

## Absorbed Mueller-Regge model for the single-particle-inclusive production of forward nucleons from a nucleon beam

K. J. M. Moriarty\*

*Department of Physics, Technion—Israel Institute of Technology, Haifa, Israel, 32000*

J. P. Rad

*Department of Physics, University of Shiraz, Shiraz, Iran*

J. H. Tabor

*Department of Mathematics, Lanchester Polytechnic, Coventry, CV1 5FB, United Kingdom*

A. Ungkitchanukit

*Department of Physics, Faculty of Science, Mahidol University, Rama 6 Road, Bangkok 4, Thailand*

(Received 1 October 1979)

The inclusive production of nucleons is investigated in a parameter-free Mueller-Regge model with absorption corrections. The inclusive cross sections and the recoil-nucleon polarization are determined and are compared with existing experimental data.

### I. INTRODUCTION

From two-body phenomenology the importance of Regge cuts in strong interactions is well established. It is then a natural extension to consider such effects in single-particle-inclusive processes. Further, to resolve the inconsistency posed by the decoupling theorems in inclusive reactions, Regge cuts are essential.<sup>1</sup> Following these developments there have been several phenomenological treatments of Regge cuts in inclusive reactions.<sup>2-4</sup> Although the different models have different derivations, the results are essentially related to the absorption model, and, apart from minor differences in detail, they are for practical purposes the same when used in phenomenology.

In a previous paper<sup>5</sup> the charge-exchange reactions  $p(n) + p \rightarrow n(p) + X$  were treated in a Mueller-Regge pole model. This model has all of its parameters fixed and, in particular, the coupling constant of the pion to two nucleons. Since pion exchange is the dominant production mechanism in the triple-Regge region for the processes  $p(n) + p \rightarrow n(p) + X$ , this coupling constant fixes the normalization of the inclusive cross section. On comparison with the experimental data this was found to be overestimated by the model. This result was also found in another calculation<sup>6</sup> of the reaction  $p + p \rightarrow n + X$ . In the present paper we consider absorptive cut corrections as a remedy for this problem. In addition, the calculation of Ref. 6 did not consider the recoil-nucleon polarization which is a critical prediction of the model. This calculation is carried out in the

present paper.

In Sec. II we present the formalism for evaluating the absorption corrections to our Mueller-Regge amplitudes. We conclude in Sec. III with a discussion of the results of our model calculations.

### II. FORMALISM

Following the model of Ref. 4, the Mueller-Regge helicity amplitudes for the reactions  $p(n) + p \rightarrow n(p) + X$  are given by (see Fig. 1)

$$H_{\lambda_a K, \lambda'_a K'}^{\lambda_b \lambda_c, \lambda'_b \lambda'_c} = (J_5^{\lambda_b \lambda_c} \Pi \Gamma_5^{\lambda_a K}) (J_5^{\lambda'_b \lambda'_c} \Pi \Gamma_5^{\lambda'_a K'})^\dagger$$

and

$$H_{\lambda_a K, \lambda'_a K'}^{\lambda_b \lambda_c, \lambda'_b \lambda'_c} = (J_\mu^{\lambda_b \lambda_c} \Pi_{\mu\nu} \Gamma_\nu^{\lambda_a K}) (J_{\mu'}^{\lambda'_b \lambda'_c} \Pi_{\mu'\nu'} \Gamma_{\nu'}^{\lambda'_a K'})^\dagger,$$

for the pseudoscalar and vector exchanges, respectively, where the  $\lambda$ 's are the helicity indices, the  $J$ 's are the currents at the particle-particle-Reggeon vertices ( $b$ - $c$ - $R$ ), the  $\Pi$ 's are the Regge-

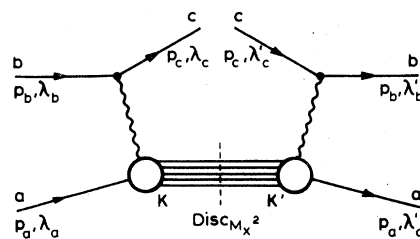


FIG. 1. The Mueller-Regge diagram for the single-particle-inclusive scattering process  $p(n) + p \rightarrow n(p) + X$ .

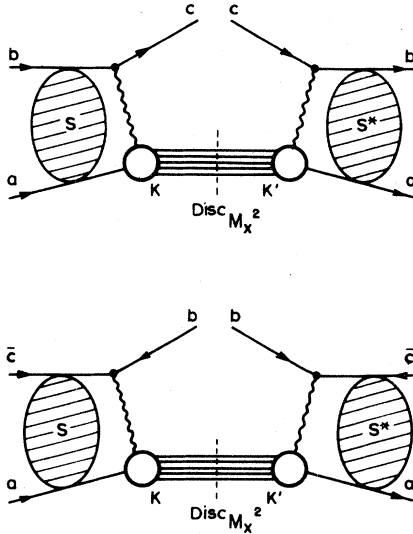


FIG. 2. Absorptive corrections in the  $ab$  and  $\bar{c}b$  channels for the reaction  $p(n) + p \rightarrow n(p) + X$ .

