Gravitational field of vacuum domain walls and strings

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The gravitational properties of vacuum domain walls and strings are studied in the linear approximation of general relativity. These properties are shown to be very different from those of regular massive planes and rods. It is argued that the domain walls are gravitationally unstable and collapse at a certain time $\sim t_c$ after their creation. If the vacuum walls ever existed, they must have disappeared at $t < t_c$.

I. INTRODUCTION

In gauge theories with spontaneous symmetry breaking, the symmetry can be restored[1] at a sufficiently high temperature, $T > T_c$. The phase transition at $T = T_c$ can have important cosmological consequences; in particular, it can give rise to a vacuum domain structure.[1-3] As the universe cools below T_c, the Higgs field ϕ acquires a nonzero expectation value $\langle\phi\rangle$. The direction of $\langle\phi\rangle$ in the manifold of degenerate vacuum states M can be different in different regions of space, and we can certainly expect it to be different in causally disconnected regions. The topology of the resulting vacuum structure is related to the topology of the manifold M, as discussed by Kibble.[3] The three possible structures are vacuum domain walls, strings, and monopoles. The original symmetry can be broken in several steps; then we have a series of phase transitions each of which can produce its own cosmic structure.

Cosmological monopole production has been discussed by a number of authors[4] who have found that the estimated number of monopoles is too large to be compatible with the standard big-bang model. Possible ways out of the difficulty have also been discussed.[4] Zeldovich *et al.*[2] considered the cosmological effects of the domain walls. They found that the gravitational field of the walls is unacceptably large, since it would cause a large asymmetry in the background radiation. Therefore, if the walls have ever existed, they must have disappeared before the end of the radiation era, so that Compton scattering has enough time to even out the anisotropy of the cosmic radiation. Kibble[3] has reached similar conclusions. He has also discussed the evolution of cosmic strings and has concluded that the existence of a large-scale network of strings does not contradict observations.

In Refs. 2 and 3 the gravitational field of the cosmic vacuum structures was estimated in the Newtonian approximation. The purpose of the present paper is to study the gravitational properties of vacuum domain walls and strings in the framework of general relativity. We shall see that these properties are very different from those of regular massive planes and rods. It will also be shown that the domain walls are gravitationally unstable and collapse at a certain time $\sim t_c$ after their creation. If domain walls existed and then disappeared, they must have done so at $t < t_c$. Possible mechanisms of the disappearance of the walls will be discussed. The cosmological consequences of vacuum strings will be considered in a separate paper.

II. THE ENERGY-MOMENTUM TENSOR

Since we are interested in macroscopic effects of walls and strings, it is reasonable to approximate them by infinitely thin surfaces and curves, respectively. (The transverse dimensions of walls and strings are comparable to the Higgs Compton wavelength.) Let us consider a static wall parallel to the (y,z) plane in a flat space-time (gravitation is neglected). The wall is described by a classical solution of the field equations with the energy-momentum tensor

$$T_\mu^\nu(x) = \sum_i \frac{\partial L}{\partial \phi^{(i)}_{,\nu}} \phi^{(i)}_{,\mu} - \delta_\mu^\nu L \,, \tag{1}$$

where $L\{\phi^{(i)}, \phi^{(i)}_{,\mu}\}$ is the Lagrangian of the theory and the summation is taken over all fields $\phi^{(i)}$. In the thin-wall approximation we replace Eq. (1) by

$$\tilde{T}_\mu^\nu(x) = \delta(x-a) \int T_\mu^\nu(x)dx \,, \tag{2}$$

where $x = a$ is the position of the wall. Since all $\phi^{(i)}$ are functions only of x, it is clear from Eqs. (1) and (2) that $\tilde{T}_\mu^\nu$ has only diagonal components and that[5]

$$\tilde{T}_0^0 = \tilde{T}_2^2 = \tilde{T}_3^3 \,. \tag{3}$$

From the conservation law

$$T^{\nu}_{\mu,\nu}=0\ , \tag{4}$$

it follows that

$$\frac{d}{dx}T^1_1=0,\quad T^1_1=\text{const},$$

and, since $T^{\nu}_{\mu}=0$ at $x=\pm\infty$, we conclude that $T^1_1=0$ and $\tilde{T}^1_1=0$. Thus we can write the energy-momentum tensor of the wall as

$$\tilde{T}^{\nu}_{\mu}(x)=\sigma\delta(x-a)\times\text{diag}\,(1,0,1,1). \tag{5}$$

