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Spontaneously generated gravity

A. Zee*
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19174
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We show, following a recent suggestion of Adler, that gravity may arise as a consequence of dynamical symmetry breaking in a scale- and gauge-invariant world. Our calculation is not tied to any specific scheme of dynamical symmetry breaking. A representation for Newton's coupling constant in terms of flat-space quantities is derived. The sign of Newton's coupling constant appears to depend on infrared details of the symmetry-breaking mechanism.

I. BACKGROUND

Of the fundamental interactions in the world, the two more feeble interactions are characterized by dimensional coupling constants: one observes Fermi's coupling constant $G_F \simeq (300\ \text{GeV})^{-2}$ and Newton's coupling constant $G_N \simeq (10^{19}\ \text{GeV})^{-2}$. As was noted long ago by Heisenberg,[1] dimensional analysis implies that interactions with coupling constants with dimensions of inverse mass to some positive power are highly divergent and nonrenormalizable. Over the last two decades or so, we have come to realize that the weak interaction at a more fundamental level is actually characterized by a dimensionless coupling constant and that the dimensional nature of G_F results from a spontaneous symmetry breaking. Indeed, $G_F \simeq 1/V_w{}^2$, where $V_w \simeq 300$ GeV is the vacuum expectation value (VEV) of some scalar field (which may be elementary or composite). The weakness of the weak interaction is then understood in terms of the fact that 300 GeV is large.[2]

It is tempting to suggest, in light of the above, that gravity is also in fact characterized by a dimensionless coupling constant and that the weakness of gravity is associated with symmetry breaking at a high mass scale. We imagine that G_N, similar to G_F, is given by the inverse square of the vacuum expectation value V of some scalar field ϕ (which may be elementary or composite). Clearly, we should not push the analogy too far since the physics of the weak interaction and of gravity is quite different. Gravitation is long ranged and so V cannot be associated with the mass of a mediating particle. Indeed, $1/G_N$ appears in the Einstein-Hilbert action, $S_E = \int d^4x\sqrt{-g}R(16\pi G_N)^{-1}$, multiplying what is essentially the kinetic energy term[3] for the graviton.

It was proposed by the present author,[4] and independently by Smolin,[5] that the action S_E be replaced by the action

$$S = \int d^4x\sqrt{-g}\left[\tfrac{1}{2}\epsilon\phi^2 R + \tfrac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} - V(\phi)\right]. \quad (1)$$

The coupling constant ϵ is dimensionless. The potential $V(\phi)$ is assumed to attain its minimum value when $\phi = V$. Then

$$G_N = \frac{1}{16\pi}\left(\frac{1}{\frac{1}{2}\epsilon V^2}\right). \quad (2)$$

The introduction of scalar fields into gravity has a long history.[7,8] Here, the crucial feature is the incorporation of spontaneous symmetry breaking. As a consequence the scalar field is "anchored" in a deep potential well $V(\phi)$ and thus the physical consequences of the present theory is indistinguishable from Einstein's theory except under extreme conditions of space-time curvature.[4] This is in sharp contrast to earlier work such as that of Brans and Dicke.[9]

II. SCALE INVARIANCE

The attractiveness of scale-invariant theories has been much discussed in recent years. Such theories are apparently renormalizable by power counting. The renormalization procedure introduces a mass scale and as a result of this so-called "dimensional transmutation[10]" all dimensionless physical quantities are calculable. It was suggested by the present author in Ref. 11 that the underlying interactions of the world, including gravity, are scale invariant. (For simplicity, we do not insist on local scale invariance here since we do not need it to make our point. While it is most appealing to impose local scale invariance, we could do so only at the price of introducing additional structures not relevant to our discussion.) In that case, the Einstein-Hilbert action for gravity is not admissible. Instead, a scale-invariant action for the world would read[12]

$$S = \int d^4x\sqrt{-g}\left[\epsilon\phi^2 R + \beta R^2 + \gamma R_{\mu\nu}R^{\mu\nu} + \tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \lambda\phi^4 + \mathcal{L}(\psi, A, \phi, \eta)\right]. \quad (3)$$

Here $\mathcal{L}$ is a scale-invariant gauge-invariant Lagrangian describing the interactions between quarks and leptons ψ, gauge fields A, the scalar field ϕ, and possibly other scalar fields η. Note the presence in general of the dimension-4 terms R^2 and $R_{\mu\nu}R^{\mu\nu}$.

We note in passing that the presence of the R^2 and $R_{\mu\nu}R^{\mu\nu}$ terms, which involve four derivatives, implies that the graviton propagator would go as k^4 for large momentum k thus rendering the theory, at least formally, renormalizable. As is well known, the price to be paid for this improved convergence is the occurrence of a ghost pole in the propagator. This ghost pole, however, occurs at momentum so large, of order $\beta^{-1/2}M_{\rm Pl}$ or $\gamma^{-1/2}M_{\rm Pl}$, that it is not even clear whether local quantum field theory would continue to hold.

Coleman and Weinberg[10] had shown that in the flat-space limit the action S, with suitable choice of coupling constants, is such that ϕ has a nonzero VEV. We expect the same would hold in curved space, at least for spaces of small curvature. Thus, the $\epsilon\phi^2R$ term would then lead to effectively Einstein's theory of gravity. (In general, the VEV of ϕ will be a functional of the metric $g_{\mu\nu}$, and thus for spaces of high curvature the effective gravitational action may be quite complicated and nonlocal.)