ized propagators, the  $\Gamma$ 's are the structure functions at the inclusive vertices, and the  $K$ 's denote the summation of the helicities of the missing mass  $X$ . The currents are given by

$$J_5 = g(\bar{N}\gamma_5 N)$$

and

$$J_\mu = g[M_1(t) \frac{P_\mu}{2m_N} (\bar{N}N) + M_2(t) \bar{N}\gamma_\mu N],$$

where  $g$  is obtained from the known pion-nucleon coupling constant  $g^2/4\pi = 30.0$ ,  $m_N$  is the nucleon mass,  $M_1(t)$  and  $M_2(t)$  are related to the Sachs form factors<sup>7</sup> through

$$M_1 = F_c - F_M, \quad M_2 = \left(1 + \frac{t}{4m_N^2}\right) F_M,$$

$$\left. \frac{d\alpha_\pi(t)}{dt} \right|_{t=m_\pi^2} [-\Gamma(-\alpha_\pi(t))] \frac{1 + \tau_\pi \exp[-i\pi\alpha_\pi(t)]}{2} \left(\frac{s}{M_X^2}\right)^{\alpha_\pi(t)}$$

and

$$\left. \frac{d\alpha_\rho(t)}{dt} \right|_{t=m_\rho^2} \Gamma(1 - \alpha_\rho(t)) \frac{1 + \tau_\rho \exp[-i\pi\alpha_\rho(t)]}{2} \left(\frac{s}{M_X^2}\right)^{\alpha_\rho(t)-1},$$

respectively, where  $\alpha(t) = \alpha_0 + \alpha't$  and  $\tau$  are the appropriate signatures. Our Regge trajectories are taken as

$$\alpha_\pi(t) = -0.013 + 0.650t$$

and

$$\alpha_\rho(t) = 0.470 + 0.905t,$$

where these trajectories are derived under the assumption of exchange degeneracy between the pairs of particles  $\pi - B$  and  $\rho - A_2$ . A detailed discussion of the calculation of these expressions is contained in Ref. 5.

As in Ref. 4 we apply absorptive corrections in impact-parameter space and obtain (see Fig. 2)

TABLE I. Absorption coefficients for  $p(n) + p \rightarrow n(p) + X$ . Values are taken from Ref. 14.

$P_{\text{lab}}$ (GeV/c)	$s$ (GeV <sup>2</sup> )	$C$	$(R^{-1})$ (GeV/c)
57.05739	109.0	0.6863	0.209
151.7833	287.0	0.6505	0.206
268.3235	506.0	0.657	0.203
280.9529	529.0	0.657	0.203
401.3594	756.0	0.7165	0.212
498.027	936.36	0.6672	0.202
1059.0421	1989.16	0.6152	0.199
1495.9174	2809.00	0.617	0.198

with  $P_\mu = (p_b + p_c)_\mu$  and  $t = (p_c - p_b)^2$ . We define the usual invariant  $s = (p_a + p_b)^2$  and the missing mass squared  $M_X^2 = (p_a + p_b - p_c)^2$ . The structure functions, after averaging and summing over  $\lambda_a, K$ , become

$$\sum_{\lambda_a, K} (\Gamma_5^{\lambda_a K})^\dagger = 2\Delta^{1/2}(M_X^2, t, m_a^2) \sigma_{\text{tot}}(\pi a \rightarrow \pi a)$$

and

$$\sum_{\lambda_a, K} (\Gamma_v^{\lambda_a K})(\Gamma_v^{\lambda_a K})^\dagger = p_{av} p_{av'} V,$$

where

$$V = \frac{8m_\rho^2}{M_X^2} \sigma_{\text{tot}}(\rho a \rightarrow \rho a).$$

Then we have

$$\sum_{\lambda_a, K} H_{\lambda_a K}^{\lambda_b \lambda_c, \lambda_b' \lambda_c'} = H^{\lambda_b \lambda_c, \lambda_b' \lambda_c'}.$$

The propagators for pseudoscalar and vector exchange, after Reggeization, take the form

$$H_{abs}^{\lambda_b \lambda_c, \lambda'_b \lambda'_c}(b, b') = S(b) H^{\lambda_b \lambda_c, \lambda'_b \lambda'_c}(b, b') S^*(b'),$$

where  $S(b) = 1 - C \exp(-\lambda b^2)$ ,  $b$  is the impact parameter, and  $C$  and  $\lambda$  are the opacity and the inverse square of the radius of interaction (see Table I). The absorbed amplitudes are then given by

$$H_{abs}^{\lambda_b \lambda_c, \lambda'_b \lambda'_c}(s, \tau, M_X^2) = \int_0^\infty \tau' d\tau' \int_0^\infty \tau'_1 d\tau'_1 H^{\lambda_b \lambda_c, \lambda'_b \lambda'_c}(s, \tau', \tau'_1, M_X^2) \\ \times \left\{ \frac{1}{\tau} \delta(\tau - \tau') - \frac{C}{2\lambda} \exp\left[-\left(\frac{\tau^2 + \tau'^2}{4\lambda}\right)\right] I_\nu\left(\frac{\tau\tau'}{2\lambda}\right) \right\} \\ \times \left\{ \frac{1}{\tau} \delta(\tau - \tau'_1) - \frac{C}{2\lambda} \exp\left[-\left(\frac{\tau^2 + \tau_1'^2}{4\lambda}\right)\right] I_{\nu'}\left(\frac{\tau\tau'_1}{2\lambda}\right) \right\},$$

where  $\nu = |\lambda_c - \lambda_b|$ ,  $\nu' = |\lambda'_c - \lambda'_b|$  are the helicity flip at the  $bc$ ,  $b'c'$  vertices, respectively, with  $\nu = 0, 1$ ;  $\nu' = 0, 1$ , and  $\tau = 2k \sin(\theta/2)$ , with  $\theta$  the c.m. scattering angle and  $I_n$  the modified Bessel function of the first kind.

