PHYSICAL REVIEW D VOLUME 23, NUMBER 3 1 FEBRUARY 1981

Constraints from jet calculus on quark recombination

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(Received 12 October 1979; revised manuscript received 21 July 1980)

Within the quantum-chromodynamic jet-calculus formalism, we deduce an equation describing recombination of quarks and antiquarks into mesons within a quark or gluon jet. This equation relates the recombination function $R(x_1,x_2,x)$ used in current literature to the fragmentation function for producing that same meson out of the parton initiating the jet. We submit currently used recombination functions to our consistency test, taking as input mainly the u-quark fragmentation "data" into π^+ mesons. The $q\bar{q}\to\pi$ recombination functions popular in the literature are consistent with measured fragmentation functions, but they must be supplemented by other contributions to provide the full $D_u^{\pi^+}$. We also discuss the Q^2 dependence of the resulting fragmentation functions.

I. INTRODUCTION

Over the past few years, the success of parton-model calculations in current-initiated reactions has led to general acceptance of quarks and gluons as useful calculational descriptions of hadronic structure functions. Clearly the next step is to use these same quark-content functions to predict purely hadronic reactions. Use in high-transverse-momentum (p_t) reactions for prediction of, e.g., jet cross sections, is well advanced; however, the application of these ideas to other regions of multiparticle phase space is still in its infancy.

A very intuitive approach applying parton-model concepts to low-p_t inclusive reactions was suggested by Das and Hwa[1] to explain the empirical observation of Ochs[2] that inclusive pion production at large x and small p_t in proton-proton collisions reflected the proton structure function. Various versions and applications of this recombination model, as it has come to be called, have appeared[3-8]; most of these resemble the original Das-Hwa paper in essential concept although there are differences in implementation. The basic idea is: Forward nondiffractive meson production can be calculated by assuming that most of the momentum in the meson came from a valence quark in the beam particle. This quark recombined with a sea quark of low momentum, also from the beam, to create a meson.

To compute the x distribution of the mesons from this process, two hitherto unstudied functions were introduced: $F(x_1, x_2)$, describing the probability that the incident beam has a quark of the correct flavor at x_1 and an antiquark of the correct flavor at x_2, and a recombination function $R(x_1, x_2, x)$ which tells how the two constituents join to create a meson of momentum $x_1+x_2=x$. Originally, both functions were created from various physical arguments; a fair number of the improvements of subsequent papers are directed towards refining these arguments.

One should be careful about specifying parton-model functions entering in such a calculation, however, since the functions are not completely arbitrary. The Q^2 dependence of hadron structure functions, and presumably of other quantities related to hadronic wave functions, is known. According to quantum chromodynamics, which predicts the various couplings among quarks (q), antiquarks ($\bar{q}$), and gluons (g), a very energetic quark generates a cloud of partons (q, $\bar{q}$, and g) that accompany it. The development of this cloud (with the Q^2 of the probing current) is described by the Altarelli-Parisi[9] evolution equations. The application of the physics in these equations to the description of jet production from quarks was formalized by Konishi, Ukawa, and Veneziano[10,11] (KUV). With their jet calculus, it is possible to calculate quickly various properties of a jet including the distribution of multiple constituents in longitudinal momentum fraction.

We adopt the basic ideas of jet calculus to get

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an expression for the probability of finding a quark at x_1 and an antiquark at x_2 and then require that they recombine to make the final-state meson with momentum fraction x. The resulting function must be an important contribution to the fragmentation function for the quark that produced the jet, that is, the probability that a given quark will materialize into a specific hadron with momentum fraction x. Such quark fragmentation functions have been phenomenologically determined by Field and Feynman from electron-positron jet production and hadron leptoproduction data.[12]

Hence, as a matter of principle, the recombination function R is *not* completely independent of other functions used in parton-model phenomenology. In fact, it is related to the fragmentation function D. This relationship will be derived in Sec. II, and although the general solution for R in terms of D is difficult, with present popular forms for R the moments of D can be written in terms of the moments of R (Sec. III). The resulting D has a Q^2 evolution somewhat different from that proposed by Owens,[13] especially at small Q^2. Results of our calculations are presented in Sec. IV where a detailed comparion with the fragmentation function of the u quark into positive pions is made. Section V contains our conclusions and discussion of implications of our results.

