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Hyperon magnetic moments and the $1^{--} \to 0^{-+} + \gamma$ decays

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Recent precision measurements of hyperon magnetic moments reveal that the moments of the Λ, Σ^+, and Ξ^0 are significantly smaller than the values predicted by the Coleman-Glashow formula or by the naive quark model. In the theoretical framework of SU(3) charge-current equal-time commutation relations and asymptotic SU(3) symmetry, the spin-nonflip sum rules imply that the usual SU(3) parametrization should apply not for the hyperon magnetic moments but for the product (anomalous magnetic moment)×(hyperon mass). Thus, the gross features of the observed hyperon magnetic moments are explained in terms of these derived mass-scale factors. The consistency of the spin-nonflip and -flip sum rules also implies that the remaining discrepancies should be attributed to the SU(3) mixings between the hyperons and their higher-lying excited $1/2^+$ states. In the same theoretical framework, the status of the similar magnetic-moment interactions, $1^{--} \to 0^{-+} + \gamma$, is also reviewed, by studying the consistency of the asymptotic SU(3) sum rules. It is concluded that the usual consideration of the related SU(3) mixing, especially the ones involving the η and η', is insufficient.

Recent progress in the precision measurements of hyperon magnetic moments and also the rates of $1^{--} \to 0^{-+} + \gamma$ decays indicate that the theoretical predictions, based on the nonrelativistic additive quark model or the Coleman-Glashow formula, are not as successful as one had hoped. The assumption that the mass scale is the same for the u and d quark magnetic moments and the moments are proportional to the quark charges is successful for the ratio of the neutron and proton magnetic moments. The value of the Λ magnetic moment μ_Λ can also be accommodated[1] if we introduce another quark-mass-scale factor $\xi = m_u/m_s$. However, the values of the newly measured Ξ^0 and Σ^+ magnetic moments are smaller in magnitude by about 15% from the predicted values based on the above picture.

Lipkin[2] recently pointed out that, to remedy the situation, one may introduce another assumption that the magnetic moment of a quark of a given flavor has a mass scale proportional to the mass of the *hadron* which contains the quark. Then the predicted value for μ_{Σ^+} is reduced from $2.65\mu_p$ to $2.15\mu_p$ and for μ_{Ξ^0} from $-1.43\mu_p$ to $-1.13\mu_p$ which are in agreement within one standard deviation of the new experimental result. Teese and Settles[3] and also Böhm[4] introduced a similar mass scale *phenomenologically* by using the spectrum-generating SU(3) symmetry. Tomozawa[5] also introduced the same baryon-mass factor [in addition to his SU(3)-breaking parameter ξ] by *assuming* that the broken-SU(6) prediction is to be used for the product (magnetic moment)×(baryon mass).

Since the quark-confinement mechanism should manifest itself somewhere in the naive quark-model calculation, the introduction of some phenomenological *hadronic* scale may be taken to be inevitable. However, then one may also wonder whether one should deal directly with hadrons without using the *unobservable* specific knowledge of confined quarks (i.e., quark masses, quark magnetic moments, and quark anomalous magnetic moments, etc.). In this paper we pursue the problem along the latter point of view.

We also note that the usual treatment of the hyperon magnetic moments may, by no means, be complete. Since SU(3) symmetry is certainly broken, there is no reason to believe that the SU(3) mixings (or in the quark model the configuration mixings) between the ground-state baryons and the higher-lying, for example, $(70, 0^+)\frac{1}{2}^+$ baryons do not take place.[6,7]

A somewhat similar discrepancy seems to persist also for the $1^{--} \to 0^{-+} + \gamma$ decays, although the recent determination of the $\rho \to \pi\gamma$ decay width[8] has considerably narrowed the gap between the experiments and the predictions based on the quark model or on the usual recipe of exact SU(3) symmetry plus mixing. Recently, it has been proposed[9] that one may consider the effect of possible anomalous magnetic moments of quarks to improve the agreement in the quark-model calculation. In the SU(3) approach, there is also *no* reason to believe that the problem should be completely solved within the realm of the ground-state 1^{--} and 0^{-+} mesons.