The single-particle-inclusive cross section is given by

$$\frac{s}{\pi} \frac{d^2\sigma}{dt dM_X^2} = \frac{1}{64\pi^2 k^2} \frac{1}{(2S_b + 1)} \sum_i \sum_{\lambda_c} H_{ats}^{\lambda_c \lambda_c},$$

where the summation over  $i$  signifies the summation over all possible exchanges,  $S_b$  is the spin of particle  $b$ , and the polarization of the recoil nucleon is given by

$$P(t) = \frac{H_{abs}^{+,+,+-} - H_{abs}^{+,-,++}}{H_{abs}^{+,+,++} + H_{abs}^{+,-,+-}}.$$

All the calculations have been carried out by com-

puter<sup>8</sup> and for accuracy all the graphs have been plotted by computer.<sup>9</sup>

### III. DISCUSSION

We have assumed that the principal trajectories exchanged in the single-particle-inclusive production processes ( $p \frac{1}{2} n$ ) and ( $n \frac{1}{2} p$ ) correspond to the  $\pi$ ,  $\rho$ , and  $A_2$  mesons. It must be kept in mind that since, for the proton-proton scattering reaction, it is not possible, in principle, to distinguish from which vertex a particular final state emerges, the total amplitudes are obtained by antisymmetrizing the amplitudes obtained from Fig. 1. However, at the high energies we are considering no error is introduced by simply multiplying the amplitudes by a factor of 2. These can then be used to compare with data which has been folded about  $x=0$ .

Figure 3 shows the energy dependence of the  $M_X^2/s$  distribution for ( $p \frac{1}{2} n$ ) for two four-momentum transfer values. We see that, although the absorptive cut correction brings down the overall

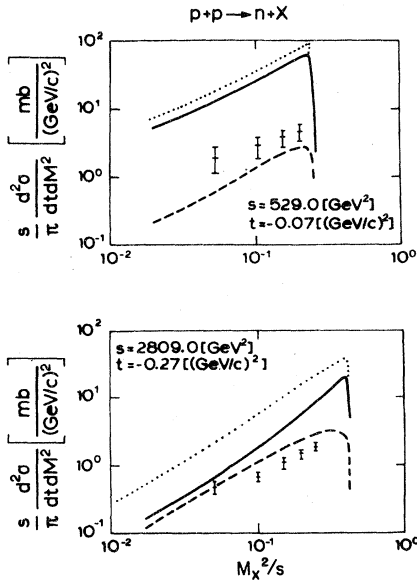


FIG. 3. The invariant cross sections for ( $p \frac{1}{2} n$ ) at two ISR energies plotted against  $M_X^2/s$  for fixed  $-t$ . Data from Ref. 10. Dotted curve indicates the pole contribution. Dashed curve indicates the cut contribution. Solid curve indicates the pole and cut contribution.

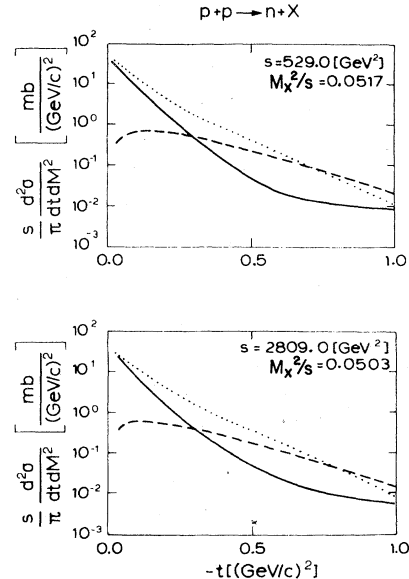


FIG. 4. The invariant cross section for ( $p \frac{1}{2} n$ ) at two ISR energies plotted against  $-t$  for fixed  $M_X^2/s$ . See caption of Fig. 3.

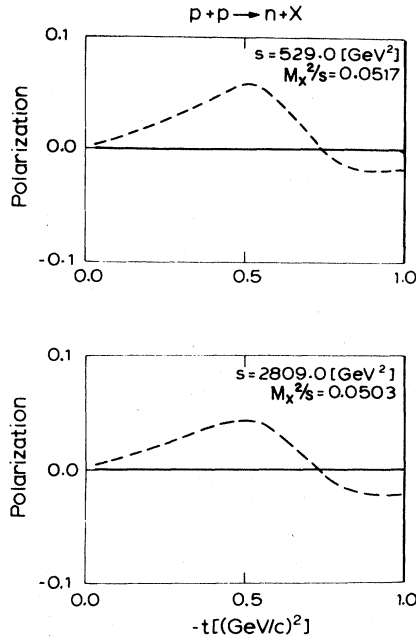


FIG. 5. The polarization of the recoil neutron for ( $p \perp n$ ) at two ISR energies plotted against  $-t$  for fixed  $M_X^2/s$ . See caption of Fig. 3.

normalization of the invariant cross section, the theoretical results still overestimate the experimental data by factors up to 10. The data<sup>10</sup> are not of such good quality that we can make any judgment of whether the  $M_X^2/s$  dependence of the Mueller-Regge pole amplitudes is improved by the addition of absorptive cuts. Data with improved statistics will be necessary to test this point.

