Energy dependence of inclusive and semi-inclusive processes in the quark-cascade jet model

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(Received 25 June 1980)

We consider here the energy dependence of different inclusive and semi-inclusive processes in the quark-cascade jet model for hadron production. The corresponding integral equations for inclusive quark fragmentation functions are derived. The energy dependence of individual multiplicities for quark-antiquark jets is also discussed. The results are calculated with different primordial quark fragmentation functions and compared with experiments. It is further suggested that the diffractive dissociation of mesons may also be possible examples of hadronization of high-energy quark-antiquark pairs corresponding to the invariant mass of the dissociated hadrons and this hypothesis is examined in the context of the present model. There is general agreement with the experimental results when the energies are not too high.

I. INTRODUCTION

The quark-cascade jet-production model[1-4] is based on repeated quark fragmentations $Q_i \to M + Q_j$ with a sharing of the longitudinal momentum of the original quark by the resulting quark and the meson. This is described by primordial quark fragmentation functions $f^M_{ij}(x)$ as phenomenological inputs.[3,4] The model is usually formulated in the high-energy limit. The energy dependence at finite energies is reproduced by the kinematics of the particles generated[3] or by assuming that when quarks lose momentum they finally stop fragmenting.[4] With the latter point of view, we shall analyze here the energy dependence of the quark jets regarding exclusive multiplicities as well as regarding the final-meson fragmentation functions. For this purpose in Sec. II we give the description of conventional quark cascades with an explicit energy dependence assuming that the quarks stop fragmenting when a momentum μ is ultimately reached.[4,5] In the same manner as some earlier investigations[5] only quark jets which end in soft quarks are explicitly considered and the corresponding probabilities are examined. We include the flavor degree of freedom as well as the possibility of symmetry breaking. In Sec. III we write down the energy-dependent integral equations and obtain the formal solutions. In Sec. IV we apply these results to pion production from u and d quark-antiquark pairs and compare the calculated results with the experimental observations. For comparison with experiments we assume here that in the diffractive dissociation process $\pi^- p \to Xp$, the hadronic system X is effectively the hadronization of $d\bar{u}$ quark-antiquark pairs. The results appear to be broadly consistent with such a picture. In Sec. V we discuss the general nature of the results.

II. GENERAL THEORY

Our motivation here is to modify the conventional ideas of the quark-cascade model to include an energy dependence for the quark jets. As mentioned, the fragmentation process

$$Q_i \to M + Q_j \tag{2.1}$$

is described by the primordial quark fragmentation function $f^M_{ij}(x)$, where x is the longitudinal fraction of the momentum carried by the meson. Also we adopt here the normalization

$$\int_0^1 f^M_{ij}(x)dx = 1 \tag{2.2}$$

whenever the above function does not vanish identically. We next define *matrices* $g^M(y)$ and $g(y)$ such that

$$g^M_{ij}(y) = p(Q_i, MQ_j) f^M_{ij}(1-y) \tag{2.3}$$

and

$$g_{ij}(y) = \sum_M g^M_{ij}(y). \tag{2.4}$$

In (2.3) $p(Q_i, MQ_j)$ is the probability that the quark Q_i fragments to M and Q_j. Clearly in contrast to (2.2) we have

$$\sum_{j,M} \int g^M_{ij}(y)dy = 1. \tag{2.5}$$

We shall now consider the quark Q_i of momentum P fragmenting n times and leaving the quark Q_j of momentum μ. Then the corresponding probability is proportional to[5]

$$[g(y_1)g(y_2)\cdots g(y_n)]_{ij}\delta(y_1y_2\cdots y_n - \lambda)dy_1dy_2\cdots dy_n, \tag{2.6}$$

where we have considered the probability in the differential form. Also, $\lambda=\mu/P$ and the δ function gives the constraint regarding the momentum of the residual quark.[5] Now we define the function

$$F^M_{ij}(n,k,x,\lambda) = \int [g(y_1)\cdots g(y_{k-1})g^M(y_k)\cdots g(y_n)]_{ij}dy_1\cdots dy_n \times \delta(y_1\cdots y_n-\lambda)\delta(y_1\cdots y_{k-1}(1-y_k)-x). \tag{2.7}$$

Clearly (2.7) will be proportional to the probability that the quark Q_i fragments n times leaving the residual quark Q_j with longitudinal momentum fraction λ and that a kth-rank meson M is produced with momentum fraction x of the original quark Q_i. We further define

$$C_{ij}(n,\lambda)=\sum_M \int F^M_{ij}(n,k,x,\lambda)dx = \int [g(y_1)\cdots g(y_n)]_{ij} \times \delta(y_1\ldots y_n-\lambda)dy_1\ldots dy_n. \tag{2.8}$$

We obviously take the total probability for hadronization as unity. This leads to the normalization constants

$$C_{ij}(\lambda)=\sum_n C_{ij}(n,\lambda) \tag{2.9a}$$

and

$$C_i(\lambda)=\sum_j C_{ij}(\lambda). \tag{2.9b}$$

Hence, as is conventional, ignoring the residual quark, the probability that the quark Q_i gives mesons with multiplicity n is given by

$$p_i(n,\lambda)=\sum_j C_{ij}(n,\lambda) \Big/ C_i(\lambda). \tag{2.10}$$

Also, the energy-dependent final-meson fragmentation function $D^M_i(x,\lambda)$ is now given as

$$D^M_i(x,\lambda)=\sum_{n,k,j} F^M_{ij}(n,k,x,\lambda) \Big/ C_i(\lambda). \tag{2.11}$$

The energy dependence above is obtained through the assumption that the quark stops fragmenting once the momentum fraction λ is reached. The parameter μ introduced above is irrelevant in the high-energy limit, but at finite energies it sets up a suitable scale for the approach to the high-energy limit.