Here σ is the surface energy density. In the general case, the energy-momentum tensor of a homogeneous massive plane has the form

$$\tilde{T}^{\nu}_{\mu}(x)=\delta(x-a)\times\text{diag}(\sigma,0,-p,\ -p)\ , \tag{6}$$

where p is the pressure ($-p$ is the surface tension). We see that for a vacuum domain wall

$$p=-\sigma\ . \tag{7}$$

Zel'dovich *et al.*[2] obtained Eq. (5) by direct calculation in a simple model of a single scalar field with quartic self-interaction. They noted also that $\tilde{T}^{\nu}_{\mu}$ is invariant under Lorentz transformations with velocity parallel to the wall. This means that tangential motion of the wall is unobservable.

The energy-momentum tensor of a string can be constructed in a similar way. Consider a static string parallel to the z axis. Defining

$$\tilde{T}^{\nu}_{\mu}(x,y)=\delta(x-a)\delta(y-b)\int T^{\nu}_{\mu}(x,y)dx\,dy \tag{8}$$

and assuming that all the fields $\phi^{(i)}$ are functions only of x and y, we find that

$$\tilde{T}^0_0=\tilde{T}^3_3 \tag{9}$$

and that all other components are equal to zero except, perhaps, $\tilde{T}^k_i$ with $i,\ k=1,2$. To show that these remaining components are also equal to zero, we can use the conservation law (4) and write

$$\int T^j_{i,j}\,x^k dx\,dy=0,$$

where all indices take values 1 or 2. Integration by parts gives

$$\tilde{T}^k_i\sim\int T^k_i\,dx\,dy=0\quad(i,k=1,2). \tag{10}$$

Thus, the energy-momentum tensor of the string is given by

$$\tilde{T}^{\nu}_{\mu}(x,y)=\mu\,\delta(x-a)\delta(y-b)\times\text{diag}(1,\ 0,\ 0,\ 1)\ , \tag{11}$$

where μ is the linear energy density.

The magnitudes of σ and μ depend on masses and coupling constants of the theory. If m and α are typical boson mass and coupling constant, respectively, then[3]

$$\sigma\sim\alpha^{-1}m^3\ ,\qquad \mu\sim\alpha^{-1}m^2\ , \tag{12}$$

where it is assumed that all relevant masses and coupling constants have the same order of magnitude (m and α can be different on different levels of symmetry breaking). For numerical estimations below we shall take $\alpha\sim10^{-2}$. Then on the electroweak scale ($m\sim10^2$ GeV) Eq. (12) gives $\sigma\sim10^{12}$ g/cm^2 and $\mu\sim10^{-4}$ g/cm. For grand unification walls and strings ($m\sim10^{15}$ GeV) we get $\sigma\sim10^{51}$ g/cm^2 and $\mu\sim10^{22}$ g/cm.

III. WEAK-FIELD APPROXIMATION

Let us now find the gravitational field of walls and strings in the linear approximation of general relativity. Representing the metric tensor as