The invariant cross section for ( $p \perp n$ ) as a function of the four-momentum transfer for almost the same value of  $M_X^2/s$  is shown in Fig. 4. The dominance of  $\pi$  exchange is clearly seen in the diagram with the strong forward peak as  $t$  tends to  $t_{\min}$ . The effect of the absorptive cut correction is to lower the forward normalization slightly and to considerably increase the rate of falloff of the cross section. The scaling of the cross section is evident in this diagram.

In Fig. 5 we have plotted the polarization of the recoil neutron as a function of the four-momentum transfer for fixed  $M_X^2/s$ . Again the scaling behavior of the amplitudes is clearly manifesting itself. The polarization vanishes at  $t_{\min}$  by angular momentum conservation, rises to a maximum of about 6% positive polarization at  $-t \approx 0.50$  ( $\text{GeV}/c^2$ ), goes through zero at about  $-t \approx 0.70$  ( $\text{GeV}/c$ ), and attains a negative value of about 2%. This behavior of the polarization is quite dramatic and would be an interesting test of the model.

Figure 6 shows the polarization as a function of  $M_X^2/s$  for fixed momentum transfer. The polariza-

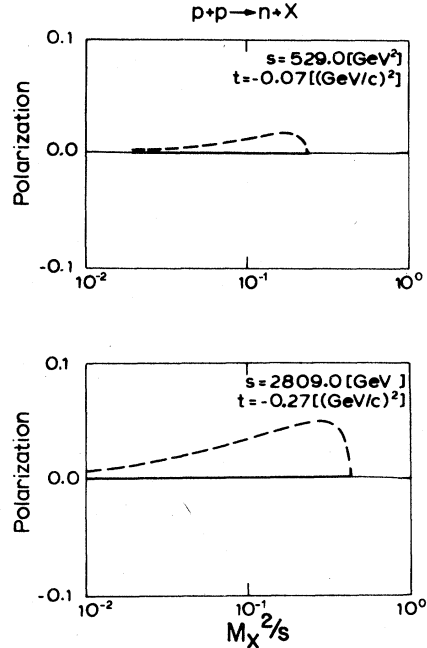


FIG. 6. The polarization of the recoil neutron for ( $p \perp n$ ) at two ISR energies plotted against  $M_X^2/s$  for fixed  $-t$ . See caption of Fig. 3.

tion, as shown in Fig. 5, only reaches appreciable values for large  $-t$  values.

The predictions of our model are compared in Fig. 7 with the data from Fermilab<sup>11</sup> on the reaction ( $n \perp p$ ) at four different incident momenta in the range  $s = 109 \text{ GeV}^2$  to  $s = 756 \text{ GeV}^2$ . These data are of much higher quality than the CERN ISR data shown previously and, as such, provide a much more stringent test of our model calculations. We observe that the  $s$ ,  $t$ , and  $M_X^2/s$  dependence of the theoretical distributions are consistent with the data. The modified Mueller-Regge pole amplitudes can be seen to interpolate the experimental data and represents a truly remarkable parameter-free fit to the data, while the unmodified amplitudes overestimate the experimental data by factors of about 3. Thus, we see that our conjecture that absorptive cut corrections could remedy the normalization problem for the invariant cross section is correct. This provides yet another stimulus to studying cut corrections to the Mueller-Regge model with simple poles, in addition to those already mentioned in the Introduction.

Our results for the invariant cross section for ( $n \perp p$ ) as a function of the momentum transfer, for fixed  $M_X^2/s$ , are compared with the data in Fig. 8. Again the agreement of the modified amplitudes with the experimental data is remarkable.

In Fig. 9 we present a plot of the polarization for the proton for ( $n \perp p$ ) at fixed  $s$ , as a function of