Parallel to (2.4) we may define

$$D_i(x,\lambda)=\sum_M D^M_i(x,\lambda), \tag{2.12}$$

which is the resultant fragmentation function for any meson to be produced with momentum fraction x. We then have the average multiplicity as given by

$$M_i(\lambda)=\sum_n np_i(n,\lambda)=\int D_i(x,\lambda)dx, \tag{2.13}$$

as is conventional. However, here we have also

$$\int D_i(x,\lambda)x\,dx=1-\lambda. \tag{2.14}$$

Equation (2.14) merely reflects the fact that the residual quark carries a momentum fraction λ, which goes to zero in the high-energy limit.

We note that in the present analysis we are retaining the energy dependence of the quark-cascade jet-production model through the variable μ. Thus our calculations will be reliable when μ behaves like an average value and sets up a scale, but will be unreliable otherwise.[5]

III. EXCLUSIVE MULTIPLICITIES AND THE MESON FRAGMENTATION FUNCTIONS

We shall now need to evaluate $C_i(\lambda)$ and $C_{ij}(\lambda)$. From (2.8) we note that for $n\geq 1$ we have the matrix recurrence equation

$$C(n+1,\lambda)=\int_\lambda^1 \frac{dy}{y}g(y)C\left(n,\frac{\lambda}{y}\right). \tag{3.1}$$

We also note that by (2.9a) the above equation implies the integral equation

$$C(\lambda)=g(\lambda)+\int_\lambda^1 \frac{dy}{y}g(y)C\left(\frac{\lambda}{y}\right), \tag{3.2}$$

since $C(1,y)=g(y)$. As earlier[3] we now consider the Mellin transforms[6]

$$\tilde{g}(\omega)=\int_0^1 g(y)y^{\omega-1}dy, \tag{3.3}$$

which has the inverse with $c>0$ as[6]

$$g(y)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} y^{-\omega}\tilde{g}(\omega)d\omega. \tag{3.4}$$

The other Mellin transforms are also defined in the same manner. We thus obtain from (3.1) the matrix equation

$$\tilde{C}(n,\omega)=[\tilde{g}(\omega)]^n. \tag{3.5}$$

Equation (3.2) also yields

$$\tilde{C}(\omega)=[I-\tilde{g}(\omega)]^{-1}\tilde{g}(\omega). \tag{3.6}$$

For specific examples we may take the inverse Mellin transforms of (3.5) and (3.6) and then obtain the energy dependence of fixed multiplicities by using (2.10).

We now note that from the normalization (2.5) we have

$$\sum_j \tilde{g}_{ij}(\omega)\Big|_{\omega=1} = 1\,. \tag{3.7}$$

Hence, writing (3.6) as

$$\tilde{C}(\omega) = \tilde{g}(\omega) + \tilde{C}(\omega)\tilde{g}(\omega)\,,$$

we obtain from (2.9b) *when* $\omega = 1$

$$\tilde{C}_i(\omega) = 1 + \tilde{C}_i(\omega)\,. \tag{3.8}$$

The contradiction in (3.8) implies that $\tilde{C}_i(\omega)$ has a singularity at $\omega = 1$. Assuming this to be a pole[3] we write

$$\tilde{C}_i(\omega) \approx R_i/(\omega - 1) \tag{3.9}$$

in the neighborhood of $\omega = 1$. In the cases we have examined, the contribution from (3.9) appears to give the dominant contribution for the inverse Mellin transforms in (3.4) for small λ. In fact, we then have, *for small* λ,

$$C_i(\lambda) = R_i/\lambda\,. \tag{3.10}$$

It was earlier noted that in case there is only one flavor, R^{-1} is given by[3]

$$R^{-1} = \int g(y)\ln(1/y)dy\,. \tag{3.11}$$

We shall now consider the inclusive fragmentation functions for mesons. We thus define

$$F_{ij}(n,k,x,\lambda) = \sum_M F^M_{ij}(n,k,x,\lambda)\,. \tag{3.12}$$

We then have the recurrence matrix equations

$$F(n+1,k+1,x,\lambda) = \int \frac{dy}{y^2} g(y) F\left(n,k,\frac{x}{y},\frac{\lambda}{y}\right) \tag{3.13}$$

and

$$F(n+1,k,x,\lambda) = \int \frac{dy}{y} g(y) F\left(n,k,x,\frac{\lambda}{y}\right). \tag{3.14}$$

Introducing the double Mellin transforms

$$\tilde{F}(n,k,\omega_1,\omega_2) = \int\int F(n,k,x,\lambda) x^{\omega_1 - 1}\lambda^{\omega_2 - 1} dx\, d\lambda\,, \tag{3.15}$$

we then obtain from (3.13) and (3.14) that

$$\tilde{F}(n+1,k+1,\omega_1,\omega_2) = \tilde{g}(\omega_1+\omega_2-1)\tilde{F}(n,k,\omega_1,\omega_2) \tag{3.16}$$

and

$$\tilde{F}(n+1,k,\omega_1,\omega_2) = \tilde{g}(\omega_2)\tilde{F}(n,k,\omega_1,\omega_2)\,. \tag{3.17}$$

We now note that

$$\tilde{F}(1,1,\omega_1,\omega_2) = \int g(\lambda)(1-\lambda)^{\omega_1-1}\lambda^{\omega_2-1}d\lambda\,. \tag{3.18}$$