Recently, Adler[13] has gone one step further; he proposed in a very interesting paper that elementary scalar fields should not be present in the fundamental action in Eq. (3). Elementary scalar fields are generally regarded with repugnance by the particle physics community. It is generally believed that the elementary scalar fields needed in present-day theories are merely the phenomenological manifestations of some composite scalar field such as $\bar\psi\psi$. In the present context, Adler[13] made the important observation that in that case scale and gauge invariance combine to forbid terms proportional to R. The terms R and $\bar\psi\psi R$ have mass dimension 2 and 5, respectively, while $A_\mu A^\mu R$, although of dimension 4, is not gauge invariant. However, if dynamical symmetry breaking occurs, such that $\bar\psi\psi$ has some nonzero VEV, then, in general, we expect that a term such as $\langle\bar\psi\psi\rangle^{2/3}R$ would be effectively induced in the action. Thus, we are led to the rather amusing view that gravity may be, at least in some sense, an inevitable consequence[14] of the dynamical breaking of a grand unified symmetry describing the strong, weak, and electromagnetic interactions.

Thus, the roots of gravity may well lie in Lorentz invariance. To write down Lorentz-invariant interactions between fields we have to introduce the Minkowski metric $\eta_{\mu\nu}$. Once we admit the possibility of $\eta_{\mu\nu}$ depending on space-time, we promote the metric to a field $g_{\mu\nu}(x)=\eta_{\mu\nu}+h_{\mu\nu}(x)$. In field-theory language, $h_{\mu\nu}$ is then a field without a proper kinetic energy term. In the view discussed here, the appropriate kinetic energy term arises as a consequence of dynamical scale-symmetry breaking.

We suggest that this view leads to a severe constraint on dynamical symmetry breaking. Scale invariance forbids the appearance of a cosmological constant term $\sim\langle\bar\psi\psi\rangle^{4/3}\int d^4x\sqrt{-g}$ in the action which would in general appear[15] upon symmetry breaking. At the moment, no one knows how to avoid generating this undesirable term. This is perhaps the weakest point[4] in the program to generate gravity spontaneously, and, indeed, this problem afflicts all current theories in particle physics which utilize the notion of spontaneous symmetry breaking. We imagine that the ultimate correct theory of dynamical symmetry breaking will not produce a cosmological term. (In a soluble two-dimensional model[16] of dynamical symmetry breaking a "cosmological" term does appear. However, it is not clear how this may be relevant since gravity does not exist in two dimensions.)

As a crucial test of this idea of gravity as a consequence of dynamical symmetry breaking, we have to perform a calculation to see whether the sign of the induced R term is positive, leading to attractive gravity. It is the purpose of this paper to do such a calculation. As was announced in the abstract, we will be led to conclude that this crucial sign depends on the infrared details of the symmetry-breaking mechanism.

III. PHILOSOPHY BEHIND OUR CALCULATION

In view of the fact that a precise understanding of dynamical symmetry breaking is lacking at present, it appears to this author that it may be better not to perform calculations[17] tied to any specific scheme of dynamical symmetry breaking. Instead, we propose a calculation which retains some general and essential features of dynamical symmetry breaking, most notably that effective masses thus generated are expected to decrease rapidly at large momentum.

Before we embark on a calculation, it may be illuminating to consider a simpler analog problem. It turns out that the problem of "spontaneously" generating a graviton in four dimensions shares some common features with the problem of generating a photon in six dimensions. Consider a six-dimensional world with local U(1) invariance. Thus, the fermion kinetic energy term in the Lagrangian would read $\bar\psi i(\partial\!\!\!/ - iA\!\!\!/)\psi$. We note that ψ has dimension $\frac{5}{2}$ while the gauge field A has dimension +1 (as is the case in any space-time dimension). Thus, if we insist on a scale-invariant Lagrangian, the "photon" kinetic energy term $F_{\mu\nu}F^{\mu\nu}$ with dimension 4 would not be allowed. Dimension-6 terms, such as $\partial_\lambda F_{\mu\nu}\partial^\lambda F^{\mu\nu}$ or F^3, are allowed. (The F^3 term may be eliminated by invoking charge conjugation invariance.) The $(\partial F)^2$ terms, involving four derivatives, are analogous to the R^2 and $R_{\mu\nu}R^{\mu\nu}$ terms in the gravity problem. Also note that, were an elementary scalar field ϕ present, the dimension-6 term $\phi^2 F_{\mu\nu}F^{\mu\nu}$ would be allowed. To summarize, scale invariance combined with gauge invariance forbids, in the absence of elementary scalar fields, a photon kinetic energy term in six dimensions and a graviton kinetic energy term in four dimensions.

With dynamical symmetry breaking, we expect an $F_{\mu\nu}F^{\mu\nu}$ term induced by, among others, a graph such as the one in Fig. 1. The dark "blobs" on the fermion line indicate the fermion condensate $\langle\bar\psi\psi\rangle$. In effect, a soft fermion mass is generated, so that the chiral-noninvariant part of the fermion propagator vanishes rapidly at large momentum. Thus, we argue that we should calculate the vacuum polarization graph with suitably softened fermion propagators. Of course, we cannot arbitrarily soften the propagators; we must maintain gauge invariance. Of the methods available, we prefer the method invented by Pauli and Villars.[18] In this application, the Pauli-Villars cutoffs are to be interpreted as perfectly physical, reflecting the characteristic damping of dynamical symmetry breaking.