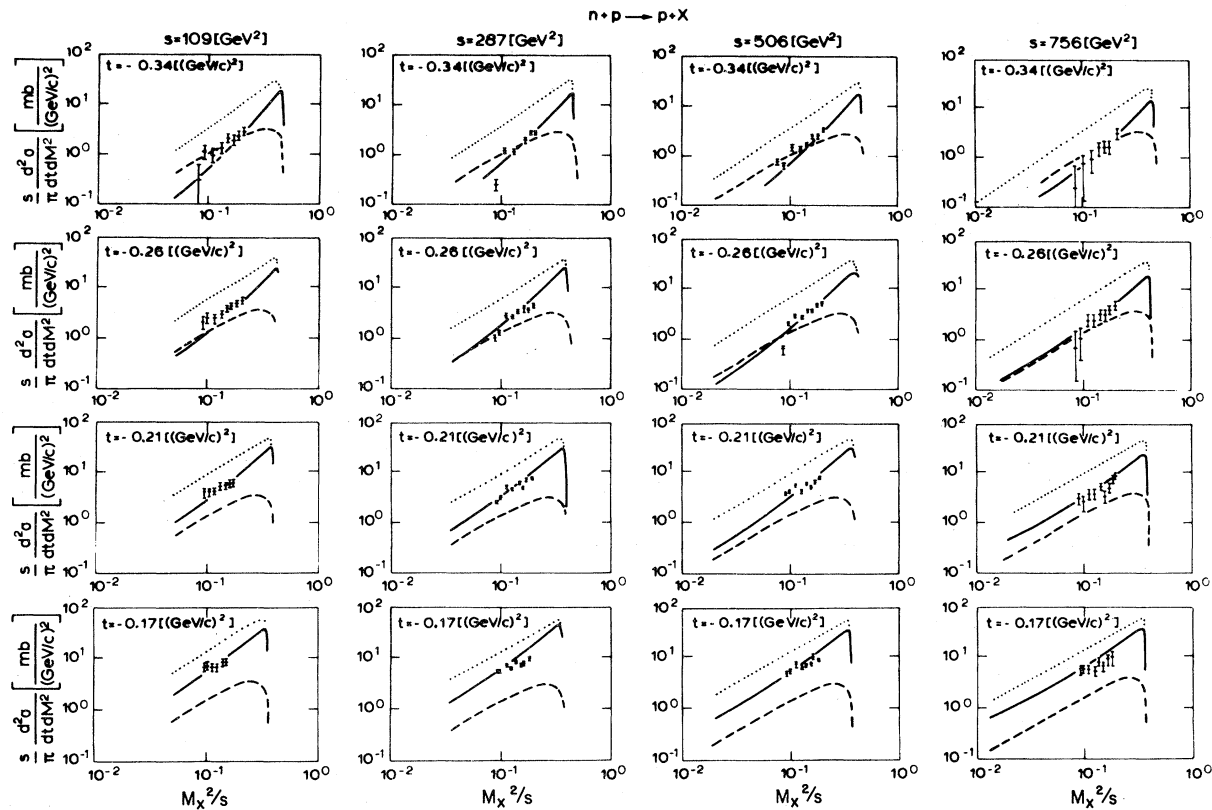


FIG. 7. The invariant cross section for  $(n \rightarrow p)$  at some Fermilab energies plotted against  $M_X^2/s$  for fixed  $-t$ . Data from Ref. 11. See caption of Fig. 3.

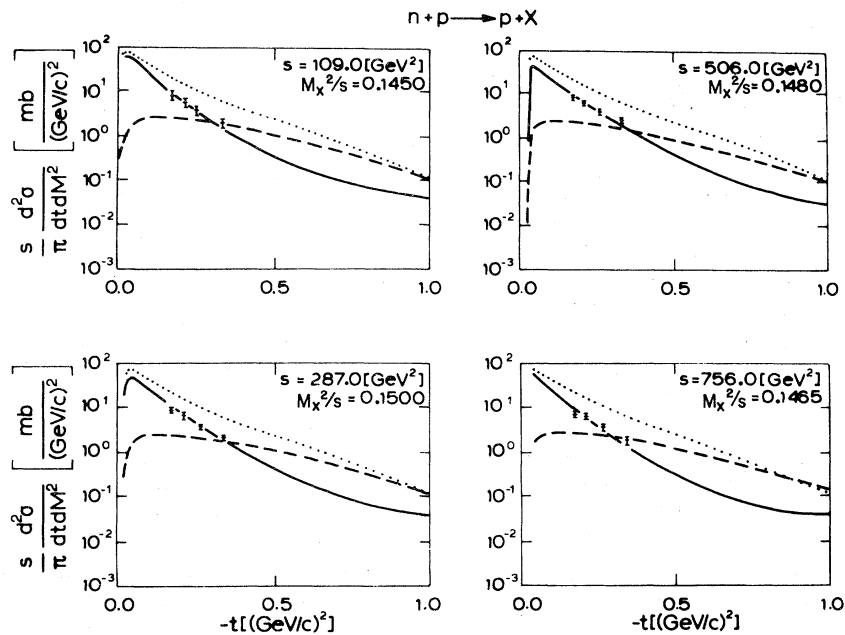


FIG. 8. The invariant cross section for  $(n \rightarrow p)$  at some Fermilab energies plotted against  $-t$  for fixed  $M_X^2/s$ . Data from Ref. 11. See caption of Fig. 3.

