μ^+ polarization in proton decay: A probe of flavor mixing in unified models

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Measurements of μ^+ polarizations from proton decay will give valuable insight into the nature of unification theories. We discuss the μ^+ polarizations expected in a class of O(10) models for two plausible flavor-mixing schemes.

Proponents of the unification of electroweak and strong interactions await the experimental developments of the next several months with great anticipation. Most of the theories proposed for electroweak-chromodynamics unification predict the existence of baryon-number-violating processes. Several experiments which look carefully for such a baryon-number violation in proton decay are either currently or soon to be underway. Furthermore, if the existing fermion families belong to equivalent representations of the gauge group of unification such that some version of O(10) or SU(5) is likely to be correct, there is an excellent chance that the present experiments will see proton decay with rates substantial enough to study the process in detail. It is well known that much of the information obtained from gross measurements of branching ratios in proton decay tests only the nature of low-energy hadronic physics, such as strong-isospin conservation, SU(3) $\times$ SU(2) $\times$ U(1) symmetry, or a particular approximation scheme for the evaluation of hadronic matrix elements. The investigation of parameters which distinguish between various models of unification and fermion-mass generation, such as mixing angles and superheavy-particle mass ratios, require more refined measurements. Recently DeRújula, Georgi, and Glashow[1] found that relative two-body branching ratios such as $e^+K/(e^+K+\mu^+K)$ and $\mu^+\pi/(e^+\pi+\mu^+\pi)$ are very sensitive to the particular unification model and flavor mixing employed. They studied these branching ratios in detail in O(10) models for two plausible flavor-mixing schemes. There are, however, still many parameters which need to be determined in any given model. Thus, it is useful to look for as many additional experimental probes of the structure of unification as possible. One such, suggested on general grounds by Weinberg[2] and Wilczek and Zee,[3] is the measurement of lepton polarizations. In this comment we investigate the most accessible of these, μ^+ polarizations, for the O(10) models and flavor-mixing schemes considered in Ref. 1, and find again that the differences between models are striking.

We assume, as in Ref. 1, that proton decay is gauge-particle dominated. The gauge particles responsible for proton decay in O(10) belong to two SU(3) triplet–SU(2) doublets with charges $(-\frac{1}{3}, -\frac{4}{3})$ and $(\frac{2}{3}, -\frac{1}{3})$, and masses m_A and m_B, respectively. As before, SU(5) is obtained from O(10) in the limit $m_A/m_B \to 0$. We then consider the effect of two mixing schemes, F and J mixing, on muon polarizations as a function of the ratio of gauge-particle masses. In the simplest scheme, F mixing, fermion masses arise from Yukawa couplings between the fermions and two complex O(10) $\underline{10}$'s of Higgs scalars, while in the more phenomenologically successful scheme, J mixing, fermion masses arise from Yukawa couplings between fermions and a $\underline{10}$ and $\underline{126}$ of Higgs scalars. The details of these mixing schemes are given in Ref. 1. We further consider only mixing between the two lightest families since universality requires that mixing to the third family be small. Finally, the outgoing muons are assumed to be fully relativistic.

The Lagrangian, including mixing terms, responsible for the emission of μ^+ in proton decay is

$$\frac{g^2}{m_A{}^2}\epsilon^{ijk}[(\bar{s}_i^c\gamma^\nu\mu^- - \bar{\mu}^+\gamma^\nu s_i)(\bar{u}_j^c\gamma_\nu u_k) + s_\theta(\bar{\mu}^+\gamma^\nu u_i)(c_\theta\bar{u}_j^c\gamma_\nu d_k + s_\theta\bar{u}_j^c\gamma_\nu s_k)]$$

$$+\frac{g^2}{m_B{}^2}\epsilon^{ijk}[s_\theta(\bar{u}_i^c\gamma^\nu\mu^-)(c_\theta\bar{d}_j^c\gamma_\nu u_k + s_\theta\bar{s}_j^c\gamma_\nu u_k)] \tag{1}$$