The calculation is then simply done, and we find an induced photon kinetic-energy term:

$$-\left(\sum_{i=0}^{2} c_i m_i^2 \ln m_i^2\right)\int d^4x\, F_{\mu\nu}F^{\mu\nu}, \qquad (4)$$

where a is a positive number.[19] Two regulator masses m_1 and m_2 are needed. We define $c_0=1$, $m_0=m=$ the physical fermion mass actually generated, and $\sum c_i=0$, $\sum c_i m_i^2=0$. [The solution reads $c_1=-(m_2^2-m^2)/(m_2^2-m_1^2)$ and $c_2=(m_1^2-m_2)/(m_2^2-m_1^2)$. Note that with the ordering $m^2<m_1^2<m_2^2$, $c_1<-1$.] The expression $\sum c_i m_i^2 \ln m_i^2$ is positive for $m<m_1, m_2$.

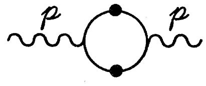

FIG. 1. Photon vacuum polarization.

IV. GENERATING THE GRAVITON

We finally turn to the problem of generating the graviton. Adopting the philosophy outlined above, we are to calculate the graviton vacuum polarization graph[20] [Fig. 2(a)] with Pauli-Villars cutoffs, the cutoff masses being thought of as a rather crude way of incorporating the soft mass generated by dynamical symmetry breaking. Thus, we calculate the graph in Fig. 2(a) with a momentum-independent mass m, but with the understanding that the momentum dependence of m is to be mocked up by the cutoff. We are to extract the order-p^2 term corresponding to the term in R quadratic in h. By dimension counting, we find that these order-p^2 terms are quadratically divergent and thus two Pauli-Villars masses m_1 and m_2 are needed. Thus proceeding, we would find that these order-p^2 terms correspond to an effective action of the form

$$S_{\text{eff}}=K\left(\sum_{i=0}^{2} c_i m_i^2 \ln m_i^2\right)\int d^4x \sqrt{-g}R\,. \qquad (5)$$

Here as before $m_0=m$ and $c_0=1$, $\sum c_i=0$, $\sum c_i m_i^2=0$, and K is an overall constant to be determined by calculating Fig. 2(a); its sign is the object of our interest. In Fig. 2(a), the internal line is taken to be a fermion line. (In general, we should

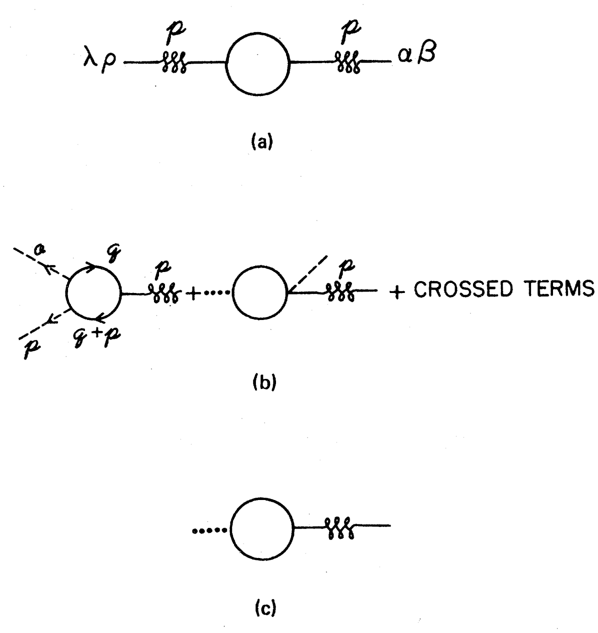

FIG. 2. (a) Graviton vacuum polarization. (b) Graviton going into two scalars. (c) Graviton going into a scalar.

also include the contribution from gauge bosons.)

The correct[21] graviton-fermion[22] coupling reads[23] (Fig. 3)

$$-i\tau_{\mu\nu} = -\frac{i}{2}\left(\gamma_\mu(2q+p)_\nu - \eta_{\mu\nu}(2\not{q}+\not{p}-2m) - \frac{i}{2}p_\lambda\{\gamma^\lambda, \sigma^{\mu\nu}\}\right). \tag{6}$$

Notice the presence of the third term, a "spin" term which can be traced to the local variation of the vierbein frame.[21] Also note that neither the first nor the third terms are symmetric in $\mu\nu$. However, when, and only when, $\tau_{\mu\nu}$ is sandwiched between Dirac spinors and the equation of motion is used, the antisymmetric parts of $\tau_{\mu\nu}$ cancel.[24]

To save ourselves considerable computational labor, we utilize a trick[25] based on exploiting the fact that gravitational gauge invariance (or, if one prefers, general coordinate invariance) relates processes involving n gravitons and processes involving $n+1$ gravitons. Under an infinitesimal coordinate transformation

$$\begin{aligned} x^\mu &\to x^\mu + \epsilon^\mu , \\ h_{\lambda\rho} &\to h_{\lambda\rho} - \partial_\rho\epsilon_\lambda - \partial_\lambda\epsilon_\rho \\ &\quad - h_{\lambda\nu}\partial_\rho\epsilon^\nu - h_{\rho\nu}\partial_\lambda\epsilon^\nu + O(\epsilon^2) . \end{aligned} \tag{7}$$

Thus, the order-h and order-h^2 term in R are necessarily related. Of course, because of momentum conservation we cannot compute a graph with just one graviton. To get around this we use the trick of coupling a scalar field ϕ to the fermions. We add to the Lagrangian the terms $-f\phi\bar{\psi}\psi$. Thus, we make the shift $m_i \to m_i + f$ in Eq. (5) and extract the term quadratic in ϕ^2, which reads

$$K\left(\sum_{i=0}^{2} c_i \ln m_i^2\right)\int d^4x\sqrt{-g}f^2\phi^2R . \tag{8}$$

In other words, to extract K we may do the much simpler calculation of the one-graviton–two-scalar Green's function given in one-loop order by the graphs in Fig. 2(b). We stress that the scalar fields are introduced as a trick in calculating the graph in Fig. 2(a). They are not to be thought of as present in the theory.