II. DERIVATION OF THE RELATION BETWEEN FRAGMENTATION AND RECOMBINATION FUNCTIONS

We briefly sketch the logic behind our relation: R measures the probability that two quarks will stick together to form a meson and D measures the probability that a meson will be found in a particular quark jet. If we can compute from first principles the probability that the quark jet in question contains the two partons necessary to form the meson, we can multiply this probability by R to obtain D. (This fragmentation function will be denoted $D_{M,i}$.)

The jet calculus[10,11] leads to the probability for finding two partons a_1, a_2 with momentum between x_1 and x_1+dx_1, x_2 and x_2+dx_2, respectively, when they originated from parton i,

$$\frac{1}{\sigma_{i\to\text{jet}}}\frac{d\sigma(i\to a_1a_2+X)}{dx_1dx_2}\equiv D_{a_1a_2;i}(x_1,x_2,Q^2)$$

$$=\sum_{b_1,b_2,j}\int_0^Y dy\int_0^1 dx\,dz\,dw_1dw_2D_{a_1b_1}(w_1,y)D_{a_2b_2}(w_2,y)$$

$$\times\hat{P}_{j\to b_1b_2}(z)D_{ji}(x,Y-y)\delta(x_1-xzw_1)\delta(x_2-x(1-z)w_2). \tag{2.1}$$

The cross section for parton i to produce a jet is $\sigma_{i\to\text{jet}}$, with $D_{a_1a_2;i}(x_1,x_2,Q^2)$ the joint probability distribution for parton i with four-momentum squared Q^2 to produce in its jet partons a_1 and a_2 at longitudinal momenta x_1 and x_2. The functions D_{ab} play the role of parton "propagators" in the variable $Y\simeq\ln(\ln Q^2)$ of the Altarelli-Parisi equations, the vertices P are the fundamental quantum-chromodynamics (QCD) vertices (quark→quark+gluon, etc.) as used in those equations. The partons a_1 and a_2 can then be dressed into hadrons of interest by use of phenomenological functions; the resulting dihadron distributions possess reasonable size and properties.[14] The δ functions impose momentum conservation upon the momentum fraction variables x, z, w_1, and w_2. The KUV variable[11] is

$$Y=(2\pi b)^{-1}\ln[1+\alpha_0 b\ln(Q^2/\Lambda^2)], \tag{2.2}$$

where $12\pi b=11N_c-2N_f$ for N_c colors and N_f flavors, and $\Lambda'^2=\Lambda^2\exp(-1/b\alpha_0)$ is a constant determining the strength and scale of the QCD coupling, $\alpha_s=1/b\ln(Q^2/\Lambda'^2)$.

If we then require that partons a_1 and a_2 recombine to make a meson, we obtain an expression for the fragmentation function $D_{M,i}$ for quark i making meson M,

$$D_{M,i}(x,Y)=\sum_{a_i,b_i,j}\int_{Y_0(Q_0^2)}^{Y}dy\int_0^1 d\xi\,dz\,dw_1dw_2dx_1dx_2R^M_{a_1a_2}(x_1,x_2,x)D_{a_1b_1}(w_1,y-Y_0)D_{a_2b_2}(w_2,y-Y_0)$$

$$\times P_{j\to b_1b_2}(z)D_{ji}(\xi,Y-y)\delta(x_1-\xi zw_1)\delta(x_2-\xi(1-z)w_2). \tag{2.3}$$

The integral over y in (2.3) has a lower limit Y_0 determined by $Q^2=Q_0^2$ in Eq. (2.2), consistent with the requirement that some minimum energy is needed to make meson M. A pictorial representation of Eq. (2.3) is given in Fig. 1.