The above equations yield that

$$\tilde{F}(n,k,\omega_1,\omega_2) = [\tilde{g}(\omega_1+\omega_2-1)]^{k-1}[\tilde{g}(\omega_2)]^{n-k} \times \tilde{F}(1,1,\omega_1,\omega_2)\,, \tag{3.19}$$

where (3.18) is to be used. Hence, substituting

$$F(x,\lambda) = \sum_{n=1}^{\infty}\sum_{k=1}^{n} F(n,k,x,\lambda) = \sum_{k=1}^{\infty}\sum_{n=k}^{\infty} F(n,k,x,\lambda)\,, \tag{3.20}$$

we obtain that

$$\tilde{F}(\omega_1,\omega_2) = [I - \tilde{g}(\omega_1+\omega_2-1)]^{-1}[I - \tilde{g}(\omega_2)]^{-1} \times \tilde{F}(1,1,\omega_1,\omega_2)\,. \tag{3.21}$$

Hence, we now obtain that the energy-dependent one-meson fragmentation function of (2.12) is obtained as

$$D_i(x,\lambda) = \sum_j F_{ij}(x,\lambda)\Big/C_i(\lambda)\,, \tag{3.22}$$

where $F_{ij}(x,\lambda)$ and $C_i(\lambda)$ are obtained from the inverse Mellin transforms using (3.21) and (3.6), respectively.

We may note, however, that the inverse Mellin transform in (3.21) is plagued with double or repeated contour integration, which is difficult and often mathematically ambiguous. Hence, we shall now proceed to obtain an energy-dependent integral equation for $D_{ij}(x,\lambda)$.

Parallel to (3.20) we also define

$$F(n,x,\lambda) = \sum_{k=1}^{n} F(n,k,x,\lambda)\,. \tag{3.23}$$

We then have the matrix recurrence relation

$$F(n+1,x,\lambda) = \frac{g(1-x)}{(1-x)} C\left(n,\frac{\lambda}{(1-x)}\right) + \int \frac{dy}{y^2} g(y) F\left(n,\frac{x}{y},\frac{\lambda}{y}\right). \tag{3.24}$$

The above equation then yields the integral equation for $F(x,\lambda)$ as

$$F(x,\lambda) = g(1-x)\delta(1-x-\lambda) + \frac{g(1-x)}{(1-x)} C\left(\frac{\lambda}{1-x}\right) + \int \frac{dy}{y^2} g(y) F\left(\frac{x}{y},\frac{\lambda}{y}\right). \tag{3.25}$$

From the above integral equation using (3.22) we now obtain the equation for $D_i(x,\lambda)$ as

$$D_i(x,\lambda)=\sum_j g_{ij}(1-x)\delta(1-x-\lambda)C_i^{-1}(\lambda)+\sum_j g_{ij}(1-x)(1-x)^{-1}C_j\left(\frac{\lambda}{1-x}\right)C_i^{-1}(\lambda)+\sum_j\int\frac{dy}{y^2}g_{ij}(y)D_j\left(\frac{x}{y},\frac{\lambda}{y}\right)C_j\left(\frac{\lambda}{y}\right)C_i^{-1}(\lambda)\,. \tag{3.26}$$

In (3.26) we shall now consider the limit λ approaching zero and utilize (3.10). We then obtain the equation in the high-energy limit as

$$D_i(x,0)=\sum_j g'_{ij}(1-x)+\sum_j\int\frac{dy}{y}g'_{ij}(y)D_j\left(\frac{x}{y},0\right), \tag{3.27}$$

where we have substituted

$$g'_{ij}(y)=g_{ij}(y)R_i^{-1}R_j\,. \tag{3.28}$$

We now note that from (3.6) we also have

$$\tilde{C}_i(\omega)=\sum_j\tilde{g}_{ij}(\omega)+\sum_j\tilde{g}_{ij}(\omega)\tilde{C}_j(\omega), \tag{3.29}$$

such that

$$R_i=\sum_j\tilde{g}_{ij}(1)R_j\,. \tag{3.30}$$

The above equation implies that the matrix $g'_{ij}(y)$ in (3.27) is also normalized as

$$\sum_j\int g'_{ij}(y)dy=1, \tag{3.31}$$

which is the same as (2.5). Equation (3.27), being the high-energy limit, is the usual integral equation which is quoted.[3,4] However, we note that particularly for heavy quark jets, the approach to high energy as described by (3.26) can be quite relevant since in such cases the symmetry violations are known to be quite large.

IV. APPLICATIONS

We shall now consider some applications of the results developed in Sec. III. We first confine our attention to only pion production of a u and d quark-antiquark system. Including isotopic-spin invariance, in such a case the matrix (2.4) is given as

$$g(y)=(\tfrac{1}{3}+\tfrac{2}{3}\tau_1)g_\pi(y), \tag{4.1}$$

where $g_\pi(y)$ is an ordinary function, and the matrix part is factored out. We note in particular that $f_u^{\pi^+}(x)=g_\pi(1-x)=f_\pi(x)$ and that $p(u,\pi^+d)=\frac{2}{3}$, whereas $p(u,\pi^0u)=\frac{1}{3}$. Also we have the normalization

$$\int g_\pi(y)dy=1\,. \tag{4.2}$$

On taking the Mellin transform we obtain from (4.1) that

$$\tilde{g}(\omega)=(\tfrac{1}{3}+\tfrac{2}{3}\tau_1)\tilde{g}_\pi(\omega)\,. \tag{4.3}$$