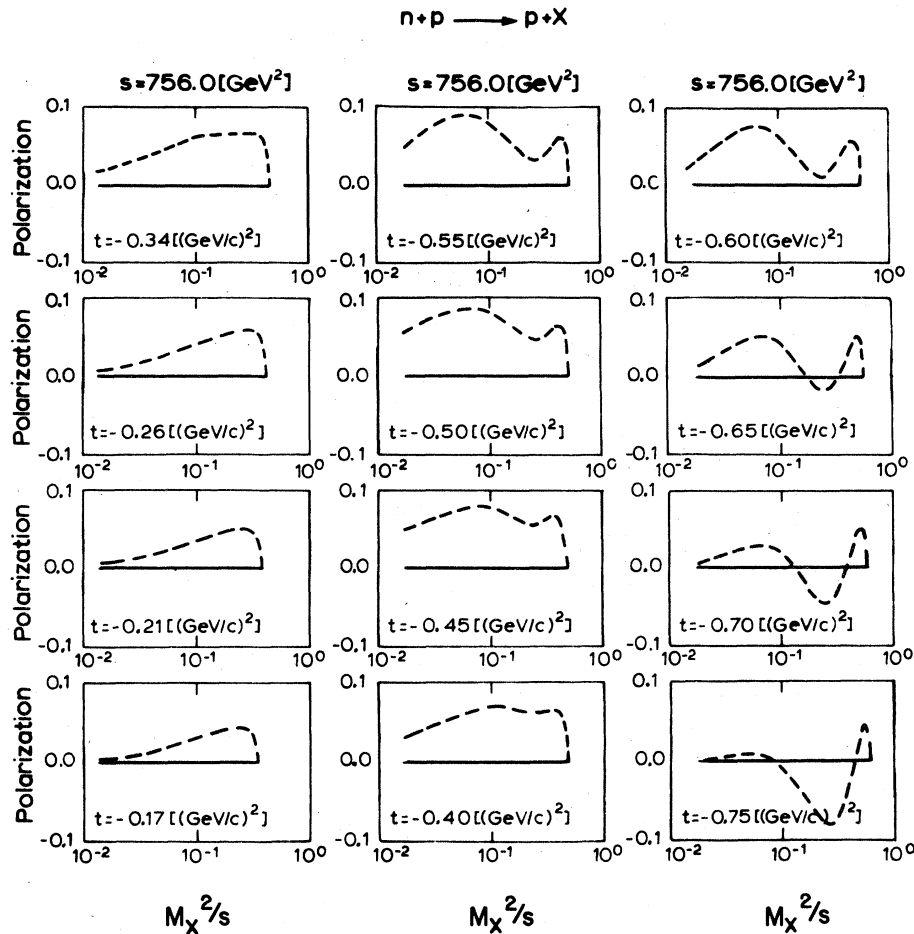


FIG. 9. The polarization of the recoil proton for  $(n \rightarrow p)$  at one energy plotted against  $M_x^2/s$  for a range of fixed  $-t$  values. See caption of Fig. 3.

$M_x^2/s$ , for twelve momentum transfers  $-t$  in the range 0.17 to 0.75  $(\text{GeV}/c)^2$ . This shows that the polarization can be expected to be positive, with a value up to 5%, at the upper end of this range and to be negative, with a value of up to 5%, at the lower end of this range.

It is easy to extend our model to the process  $(p \rightarrow \Lambda)$ . This is carried out by replacing the  $\pi$ ,  $\rho$ , and  $A_2$  trajectories by the  $K$ ,  $K^*(890)$ , and  $K^*(1420)$  trajectories, respectively, and changing the coupling constants accordingly. This reaction is now of interest because the polarization of the recoil  $\Lambda$  reveals itself through its decay. It is therefore an easy experimental quantity to measure. In a recent paper by Overseth<sup>12</sup> it was shown that the polarization of the  $\Lambda$  had been measured in an experiment at  $s=300 \text{ GeV}^2$ . In the range  $0 \leq p_T \leq 1.8(\text{GeV}/c)^2$  the polarization was shown to rise from zero to a value of about 10%. In fact, the polarization of the  $\Lambda$  looks very much

like our Fig. 5 in the range  $-t_{\min} \leq -t \leq 0.5 (\text{GeV}/c)^2$ . The theoretical calculation of the  $\Lambda$  polarization has been successfully carried out.<sup>13</sup>

#### ACKNOWLEDGMENTS

We wish to thank Professor H. G. Eggleston for encouragement in this work. One of the authors (K.J.M.M.) wishes to express his thanks to the Lady Davis Trust for the award of a Lady Davis Professorial Fellowship which made his sabbatical visit to the Technion possible. The financial support of Royal Holloway College is gratefully acknowledged by two of the authors (J. P. R. and A. U.), further financial support by the Reza Pahlavi Cultural Foundation is acknowledged by one of the authors (J. P. R.), while one of the authors (J. H. T.) wishes to thank the Science Research Council of Great Britain for a Research Studentship.

## APPENDIX: HELICITY AMPLITUDES

We first write the unmodified amplitudes, given in Appendix B of Ref. 5, in the form of products of exponentials and powers of  $\tau$ . We can then perform the Fourier-Bessel transformations analytically.

We can write our amplitudes in the following compact form which explicitly exhibits the  $\tau$  dependence:

$$\begin{aligned}\phi_{\pm\pm}^r(s, \tau, M_X^2) &= \sum_{i=1}^4 P_{\pm\pm}^i (1 - \tau^2/8k^2) [\exp(-b_{1i}\tau^2) + \eta_r \exp(-b_{2i}\tau^2)], \\ \phi_{\pm\pm}^r(s, \tau, M_X^2) &= \sum_{i=1}^4 P_{\pm\pm}^i \left(\frac{\tau}{2k}\right) [\exp(-b_{1i}\tau^2) + \eta_r \exp(-b_{2i}\tau^2)], \\ \phi_{\pm\pm}^o(s, \tau, M_X^2) &= \left(R_{\pm\pm}^1 - R_{\pm\pm}^2 \frac{\tau^2}{2k^2}\right) (1 - \tau^2/2k^2) [\exp(-d_1\tau^2) + \eta_o \exp(-d_2\tau^2)],\end{aligned}$$

and

$$\phi_{\pm\pm}^o(s, \tau, M_X^2) = \left(R_{\pm\pm}^1 - R_{\pm\pm}^2 \frac{\tau^2}{2k^2}\right) \left(\frac{\tau}{2k}\right) [\exp(-d_1\tau^2) + \eta_o \exp(-d_2\tau^2)],$$