This equation has a flaw, however. It requires $D_{M,i}(x,Y_0)=0$, where Y_0 is the lower limit of a region $Y_0\leq y\leq Y$ in which leading-logarithm QCD has been assumed to describe the jet evolution. While it is possible that there is a Y_0 at which

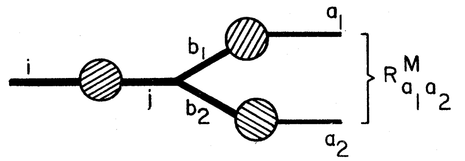

FIG. 1. Schematic representation in the jet calculus for the fragmentation function of parton i into meson M as given by Eq. (2.3) of the text. The circular blobs on the parton lines i, b_1, b_2 stand for the complete QCD evolution of a parton cloud out of which partons j, a_1, a_2, respectively, are selected. The partons a_1 and a_2 are recombined to make the diparton state M through use of R.

the fragmentation functions vanish, there is no reason why this Y_0 should be large enough for the KUV assumptions about jet evolution (α_s small) to hold. We must therefore modify Eq. (2.3) to allow for this.

To improve our understanding of the physics involved, let us look at the differential change in probability as the Q^2 interval under consideration is lengthened. As shown in Fig. 2, there are two possible contributions to a change in D as the interval in Q^2 is lengthened. The contribution of Fig. 2(a) is that discussed by Owens.[13] Here the meson was already formed at $Q_1{}^2$, and the additional vertex added with the lengthening of the Q^2 interval is just the familiar Altarelli-Parisi "bremsstrahlung" matrix A_{ij}. Figure 2(b), on the other hand, contains the basic process discussed above. In this case, the vertex added by lengthening the interval produces two partons whose products later recombine according to R.

Thus we have a differential equation for the evolution of the jet. For each moment,

$$\frac{dD^n_{M,i}}{dY} = A^n_{ji} D^n_{M,j} + f^n_{M,i}\,, \tag{2.4a}$$

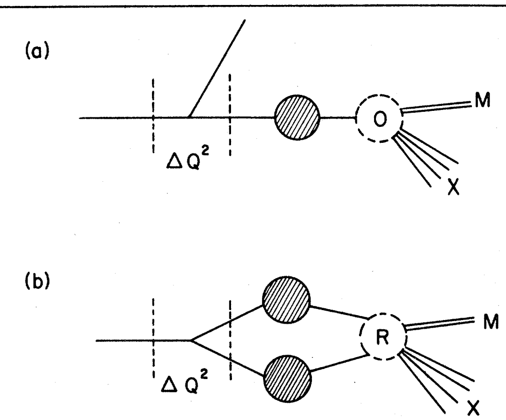

FIG. 2. Pictorial representation of contributions to changes in the fragmentation function as the Q^2 interval is lengthened: (a) Owens-type contribution and (b) the recombination process discussed in the text.

where

$$f_{M,i}(t,y) = \int dz\, dw_1 dw_2 R^M_{a_1 a_2}(zw_1, (1-z)w_2, t) \times D_{a_1 b_1}(w_1, y - Y_0) D_{a_2 b_2}(w_2, y-Y_0) \times P_{j\to b_1 b_2}(z)\,. \tag{2.4b}$$

The solution then becomes

$$D^n_{M,i}(Y) = D^n_{M,j}(Y_1)[e^{A^n(Y-Y_1)}]_{ji} + [e^{A^n(Y-Y_1)}]_{ji} \int_{Y_1}^{Y} f^n_{M,k}[e^{-A^n(y-Y_1)}]_{kj}\, dy\,. \tag{2.5}$$

We see that in general we will have two contributions to the fragmentation function. One, with Owens-type behavior, $D(Y_1)e^{A^n(Y-Y_1)}$, arises from single Altarelli-Parisi evolution of the "boundary value" $D(Y_1)$. The other term comes from recombination. It is very similar to the form derived in Eq. (2.3). In fact it becomes exactly Eq. (2.3) provided the recombination functions satisfy the "scaling" law

$$R^M_{a_1a_2}\left(\frac{x_1}{z}, \frac{x_2}{z}, \frac{x}{z}\right) = zR^M_{a_1a_2}(x_1, x_2, x)\,. \tag{2.6}$$

This property is possessed by the Das-Hwa function discussed below.