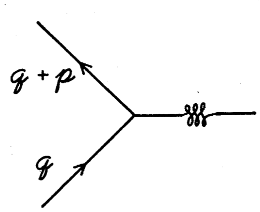

FIG. 3. Graviton-fermion vertex.

Since $\sqrt{-g}R = \partial^2 h - \partial^\mu\partial^\nu h_{\mu\nu} + O(h^2)$ the sum of the graphs in Fig. 2(b) must be proportional to $(p^\mu p^\nu - \eta^{\mu\nu}p^2)$. Actually the calculation can be simplified even further[26] by letting one of the scalar lines in Fig. 2(b) carry momentum p (the incoming momentum of the graviton) and the other carry momentum zero. In that case, we need actually only compute the graph[27] in Fig. 2(c). The graphs in Fig. 2(b) can then be obtained by differentiating with respect to the fermion mass. The calculation of Fig. 2(c) is now quite simple and is presented in Appendix B. These different ways of doing the calculation can, of course, be exploited to provide computational checks.

The result, when expressed in the language of Eq. (5), is that an effective action

$$S_{\text{eff}} = K\left(\sum_{i=0}^{2} c_i m_i^2 \ln m_i^2\right)\int d^4x\sqrt{-g}R$$

is induced with the overall coefficient K given by

$$K = 2\pi^2/3(2\pi)^4 . \tag{9}$$

As remarked before, the expression

$$\sum c_i m_i^2 \ln m_i^2 = m_2^2\left(\frac{m_1^2 - m^2}{m_1^2 - m_2^2}\right)\ln\frac{m_1^2}{m_2^2} - m^2\ln\frac{m_1^2}{m^2} \tag{10}$$

is positive as long as $m^2 < m_1^2, m_2^2$. The cutoff masses m_1 and m_2 mock up the vanishing of the dynamical mass m with increasing momentum and may be expected to be some multiples of m: $m_i = \beta_i m$, $i = 1, 2$ with $\beta_i > 1$. The expression in Eq. (10) then becomes

$$m^2(c_1\beta_1^2\ln\beta_1^2 + c_2\beta_2^2\ln\beta_2^2) , \tag{11}$$

where $c_1 + c_2 = c_1\beta_1^2 + c_2\beta_2^2 = -1$.

Numerically, the expression in Eq. (10) is largest and equal to M^2 when m_1^2 and m_2^2 are large and comparable: $m_1^2 \sim m_2^2 \sim M^2$. The factor K, characteristic of a one-loop calculation, is of order 10^{-2}. Thus, in order to produce Newton's coupling, the scale M of the relevant symmetry breaking has to be of order $\sim\pi M_{\text{Pl}}$. It is amusing that the striking feebleness of gravity may be linked[6] with the large mass scale of symmetry breaking necessary in the ultimate grand unified theory and indirectly with the longevity of the proton.

V. A QUESTION OF SIGN

Before we conclude from the preceding calculation that the correct sign for gravity is obtained, we must examine in closer detail our cutoff procedure. At issue is the question of how sensitively the sign of gravity depends on the precise momentum dependence of the dynamically generated mass. Unfortunately, we are led to conclude by the following discussion that the dependence is quite sensitive.

Let us consider the quadratically divergent integral $I(m^2) \equiv i\int d^4q(q^2 - m^2 + i\epsilon)^{-1}$. This is a typical integral we encounter after extracting the p^2 terms when calculating the graph in Fig. 2(a). What we did in Sec. IV amounts to replacing $I(m^2)$ by

$$I^{\rm PV}(m^2) = \sum_{i=0}^{2} c_i I(m_i{}^2) = \pi \sum_{i=0}^{2} c_i m_i{}^2 \ln m_i{}^2$$

$$= \pi m^2 \sum_{i=1}^{2} c_i \beta_i{}^2 \ln \beta_i{}^2 > 0\,. \qquad (12)$$

By power counting, we see that the coefficient of the order-p^2 terms in the Feynman graph in Fig. 2(a) involves quadratically divergent integrals such as $I(m^2)$. However, the Einstein-Hilbert action is not allowed in the absence of dynamical symmetry breaking. Thus, we may argue that we should have subtracted off from the graph in Fig. 2(a) the corresponding Feynman expression in the absence of scale-symmetry breaking. In other words, an integral such as $I(m^2)$ should have been replaced by

$$I(m^2) - I(0) = i\int d^4q\left(\frac{1}{q^2 - m^2} - \frac{1}{q^2}\right). \qquad (13)$$

Adopting this procedure, we would encounter only logarithmically divergent integrals. [Note that the integral subtracted off, $I(0)$, is in fact identically zero in dimensional regularization.[28]]

In the presence of dynamically symmetry breaking, m^2 would be replaced by a (presumably) rapidly vanishing function of momentum $m^2(q^2)$. Thus, the integral $I(m^2) - I(0)$ would effectively converge and is equal to

$$I(m^2) - I(0) = -\pi^2 \int_0^\infty dq^2 \frac{m^2(q^2)}{q^2 + m^2(q^2)}\,. \qquad (14)$$

A rotation to Euclidean space has been performed. This integral is negative if $m^2(q^2)$ is positive. In particular, if we had regulated the logarithmic integral in Eq. (13) by introducing a cutoff we would have obtained a negative value. In general, the inverse fermion propagator has the form $\not{q}A(q^2) - B(q^2)$.