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}, \tag{13}$$

where $\eta_{\mu\nu}=\text{diag}\,(1,-1,-1,-1)$ is the Galilean metric and $|h_{\mu\nu}|\ll1$, we can write the Einstein equations as[6]

$$(\nabla^2-\partial_t{}^2)h_{\mu\nu}=16\pi G(T_{\mu\nu}-\tfrac{1}{2}\eta_{\mu\nu}T)\ , \tag{14}$$

with the harmonic coordinate conditions

$$\partial_{\nu}(h^{\nu}_{\mu}-\tfrac{1}{2}\delta^{\nu}_{\mu}h)=0\ . \tag{15}$$

The remaining coordinate freedom is restricted to the transformations

$$h'_{\mu\nu}=h_{\mu\nu}-\xi_{\mu,\nu}-\xi_{\nu,\mu}\ , \tag{16}$$

with

$$(\nabla^2-\partial_t{}^2)\xi_{\mu}=0\ . \tag{17}$$

The solution of Eqs. (14) and (15) for a static massive plane with T^{ν}_{μ} given by Eq. (6) is easily found:

$$\begin{aligned}h_{00}&=4\pi G(\sigma+2p)|x|\ ,\\ h_{11}&=4\pi G(\sigma-2p)|x|\ ,\\ h_{22}&=h_{33}=4\pi G\sigma|x|\ ,\end{aligned} \tag{18}$$

and all other $h_{\mu\nu}$ are equal to zero. (Here I have set $a=0$, so that the plane is situated at $x=0$.) For a vacuum domain wall, $p=-\sigma$ and

$$\begin{aligned}h_{00}&=-h_{22}=-h_{33}=-4\pi G\sigma|x|\ ,\\ h_{11}&=12\pi G\sigma|x|\ .\end{aligned} \tag{19}$$

Similarly, we can find the field of a massive string situated at the z axis:

$$h_{00}=h_{33}=4G(\mu+p)\ln(r/r_0)\ ,$$
$$h_{11}=h_{22}=4G(\mu-p)\ln(r/r_0)\ . \tag{20}$$

Here p is the pressure in the z direction, $r=(x^2+y^2)^{1/2}$, and r_0 is a constant which we can set to be equal (approximately) to the radius of the string. For a vacuum string, $p=-\mu$ and

$$h_{00}=h_{33}=0,$$
$$h_{11}=h_{22}=8G\mu\ln(r/r_0)\ . \tag{21}$$

From Eqs. (19) and (21) we see that the weak-field approximation breaks down at large distances from walls and strings where $|h_{\mu\nu}|$ become comparable to or greater than one. We shall return to this question in the next section. At the moment let us discuss the properties of the gravitational fields (19) and (21) in the region where our approximation is valid.

The equation of motion of a nonrelativistic test particle in a weak gravitational field is[6]

$$\ddot{x}^i=-\frac{1}{2}\frac{\partial h_{00}}{\partial x^i}\ . \tag{22}$$

From Eqs. (19) and (22) we see that domain walls repel particles with acceleration $g=2\pi G\sigma$. Walls of pressureless dust (for brevity we shall call them "regular" walls) attract particles with the same acceleration. Vacuum domain walls do not deflect light (while regular walls do). This can be easily seen if we note that the coordinate transformation (16) with $\xi_1=2\pi G\sigma x^2\,\mathrm{sgn}x$ and $\xi_2=\xi_3=\xi_0=0$ brings the metric (19) to a conformally flat form

$$ds^2=(1-4\pi G\sigma|x|)(dt^2-dx^2-dy^2-dz^2)\ . \tag{23}$$

Let us now derive the equation of motion of a massive plane in a weak external gravitational field. For simplicity we shall consider a nonrelativistic motion. The energy-momentum tensor of a plane moving with velocity v in the x direction is obtained from Eq. (6) by a coordinate transformation:

$$T^{00}=\sigma\delta(x-vt),$$
$$T^{22}=T^{33}=p\delta(x-vt), \tag{24}$$
$$T^{10}=T^{01}=\sigma v\,\delta(x-vt)\ ,$$

and all other $T^{\mu\nu}=0$. The energy-momentum conservation law can be written as[6]

$$\partial_\nu(T^\nu_\mu\sqrt{-g})=\tfrac{1}{2}\sqrt{-g}\ \frac{\partial g_{\sigma\tau}}{\partial x^\mu}T^{\sigma\tau}\ . \tag{25}$$