In principle, the two parameters Y_0 and Y_1 appearing in Eqs. (2.4b) and (2.5) are unrelated. Equation (2.5) has the property that if D satisfies it for any Y_1, it also satisfies it for $Y_2 > Y_1$, with Y_0 fixed. The parameter Y_1 is thus an initial point for the solution of the equation, whereas Y_0 is a measure of how far off shell the recombination occurs.

While we have focused out attention on the "two-particle" recombination of $q\bar{q}$ into a meson, it is clear that there must also be "higher-order" recombinations such as $q\bar{q}g$, $q\bar{q}gg$, etc. Also, we should presumably allow for the possibility that $q(x_1) + g(x_2) \to M(x) + Z$, where $x_1 + x_2 \geq x$, and Z is undetected. This is implicitly included in Eq. (2.4b); the region of integration there is $zw_1 + (1-z)w_2 \geq t$.

As pointed out by KUV, the average $P_\perp{}^2$ between the two partons calculated in their jet calculus grows with Q^2 like $Q^2/(\ln Q^2)$. This might appear to be a barrier to recombination of these partons into a pion, with its small mass. To have a theoretically correct model, we should be sure that we are recombining $q\bar{q}$ ($+n$ gluons) in a color singlet with small invariant mass. This can in fact be done, using the extended jet calculus of Bassetto, Ciafaloni, and Marchesini,[15] and these calculations are underway.

If we ignore the color-singlet problem, and limit ourselves to the KUV color-averaged formalism, we would in principle have a formula of the form

$$\int dp_\perp{}^2 D_{a_1a_2;i}(x_1, x_2, p_\perp{}^2, Q^2) R(x_1, x_2, x, p_\perp) dx_1 dx_2 , \tag{2.7}$$

where $R(x_1, x_2, x, p_\perp)$ peaks at small $p_\perp$. This would constrain the invariant mass to be small. However, the function $D_{a_1a_2;i}(x_1, x_2, p_\perp{}^2, Q^2)$ is plotted by KUV in their Fig. 8. We see that it peaks at small $p_\perp$, so one does not need much additional peaking in $R(x_1, x_2, x, p_\perp)$ to ensure that the important contributions to Eq. (2.7) come from a region where the invariant mass of the pair is limited. Since nothing in the applications to strong interactions constrained the invariant mass of the pair to be recombined, the $p_\perp$ dependence of R is not measured. Thus, we take the point of view that the R of Das and Hwa is a phenomenological average over $p_\perp$ which includes the desired peaking in such a way that we can limit ourselves in Eqs. (2.3) to integrals over x.

Our attitude here has some precedent. KUV need a two-hadron fragmentation function $D_a^{h_1h_2}(x_1, x_2)$ describing the fragmentation of a single quark into two hadrons. As shown in their Eq. (5.8), this has an implicit $p_\perp{}^2$ dependence which in fact is quite important for its physical interpretation. The contribution must be important only when the invariant mass of the hadron pair is small. However, this dependence is not displayed or used in their equations (2.15) and (2.16) where this function figures prominently. The $p_\perp{}^2$ damping of our recombination function is of exactly the same nature.