Hence, (3.5) gives

$$\tilde{C}(n,\omega)=(A_n^e+A_n^0\tau_1)[\tilde{g}_\pi(\omega)]^n, \tag{4.4}$$

where we have substituted

$$A_n^e=\frac{1}{2}\left(1+\frac{(-1)^n}{3^n}\right) \tag{4.5}$$

and

$$A_n^0=\frac{1}{2}\left(1-\frac{(-1)^n}{3^n}\right). \tag{4.6}$$

We note that in the above A_n^e and A_n^0 are the probabilities that after n fragmentations the quark does or does not change its flavor. From (4.4) we have

$$\tilde{C}_i(n,\omega)=\sum_j\tilde{C}_{ij}(n,\omega)=[\tilde{g}_\pi(\omega)]^n\,. \tag{4.7}$$

Hence, corresponding to (2.9a) and (2.9b), we have in (3.6)

$$\tilde{C}_i(\omega)=\tilde{g}_\pi(\omega)/[1-\tilde{g}_\pi(\omega)]\,. \tag{4.8}$$

From (4.8) we note that as expected, the normalization constant (2.9b) is independent of the flavor of the quark. We may take the inverse Mellin transforms of (4.7) and (4.8) and by (2.10) write the exclusive probability for pion production with a fixed multiplicity to be given as, with $C_i(n,\lambda)=C_\pi(n,\lambda)$ and $C_i(\lambda)=C_\pi(\lambda)$,

$$p_i(n,\lambda)=C_\pi(n,\lambda)/C_\pi(\lambda)\,. \tag{4.9}$$

We note that (4.9) is a function of the quark energy through $\lambda=\mu/P$ with μ as a fixed parameter to be determined from experiments. Also here parallel to (3.10) and (3.11) we have in the high-energy limit when λ is small, by (4.8),

$$C_i(\lambda)=R_\pi/\lambda\,, \tag{4.10}$$

where[3]

$$R_\pi^{-1}=\int g_\pi(y)\ln(1/y)dy\,. \tag{4.11}$$

We now note that the total-multiplicity analysis of a single-quark jet is not experimentally very useful since it is difficult to precisely separate the hadrons belonging to single-quark jets. Due to color confinement the overlap between different jets will be inevitable and the individual jets will be necessarily ill defined. In the above analysis we have also ignored the residual quark. Hence, instead of considering the jet due to a single quark, we shall consider the hadronization of the quark-antiquark pair.

To fix our ideas let us consider the $u\bar{u}$ quark-antiquark pair and let n and m be the respective multiplicities of the u jet and $\bar{u}$ jet with the residual quark and antiquark being i and j. On the basis of the earlier analysis of quark jets, the probability for the above is *proportional to*

$$[(\tfrac{1}{3}+\tfrac{2}{3}\tau_1)^n]_{1i}C_\pi(n,\lambda)[(\tfrac{1}{3}+\tfrac{2}{3}\tau_1)^m]_{j1}C_\pi(m,\lambda). \quad (4.12)$$

In the above we have set $\lambda=\mu/(\sqrt{s}/2)$, where $\sqrt{s}$ is the c.m. energy of the $u\bar{u}$ pair. Here clearly $q_i\bar{q}_j$ will recombine to yield a pion. Now from the quark content of the pion and isotopic-spin invariance the probability for the recombination of $q_i\bar{q}_j$ to form a pion will be *proportional* to $(\frac{1}{3}+\frac{2}{3}\tau_1)_{ij}$. Hence in (4.12), taking further this probability of recombination into account, the probability that the u jet has n pions, the $\bar{u}$ jet has m pions, and there is a pion due to recombination becomes *proportional to*

$$[(\tfrac{1}{3}+\tfrac{2}{3}\tau_1)^{n+m+1}]_{11}C_\pi(n,\lambda)C_\pi(m,\lambda)=A_N^eC_\pi(n,\lambda)C_\pi(m,\lambda). \quad (4.13)$$

In the above, $N=n+m+1$ is the total multiplicity, with $N\geq 3$. We are obviously taking the energy $\sqrt{s}$ to be large enough so that the above assumption is valid. We shall now examine the individual charged multiplicities during pionization of the $u\bar{u}$ pair as a function of energy. From charge conservation, clearly here we shall have only an even number of charged mesons. We shall now obtain the individual probabilities by normalizing the total probability to unity. Clearly the normalization constant from (4.13) becomes

$$C_e(\lambda)=\sum_{N\geq 3}A_N^eB_N(\lambda), \quad (4.14)$$

where we have substituted

$$B_N(\lambda)=\sum_{\substack{n,m\\ n+m+1=N}}C_\pi(n,\lambda)C_\pi(m,\lambda). \quad (4.15)$$

Obviously from (4.14) we obtain, parallel to (2.10),

$$P_e(N,\lambda)=A_N^eB_N(\lambda)/C_e(\lambda). \quad (4.16)$$

To obtain the signal of charged pions, we note that in the binomial expansion of $(\frac{1}{3}+\frac{2}{3}\tau_1)^N$, the power of τ_1 will exactly correspond to the number of charged pions. Now, the probability that there are $2l$ charged pions with the total multiplicity being N is given by

$$\frac{2^{2l}}{3^N}{}^NC_{2l}/A_N^e. \quad (4.17)$$

Hence, the inclusive probability that there are $2l$ charged pions (and an arbitrary number of neutral pions) is given as

$$P_c(2l,\lambda)=\sum_{N\geq 2l}P_e(N,\lambda)\frac{2^{2l}}{3^N}{}^NC_{2l}\Big/A_N^e. \quad (4.18)$$

We also note that the exclusive probability that there are *only* $2l$ charged pions is given as

$$P_c^{\text{excl}}(2l,\lambda)=(\tfrac{2}{3})^{2l}B_{2l}(\lambda)/C_e(\lambda). \quad (4.19)$$

We note that all the above statements are also true for the $d\bar{d}$ quark-antiquark pair.