Following the Pauli-Villars procedure, we would have obtained $I^{\rm PV}(m^2) - I^{\rm PV}(0) = I^{\rm PV}(m^2)$ [cf. Eqs. (11) and (12)]. When one regulates a quadratically divergent integral in the Pauli-Villars method, c_1, the coefficient associated with the first regulator is less than -1. Thus the first regulator "oversubtracts".

This discussion suggests that the sign of the induced gravitational constant is sensitive to the infrared details of dynamical symmetry breaking. Indeed, aside from the quadratically divergent integral $I(m^2)$, we also encounter, when calculating the graph in Fig. 2(a), subdominant pieces of the Feynman integral which may be only logarithmically divergent or even convergent. For instance, consider the convergent integral

$$J(m^2) \equiv i\int d^4q \frac{m^4}{(q^2 - m^2)^3}\,.$$

In the Pauli-Villars calculation of Sec. IV this term is regularized to zero: $\sum_{i=0}{}^2 c_i J(m_i{}^2) = 0$. On the other hand, following the procedure outlined above, we would have obtained $J(m^2) - J(0) = J(m^2)$ = a finite number. In contrast, we obtained $I(m^2) - I(0) = -\ln(M^2/m^2)$. Recalling that M^2 is to be interpreted as the scale of decrease of the dynamically generated mass, we believe that M^2 may not be much larger than m^2 so that terms such as $I(m^2) - I(0)$ may well be overwhelmed by a "subdominant" term like $J(m^2)$.

We are thus led to conclude that, within the present framework, an understanding of the sign of gravity may well have to wait until after one achieves a detailed understanding of dynamical symmetry breaking. This is not an entirely unwelcome conclusion in that gravity may eventually provide a way of discriminating between competing theories of dynamical symmetry breaking.

VI. A REPRESENTATION FOR NEWTON'S CONSTANT

The graph in Fig. 2(a) suggests[29] a representation of the induced gravitational constant in terms of the two-point function of the stress-energy tensor $T_{\mu\nu}$. Starting with the Lagrangian $\mathcal{L} = \int d^4x(-\frac{1}{2}T^{\mu\nu})h_{\mu\nu} + \ldots$ (which defines $T_{\mu\nu}$) we expand $\langle 0|Te^{+i\int\mathcal{L}d^4x}|0\rangle$ to extract the term quadratic in h. We treat $h_{\mu\nu}$ as a c-number classical field and thus the effective order-h^2 Lagrangian is given by

$$i\int d^4x\,\mathcal{L}_{\rm eff}(x)=\frac{1}{2!}\left(\frac{-i}{2}\right)^2\int d^4x\,d^4y\,h_{\mu\nu}(x)h_{\lambda\rho}(y)\times\langle 0|T\,T^{\mu\nu}(x)T^{\lambda\rho}(y)|0\rangle\,. \tag{15}$$

We specialize to the form $h_{\mu\nu}=\frac{1}{4}\eta_{\mu\nu}h$ and choose $h(x)$ to be a rapidly vanishing, sharply peaked function concentrated around some point so that one can expand

$$h(x)=h(y)+z^{\mu}\partial_{\mu}h(y)+\frac{z^{\mu}z^{\nu}}{2!}\,\partial_{\mu}\partial_{\nu}h(y)+\ldots\,, \tag{16}$$

with $x=y+z$. Defining $T(x)=\eta_{\mu\nu}T^{\mu\nu}(x)$ we find

$$i\mathcal{L}_{\rm eff}(x)=-\frac{1}{2^7}h^2(x)\int d^4z\langle 0|T\,T(z)T(0)|0\rangle+\frac{1}{2^{10}}[\partial h(x)]^2\int d^4z\,z^2\langle 0|T\,T(z)T(0)|0\rangle\,. \tag{17}$$

A simple computation (see Appendix A) shows the order-h^2 term in $\sqrt{-g}R$ to be $-\frac{3}{32}(\partial h)^2$. Thus we find the following representation for Newton's coupling constant:

$$\frac{1}{16\pi G}=\frac{i}{96}\int d^4x\,x^2\langle 0|T\,T(x)T(0)|0\rangle\,. \tag{18}$$

This representation was first derived by Adler[29] using a slightly different formalism. If the two-point function is evaluated in a free-field theory we recover the one-loop graph shown in Fig. 2(a).

VII. PHYSICAL CONSEQUENCES

The type of gravity theory considered here and Einstein's theory can differ substantially only with ultrahigh temperatures and/or curvature, in particular near the initial (or final) singularity. Unfortunately, this is also the regime in which quantum gravity effects presumably become important.

