outside the black hole, which in principle could compensate the excess of light baryons, is negligible because the particle flux from the surface of the black hole is small.

The following example explicitly confirms the above statement. Let $m_A\simeq m_H\gg m_L$ and the black-hole temperature be small enough so that $m_H\gg T\simeq m_L$. The wave equation governing (for simplicity spinless) particle propagation in the gravitational field of a black hole is of the form (see, e.g., Ref. 5)

$$[\partial_\xi{}^2+\epsilon^2-V(\xi;l)]R^{l,l_3}(\xi;\epsilon)=0\,, \tag{2}$$

where $R^{l,l_3}(\xi;\epsilon)$ is the particle radial wave function with orbital momentum l and its third component l_3. The total wave function is decomposed in terms of R^{l,l_3} as follows:

$$\psi(\vec{r},\epsilon)=r^{-1}\sum_{l,l_3}Y_{l,l_3}(\theta,\varphi)R^{l,l_3}(\xi;\epsilon)\,,$$

where ϵ is the particle energy in units of inverse gravitational radius ($\epsilon=Er_g=E2MG$), M is the black-hole mass, and $G=0.6\times10^{-38}m_{\rm p}{}^{-2}$ is the gravitational (Newton) constant. The potential V has the form

$$V(\xi;l)=(1-\rho^{-1})[l(l+1)\rho^{-2}+(mr_g)^2+\rho^{-3}]\,, \tag{3}$$

where m is the particle mass, $\rho=r/r_g$, and r is the usual radius vector related to ξ by the equation

$$\xi = \rho + \ln(\rho - 1) . \tag{4}$$

It is essential that $V(\xi, l) \to 0$ as $\xi \to -\infty$ $(r \to r_g)$ and $V(\xi, l) \to (mr_g)^2$ as $\xi \to +\infty$ $(r \to +\infty)$. This means that in the vicinity of the black hole all evaporated particles are effectively massless and have the same (thermal)energy distribution ($\sim\exp(-E/T)[1 \pm \exp(-E/T)]^{-1}$). However, only those particles which have sufficiently high energy ($E > m$) can propagate to infinity. So in the case considered the flux of A and H particles at large distances from the black hole is exponentially [$\sim\exp(-m/T) \ll 1$] suppressed whereas the back capture of L particles is not so overwhelmingly large. The flux of L and $\bar{L}$ particles at infinity is thus not small and because of larger amounts of L produced, as compared to that of $\bar{L}$ [see inequality (1)], the net flux of baryonic charge is nonzero. There is, of course, some suppression of A decays due to the slowing down of time in the vicinity of a black hole but it results only in a power-law suppression and not in an exponential one. To make this more precise, consider the wave function of L and $\bar{H}$ produced by the A decay in the gravitational field of the black hole:

$$\psi_{L\bar{H}}(\vec{r}, \vec{r}', \epsilon_L, \epsilon_H) = \frac{1}{rr'} \sum_{\substack{l, l_3 \\ l', l_3'}} Y_{l l_3}(\theta, \varphi) Y_{l' l_3'}(\theta', \varphi') \times R_{L\bar{H}}^{l l_3; l' l_3'}(\xi, \xi'; \epsilon_L, \epsilon_H) , \tag{5}$$

where ξ is related to r through expression (4). It can be shown that $R_{L\bar{H}}$ satisfies the equation

$$[\partial_\xi{}^2 + \epsilon_L{}^2 - U_L(\xi; l)][\partial_{\xi'}{}^2 + \epsilon_H{}^2 - U_H(\xi', l')] R_{L\bar{H}}^{l l_3; l' l_3'}(\xi, \xi'; \epsilon_L, \epsilon_H) = 2 i f r_g{}^2 \frac{r_g}{r}\left(1 - \frac{r_g}{r}\right) \delta(\xi - \xi') \sum_{l_A, l_{3A}} R_A^{l_A, l_{3A}}(\xi', \epsilon_L + \epsilon_H) \mathfrak{D}(l_A, l_{3A}, l, l_3; l', l_3') , \tag{6}$$

where f is the coupling constant of the $AL\bar{H}$ transition, and R_A is the wave function of the A meson. R_A satisfies Eq. (2) with the substitution $m_A \to m_A - i\Gamma_A/2$, Γ_A being the total decay width of the A meson. The derivation and solution of the coupled equations (2) and (6) is discussed in a longer paper[6] where the following estimate for the baryon charge produced by the black-hole evaporation in the case of $m_{A,H} r_g > 1$ and $m_L r_g < 1$ was obtained:

$$B = N_L - N_{\bar{L}} \simeq \Delta \frac{\Gamma_A}{m_A} \left(\frac{M_0}{m_{\mathcal{P}}}\right)^2 N_{\rm eff}{}^{-1} . \tag{7}$$