We next consider pionization of the $u\bar{d}$ quark-antiquark pair. As before we obtain that in this case the probability that the u jet has n pions, the $\bar{d}$ jet has m pions, and the total multiplicity is $N=n+m+1$ with one recombination meson is proportional to

$$[(\tfrac{1}{3}+\tfrac{2}{3}\tau_1)^N]_{12}C_\pi(n,\lambda)C_\pi(m,\lambda)=A_N^0C_\pi(n,\lambda)C_\pi(m,\lambda), \quad (4.20)$$

which is the parallel of (4.13). Parallel to (4.14), the present normalization constant becomes, with (4.15),

$$C_0(\lambda)=\sum_{N\geq 3}A_N^0B_N(\lambda). \quad (4.21)$$

We also obtain, corresponding to (4.16), that the probability that the $u\bar{d}$ pair gives rise to exactly N pions is given as

$$P_0(N,\lambda)=A_N^0B_N(\lambda)/C_0(\lambda). \quad (4.22)$$

In the present case the total number of charged pions produced will always be odd. As before, we now obtain the inclusive probability for the production of $2l+1$ charged pions with any total multiplicity as

$$P_c(2l+1,\lambda)=\sum_{N\geq 2l+1}P_0(N,\lambda)\frac{2^{2l+1}}{3^N}{}^NC_{2l+1}\Big/A_N^0. \quad (4.23)$$

Also, we get the exclusive probability for the production of exactly $(2l+1)$ charged pions as

$$P_c^{\text{excl}}(2l+1,\lambda)=(\tfrac{2}{3})^{2l+1}B_{2l+1}(\lambda)/C_0(\lambda). \quad (4.24)$$

We shall now explicitly examine the energy dependence of the above probability with calculations and compare the same with experimental results as are available. For this purpose we need to make specific assumptions regarding $g_\pi(y)$ in (4.1). Before doing that, let us first see the type of experimental data with which we may be able to compare our calculations.

We note that e^+e^- annihilation yields hadronization of quark-antiquark pairs. This, however, contains mixed information since even above 1 GeV $s\bar{s}$ quark-antiquark pairs will be formed, and also above 3–4 GeV, charm signals will constitute a major fraction of the hadrons produced. The decay channels of these charmed mesons are not adequately known[7] for us to be able to use the experimental results in the background of the present analysis regarding individual multiplicity signals of specific quark-antiquark jets.

We may note, however, that another class of result has existed for quite some time which we may imagine as hadronization of quark-antiquark pairs: the diffractive dissociation of pions in $\pi^- p$ scattering.[8] For the pion dissociation in $\pi^- p$ diffractive dissociation, we imagine that the pion dissociates to the $d\bar{u}$ pair, which subsequently hadronizes. In the context of quark-parton ideas[1,9] the above appears to be a simple but inevitable picture for pion dissociation, where in the present context we need not worry about the dynamical mechanism[10] which causes the "dissociation" of π^- to $d\bar{u}$, since in the conventional picture the hadrons produced will be independent of this. Assuming the above to be the mechanism for the dissociation process as a two-stage process,[10] here we shall have a fairly clean instance of hadronization of a nonstrange-light-quark–antiquark pair. We shall thus utilize our present analysis to calculate, e.g., the charge multiplicity of the system X for the diffractive-dissociation process $\pi^- p \to Xp$ as a function of $M_X = M$. In this analysis $\lambda = \mu/(M/2)$, where we shall choose the parameter μ as 500 MeV as we had estimated earlier,[5] this also being generally the region for nonperturbative infrared cutoff in quantum chromodynamics. In order to justify such a picture we shall analyze, e.g., the total charged multiplicity for the process $\pi^- p \to Xp$, as in Ref. 8, as a function of M_X against the same for e^+e^- annihilation.[11] We have plotted them in Fig. 1 and may notice that with $\sqrt{s} = M_X$, from 2 to 10 GeV, the points are close to each other. We have also verified that in the quark fragmentation models[2-5] *at such energies* the charged quark-antiquark system $d\bar{u}$ and the neutral $u\bar{u}$ or $d\bar{d}$ systems have the same charged multiplicities. The disagreement between the two in Fig. 1 around 2 GeV and below is expected, as may be clear from the careful experimental analysis of e^+e^- annihilation data below charm threshold,[12] and is apparently due to the tail of the ρ' production. We regard the agreement of the points for the two systems in Fig. 1 as evidence that the π^--dissociation charged multiplicity is also the result of the hadronization of the $d\bar{u}$ system. We shall use this conjecture in our subsequent calculations for comparison with the data.