III. REDUCTION TO MOMENTS

We now need a form for R^M. In this paper we concentrate our attention on the form used by previous workers in the field, although it is a very special one and perhaps should be generalized or altered as we discuss later. The recombination function of Das and Hwa,[1] and the more general form of Van Hove,[7] can be written as

$$R(x_1, x_2, x) = x\int_0^1 d\eta\, \tilde{P}_{a_1a_2\to M}(\eta)\delta(x_1 - \eta x) \times \delta(x_2 - (1-\eta)x) . \tag{3.1}$$

When this is substituted into Eq. (2.5) and the δ functions removed, we find

$$D_{M,i}(x, Y) = \sum_j \int_{Y_1}^{Y} dy \int_x^1 \frac{dt}{t} D_{ji}\left(\frac{x}{t}, Y - y\right) f_j(t, y) + \tilde{D}_{M,i}(x, Y) , \tag{3.2}$$

where

$$f_j(t, y) = t\int_0^1 d\eta\, \tilde{P}_{a_1a_2\to M}(\eta) \times \int_{\eta t}^{1-(1-\eta)t} \frac{dz}{z(1-z)} D_{a_1b_1}\left(\frac{\eta t}{z}, y - Y_0\right) \times D_{a_2b_2}\left(\frac{(1-\eta)t}{(1-z)}, y - Y_0\right) \times P_{j\to b_1b_2}(z) . \tag{3.3}$$

We find it most convenient to work with moments of these functions rather than with the functions themselves, since for the simple R under study this removes all integrations over the x variables. If we define the moments as

$$D^n_{M,i}(Y) = \int_0^1 x^n D_{M,i}(x, Y) dx , \tag{3.4}$$

then (2.5) becomes

$$D^n_{M,i}(Y) = \sum_j \int_{Y_1}^{Y} dy\, D^n_{ji}(Y - y) f^n_j(y) + D^n_{M,i}(Y_1) D^n_{j,i}(Y - Y_1) \tag{3.5}$$

with

$$f^n_j(y) = \int_0^1 \int_0^1 d\xi_1 d\xi_2\, \theta(1 - \xi_1 - \xi_2)(\xi_1 + \xi_2)^n \times \tilde{P}_{a_1a_2M}\left(\frac{\xi_1}{\xi_1 + \xi_2}\right) \psi_{a_1a_2j}(\xi_1, \xi_2, y), \tag{3.6}$$

and

$$\psi_{a_1a_2j}(\xi_1, \xi_2, y) = \sum_{b_1b_2} \int_{\xi_1}^{1-\xi_2} \frac{dz}{z(1-z)} D_{a_1b_1}\left(\frac{\xi_1}{z}, y - Y_0\right) \times D_{a_2b_2}\left(\frac{\xi_2}{1-z}, y - Y_0\right) \times P_{j\to b_1b_2}(z) . \tag{3.7}$$

The special case proposed by Das and Hwa and used by most other workers in this field has

$$\tilde{P}_{a_1a_2\to M}\left(\frac{\xi_1}{\xi_1 + \xi_2}\right) = \frac{\xi_1\xi_2}{(\xi_1 + \xi_2)^2} C_{a_1a_2\to M} . \tag{3.8}$$

This has the virtue, for our purposes, that the moments f^n reduce to double moments of ψ:

$$f^n_j(y) = \sum_{\substack{m \\ a_1a_2}} \binom{n-2}{m} \int_0^1 d\xi_1 \int_0^1 d\xi_2\, \theta(1 - \xi_1 - \xi_2)\xi_1{}^{n-1-m}\xi_2{}^{m+1} \psi_{a_1a_2j}(\xi_1, \xi_2, y) C_{a_1a_2\to M}, \tag{3.9}$$

which can be calculated explicitly,

$$f_j^n(y)=\sum_{\substack{m\\a_1a_2}}\binom{n-2}{m}C_{a_1a_2\to M}D_{a_1b_1}^{n-1-m}(y-Y_0)\times D_{a_2b_2}^{m+1}(y-Y_0)P_{j\to b_1b_2}^{n-1-m,\,m+1}\,. \tag{3.10}$$

Thus the moments of the fragmentation functions can be calculated from Eq. (3.5) using only the known moments of basic D_{ij} parton propagators and the known vertices P, plus knowledge of which quarks combine to make which mesons. We also need the boundary values $D_{M,i}^n(Y_1)$.