The purpose of this paper is to point out that in the theoretical framework of SU(3) charge-current algebra and asymptotic SU(3) symmetry (which we believe to reflect an important aspect of confined quark dynamics), one can derive broken-SU(3) anomalous-magnetic-moment sum rules which modify the Coleman-Glashow formulas through the appearance of the baryon-mass-scale factors mentioned above. Therefore, the theory reproduces, at least, a qualitative feature of the hyperon magnetic moments—the magnetic moments of the Λ, Σ^+, and Ξ^0 are consistently smaller than the exact-SU(3) values. However, the overall consistency of the theory also suggests that the treatment is not complete and we need to consider the effect of SU(3) mixings between the hyperons and the higher-lying $\frac{1}{2}^+$ states.

We follow the procedure of Slaughter and Oneda cited in Ref. 6. The SU(3) electromagnetic current is given by

$$j^\mu(x) \equiv j_3^\mu(x) + \frac{1}{\sqrt{3}} j_8^\mu(x) .$$

With the SU(3) generators V_i ($V_{\pi^+} = V_1 + iV_2$, $V_{K^+} = V_4 + iV_5, \ldots$), the current satisfies the following equal-time commutator:

$$[[j^\mu(x), V_{K^+}], V_{K^-}] + [[j^\mu(x), V_{\pi^+}], V_{\pi^-}] = 3j^\mu(x) , \tag{1}$$

which is *valid* in broken SU(3) symmetry and is convenient for making the broken-SU(3) parametrization of the asymptotic matrix elements of the current $j^\mu(x)$. We define a matrix element

$$\langle B_\alpha^\mu(s, s')\rangle \equiv \langle B_\alpha(\vec{p}, s)| j^\mu(0) | B_\alpha(\vec{p}', s')\rangle , \tag{2}$$

where $\alpha = p, n, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-$ and s denotes the helicity. Except for the appropriate normalization factor,[10] $\langle B_\alpha^\mu(s, s')\rangle$ is given by

$$\langle B_\alpha^\mu(s, s')\rangle = \bar{u}_\alpha(\vec{p}, s)\left[e\gamma^\mu F_1^\alpha(q^2) + k_\alpha\left(\frac{e}{2m_p}\right)F_2^\alpha(q^2) i\sigma^{\mu\nu}q_\nu\right] \times u_\alpha(\vec{p}', s') . \tag{3}$$

Here $q^2 = (p - p')^2$. $F_1^\alpha(0) = Q_\alpha/e$, where Q_α is the charge of the B_α and $F_2^\alpha(0) \equiv 1$. (From now on, we always use nuclear magnetons as units.) Therefore, k_α ($\alpha = p, n, \ldots$) denotes the anomalous magnetic moment of the B_α measured in *nuclear magnetons*. The total magnetic moment μ_α is thus given by $\mu_\alpha = (m_p/m_\alpha)F_1^\alpha(0) + k_\alpha$. We take the limit $\vec{p} \to \infty$ and $\vec{p}' \to \infty$ with $p_z = \lambda|\vec{p}'|$ and $\lambda > 0$. $\vec{p}'$ is taken along the z axis and $\vec{p}$ lies in the zx plane, i.e., $p_y = 0$. The four-momentum transfer squared q^2 is given by $q^2 = -(1-\lambda)^2\lambda^{-1}m_\alpha^2 - p_x^2\lambda^{-1}$. $q^2 \to 0$ when $\lambda \to 1$ and $p_x \to 0$. By sandwiching Eq. (1) between the states $\langle B_\alpha(p, s)|$ and $|B_\alpha(\vec{p}', s')\rangle$ with $\vec{p} \to \infty$ and $\vec{p}' \to \infty$ and using asymptotic SU(3) symmetry,[6] we then obtain the Coleman-Glashow relation *not* for the hyperon magnetic moments but for the asymptotic matrix elements, Eq. (2), i.e., $\langle B_\alpha^\mu\rangle \equiv \langle B_\alpha^\mu(s, s')\rangle$ with $\vec{p} \to \infty$ and $\vec{p}' \to \infty$:

$$\begin{aligned} &\langle B_\Lambda^\mu\rangle = \tfrac{1}{2}\langle B_n^\mu\rangle , \quad \langle B_{\Sigma^+}^\mu\rangle = \langle B_p^\mu\rangle, \langle B_{\Sigma^0}^\mu\rangle = -\tfrac{1}{2}\langle B_n^\mu\rangle, \\ &\langle B_{\Sigma^-}^\mu\rangle = -\langle B_p^\mu\rangle - \langle B_n^\mu\rangle, \\ &\langle B_{\Xi^0}^\mu\rangle = \langle B_n^\mu\rangle, \quad \langle B_{\Xi^-}^\mu\rangle = -\langle B_p^\mu\rangle - \langle B_n^\mu\rangle, \\ &\langle B_{\Sigma\Lambda}^\mu\rangle = \tfrac{1}{2}(\sqrt{3})\langle B_n^\mu\rangle . \end{aligned} \tag{4}$$