Here $N_{L(\bar{L})}$ is the total amount of light baryons (antibaryons) evaporated by the black hole, Δ is defined by expression (1), $m_{\mathcal{P}} \simeq 10^{19}$ GeV is the Planck mass, $N_{\rm eff}$ is the effective number of different particle species evaporated by the black hole ($N_{\rm eff} \simeq 10-100$), and M_0 is the initial value of the black-hole mass, with the following condition being valid: $T = m_{\mathcal{P}}{}^2/8\pi M_0 > m_L$.

To evaluate the average baryon number density in the Universe we proceed as follows. The energy density in the early Universe is

$$\rho(t) = \frac{3}{32\pi} \frac{m_{\mathcal{P}}{}^2}{t^2} . \tag{8}$$

If the contribution of the primordial black holes with mass M into ρ is equal to $\kappa\rho$ ($\kappa < 1$) then the number density of such black holes (BH) is

$$n_{\rm BH} = \kappa\rho(t)/M . \tag{9}$$

The value of κ is unknown; in what follows we assume that it is of the order of unity. The baryon number density to the moment of the black-hole evaporation[7]

$$t = \tau_{\rm BH} = (10^4/3N_{\rm eff}) M^3 m_{\mathcal{P}}{}^{-4}$$

is

$$n_B = n_{\rm BH} B = \kappa B \rho(\tau_{\rm BH}) M^{-1} . \tag{10}$$

After the black-hole evaporation thermodynamic equilibrium is established in the primeval plasma, with the temperature being defined by the equation

$$\rho = \frac{\pi^2 N}{15} T^4 , \tag{11}$$

where N is the number of different particle species present in the plasma. In what follows we assume that $N \simeq N_{\rm eff}$ [see Eq. (7)]. Now the following result for the inverse specific entropy per baryon can be obtained:

$$\beta = \frac{n_B}{(\rho/T)} = \frac{\kappa B}{T}\left(\frac{15\rho(\tau_{\rm BH})}{\pi^2 N}\right)^{1/4} \simeq 0.1\kappa\Delta \frac{\Gamma_A}{m_A} N^{-3/4} \left(\frac{m_{\mathcal{P}}}{M}\right)^{1/2} . \tag{12}$$

The increase of β for small M is connected with the assumption that $n_{\rm BH} \sim M^{-1}$ [see Eq. (9)]. A reasonable order-of-magnitude estimate of the parameters in the right-hand side of Eq. (12), i.e., $(\Gamma_A/m_A) N^{-3/4} \simeq 10^{-2}$, $\Delta \simeq 10^{-4}$ (probably smaller), gives $\beta = 10^{-7}\kappa(m_{\mathcal{P}}/M)^{1/2}$. Comparing this with with the known value, $\beta = 10^{-9\pm1}$, we conclude that the discussed mechanism can provide the observed baryonic asymmetry of the Universe if primordial black holes with the mass $M = 10^{4\pm2} m_{\mathcal{P}} \simeq (10^{-3}-1)g$ give noticeable contribution to total energy density. Remember, however, that expression (7) was derived under the assumption that $r_g m_{A,H} > 1$; this means that the results obtained could be valid if there existed an A meson and H baryon heavier than $10^{-4\pm 2} m_{\varphi} \simeq 10^{+15}$ GeV.

It is easy to see that in the case of large $m_A r_g$ and $m_H r_g$ and small $m_L r_g$, considered up until now, the resulting baryonic asymmetry proved to be the largest. If $m_H r_g < 1$, then the value of B [Eq. (7)] would be suppressed by the extra factor $(m_H - m_L) r_g$, and to get the desired value of β the heavy baryon H should be heavier than $10^{-(4\pm 2)} m_{\varphi}$. The case of small $m_A r_g$ is even less favorable because the baryonic charge generation is further suppressed by the slowing down of the A decay rate due to the γ factor, where $\gamma = m_A/E_A \simeq m_A/T \simeq m_A r_g$.

In conclusion I would like to note that the discussed mechanism of the explanation of the baryonic asymmetry of the Universe is seemingly not so beautiful as the possibility which naturally arises in grand unified theories with baryonic charge nonconservation. But if proton instability is not discovered in the near future, the model considered here will look much more attractive. There is a possibility, of course, that both mechanisms are operative.

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