Assuming that $h_{\mu\nu}$ are functions only of x, setting $\mu=1$, and integrating over x, we find in the weak-field approximation

$$\frac{dv}{dt}=-\frac{1}{2}\frac{d}{dx}\left[h_{00}+\frac{p}{\sigma}(h_{22}+h_{33})\right]\ . \tag{26}$$

Note that a regular wall ($p=0$) moves just like a test particle [compare with Eq. (22)]. The interaction of two walls can now be easily described using Eqs. (18) and (26). We find that two parallel domain walls are repelled from each other with acceleration $6\pi G\sigma$, a domain wall and a regular wall are also repelled with acceleration $2\pi G\sigma$, and, of course, two regular walls are attracted with acceleration $2\pi G\sigma$. (In all cases I assume that both walls have the same density σ.)

Turning now to the case of strings, we find from Eqs. (21) and (22) that in our approximation the gravitational field of the strings does not couple to nonrelativistic matter. It can also be shown that strings at rest (or slowly moving) are not affected by the gravitational field of nonrelativistic bodies.[7]

To get a better insight in the geometry of the metric (21), let us rewrite it in cylindrical coordinates:

$$ds^2=dt^2-dz^2-(1-\lambda)(dr^2+r^2d\phi^2)\ , \tag{27}$$

where

$$\lambda=8G\mu\ln(r/r_0)\ . \tag{28}$$

Introducing a new radial coordinate r' as

$$(1-\lambda)r^2=(1-8G\mu)r'^2\ , \tag{29}$$

$$(1-\lambda)dr^2=dr'^2\ , \tag{30}$$

we get

$$ds^2=dt^2-dz^2-dr'^2-(1-8G\mu)r'^2d\phi^2\ . \tag{31}$$

With a new angular coordinate

$$\phi'=(1-4G\mu)\phi\ , \tag{32}$$

the metric takes a Galilean form

$$ds^2=dt^2-dz^2-dr'^2-r'^2d\phi'^2\ . \tag{33}$$

This metric, however, does not describe a Euclidean space, since ϕ' changes from 0 to $(1-4G\mu)2\pi$. Such space can be called conical. The trajectories of light and particles in coordinates (33) are straight lines. As the radial coordinate changes from a very large distance $R\gg\rho$ to the point closest to the string ($r'=\rho$) and again to R, the corresponding change in ϕ' is $\Delta\phi'=\pi$ and

$$\Delta\phi=\pi(1+4G\mu)\ . \tag{34}$$

Thus the light deflection is $\delta\phi=4\pi G\mu$ and is independent of the impact parameter ρ. This effect can give rise to double images of cosmic objects situated behind the string within the angle of order

$\delta\phi$ from the string. The angular separation of the images is $\lesssim \delta\phi$. This separation is negligible for electroweak strings ($\delta\phi \sim 10^{-31}$ rad), but is quite observable for grand unification strings ($\delta\phi \sim 10^{-5}$ rad). The effect of double images may be relevant to the double quasar.[8]

It should be noted that the results of this section are not directly applicable to curved walls and strings, in particular to closed bubbles and loops. Because of the surface tension, portions of curved walls and strings can develop relativistic speeds, retardation will be important, and the static solutions will not be relevant. For example, a closed bubble of radius R develops a relativistic speed in time $t \sim R$. The gravitational field outside the bubble is that of Schwarzschild and is attractive, since the total energy of the wall is positive.

IV. COLLAPSE OF THE DOMAIN WALLS

As was noted in the previous section, the weak-field approximation breaks down at large distances from vaccum domain walls and strings. In this section we shall examine the exact vacuum solutions of the Einstein equations with planar and cylindrical symmetry.