In Eq. (4), we first consider the *spin-nonflip* case, i.e., $s = s' = \frac{1}{2}$. In the asymptotic expansion (in the limit $|\vec{p}| \to \infty$), we have to deal with the two possible *leading* terms. One is the term proportional to p_x (however, we eventually let $p_x \to 0$ to achieve the limit $q^2 = 0$) obtained in the *noncollinear* limit. From their intrinsic kinematical structures, information about the magnetic-moment form factors can *only* be derived through the noncollinear sum rules. The other is the leading term independent of p_x obtained in the collinear limit. For the case $\mu = x$, z, and t, the implication of the sum rules thus obtained from the leading terms (noncollinear for $\mu = x$ and collinear for $\mu = z$ and t) is nothing but the conservation of charges. However, for $\mu = y$ (which produces only the noncollinear sum rules) we derive sum rules which imply that SU(3) parametrization should apply for the product $m_B k_B$. Let us, for illustration, consider the implication of the sum rule $\langle B_{\Sigma^+}^\mu\rangle = \langle B_p^\mu\rangle$ in Eq. (4). For $\mu = x$, z, and t we obtain the identity, $F_1^{\Sigma^+}(0) = F_1^p(0) = 1$. For $\mu = y$, the leading terms in the asymptotic expansion are proportional to $p_x/\lambda|\vec{p}'|$ for $\lambda \to 1$ and $|\vec{p}| \to \infty$ and imply that (in the limit $B_x \to 0$)

$$F^{\Sigma^+}(0) + 2m_\Sigma\left(\frac{k_{\Sigma^+}}{2m_p}\right) = F_1^p(0) + 2m_p\left(\frac{k_p}{2m_p}\right),$$

i.e., $m_\Sigma k_{\Sigma^+} = m_p k_p$. Therefore,

$$\mu_{\Sigma^+} = \left(\frac{m_p}{m_\Sigma}\right) + \left(\frac{m_p}{m_\Sigma}\right)k_p = \left(\frac{m_p}{m_\Sigma}\right)\mu_p .$$

In this way, we obtain for the spin-nonflip case the following modified Coleman-Glashow formulas from Eq. (4):

$$\begin{aligned} &\mu_\Lambda = \frac{1}{2}\left(\frac{m_n}{m_\Lambda}\right)\mu_n , \quad \mu_{\Sigma^+} = \left(\frac{m_p}{m_\Sigma}\right)\mu_p , \quad \mu_{\Sigma^0} = -\frac{1}{2}\left(\frac{m_n}{m_\Sigma}\right)\mu_n , \\ &\mu_{\Sigma^-} = -\left(\frac{m_p}{m_\Sigma}\right)(\mu_p + \mu_n), \quad \mu_{\Xi^0} = \left(\frac{m_n}{m_{\Xi^0}}\right)\mu_n , \\ &\mu_{\Xi} = -\left(\frac{m_p}{m_\Xi}\right)(\mu_p + \mu_n), \quad k_{\Sigma\Lambda} = \frac{\sqrt{3}}{2}\left(\frac{2m_n}{m_\Sigma + m_\Lambda}\right)\mu_n . \end{aligned} \tag{5}$$

Choosing as input the proton and neutron magnetic moments $\mu = 2.793$ and $\mu_n = -1.913$, we obtain from Eq. (5),

$$\mu_\Lambda = -0.80,\quad \mu_{\Sigma^+} = 2.20,\quad \mu_{\Sigma^0} = 0.75,$$
$$\mu_{\Sigma^-} = -0.69,\quad \mu_{\Xi^0} = -1.37, \tag{6}$$
$$\mu_{\Xi^-} = -0.63,\quad k_{\Sigma\Lambda} = -1.35\,.$$