for F mixing, and

$$\frac{g^2}{m_A{}^2}\epsilon^{ijk}\{\eta[s_e(s_d\bar{s}_i^c - c_d\bar{d}_i^c)\gamma^\nu\mu^- + s_e\bar{\mu}^+\gamma^\nu(c_d d_i + s_d s_i) - c_e(s_d\bar{d}_i^c + c_d\bar{s}_i^c)\gamma^\nu\mu^- - c_e\bar{\mu}^+\gamma^\nu(-s_d d_i + c_d s_i)](\bar{u}_j^c\gamma_\nu u_k)$$
$$+(c_u s_e + \eta c_e s_u)(\bar{\mu}^+\gamma^\nu u_i)[-\eta c_u\bar{u}_j^c\gamma_\nu(c_d d_k + s_d s_k) + s_u\bar{u}_j^c\gamma_\nu(-s_d d_k + c_d s_k)]\}$$
$$-\frac{g^2}{m_B{}^2}\epsilon^{ijk}\{(\eta c_u s_e + s_u c_e)(\bar{u}_i^c\gamma^\nu\mu^-)[c_u(-c_d\bar{d}_j^c + s_d\bar{s}_j^c)\gamma_\nu u_k - \eta s_u(s_d\bar{d}_j^c + c_d\bar{s}_j^c)\gamma_\nu u_k]\} \quad (2)$$

for J mixing, where all fields are left-handed, i, j, k are color indices, s_θ (c_θ) denotes the sine (cosine) of the Cabibbo angle for F mixing, and s_e, s_d, s_u (c_e, c_d, c_u) and η denote the sines (cosines) of the mixing angles and phase, respectively, which appear for J mixing. We define the μ^+ polarization P to be

$$P(p \to \mu^+ H) = \frac{\Gamma(p \to \mu_L^+ H) - \Gamma(p \to \mu_R^+ H)}{\Gamma(p \to \mu_L^+ H) + \Gamma(p \to \mu_R^+ H)}, \quad (3)$$

where μ_L^+ (μ_R^+) are left- (right-) handed muons, and H is given final hadronic state. If experimentally the final hadronic state can be separated into strange versus nonstrange modes, the polarization P is independent of the final hadronic matrix elements. This is particularly useful since it is precisely in the evaluation of these matrix elements where our ignorance of 1-GeV physics confuses the picture of unification we can extract from proton-decay experiments. For F mixing, the μ^+ polarizations into nonstrange (X_N) and strange (X_S) final states are

$$p(p \to \mu^+ X_N) = \frac{m_A{}^{-4} - m_B{}^{-4}}{m_A{}^{-4} + m_B{}^{-4}} \quad (4)$$

and

$$P(p \to \mu^+ X_S) = \frac{m_A{}^{-4}(1 + s_\theta{}^2)^2 - (m_A{}^{-2} + s_\theta{}^2 m_B{}^{-2})^2}{m_A{}^{-4}(1 + s_\theta{}^2)^2 + (m_A{}^{-2} + s_\theta{}^2 m_B{}^{-2})^2}. \quad (5)$$

For J mixing the results are somewhat more complicated. If we use the phenomenologically acceptable estimate that the up and lepton mixing angles are approximately equal, i.e., $\theta_u \sim \theta_e$ and define $x = m_A{}^2/m_B{}^2$, $s_\pm = \sin(\theta_e + \theta_d)$ and $c_\pm = \cos(\theta_e + \theta_d)$, the μ^+ polarizations predicted by J mixing are

$$P(p \to \mu^+ X_N) = \frac{[s_+ + s_e c_e(1 + \mathrm{Re}\eta)c_-]^2 - [s_+ + x s_e c_e(1 + \mathrm{Re}\eta)c_-]^2 + (1 - x^2)(s_e c_e \,\mathrm{Im}\eta c_+)^2}{[s_+ + s_e c_e(1 + \mathrm{Re}\eta)c_-]^2 + [s_+ + x s_e c_e(1 + \mathrm{Re}\eta)c_-]^2 + (1 + x^2)(s_e c_e \,\mathrm{Im}\eta c_+)^2} \quad (6)$$