The coefficients C are typically determined by recombination modelists as follows: Only the valence quarks in the produced meson are considered as it is believed that the sea develops later from gluons spawned by these valence quarks. Then, the constraint

$$\int_0^1 d\xi_1\int_0^1 d\xi_2 R(\xi_1,\xi_2)=1 \tag{3.11}$$

is either imposed or approximated. This forces all such pairs to make pseudoscalars rather than the other possible mesons which could be formed, e.g., vector or tensor mesons. Hence, the size is fixed. Typically, therefore,

$$C_{u\bar d\to\pi^+}=C_{\bar d u\to\pi^+}\le 6\,. \tag{3.12}$$

Most applications choose 6 for this, to satisfy Eq. (3.11), although Van Hove[7] chooses 4.0. More recently, Hwa, using different arguments, has settled on 1.0,[16] but he has also allowed for the more general recombination functions R_{qg} and R_{gg} with $x_1+x_2\ge x$. We discuss the implications of these values in the next section.

IV. RESULTS FOR QUARK FRAGMENTATION

A. Momentum conservation

The recombination term in (2.5) has a different Q^2 dependence from the first (Owens-type) term. The consequences of this are perhaps most clear when the fraction of the momentum of the quark that appears in hadrons is calculated. Owing to the properties of the Altarelli-Parisi matrix, if

$$\sum_M D_{M,i}^1(Y_1)=\beta<1\,,\quad \text{independent of } i$$

then the Owens-type term preserves the net momentum appearing in hadrons as a function of Q^2,

$$\sum_M\int_0^1 dx\,x\int_x^1\frac{dz}{z}D_{M,j}\left(\frac{x}{z},Y_1\right)D_{ji}(z,Y-Y_1)=\beta\,. \tag{4.1}$$

The momentum of the recombination contribution, on the other hand, increases with Q^2 as Y increases from the boundary value Y_1. Hence the net fraction of the incident parton's momentum which appears in hadrons increases as a function of Q^2; this may be interpreted as a "pumping" of momentum into the hadrons from the extra recombination mechanism.

Needless to say, the momentum fraction which appears ultimately in hadrons must not exceed unity; hence the increase with Q^2 of the first moment of the (sum over hadron) D functions cannot go on indefinitely. The Q^2 dependence of the recombination term is shown for a typical recombination function in Fig. 6 (Ref. 17); we see that it has the expected behavior. More quantitatively, if

$$R_{a_1a_2;M}(x_1,x_2,x)=\frac{x_1x_2}{x^2}C_M^{\bar q q}(\delta_{a_1\bar q}\delta_{a_2q}+\delta_{a_1q}\delta_{a_2\bar q})\,,$$

where q is the "valence" quark in the jet, then (taking $Y_1=Y_0$ for simplicity) the momentum into hadrons from recombination is

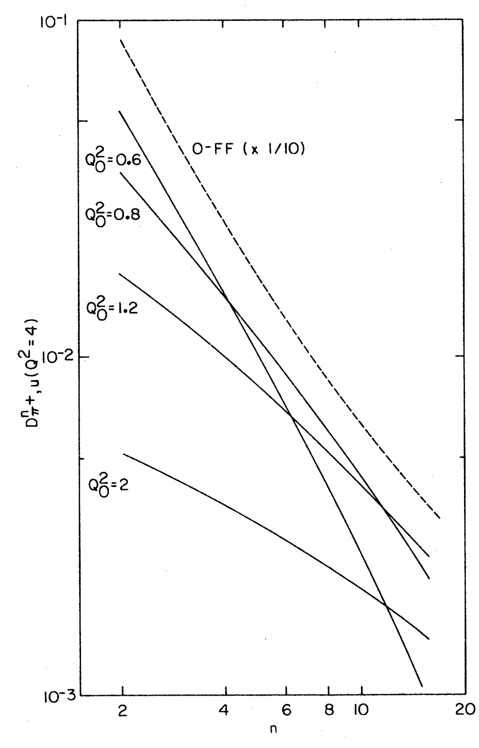

FIG. 3. The fragmentation function versus moment number n for a u quark at $Q^2=4$ GeV^2 producing a π^+ meson for various choices of $Q_0{}^2$. The dashed curve corresponds to the Feynman-Field parametrization of this fragmentation, reduced by 0.1. Note that a value $Q_0{}^2$ between 0.8 and 1.2 GeV^2 becomes parallel to the O-FF curves at large n.