In the present calculations we consider three phenomenological primordial fragmentation functions $f_\pi(x)$ as illustrations, and calculate the energy dependence of exclusive and semi-inclusive multiplicities. We shall first take $f_\pi(x)$ as[3]

$$f_\pi^a(x) = \alpha + \beta(1-x)^2\,, \qquad (4.25a)$$

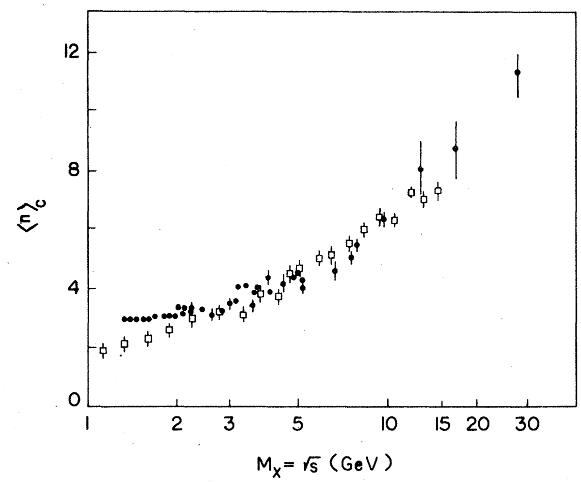

FIG. 1. The experimental average charged multiplicities for e^+e^- annihilation and π^- dissociation are plotted against M_X or $\sqrt{s}$. The data for e^+e^- annihilation from ADONNE, SPEAR–Mark I, DASP, PLUTO, and TASSO Collaborations have not been distinguished and are all represented by solid circles (Ref. 11). The open squares represent the π^--dissociation data (Ref. 8).

where we have $\alpha = 0.12$ and $\beta = 2.64$ as parametrized by Field and Feynman. Our second choice will be the simple analytic expression

$$f_\pi^b(x) = 6x(1-x)\,. \qquad (4.25b)$$

The above choice is made parallel to the parametrization of Buras and Gaemers[13] for the structure functions in deep-inelastic lepton-hadron scattering processes or for the resultant fragmentation functions $D_i^M(x)$. Many expressions can be analytically written down when we make the choice (4.25b). The alternate choice of $Ax^a(1-x)^b$ can also be made, but it becomes more cumbersome.

Our third choice of $f_\pi(x)$ is, as was derived from the ratios of cross sections in Ref. 5, given as

$$f_\pi^c(x) = \frac{d\sigma(d\tilde{Q} \to \pi^- u\tilde{Q})}{dx} \Big/ \sigma_t(d\tilde{Q} \to \pi^- u\tilde{Q})\,. \qquad (4.25c)$$

In the context of the present paper we only regard it as a phenomenological input, and the function $f_\pi^c(x)$ is plotted in Fig. 2. This fragmentation function had some general properties parallel to quark fragmentation, such as boundedness of the transverse momentum coming from the wave function of the meson and universality along with scaling. It also had some agreements with experiments[5] and was based on similar assumptions for coherent[14] and incoherent[15] processes. We may note from Fig. 2 that the function here is qualitatively similar to (4.25b).

We shall now apply Eq. (4.23) to obtain the prob-

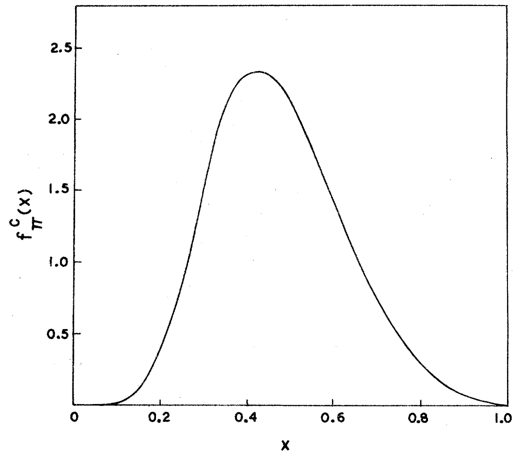

FIG. 2. The primordial fragmentation function $f^c_\pi(x)$ proposed earlier (Ref. 5) is plotted against x.

abilities for the individual charged multiplicities for π^- dissociation. Clearly the probabilities for observing two-, four-, six-, and eight-prong events for the process $\pi^- p \to Xp$ will be respectively given by $P_c(2l+1,\lambda)$ for $l=0$, 1, 2, and 3. With the experimental values of $d\sigma/dM^2(\pi^- p\to Xp)$, we now note that the respective cross sections are given as

$$\frac{d\sigma}{dM^2}(2l+2 \text{ prongs}) = \frac{d\sigma}{dM^2}(\text{total})\, P_c(2l+1,\lambda)\,. \tag{4.26}$$

For the three cases of Eq. (4.25), we have plotted the calculated values of the above cross sections against the experimental points[8] in Figs. 3(a)-3(d) for two-, four-, six-, and eight-prong events, respectively. We note that the data points are quite crude and unreliable. However, the present calculations reflect that such exclusive signals can distinguish between models if we have a more careful analysis. In (4.26) we have taken $(d\sigma/dM^2)$ (total) $\approx (524x_F/M^2)$ μb/GeV^2, corresponding to Regge phenomenology, and such that it gives the points of Ref. 8. We note that the Feynman variable $x_F = 1 - M^2/s$, and that the M dependence of the results in (4.26) arises through $\lambda = \mu/(M/2)$. For (4.25a), we have chosen $\mu = 1$ GeV and for (4.25b) and (4.25c), we have chosen $\mu = 0.5$ GeV.[5] The qualitative nature of the results does not depend on μ and we have chosen this parameter so that the agreement is the best. After the choice of primordial fragmentation function this is the only parameter in the present model which fixes the scale for the energy dependence of the results at finite energies.