and

$$P(p \to \mu^+ X_S) = \frac{[c_+ - s_e c_e s_-(1 + \mathrm{Re}\eta)]^2 - [c_+ - x s_e c_e s_-(1 + \mathrm{Re}\eta)]^2 + (1 - x^2)(s_e c_e \,\mathrm{Im}\eta s_+)^2}{[c_+ - s_e c_e s_-(1 + \mathrm{Re}\eta)]^2 + [c_+ - x s_e c_e s_-(1 + \mathrm{Re}\eta)]^2 + (1 + x^2)(s_e c_e \,\mathrm{Im}\eta s_+)^2}. \quad (7)$$

The results of Eqs. (4)–(7) are displayed graphically in Figs. 1 and 2, where we have used the values $s_+ = 0.264$, $s_- = 0.129$, $1 + \mathrm{Re}\eta = 0.6$, $\mathrm{Im}\eta = 0.917$, and $s_e c_e = 0.069$ which are extracted as in Ref. 1 from known values of the lepton masses and the observed value of the Cabibbo angle. The horizontal scales of Figs. 1 and 2 are again linear in $(m_A/m_B)^{1/2}$ for ease of comparison with the results of Ref. 1. Once m_A/m_B has been fixed by the two-body branching ratios of Ref. 1, μ^+ polarizations impose clear constraints on the unification flavor mixing [with the exception, of course, of the fully O(10) symmetric case $m_A = m_B$, in which the μ^+ polarizations are each identically zero]. For example, in the SU(5) limit $m_A/m_B \to 0$, $P(p \to \mu^+ X_N)$ rapidly approaches +1 (although the rate is suppressed overall by a factor $s_\theta{}^2 c_\theta{}^2$ as expected for Cabibbo-suppressed modes) while $P(p \to \mu^+ X_S)$ approaches $s_\theta{}^2$ asymptotically for F mixing. J mixing gives dramatically different results. In this case, $P(p \to \mu^+ X_N)$ stays small and positive, approaching 0.167 asymptotically while $P(p \to \mu^+ X_S)$ is very small and negative throughout the entire region $m_A/m_B < 1$. In the anti-SU(5) limit $m_A/m_B > 1$, $P(p \to \mu^+ X_N) \to -1$ for both mixing schemes. However, the approach is much less rapid for J mixing. $P(p \to \mu^+ X_S)$ also approaches −1 asymptotically in this limit, again less rapidly for J mixing than for F mixing. The differences are even more dramatic if the mass ratio lies between these two extremes, i.e., $m_A/m_B \approx 4$. Then $P(p \to \mu^+ X_S)$ is small but *positive* (~ 0.05) for J mixing, but fairly large and negative (~ -0.5) for F mixing, which would be clearly distinguishable.

Experimentally it may be difficult to clearly identify the strangeness of the final state in proton decay. In that case the previous polarizations

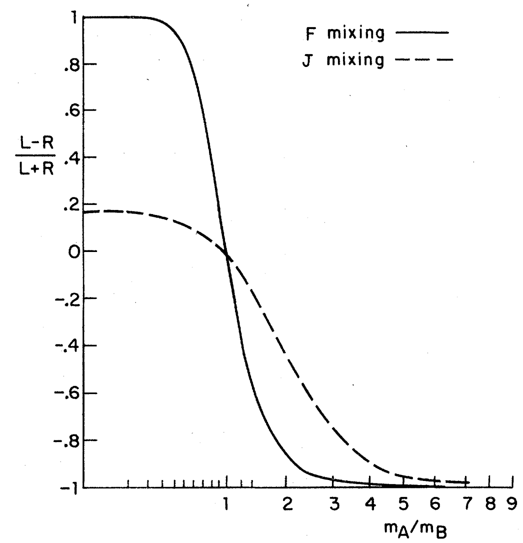

FIG. 1. μ^+ polarization from proton decay for final states containing nonstrange hadrons. The horizontal scale is linear in $(m_A/m_B)^{1/2}$. The continuous (dashed) line corresponds to F (J) flavor-mixing schemes in O(10).