In the present algebraic approach, one can also derive[11] SU(6)-like relations by using the hypothesis of asymptotic level realization of SU(3) in the algebra $[A_{\pi^+}, A_{\pi^-}] = 2V_3$ and $[[j^\mu(x), A_{\pi^+}], A_{\pi^-}] = 2j_3^\mu(x)$, etc. The relation corresponding to the SU(6) relation $\mu_p = (-3_\mu/2_n)$ is $k_p = -k_n$. With this relation and with $\mu_p = -2.973$ we predict

$$\mu_n = -1.793,\quad \mu_\Lambda = -0.75,\quad \mu_{\Sigma^+} = 2.20,$$
$$\mu_{\Sigma^0} = 0.70,\quad \mu_{\Sigma^-} = -0.78,\quad \mu_{\Xi^0} = -1.28, \tag{7}$$
$$\mu_{\Xi^-} = -0.72,\quad k_{\Sigma\Lambda} = -1.27\,.$$

Recent precision measurements give $\mu_\Lambda = -0.61 \pm 0.01$,[12] $\mu_{\Sigma^+} = 2.33 + 0.13$,[13] $\mu_{\Xi^0} = 1.236 \pm 0.014$,[14] and $\mu_{\Xi^-} = 0.75 \pm 0.07$.[15] Although the baryon-mass factors which appear in Eq. (5) significantly improve the agreement between the theory and experiments, the improvement is apparently not perfect. As a matter of fact, there is a reason for it. If we consider, as in Ref. 6, the *spin-flip* ($s = \frac{1}{2}$ and $s' = -\frac{1}{2}$) sum rules of Eq. (4), we do not obtain the baryon-mass factor.[16] However, this need not be the contradiction of the theory. In the hypothesis of asymptotic SU(3) symmetry, SU(3) breaking manifests itself as the SU(3)-multiplet mass splitting and also the SU(3) particle mixing. Even in the naive quark model, one has to consider, in addition to the quark-mass breaking, the particle mixing at the hadronic level. In the present formulation, the asymptotic SU(3) sum rules are explicitly compatible[17] with the Gell-Mann–Okubo mass formula for the hyperons. However, the formula for the octet hyperons is known to involve an error of the order of 10%. In the framework of asymptotic SU(3) symmetry, *this* discrepancy must be attributed to the SU(3) mixings between the ground-state $\frac{1}{2}^+$ hyperons and their higher-lying excited $\frac{1}{2}^+$ states. At the level where we consider these SU(3) mixings, the spin-flip and nonflip sum rules will become compatible. Thus, the spin-*nonflip* sum rules, Eq. (4), can be regarded as the sum rules which are *less* sensitive to the SU(3) mixings and provide us directly the most important features of SU(3) breaking, while the spin-flip sum rules require the *full* treatment of complicated SU(3) mixings. In Ref. 6, a model calculation has been carried out for the Λ magnetic moment using the *spin-flip* sum rules, by introducing a Λ-Λ' mixing. There, the Λ' was assumed to be the SU(3)-singlet ninth $\frac{1}{2}^+$ baryon, which may be the 70 $L^P = 0^+$ $(1, \frac{1}{2}^+)$ baryon. A reasonable result has been obtained. As remarked in Ref. 6, since the values of the Σ^+ and Ξ^0 magnetic moments are now also known to deviate significantly from their SU(3) values, in the spin-flip sum rules we have to take into account the full effect of SU(3) mixing besides the Λ-Λ' mixing. It has been argued[6] that the most important mixings may be the ones with the 70 $L^P = 0^+$ $(10, \frac{1}{2}^+)$ baryons. For the octet-decuplet mixing, the mixing takes place *only* for the Σ- and Ξ-type baryons. If we include these effects of mixings, then the transition magnetic moments as well as the magnetic moments of higher-lying baryons and even the F_3 form factors come into play and the simple picture is lost. The spin-*nonflip* sum rules give a value of μ_Λ which is still somewhat larger[18] than the experimental value, although some significant improvement ($\simeq 16\%$) is already made by the mass-scale factor. The slightly different behavior of μ_Λ from those of μ_{Σ^+} and μ_{Ξ^0} in Eq. (6) or (7) may be attributed to the small difference between the octet-singlet (Λ-Λ') and the octet-decuplet (Σ-Σ' and Ξ-Ξ') mixings.[19] A relatively simple calculation [thus considering only the effect of the mixings of the Λ, Σ, and Ξ hyperons and their counterparts in the 70 $L^P = 0^+$ $(1, \frac{1}{2}^+)$ and $(10, \frac{1}{2}^+)$ multiplets] may hope to improve, for the spin-*nonflip* sum rules, the agreement between the experiments and the predictions already obtained in Eq. (6) or (7). As will be illustrated in the much simpler case of $1^{--} \to 0^{-+} + \gamma$ transitions below, the consistency of the spin-flip and -nonflip sum rules will impose certain constraints among the mass spectra of the hyperons and the higher-lying $\frac{1}{2}^+$ baryons.