As mentioned, for the case (4.25b) analytic solutions are possible. A straightforward but quite lengthy calculation ultimately gives us

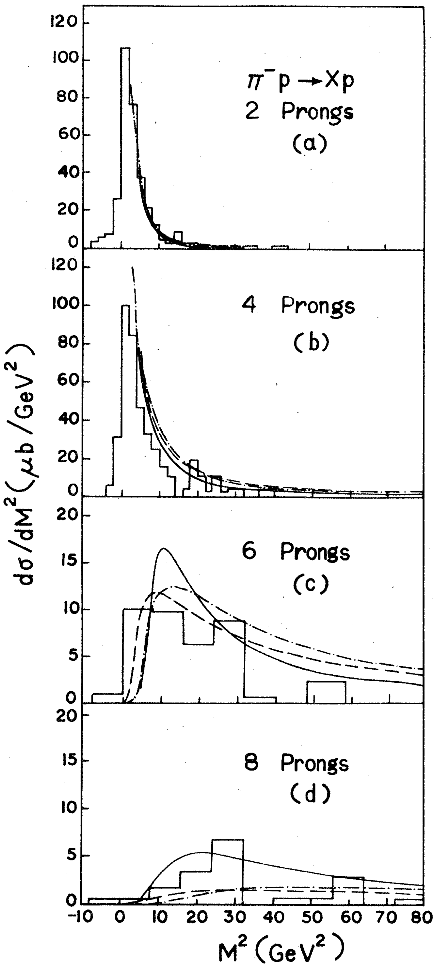

FIG. 3. In (a), (b), (c), and (d), $(d\sigma/dM^2)$ (2, 4, 6, and 8 prongs) vs M^2, respectively, are plotted. The continuous, the dashed, and the dot-dashed curves correspond to the primordial fragmentation functions $f^a_\pi(x)$, $f^b_\pi(x)$, and $f^c_\pi(x)$, respectively. The experimental data are taken from Ref. 8 after background subtraction.

$$C_\pi(n,\lambda) = \sum_{k=2n-1}^{\infty} \frac{(-1)^{k-1}}{k!}\left(\ln\frac{1}{\lambda}\right)^k \frac{6^n}{(n-1)!}\,\lambda \times (k-2n+2)(k-2n+3)\cdots(k-n) \tag{4.27}$$

and

$$C_\pi(\lambda) = \frac{6}{5}\left(\frac{1}{\lambda} - \lambda^4\right). \tag{4.28}$$

One may check (4.28) with the approximation (4.10) in which in fact we have R_π as $\frac{6}{5}$.

In the present case the charged multiplicity is given by

$$M_c(\lambda) = \sum_l (2l+1) P_c(2l+1, \lambda) , \qquad (4.29)$$

where (4.23) is to be again used. We have plotted in Fig. 4 $M_c(\lambda)$ as a function of M^2 against the experimental points[8] for the three primordial fragmentation functions of (4.25). As earlier, the identification is through $\lambda = \mu/(M/2)$. We notice that the predicted curve for large M lies below the experimental points for the cases (4.25b) and (4.25c). This feature may be compared with the recently observed increase in the multiplicity structure for e^+e^- annihilation.[11] We may identify these with the effect of gluon production[11] directly or indirectly as due to scaling violations.[16] The curve due to (4.25a) appears to lie above the experimental points.

We next consider some exclusive charged-pion signals from e^+e^- annihilations. We may believe that such exclusive pion production may come only from u and d quark-antiquark pairs. Here with (4.19), we write the energy-dependent cross section as, with $\lambda = \mu/(\sqrt{s}/2)$,

$$\sigma(e^+e^- \to 2l \text{ charged pions}) = (4\pi\alpha^2/s)\tfrac{5}{9} P_c^{\text{excl}}(2l, \lambda) . \qquad (4.30)$$

With (4.30), we have plotted the energy dependence of the total cross sections for $2l = 4$ and 6 against the corresponding experimental points[17] in Figs. 5(a) and 5(b), respectively, again using all three primordial fragmentation functions in (4.25). We note that the predictions of the different models are different, but confirmation of these experiments would be desirable before any conclusion can be drawn, since observation of such exclusive processes is inherently difficult. In contrast to Fig. 4, here the agreement for (4.25a) appears to be the best. We may further note that in Fig. 5(a) all three curves decrease when $\sqrt{s} \lesssim 2.5$ GeV, whereas the experimental points continue to show an increase. This may be due to the ρ' tail as indicated in the recent careful analysis of $e^+e^- \to$ hadrons below the J/ψ resonance.[12] The same effect of resonance production may also be operative in Fig. 5(b).

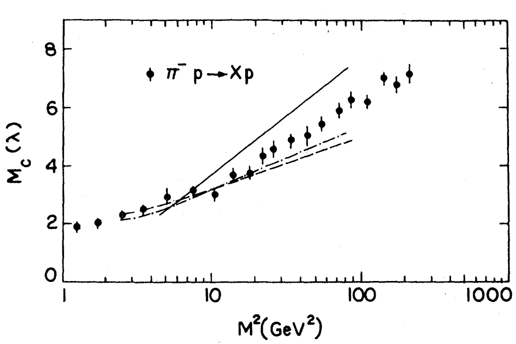

FIG. 4. The average charged multiplicity corresponding to π^- dissociation vs M^2 is plotted as calculated from $f_\pi^a(x)$, $f_\pi^b(x)$, and $f_\pi^c(x)$ as continuous, dashed, and dot-dashed curves, respectively, against the observed points from Ref. 8.