At temperatures comparable to the Planck mass, such as may have existed in the very early Universe, we expect $\langle\bar\psi\psi\rangle$ or $\langle\phi^2\rangle$ to change as a temperature-dependent function. In Ref. 11 the case of $\langle\bar\psi\psi\rangle$ increasing was discussed and was shown to have some relevance to the horizon problem. The opposite case, in which $\langle\bar\psi\psi\rangle$ or $\langle\phi^2\rangle\to 0$ as temperature increases, is also of some speculative interest. Gravity, as controlled by the $\int d^4x\sqrt{-g}R$ term in the action, may be "cooked away," so that the early evolution of the Universe may be controlled[30] by the $\int d^4x\sqrt{-g}R^2$ and $\int d^4x\sqrt{-g}R_{\mu\nu}R^{\mu\nu}$ terms.

The gravity theory considered here is, in some sense, softer than Einstein's theory. Whether this softening would modify the physics of the initial singularity, of black holes, etc., is a question which deserves further investigation.

ACKNOWLEDGMENTS

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APPENDIX A

In this appendix we outline our notation and convention, paying special attention to signs which might affect our final conclusion. We use the Minkowski metric $\eta_{\mu\nu}=(+1,-1,-1,-1)$, and also $g\equiv\det g_{\mu\nu}$. With this convention the action (defined by $S=\int d^4x\,\mathcal{L}$) for gravity is

$$S=+(16\pi G)^{-1}\int d^4x\sqrt{-g}R\,. \tag{A1}$$

[Note that since $\Gamma^{\lambda}_{\mu\nu}\equiv\frac{1}{2}g^{\lambda\kappa}(\partial_{\mu}g_{\kappa\nu}+\partial_{\nu}g_{\kappa\mu}-\partial_{\kappa}g_{\mu\nu})$, $R^{\lambda}_{\mu\nu\kappa}\equiv(\partial_{\kappa}\Gamma^{\lambda}_{\mu\nu}+\Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\kappa\eta})-(\nu\leftrightarrow\kappa)$, and $R\equiv R^{\lambda}_{\mu\lambda\kappa}g^{\mu\kappa}$, $\Gamma^{\lambda}_{\mu\nu}$ and $R^{\lambda}_{\mu\nu\kappa}$ do not change while R changes sign under $g_{\mu\nu}\to -g_{\mu\nu}$.]

In our convention, the stress-energy tensor $T_{\mu\nu}$ is defined by varying the matter action:

$$\delta S_{\rm matter}=\frac{1}{2}\int d^4x\sqrt{-g}(-T^{\mu\nu})\delta g_{\mu\nu}\,. \tag{A2}$$

The minus sign is necessary so that T_{00} would correspond to a positive energy. [Recall in the variation in Eq. (A2) x^{μ} and therefore ∂_{μ}, A_{μ} are to be fixed.]

As a check on the sign in Eq. (A1) we can deduce from Eq. (A1) and Eq. (A2) the field equation

$$R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R=-8\pi GT^{\mu\nu}\,, \tag{A3}$$

which we can solve in the weak-field limit for a small mass source. We then verify that the motion of a point test particle is described correctly. [It is most efficient to obtain the equation of motion for a test particle by a variational (least proper time) principle, which also yields the above-mentioned relation between $\Gamma^\lambda_{\mu\nu}$ and $g_{\mu\nu}$.]

In terms of the graviton field $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$, we have $\Gamma \sim \partial h(1+h+\ldots)$ and $R \sim \partial^2 h + h\partial^2 h + h^2\partial^2 h + \ldots$. An easy computation gives

$$R = \eta^{\mu\nu}\partial^2 h_{\mu\nu} - \partial^\mu\partial^\nu h_{\mu\nu} + O(h^2)\,. \tag{A4}$$

To identify the coefficient of $\int d^4x\sqrt{-g}R$ by computing the graph in Fig. 2(a) we need to expand $\sqrt{-g}R$ up to order h^2. This may be done by brute force.[30] However, it is much easier to note that $\int d^4x\,\delta(\sqrt{-g}R) = \int d^4x\sqrt{-g}(\frac{1}{2}g^{\mu\nu}R - R^{\mu\nu})\delta g_{\mu\nu}$ so that we only need to expand $R^{\mu\nu}$ and R to first order in h. We find, in order h^2,

$$\int d^4x(\sqrt{-g}R)^{(2)} = \tfrac{1}{2}\int d^4x(-\tfrac{1}{2}\partial^\nu h\partial_\nu h + \partial_\mu h\partial^\nu h_{\mu\nu} + \tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} - \partial_\alpha h_{\alpha\beta}\partial_\lambda h^{\lambda\beta})\,. \tag{A5}$$

This expression is gauge invariant.

APPENDIX B

We outline here the computation of the graphs in Fig. 2(b). The first graph is given by

$$(-if)^2(-i)i^3(-)\sum c_i\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\frac{1}{\not q - m_i}\frac{1}{\not q - m_i}\times\frac{1}{(\not q+\not p - m_i)}\tau_{\mu\nu}(q,p)\,. \tag{B1}$$

The factors in front of the integral came from the vertices, the propagators, and the Pauli sign for a closed loop. A typical integral we encounter is then

$$I_{\mu\nu} = \sum c_i\int d^4q\,\mathrm{Tr}\frac{1}{\not q - m_i}\frac{1}{\not q - m_i}\frac{1}{\not q+\not p - m_i}\gamma_\mu q_\nu\,.$$