The static solution of the Einstein equations with $T_\mu^\nu = 0$ everywhere except in the (y,z) plane and having planar and reflectional symmetry is uniquely given by[9] (up to a coordinate transformation)

$$ds^2 = (1 + K|x|)^{-1/2}(dt^2 - dx^2) - (1 + K|x|)(dy^2 + dz^2) , \tag{35}$$

where $K = \text{const}$. We note that Eq. (35) is related, by a coordinate transformation, to the Kasner solution[6]

$$ds^2 = x^{2p_1} dt^2 - dx^2 - x^{2p_2} dy^2 - x^{2p_3} dz^2, \quad p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1 , \tag{36}$$

with $p_1 = -\frac{1}{3}$ and $p_2 = p_3 = \frac{2}{3}$. The Kasner metric (36) is the general solution of vacuum equations depending on one (spatial) coordinate.

In the weak-field region ($K|x| \ll 1$), Eq. (35) gives

$$h_{00} = -h_{11} = -\tfrac{1}{2}K|x| , \quad h_{22} = h_{33} = -K|x| . \tag{37}$$

These $h_{\mu\nu}$ cannot be directly compared with Eq. (19), since they do not satisfy the harmonic coordinate condition (15). A coordinate transformation (16) with $\xi_1 = \frac{1}{2}Kx^2 \operatorname{sgn} x$ and $\xi_0 = \xi_2 = \xi_3 = 0$ brings the metric (37) to a harmonic form

$$h_{00} = -\tfrac{1}{2}K|x| , \quad h_{11} = -\tfrac{3}{2}K|x| , \quad h_{22} = h_{33} = -K|x| . \tag{38}$$

Comparing Eqs. (18) and (38) we find

$$K = -4\pi G\sigma, \quad p = -\tfrac{1}{4}\sigma .$$

We see that the exact vacuum solution (35) does not match with the weak-field approximation (19) corresponding to $p = -\sigma$. We also note that the metric (35) is nonsingular only if $K > 0$, in which case it corresponds to a negative energy of the wall and thus is unphysical.

The result just obtained means that the weak-field approximation (19) does not correspond to any exact static solution of the Einstein equations. Physically, this implies that the walls are gravitationally unstable and collapse to form singularities. This result can be easily understood in the following way. Consider a sphere of radius R centered on the wall. The mass of the wall inside the sphere is $M(R) = \pi R^2\sigma$. For $R > (2\pi G\sigma)^{-1}$, the gravitational radius $2GM(R)$ is greater than R and we expect the wall to collapse.

To discuss the cosmological evolution of the walls, let us first assume for simplicity that a single plane wall is created during the phase transition at $t \sim t_0$. At $t > t_0$, the gravitational field of the wall propagates with the velocity of light and settles down to the weak-field metric (19) at distances smaller than t from the wall. The weak-field approximation breaks down at $t \sim t_c$, where

$$t_c \sim (G\sigma)^{-1} \tag{39}$$

[see Eq. (19)]. At this point the nonlinearity of the Einstein equations becomes important. We shall assume that the collapse of the wall starts at about the same time. The critical time t_c is of order 10^6 sec for electroweak walls and $t_c \sim 10^{-33}$ sec for grand unification walls. If the wall is to disappear, it has to do so at $t < t_c$, since we cannot get rid of the wall after it has formed a singularity.

In a more realistic case of many domain walls of irregular shapes, two new effects come into play: (i) the interference of the gravitational fields of different walls and (ii) curved walls move with relativistic speeds under the action of the surface tension. However, the intuitive argument given above still suggests that at $t \sim t_c$ the walls "know" that their size is greater than $(G\sigma)^{-1}$ and start to collapse. According to Kibble,[3] the cosmic domain structure evolves in such a way that the typical separation of the walls, as well as the typical curvature radius of each wall, is always of order t. Then the average mass density due to the walls is $\rho_d \sim \sigma t^{-1}$. The total density of the universe is $\rho \sim (Gt^2)^{-1}$ and thus

$$\rho_d / \rho \sim G\sigma t \sim t / t_c . \tag{40}$$

If we require that the walls disappear at $t < t_c$, then the universe never becomes domain-wall dominated.