where

$$\begin{aligned}P_{\pm\pm}^i &= -\frac{1}{2} A_{\pm\pm}^r \left(\frac{s}{M_X^2}\right)^{\alpha_{\pm\pm}^r} [a_i \exp(b_i t_{\min})], \\ A_{\pm\pm}^r &= \frac{[q(E_2 + m_2) \mp k(E_4 + m_4)]}{[(E_2 + m_2)(E_4 + m_4)]^{1/2}} g \Gamma_5^{\lambda K}, \\ b_{1i} &= \left[\alpha_{\pm\pm}^r \ln\left(\frac{s}{M_X^2}\right) + b_i\right] \frac{q}{k}, \quad b_{2i} = b_{1i} - i\pi \alpha_{\pm\pm}^r \frac{q}{k}, \\ R_{\pm\pm}^1 &= \frac{Z_1 \left(\frac{s}{M_X^2}\right)^{\alpha_{\pm\pm}^o - 1}}{2} (A_{\pm\pm}^o + B_{\pm\pm}^o) \exp(Z_2 t_{\min}), \\ R_{\pm\pm}^2 &= \frac{Z_1 \left(\frac{s}{M_X^2}\right)^{\alpha_{\pm\pm}^o - 1}}{2} B_{\pm\pm}^o \exp(Z_2 t_{\min}), \\ A_{\pm\pm}^o &= \{C_1 C_{\pm} [E_1(E_2 + E_4) + k^2] \mp 2C_2 C_{\pm} kq(E_1 + E_2)\} V^{1/2}, \\ B_{\pm\pm}^o &= [C_1 C_{\mp} + 2C_2 C_{\pm} (E_1 + E_2)] kq V^{1/2}, \\ C_1 &= \frac{1}{2m_N} F_C, \quad C_2 = \frac{1}{4m_N^2} F_M, \quad C_{\pm} = \frac{[(E_2 + m_2)(E_4 + m_4) \pm kq]}{[(E_2 + m_2)(E_4 + m_4)]^{1/2}}, \\ d_1 &= \left(\alpha_{\pm\pm}^o \ln \frac{s}{M_X^2} + Z_2\right) \frac{q}{k}, \quad d_2 = d_1 - i\pi \alpha_{\pm\pm}^o \frac{q}{k}, \\ \alpha_{0r}^r &= \alpha_{0r}^o + \alpha_{\pm}^r t_{\min}, \quad \alpha_{0o}^o = \alpha_{0o}^o + \alpha_{\pm}^o t_{\min}, \\ \alpha_{\pm}^r &= \alpha_{0r}^r - \alpha_{\pm}^r \frac{q}{k} \tau^2, \quad \alpha_{\pm}^o = \alpha_{0o}^o - \alpha_{\pm}^o \frac{q}{k} \tau^2, \\ \eta_r &= \tau_r \exp(-i\pi \alpha_{0r}^r), \quad \eta_o = \tau_o \exp(-i\pi \alpha_{0o}^o),\end{aligned}$$

where the  $\Gamma$  function for the pion propagator for the region  $|t| \leq 1$  was approximated by a quadruple exponential of the form  $\sum_{i=1}^4 a_i \exp(b_i t)$ , where  $a_1 = 22.79791$ ,  $a_2 = 35.67416$ ,  $a_3 = 10.96054$ ,  $a_4 = 0.73482$ ,  $b_1 = 99.25420$ ,  $b_2 = 23.12340$ ,  $b_3 = 3.72921$  and  $b_4 = -0.42392$ , while the function  $\Gamma(1 - \alpha_{\rho}(t))/(1 - t/m_{\rho}^2)$  for the  $\rho$  propagator for the region  $|t| \leq 1$  was approximated by an exponential of the form  $Z_1 \exp(Z_2 t)$ , where  $Z_1 = 1.518186$  and  $Z_2 = 1.924948$  and  $\tau_{\pi}$  and  $\tau_{\rho}$  are the  $\pi$  and the  $\rho$  signature factors, respectively.

These expressions can then be transformed according to the expressions given in Sec. II. We obtain

$$\phi_{\pm\pm}^{r\text{abs}}(s, \tau, M_X^2) = \phi_{\pm\pm}^r(s, \tau, M_X^2) - C \exp\left(-\frac{\tau^2}{4\lambda}\right) \left(\sum_{i=1}^4 P_{\pm\pm}^i \left\{ \left[ \frac{\exp[\tau^2/4\lambda(4\lambda b_{1i} + 1)]}{(4\lambda b_{1i} + 1)} \left(1 - \frac{4\lambda(4\lambda b_{1i} + 1) + \tau^2}{8k^2(4\lambda b_{1i} + 1)^2}\right) \right] + \eta_r [b_{1i} - b_{2i}] \right\}\right),$$

$$\phi_{+-}^{\text{rabs}}(s, \tau, M_X^2) = \phi_{+-}^{\text{r}}(s, \tau, M_X^2) - C \left( \frac{\tau}{k} \right) \exp\left(-\frac{\tau^2}{2k}\right) \left\{ \sum_{i=1}^4 P_i^+ \left[ \frac{\exp[\tau^2/4\lambda(4\lambda b_{1i} + 1)]}{(4\lambda b_{1i} + 1)^2} \right] + \eta_r(b_{1i} - b_{2i}) \right\},$$