Expanding the integrand to order p^2 we find that the relevant part of $I_{\mu\nu}$ is given by

$$\sum_i c_i\int\frac{d^4q}{(q^2-m_i^2)^5}\mathrm{Tr}\not q\not q\not q\not p\not q\not p\not q\gamma_\mu q_\nu\,. \tag{B2}$$

A tremendous simplification results from the realization that in the numerator trace we can set the masses equal to zero. For instance, a term in the numerator proportional to m_i^2 is multiplied by a convergent integral proportional to m_i^2 and thus vanishes under Pauli-Villars regularization since $\sum_i c_i = 0$. The evaluation of Eq. (B2) then proceeds along standard lines and yields

$$-\tfrac{1}{3}(p_\mu p_\nu + p^2\eta_{\mu\nu})(-i\pi^2)\sum_i c_i\ln m_i^2\,. \tag{B3}$$

We proceed in this vein. (Incidentally, the terms in $\tau_{\mu\nu}$ linear in p contribute nothing.) The four graphs in Fig. 2(b), after some work, are found to sum to

$$(-i)\frac{4\pi^2}{3(2\pi)^4}f^2(p_{\mu\nu} - \eta_{\mu\nu}p^2)\sum_i c_i\ln m_i^2\,. \tag{B4}$$

The fact that the sum total is proportional to the gauge-invariant form $p_\mu p_\nu p^2$ is of course an important check on the computation of the individual pieces.

As explained in the text, it is far less tedious to compute the graph in Fig. 2(c). Here, in the step arriving at the analog of Eq. (B2), one must keep all terms proportional to m in the numerator in the Feynman integrand.

As a check we have also considered the graph in Fig. 2(a) directly. Because of gauge invariance, the graph must be proportional to [cf. Eq. (A5)]

$$\tfrac{1}{2}i[p^2(-\eta_{\lambda\rho}\eta_{\alpha\beta} + \tfrac{1}{2}\eta_{\lambda\alpha}\eta_{\rho\beta} + \tfrac{1}{2}\eta_{\lambda\beta}\eta_{\rho\alpha}) + (\eta_{\lambda\rho}p_\alpha p_\beta + \eta_{\alpha\beta}p_\lambda p_\rho) - \tfrac{1}{2}(\eta_{\rho\beta}p_\lambda p_\alpha + \eta_{\rho\alpha}p_\lambda p_\beta + \eta_{\lambda\beta}p_\rho p_\alpha + \eta_{\lambda\alpha}p_\rho p_\beta)]\,. \tag{B5}$$

We can save a vast amount of labor by contracting the graph against $\eta^{\alpha\beta}$ and $\eta^{\lambda\rho}$. The graph is then porportional to

$$I(m) = \int d^4q\,\mathrm{Tr}\frac{1}{\not q - m}(8m - 6\not q - 3\not p)\times\frac{1}{\not q+\not p - m}(8m - 6\not q - 3\not p)\,. \tag{B6}$$

A simple calculation gives the order-p^2 term in I to be

$$\sum c_i I(m_i) = 8i\pi\left(\sum c_i m_i^2\ln m_i^2\right)p^2\,. \tag{B7}$$

To verify that we indeed get the tensor structure in Eq. (B5) we have also computed the graph in Fig. 2(a), but, in order to save labor, with the internal fermion lines replaced by a scalar-boson line. The graph is then given by

$$J_{\lambda\rho,\alpha\beta} = i^2(-\tfrac{1}{2}i)^2\int\frac{d^4q}{(2\pi)^4}\frac{\tau_{\lambda\rho}(q,q+p)\tau_{\alpha\beta}(q,q+p)}{(q^2-m^2)[(q+p)^2-m^2]}\,, \tag{B8}$$

with

$$\tau_{\mu\nu}(k,k') = k_\mu k'_\nu + k'_\mu k_\nu - \eta_{\mu\nu}(k\cdot k' - m^2)\,. \tag{B9}$$

For simplicity, we have also taken the old unimproved stress-energy tensor for the scalar field.

(Again, the graph has no p dependence and may be neglected.) For the sake of completeness, we exhibit the result here (Pauli-Villars regularization is understood):

$$J_{\lambda\rho,\alpha\beta}=\frac{-i^2}{4(2\pi)^4}\int_0^1 d\gamma\,\ln\mu^2[\tfrac{1}{2}\mu^4(I_1+I_2)+\tfrac{1}{2}\mu^2(1-2\gamma)^2(I_3-2I_4+p^2I_1)+m^2(m^2-2\mu^2)I_1+\gamma^2(1-\gamma)^2(4I_5-4p^2I_4+3p^4I_1)]\,. \quad \text{(B10)}$$

Here μ^2 denotes $m^2-\gamma(1-\gamma)p^2-i\epsilon$ and the I's denote schematically the following tensor invariants:

$$I_1=\eta_{\alpha\beta}\eta_{\mu\nu}\,,$$
$$I_2=\eta_{\alpha\mu}\eta_{\beta\nu}+(\mu\leftrightarrow\nu)\,,$$
$$I_3=\eta_{\alpha\mu}p_\beta p_\nu+\eta_{\beta\mu}p_\alpha p_\nu+(\mu\leftrightarrow\nu)\,,$$
$$I_4=\eta_{\alpha\beta}p_\mu p_\nu+\eta_{\mu\nu}p_\alpha p_\beta\,,$$
$$I_5=p_\alpha p_\beta p_\mu p_\nu\,.$$

Extracting the order-p^2 term, we find the tensor structure in Eq. (B5). Note the presence of the order-p^4 term in Eq. (B10), leading to renormalization of the coefficients of the R^2 and $R_{\mu\nu}R^{\mu\nu}$ terms in the action.