Let us now turn to vacuum strings. If we again consider a sphere of radius R centered on a straight string, then $GM(R)/R \sim G\mu$ for all R. From Eq. (12), $G\mu \sim \alpha^{-1} Gm^2 \ll 1$. This suggests that self-gravity of the strings is unimportant and cannot cause their collapse. Besides, there exists an exact static solution which matches with the weak-field approximation (33). It is just the locally flat "conical" space (33) with $0 < \phi' < (1-4G\mu)2\pi$. The general static vacuum solution with cylindrical symmetry is[10]

$$ds^2 = (r/r_0)^{4(1-K)/(K^2+3)} dt^2 - dr^2 - (r/r_0)^{-2(K-1)^2/(K^2+3)} r^2 d\phi^2 - (r/r_0)^{2(K^2-1)/(K^2+3)} dz^2 , \quad (41)$$

where $K = \text{const}$ and $r_0 = \text{const}$. Note that this metric can also be brought to the Kasner form (36). The Riemann tensor of (41) is equal to zero if and only if $K = 1$. The metric (41) can be matched with the weak-field approximation (33) only if $K - 1 = O(G^2\mu^2)$. Inside the string ($r \lesssim r_0$), $T_\mu^\nu \neq 0$ and the metric is different from (33) or (41). If we assume that gravitation does not break the Lorentz invariance of the string in the z direction, then the only allowed values of K in Eq. (41) are $K = 1$ and $K = -3$. For $K = -3$, the length of a circle, $r = \text{const}$, decreases at $r^{-1/3}$ when $r \to \infty$. We dismiss this case as unphysical. $K = 1$ corresponds to a conical space.

V. DISAPPEARANCE OF THE DOMAIN WALLS

According to the discussion in the previous section, the domain walls, if they ever existed, must have disappeared at $t < t_c \sim (G\sigma)^{-1}$. Zeldovich *et al.*[2] and Kibble[3] noted that we can get rid of domain walls by allowing a small initial bias, so that one of the vacuum states separated by the walls has slightly smaller energy density than the other: $\rho_1 - \rho_2 = \epsilon \neq 0$. This energy difference becomes dynamically important at $t \sim t_1$ when the energy excess on scale $\sim t_1$ becomes comparable to the energy of the walls on the same scale (we assume[3] that the characteristic scale of the domain structure at time t is $\sim t$): $\epsilon t_1^3 \sim \sigma t_1^2$; $t_1 \sim \sigma/\epsilon$. Requiring that $t_1 < t_c$, we get

$$\epsilon > G\sigma^2 \sim \alpha^{-2}(Gm^2)m^4 . \quad (42)$$

An asymmetry put in by hand is aesthetically unappealing and is not in the spirit of spontaneous symmetry breaking. One other possibility is to consider a small universe with nontrivial topology.[11] A simple example is a cube $|x^i| \leq a$ in which the points $x^i = a$ and $x^i = -a$ are identified. Since the expansion law is determined by the local energy density, the local properties of such a universe are identical to those of an infinite Friedmann model. Domain walls disappear when the horizon size t becomes comparable to the size of the universe. If this is to happen at $t < t_c$, then the size of the universe at $t \sim t_c$ is smaller than t_c and the present size of the universe is smaller than

$$R < t_c (t_{\text{eq}}/t_c)^{1/2} (t/t_{\text{eq}})^{2/3} , \quad (43)$$

where $t_{\text{eq}} \sim 10^{13}$ sec is the end of the radiation era and $t \sim 10^{18}$ sec is the present time. Equation (43) gives $R < 10^{23}$ cm and $R < 10^4$ cm for electroweak and grand unification walls, respectively. Observations exclude[11] $R < 400$ Mpc $\sim 10^{26}$ cm and, therefore, this mechanism is ruled out.