We now consider the similar processes $V(\vec{p}) \to P(\vec{p}\,') + \gamma$ which take place through the anomalous magnetic moments of the vector mesons. We define the coupling constant g by

$$T^\mu_{VP} \equiv \langle V(\vec{p}, s) | j^\mu(0) | P(\vec{p}\,')\rangle = g_{VP\gamma}(q^2)\epsilon^{\mu\nu\rho\delta}\epsilon_v^{(s)}(\vec{p})p_\rho p'_\delta\,. \tag{8}$$

$\epsilon_v^{(s)}(\vec{p})$ is the polarization vector of the 1^{--} meson. We now take the same asymptotic limit $\vec{p} \to \infty$ and $\vec{p}\,' \to \infty$ as the limit used for the hyperon magnetic moments. Using the commutator, Eq. (1), and asymptotic SU(3) symmetry, T^μ_{VP} can now be parametrized by the usual prescription of exact SU(3) plus mixing in the asymptotic limit. We again obtain two different sum rules, i.e., the noncollinear sum rules ($\mu = z$ and t) and the collinear sum rules ($\mu = x$ and y). The former sum rules imply that the $g_{VP\gamma}(0)$ can be parametrized by the usual recipe of exact SU(3) plus mixing, while the latter imply that the quantities $(m_V^2 - m_P^2)g_{VP\gamma}(0)$ should satisfy the same parametrizations.[19]

However, these two sets of sum rules are less contradictory [within the framework of SU(3) mixing under consideration] in comparison with the case of the hyperon magnetic moments as will be discussed below. Let us first consider only the $VP\gamma$ couplings which do not involve the η and η'. For example, we have

$$g_{\rho^+\pi^+\gamma} = g_{K^+K^+\gamma} \quad \text{(noncollinear)}, \tag{9}$$

$$(m_\rho^2 - m_\pi^2) g_{\rho^+\pi^+\gamma} = (m_{K^*}^2 - m_K^2) g_{K^{*+}K^+\gamma} \quad \text{(collinear)}, \tag{10}$$

$$g_{\rho^0\pi^0\gamma} - g_{\overline{K}^{*0}\overline{K}^0\gamma} = \sqrt{3}\,(\cos\theta g_{\phi\pi\gamma} + \sin\theta g_{\omega\pi\gamma}) \quad \text{(noncollinear)}, \tag{11}$$

$$(m_\rho^2 - m_\pi^2) g_{\rho^0\pi^0\gamma} - (m_{K^*}^2 - m_K^2) g_{\overline{K}^{*0}\overline{K}^0\gamma} = \sqrt{3}\,[\cos\theta(\phi^2 - \pi^2) g_{\phi\pi\gamma} + \sin\theta(\omega^2 - \pi^2) g_{\omega\pi\gamma}] \quad \text{(collinear)}. \tag{12}$$

Here θ denotes the $\omega - \phi$ mixing angle. The compatibility of the collinear and noncollinear sum rules, Eqs. (9)–(12), imposes the mass constraint

$$m_\rho^2 - m_\pi^2 = m_{K^*}^2 - m_K^2 \tag{13}$$

and also the mass–mixing-angle–coupling-constant constraint

$$\frac{g_{\phi\pi\gamma}}{g_{\omega\pi\gamma}} = -\tan\theta \, \frac{(m_\rho^2 - m_\omega^2)}{(m_\rho^2 - m_\phi^2)} \,. \tag{14}$$

It is gratifying to notice that both Eqs. (13) and (14) are exactly the same sum rules as the ones obtained in the totally independent calculations which are based on the asymptotic realizations of *exotic* commutators[20] and also on the asymptotic level realization of SU(3) in the algebra $[[j^\mu(x), A_{\pi^+}], A_{\pi^-}] = 2j_3^\mu(x)$ etc.[21] There is thus a surprising consistency within the present algebraic approach. Both Eqs. (13) and (14) are reasonably consistent with experiments. Experimentally Eq. (13) reads 0.225 = 0.201 GeV.[2] The main source of the above ($\simeq 10\%$) error may be due to the fact that the $L = 0$ 0^{-+} mesons can mix with their radially excited states, whereas the $L = 0$ 1^{--} mesons will mix with their $L = 2$ counterparts as well as their radially excited states. One can argue that the usual SU(3) predictions on the rates of the $V \to P\gamma$ decays which do not involve the η and η' mesons (i.e., the $\omega\pi\gamma$, $\phi\pi\gamma$, $\rho\pi\gamma$, $K^{*+}K^+\gamma$, and $K^{*0}K^0$) may be violated as much as 20%, because of the neglect of such SU(3) mixings.