$$\begin{aligned} \phi_{++}^{\text{rabs}}(s, \tau, M_X^2) = \phi_{++}^{\text{r}}(s, \tau, M_X^2) - C \exp\left(-\frac{\tau^2}{4\lambda}\right) & \left\{ \frac{\exp[\tau^2/4\lambda(4\lambda d_1 + 1)]}{(4\lambda d_1 + 1)} \right. \\ & \times \left[ R_{++}^1 - \frac{(R_{++}^1 + 4R_{++}^2)}{8k^2(4\lambda d_1 + 1)^2} [4\lambda(4\lambda d_1 + 1) + \tau^2] \right. \\ & \left. \left. + \frac{R_{++}^2}{16k^2(4\lambda d_1 + 1)^4} [\tau^2 + 16\tau^2\lambda(4\lambda d_1 + 1) + 32\lambda^2(4\lambda d_1 + 1)^2] \right] \right\} \\ & + \eta_o\{d_1 - d_2\}, \end{aligned}$$

and

$$\begin{aligned} \phi_{+-}^{\text{pabs}}(s, \tau, M_X^2) = \phi_{+-}^{\text{p}}(s, \tau, M_X^2) - C \left( \frac{\tau}{2k} \right) \exp\left(-\frac{\tau^2}{4\lambda}\right) & \left\{ \frac{\exp[\tau^2/4\lambda(4\lambda d_1 + 1)]}{(4\lambda d_1 + 1)^2} \left( R_{+-}^1 - \frac{R_{+-}^2}{2k^2} \frac{[\tau^2 + 8\lambda(4\lambda d_1 + 1)]}{(4\lambda d_1 + 1)^2} \right) \right. \\ & \left. + \eta_o\{d_1 - d_2\} \right\}. \end{aligned}$$

The values of the absorption parameters  $C$  and  $\lambda$  are given in Table I.

\*Permanent address: Department of Mathematics, Royal Holloway College, Egham, Surrey, TW20 OEX, United Kingdom.

<sup>1</sup>R. C. Brower, C. E. Detar, and J. H. Weis, *Phys. Rep.* **14C**, 257 (1974).

<sup>2</sup>J. Pumplin, *Phys. Rev. D* **13**, 1249 (1976); **13**, 2161 (1976); F. Paige and D. P. Sidhu, *ibid.* **13**, 3015 (1976); **14**, 2307 (1976); G. R. Goldstein and J. F. Owens, *Nucl. Phys.* **B118**, 29 (1977).

<sup>3</sup>N. S. Craigie, G. Kramer, and J. Körner, *Nucl. Phys.* **B68**, 509 (1974); N. S. Craigie and G. Kramer, *ibid.* **B75**, 509 (1974). There is an error in the definition of  $C$  in the preceding reference. This has been corrected in the following reference: K. Ahmed, J. G. Körner, G. Kramer, and N. S. Craigie, *ibid.* **B108**, 275 (1976).

<sup>4</sup>K. J. M. Moriarty, J. H. Tabor, and A. Ungkitchanukit, *Phys. Rev. D* **16**, 130 (1977); **D 18**, 717 (1978); K. J. M. Moriarty and H. N. Thompson, *ibid.* **21**, 2570 (1980).

<sup>5</sup>K. J. M. Moriarty, J. P. Rad, J. H. Tabor, and A. Ungkitchanukit, *Acta Phys. Austriaca* **46**, 105 (1977).

<sup>6</sup>A. Garcia Azcárate, *Phys. Rev. D* **17**, 3022 (1978).

<sup>7</sup>R. G. Sachs, *Phys. Rev.* **126**, 2256 (1962).

<sup>8</sup>K. J. M. Moriarty and J. H. Tabor, *Comput. Phys. Commun.* **12**, 277 (1976); also, in preparation.

<sup>9</sup>J. Anderson, K. J. M. Moriarty, and R. C. Beckwith, *Comput. Phys. Commun.* **9**, 85 (1975); J. Anderson, R. C. Beckwith, K. J. M. Moriarty, and J. H. Tabor, *ibid.* **15**, 437 (1978).

<sup>10</sup>J. Engler, B. Gibbard, W. Isenbeck, F. Mönning, J. Moritz, K. Pack, K. H. Schmidt, D. Wegener, W. Bartel, W. Flauger, and H. Schopper, *Nucl. Phys.* **B84**, 70 (1975).

<sup>11</sup>B. Robinson, K. Abe, J. Carr, J. Keyne, A. Pagnamenta, F. Sannes, I. Siotis, and R. Stanek, *Phys. Rev. Lett.* **34**, 1475 (1975).

<sup>12</sup>O. E. Overseth, in *New Fields in Hadronic Physics*, proceedings of the XIth Rencontre de Moriond, Flaine-Haute-Savoie, 1976, edited by J. Trân Thanh Vân (CNRS, Paris, 1976), p. 291.

<sup>13</sup>K. J. M. Moriarty, J. P. Rad, J. H. Tabor, and A. Ungkitchanukit, *Lett. Nuovo Cimento* **17**, 366 (1976); in *High Energy Physics with Polarized Beams and Targets*, proceedings of the Argonne Symposium, 1976, edited by M. Marshak (AIP, New York, 1976), p. 174; K. J. M. Moriarty and J. P. Rad, *Lett. Nuovo Cimento* **19**, 393 (1977).

<sup>14</sup>R. W. B. Ardill, P. Choudhury, K. J. M. Moriarty, and A. Ungkitchanukit, *Acta Phys. Austriaca* **46**, 27 (1976);