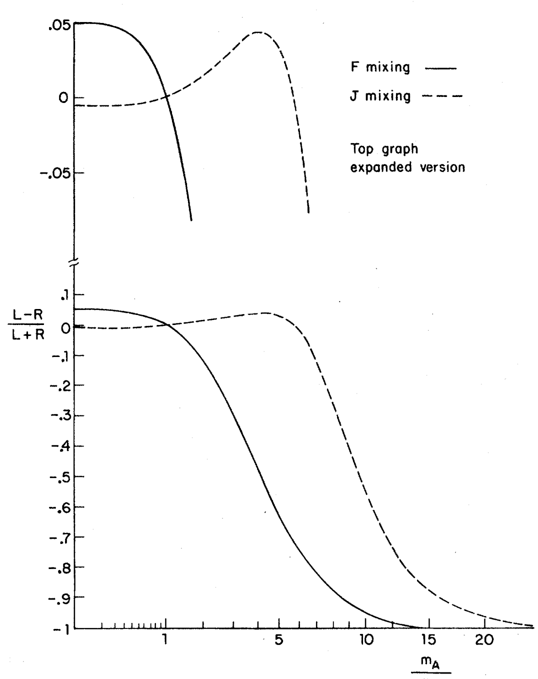

FIG. 2. μ^+ polarization from proton decay for final states containing a strange hadron, for F and J mixing (as in Fig. 1). The upper portion of the figure expands the vertical scale of the lower figure on the same horizontal scale to display more clearly the detail in this region.

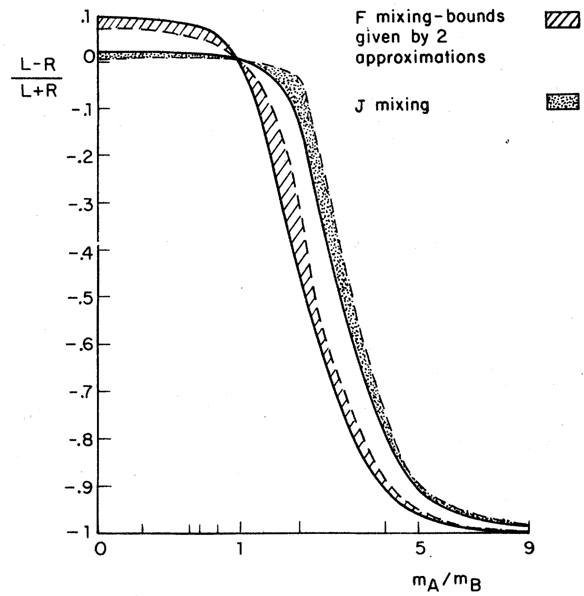

FIG. 3. μ^+ polarization from proton decay for the final state in which pions are not distinguished from kaons in the final state. The horizontal scale is linear in $(m_A/m_B)^{1/2}$. The crosshatched (dotted) band corresponds to F (J) flavor-mixing schemes in O(10), where the bounds on each band correspond to the approximation schemes for hadronic matrix elements given in Refs. 4 and 5.

into strange or nonstrange final states must be weighted by the relative branching fractions of these modes. The polarization depends on the final hadronic matrix element. In Fig. 3 we display the μ^+ polarization for final states which do not distinguish kaons as bands whose bounds are given by the approximations to the hadronic matrix elements of Refs. 4 and 5. Although similar in shape for F and J mixing, the bands are nonoverlapping. Thus, even with the uncertainties in the 1-GeV hadronic matrix elements, it should be possible to distinguish between the two mixing schemes.

In conclusion, we have shown that μ^+ polarizations from proton decay are extremely sensitive to the parameters entering grand unified models while remaining relatively insensitive to the details of low-energy approximations to hadronic matrix elements. It is hoped that in the not-too-distant future measurements of these polarizations will give valuable insight into the nature of unification, flavor mixing, and the generation of fermion masses.

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