If we now add the sum rules involving the η and η', the *overall* consistency of the collinear as well as the noncollinear sum rules requires that (i) the 1^{--} nonet should be ideal ($\rho^2 = \omega^2$, $\sin\theta = 1/\sqrt{3}$, and $g_{\phi\pi\gamma} = 0$ [see Eq. (14)], etc.) and (ii) the 0^{-+} nonet has to satisfy the Schwinger's nonet mass relation [see Eq. (17) below]. This mass relation cannot accomodate the η' (958) as the ninth 0^{-+} meson. It predicts the mass of the ninth 0^{-+} meson around 1.6 GeV. Therefore, for the processes involving the η and η', one *cannot* expect that the usual recipe of exact SU(3) plus mixing (with $|\theta_p| \simeq 11°$) works very well. The most important source of the trouble will be that there are other $I = Y = 0$ 0^{-+} mesons which can mix with the η and η' mesons. One may, for example, study whether the consideration of SU(4) (i. e., the inclusion of the η_c) improves the situation. Equation (13) is now extended to [by using the hypothesis of asymptotic SU(4) symmetry]

$$m_{D^*}^2 - m_D^2 = m_{F^*}^2 - m_F^2 = m_{K^*}^2 - m_K^2 = m_\rho^2 - m_\pi^2 \,. \tag{15}$$

The masses of the D^* and D satisfy Eq. (15) reasonably well. Therefore, we expect that the predictions[19] of the rates of $D^* \to D\gamma$ decays agree with experiment approximately to that extent. If we consider the compatibility of all the collinear as well as the noncollinear sum rules, we obtain, in addition to Eq. (15), another constraint on the mass spectrum of the mesons involved. To see this we add one simplifying (but probably very good) assumption that the 1^{--} 16-plet mesons are ideal. [We recall that in SU(3), the 1^{--} nonet was forced to be ideal.] We then obtain the following mass relation after eliminating the $g_{VP\gamma}$'s from the set of sum rules:

$$\begin{aligned} &2(\rho^2 - \psi^2)^2(\rho^2 - \phi^2)^2(2\rho^2 - \psi^2 - \phi^2 + 2\eta_c^2 + 2\eta'^2 + 2\eta^2 - 6\pi^2) \\ &\quad + 2[(\rho^2 - \psi^2)^2(\rho^2 - \phi^2) + (\rho^2 - \psi^2)(\rho^2 - \phi^2)^2][(\eta^2 - \pi^2)(\eta'^2 - \pi^2) + (\eta'^2 - \pi^2)(\eta_c^2 - \pi^2) + (\eta_c^2 - \pi^2)(\eta^2 - \pi^2)] \\ &\quad + [3(\rho^2 - \psi^2)^2 + 3(\rho^2 - \phi^2)^2 - 2(\rho^2 - \psi^2)(\rho^2 - \phi^2)](\eta_c^2 - \pi^2)(\eta'^2 - \pi^2)(\eta^2 - \pi^2) = 0 \,. \end{aligned} \tag{16}$$

If, in Eq. (16), we take an SU(3) limit ($\psi^2 \to \infty$, $\eta_c^2 \to \infty$) with $x \equiv \eta_c^2/\psi^2 = 1$ as $\psi^2 \to \infty$, we obtain the Schwinger's nonet mass relation

$$(4K^2 - \pi^2 - 3\eta^2)(4K^2 - \pi^2 - 3\eta'^2) + 8(K^2 - \pi^2)^2 = 0 \,. \tag{17}$$