Yet another possibility is that the symmetry is broken at high temperature T_{c2} and then restored at $T_{c1} < T_{c2}$. A model of this sort was discussed in a different context by Langacker and Pi.[12] A simple example of a symmetry broken at high temperatures and restored at lower temperatures was discussed by Weinberg.[13] The physics of domain walls in such models is different, since the mass density of the walls changes with temperature.

The effective mass of a scalar boson responsible for the symmetry breaking can be written as[1,13]

$$m^2(T) = m^2(0) + \gamma\alpha T^2 , \quad (44)$$

where γ is a numerical coefficient depending on the ratios of different coupling constants and on the group structure of the theory. The quantity m in Eq. (12) is of order $|m(T)|$. It is usually assumed that $m^2(0) < 0$ and $\gamma > 0$. Then the symmetry is broken at low temperatures and the phase transition occurs at $T = T_c = (\gamma\alpha)^{-1/2}|m(0)|$. At $T \ll T_c$, $m^2(T) \approx m^2(0)$ and the density of the wall (12) is independent of T. Suppose now that $m^2(0) > 0$ and $\gamma < 0$. Then $T_{c1} = (-\gamma\alpha)^{1/2} m(0)$ and the symmetry is broken at $T > T_{c1}$. When $T \gg T_{c1}$, $m^2(T) \approx \gamma\alpha T^2$ and

$$\sigma \sim \alpha^{1/2} T^3 . \quad (45)$$

At $T \sim T_{c2}$ heavy particles associated with another level of symmetry breaking come into play (we assume that $T_{c2} \gg T_{c1}$). As a result γ changes sign and the symmetry is restored at $T = T_{c2}$. In the whole interval $T_{c1} \ll T \ll T_{c2}$, the surface density of the walls is given by Eq. (45). The critical time t_c changes with temperature and

$$t/t_c \sim G\sigma t \sim \alpha^{1/2} G^{1/4} t^{-1/2} \sim (\alpha t_g/t)^{1/2} \ll 1, \quad (46)$$

where $t_g = G^{1/2} \sim 10^{-43}$ sec is the Planck time and I have used $T \sim G^{-1/4} t^{-1/2}$. We see that in this case the gravitational field of the walls is always weak and can hardly produce any significant effects.

VI. CONCLUSIONS

We have studied the gravitational field of static vacuum domain walls and strings in the weak-field approximation. We found that the domain walls repel nonrelativistic particles and each other and do not deflect light. The strings do not interact with nonrelativistic matter, but do deflect light. The deflection angle is independent of the impact parameter and is of order 10^{-5} rad ($\sim 2''$) for grand unification strings. This effect can give rise to double images of cosmic objects situated behind the strings and may be relevant to the double quasar.

We also found that, in the case of domain walls, our approximate solution of the Einstein equations does not correspond to any exact static solution. The weak-field approximation can be used only during time $t < t_c$ after the phase transition. At $t \gtrsim t_c$ the gravitational field gets into a nonlinear regime and the walls collapse. For electroweak walls $t_c \sim 10^6$ sec and for grand unification walls $t_c \sim 10^{-33}$ sec.

If vacuum domain walls ever existed, they must have disappeared at $t < t_c$. We discussed three possible mechanisms for the disappearance of the walls: (i) small initial bias favoring one of the two vacuum states; (ii) small universe with a nontrivial topology; and (iii) symmetry broken and then restored. The first possibility is not very attractive, since it is not in the spirit of spontaneous symmetry breaking. The second mechanism has been shown to contradict observations. For the third mechanism, we found that the surface density of the walls becomes a function of temperature and changes in such a way that the gravitational field of the walls is always negligible. Thus, either the walls never existed, or if they did, they had little effect on the evolution of the universe.

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$$ds^2 = (1+2U)dt^2 - (1-2U)(dx^2+dy^2+dz^2),$$

where U is the Newtonian potential. In this metric the right-hand side of Eq. (25) takes the form $\frac{1}{2}(\partial U/\partial x^\mu)(T_0^0 - T_k^k)$ and vanishes for the energy-momentum tensor of a vacuum string (11).

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