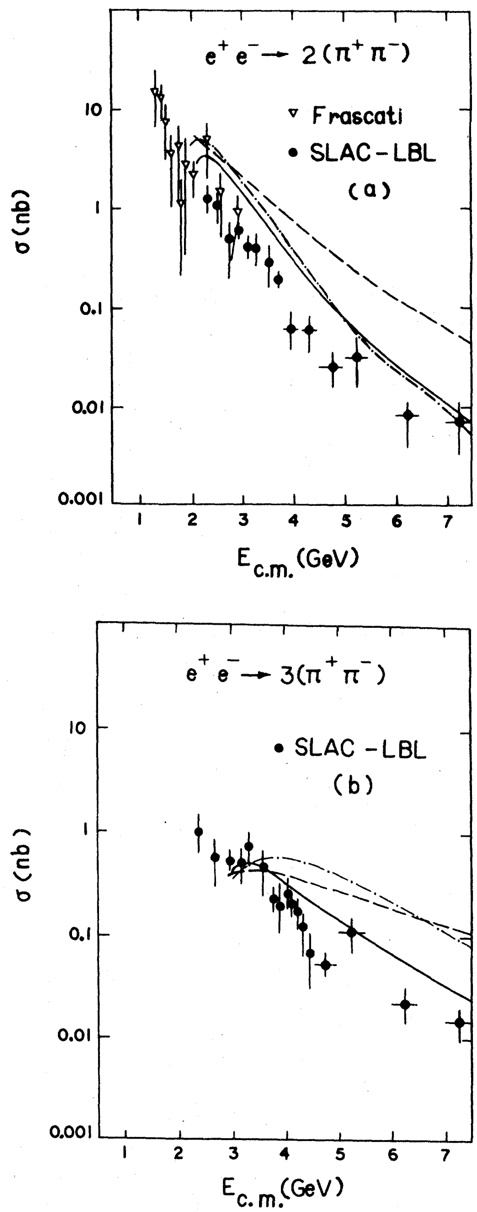

FIG. 5. (a) $\sigma(e^+e^- \to$ four charged pions) vs c.m. energy is plotted for the three functions. The experimental points are from Ref. 17. (b) $\sigma(e^+e^- \to$ six charged pions) vs c.m. energy against the corresponding experimental points (Ref. 17).

V. DISCUSSIONS

We have calculated here the energy dependence of specific hadron multiplicities in the quark-cascade jet model using Mellin transforms and have tested this against experimental observations. The same problem had been tackled earlier[18] in the context of fireball models[19] where the results are quite similar to ours. We believe that these signals will also be useful for the quark-fragmentation model in addition to the others examined earlier by many authors.[2-5] Further, Fig. 1 and Figs. 3(a)–3(d) indicate that the diffractive-dissociation process $\pi^- p \to Xp$ may be a two-stage process[10] in the quark model where the quark-antiquark pair $d\bar{u}$ subsequently hadronizes after dissociation. That a diffractive-dissociation process may be viewed as a two-stage process had been suggested earlier also in the context of the fireball model,[19] the first step being the production of a fireball. We are obviously considering here the same problem in the context of quark-parton ideas. This identification implies a new source of information regarding hadronization of quark-antiquark pairs both from experimental as well as theoretical points of view.[20] Further, this source will be more "clean" than that from e^+e^- annihilation, where, above the charm threshold, the signal of the $c\bar{c}$ pair cannot be easily separated.[7] The hadronization here should also be associated with quark-antiquark jets,[20] which appear to be observed in a recent experimental analysis[21] in the study of $\gamma p \to \pi^+\pi^-\pi^+\pi^-\pi^+\pi^- p$ with a two-jet structure for the pionic system. In the context of the vector-dominance model[22] and in the present context, we may expect here this observed two-jet structure for the pion system above as corresponding to the hadronization of $u\bar{u}$ or $d\bar{d}$ pairs resulting from the dissociation of the vector mesons ρ^0 and ω. We note that such a two-jet structure will *not* be expected to be present in the fireball model.[19] It is also difficult to visualize this with triple-Regge phenomenology,[23] which is a conventional way of understanding diffractive-dissociation processes. On the other hand, it is to be expected in the quark-model picture for diffractive dissociation which involves a phenomenological hard scattering.[10] However, it will be more satisfying to see this structure for the system X in the reaction $\pi^- p \to Xp$ directly, but naturally one has to look for this. Such a source of hadronization of quark-antiquark pairs or even of quark-diquark systems for $pp \to pX$ may be carefully experimentally studied with appropriate selection of data in purely hadronic collisions, particularly since Ref. 8 has poor statistics for such details.

We are attempting to calculate here the nonperturbative hadronization effects with specific assumptions. At very high energies, leading-logarithm approximations of the jets in perturbative quantum chromodynamics will come into play.[24] It appears from an analysis of Furmanski[25] that the energies for which the quantum-chromodynamic jets will manifest in e^+e^- annihilations in an unambiguous manner have not yet been reached. Hence, the present nonperturbative models for an analysis of data continue to be unavoidable. However, we may also note that the quark-fragmentation model will have to be ultimately supplemented or altered at high energies. E.g., quark recombination or other alternative mechanisms are needed[26] in high-energy hadronic collisions. We expect that such effects arise from hadronization of quarks and gluons as in perturbative quantum chromodynamics with leading-logarithm approximation and subsequent hadronization. The rapid increase in multiplicity in e^+e^- annihilations[27] also appears to indicate this effect. We conjecture that this will require perturbative[16,24] and nonperturbative[5,15] techniques being used in a combined manner.[28] This in the context of the present model is nontrivial. Further, we must have as much verification of the nonperturbative phenomena separately at comparatively low energies, which is being proposed here, so that at intermediate energies we may be ultimately able to predict phenomena with reasonable certainty.

ACKNOWLEDGMENTS

The authors would like to thank T. Pradhan, J. C. Pati, L. Maharana, K. Biswal, and S. K. Mishra for discussions. One of the authors (A.R.P) would also like to thank the University Grants Commission for financial assistance.

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