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*Present Address: University of Washington, Seattle, Washington 98195.

[1]W. Heisenberg, Z. Phys. 110, 251 (1938). We thank H. Primakoff for informing us of this historical fact.

[2]This is compared to the hadronic mass scale, which is presumably largely due to chiral symmetry breaking. If we believe that symmetry breaking at 300 GeV is due to heavy color [L. Susskind, Phys. Rev. D 20, 2619 (1979); S. Weinberg, *ibid.* 13, 974 (1976); 19, 1277 (1979)] in analogy to the breaking of chiral symmetry by color, then we are led, rather amusingly, to explain the weakness of the weak interaction in terms of the weakness of the strong interaction compared to the heavy-color–strong interactions.

[3]See Appendix A. $\sqrt{-g}\,R$ can be expanded in an infinite series as $\partial^2h+\partial h\,\partial h+h\,\partial h\,\partial h+\cdots$ (in a self-evident schematic notation). The term linear in h is a total divergence and can be dropped in S_E (but not in the action described in this paper). The Einstein-Hilbert action is reminiscent of the Yang-Mills action $S_{YM}=\int d^4x\,(-\frac{1}{4}F^2)(g^2)^{-1}$, where F^2 has the schematic (terminating) expansion $\partial A\partial A+AA\partial A+AAAA$. The couplings G_N and g^2 measure the "stiffness" of the graviton and the gluon field against excitation. Confinement is essentially equivalent to having a large effective g^2 outside and a small effective g^2 inside hadrons. This may be described phenomenologically by an effective action $S=\int d^4x\,\phi^2\,(-\frac{1}{4}F^2)$, where ϕ^2 varies from a large value inside hadrons to a small value outside hadrons. G. 't Hooft, Report No. CERN-1902 TH, 1974 (unpublished); J. Kogut and L. Susskind, Report No. CLNS-263, 1974 (unpublished); T. D. Lee, Columbia report (unpublished). This effective phenomenological action is similar in spirit to the action described here.

[4]A. Zee, Phys. Rev. Lett. 42, 417 (1979); 44, 703 (1980).

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[6]One is tempted to suggest that ϕ is also responsible for the breaking of a grand unified theory into strong, weak, and electromagnetic interactions (see Ref. 4). Unfortunately, it is now believed that the relevant symmetry scale of grand unification is lower than what was originally suggested [H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974)].

[7]For a list of references, see Ref. 4.

[8]Earlier work which does discuss spontaneous symmetry breaking includes Y. Fujii, Phys. Rev. D 9, 874 (1974); P. Minkowski, Phys. Lett. 71B, 419 (1977); T. Matsuki, Prog. Theor. Phys. 59, 235 (1978); A. D. Linde, Pis'ma Zh. Eksp. Teor. Fiz. 30, 479 (1979) [JETP Lett. 30, 447 (1979)]. (We thank Professor Y. Fujii and Professor A. Linde for bringing their work to our attention.)

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[12]$R_{\mu\gamma\lambda\rho}R^{\mu\gamma\lambda\rho}$ may be expressed in terms of R^2 and $R_{\mu\gamma}R^{\mu\gamma}$. B. S. DeWitt, Phys. Rev. 162, 1195 (1967); 162, 1239 (1967); G. 't Hooft and M. J. G. Veltman, Ann. Inst. Henri Poincaré 20, 69 (1974).

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[14]This represents, in some sense, the modern realization of ideas of A. D. Sakharov, Dokl. Akad. Nauk. SSSR 177, 70 (1967) [Sov. Phys.—Dokl. 12, 1040 (1968)] and of O. Klein, Phys. Scr. 9, 69 (1974).

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[16]D. Gross and A. Neveu, Phys. Rev. D 10, 3235 (1974).

[17]Very interesting calculations in the context of specific dynamical schemes are being performed by B. Hasslacher and E. Mottola (private communication from S. Adler).

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[19]a is defined by $a\equiv 3m^2\int d_E{}^6q\,(q_E{}^2+m^2)^{-4}$.

[20]The graph involving the contact coupling between two gravitons and two fermions does not contribute in order p^2.

[21]F. J. Belinfante, Physica (Utrecht) 7, 449 (1940); S. Deser and P. van Nieuwenhuizen, Phys. Rev. D 10, 411 (1974); D. Boulware, *ibid.* 12, 350 (1975).

[22]We do the calculation here for a Dirac fermion. The

calculation for a Majorana fermion should be similar.

[23]The overall minus sign in Eq. (6) is explained in Appendix A.

[24]In particular, the prescription of symmetrizing the first term "by hand" is wrong.

[25]This trick was developed in conversation with D. Boulware to whom we are grateful.

[26]We exploited the existence of three different ways of doing this calculation to check our computation.

[27]The contact graph has trivial momentum dependence.

[28]G. 't Hooft and M. Veltman, in *Particle Interactions at Very High Energies*, edited by D. Speiser, F. Halzen, and J. Weyers (Plenum, New York, 1974), Part B, p. 177 and references therein.

[29]S. Adler, IAS report, 1980 (unpublished).

[30]The consequences of this remark are under investigation.

[31]The much more tedious task of expanding about a general curved metric has been carried out. See G. 't Hooft and M. Veltman, Ref. 12.