However, if we take the same limit with $x = 0.8$, we obtain another remarkable nonet mass formula[22]

$$(4K^2 - \pi^2 - 3\eta^2)(4K^2 - \pi^2 - 3\eta'^2) + 2(K^2 - \pi^2)^2 = 0 \tag{18}$$

which is well satisfied by the 0^{-+}nonet (the η' is predicted to be around 943 MeV). Therefore, in the framework of SU(4) it is possible to accommodate[23] the unusual behavior of the 0^{-+} mesons. However, this does not seem to be the whole story. Given the mass of $\psi, \psi = 3.097$ GeV, Eq. (16) predicts the mass of η_c in terms of the masses of the ρ, ϕ, π, η, and η'. We obtain $\eta_c \simeq 2.65$–2.80 GeV corresponding to the choice of the ρ mass in the range 740–770 MeV. Again this result is very similar to the result[24] obtained by realizing the exotic commutators in the asymptotic limit. However, recently the mass of η_c seems to have finally settled[25] around 2.98 GeV. We thus see that the source of 5–10% discrepancy in the prediction of the mass of η_c lies in the insufficient handling of the mixing effects. It may be due to the neglect of the effects of either the existence of the $\bar{b}b$ and $\bar{t}t$ states or the mixing of the η and η' with their radially excited states [such as possibly the $E(1420)$ meson].

In this paper, we have, for simplicity, assumed that the SU(3) electromagnetic current is given by $j^\mu(x) = j_3^\mu + (1/\sqrt{3}) j_8^\mu$. Since there are now more than three quarks, even if we confine our attention only to the old problems of SU(3), the $j^\mu(x)$ involves an extra SU(3)-singlet current. This SU(3)-singlet current introduces an extra parameter to the (broken) SU(3) parametrizations of the hyperon magnetic moments and the $1^{--}\rightarrow 0^{-+}+\gamma$ transitions as discussed earlier by Böhm and Teese[26] and others.[19] The introduction of small isoscalar anomalous magnetic moments of quarks in the naive quark-model calculation[9,27] also produces a similar effect.

With the inclusion of the SU(3)-singlet electromagnetic current, our spin-nonflip and spin-flip SU(3) sum rules for the hyperon magnetic moments are equivalent to those of Ref. 4, if the appropriate mass factors (derived in this paper) are used. The consistency of the two types of sum rules still requires that the SU(3) mixings between the hyperons and their higher-lying $\frac{1}{2}^+$ excited states play an appreciable role. However, the spin-nonflip sum rules will be less sensitive to the above SU(3) mixings and can explain the gross features of broken SU(3) symmetry in hyperon magnetic moments.

An analogous situation persists for the SU(3) fit of the $1^{--}\rightarrow 0^{-+}+\gamma$ transitions. However, the mass formulas obtained in this paper by using the consistency argument of the collinear and noncollinear sum rules are *independent* of the presence of the SU(3)-singlet electromagnetic current. The small discrepancies between the above-obtained mass formulas and experiment imply that one *still* needs to consider the further effect of SU(3) mixings (beyond those among the ground-state 0^{-+} and 1^{--} mesons), in addition to the effect of the SU(3)-singlet electromagnetic current.

Intuitive naive quark counting certainly works well enough. However, as the recent experiments on the hyperon magnetic moments clearly demonstrate, the effect of confined quark dynamics at the hadronic level must also be taken into account. In the present paper, we have shown that the above two features of unobservable quarks may be put together in an algebraic approach to hadrons. In this approach the chiral quark algebras (which are *valid* in broken flavor symmetry) are regarded simply as the fundamental constraints imposed by confined quarks upon the world of observable hadrons. The asymptotic ansätzes which are introduced, such as the asymptotic flavor symmetry and the asymptotic level realization of certain algebras, may be taken as a (we hope) accurate and sensible abstraction of confined quark dynamics at the hadronic level. In this way one may bypass the quark-confinement problem by dealing *directly* with hadrons. The salient features of this theory are that certain mass constraints must be satisfied and there is a remarkable interplay (which produces the quark-line selection rules) among the masses, the flavor-mixing parameters and the asymptotic matrix elements of the vector and axial-vector currents, as demonstrated by Eqs. (13)–(16) of this paper.

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[17]Combined with the use of exotic commutator $[\dot{V}_K0,\ V_K0]=0$ which expresses the usual mechanism of SU(3) breaking, the hypothesis of asymptotic SU(3) symmetry implies that the Gell-Mann-Okubo mass formulas with the full inclusion of all possible SU(3) mixings are the *exact* constraints of the theory. For the recent review of the present algebraic approach, see S. Oneda, in *Lecture Note in Physics 94, Group Theoretical Methods in Physics*, proceedings of Austin Conference, 1978, edited by W. Beiglböck, A. Böhm, and E. Takasugi (Springer, Berlin, 1979), p. 334.

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