$$\sum_M \int_0^1 x\,dx \int dx_1 dx_2 D_{a_1a_2;i}(x_1, x_2, Y-Y_0) R_{a_1a_2;M}(x_1,x_2,x)$$
$$= 2\sum_{M,\bar q} C_M^{\bar q q} \int_0^1 dx_1 x_1 \int_0^{1-x_1} dx_2 D_{\bar q q;q}(x_1,x_2,Y-Y_0)\left(\frac{x_2}{x_1+x_2}\right)$$
$$\leq 2\sum_{M,\bar q} C_M^{\bar q q} \int_0^1 dx_1 x_1 \int_0^{1-x_1} dx_2 D_{\bar q q;q}(x_1,x_2,Y-Y_0) = 2\sum_{M,\bar q} C_M^{\bar q q} D_{\bar q q}^{1}(Y-Y_0)\,. \quad (4.2)$$

The last equality uses the "flavor-conservation sum rule," Eq. (3.25) of KUV. Since

$$D_{\bar q q}^{1}(Y-Y_0) = \frac{3N_f}{(16+3N_f)(2N_f)} - \frac{1}{2N_f} e^{-16/9(Y-Y_0)} + \frac{16}{(16+3N_f)(2N_f)} \exp\left[-\frac{16+3N_f}{9}(Y-Y_0)\right],$$

where N_f is the number of flavors, we see that our bound on momentum due to recombination rises from zero at Y_0 to an asymptotic value of

$$2\left(\sum_{M,\bar q} C_M^{\bar q q}\right)\frac{3N_f}{(16+3N_f)(2N_f)}\,. \quad (4.3)$$

This then gives a bound on $\sum_M C_M$. For three flavors we have, for each q,

$$2\sum_{M,\bar q} C_M^{\bar q q}\,\tfrac{3}{2}\,\tfrac{1}{25} \leq 1 \quad (4.4)$$

or $\sum_{M,\bar q} C_M^{(3)} \leq \frac{25}{3}$, whereas for two flavors $\sum_{M,\bar q} C_M^{(2)} \leq \frac{22}{3}$. In either case all the possible values suggested at the end of Sec. III are allowed, although smaller values would seem to be preferred. One should bear in mind that, if the Owens-type term gives momentum fraction β, the recombination term should tend asymptotically to $1-\beta$ [i.e., the right-hand side of Eq. (4.4) should be $1-\beta$ rather than 1; this will reduce the values allowed to the recombination parameters C].

More generally, suppose we allow for $R_{a_1a_2;M}(x_1,x_2,x)$ with $x_1+x_2 \geq x$. One example of this is the construction used by Chang and Hwa.[17] Having found that $q\bar q$ recombination alone did not suffice to fit low-energy e^+e^- data, they decided to turn all the gluons in the jet into $q\bar q$ pairs and then recombine these quarks and antiquarks with the ones already present in the jet. Their modus operandi produces effective recombination functions $R_{a_1a_2;M}(x_1,x_2,x)$ for all partons a_1 and a_2. These have the scaling property of Eq. (2.6).

It would appear that the only overall constraint on these more general recombination functions would be that of momentum conservation:

$$\sum_n \int_0^1 x\,dx\left[\int_x^1 \frac{dz}{z} D_{M,j}\left(\frac{x}{z}, Y_0\right) D_{ji}(z, Y-Y_0) + \int\int_{x\leq x_1+x_2} dx_1 dx_2 D_{a_1a_2;i}(x_1,x_2,Y-Y_0) R_{a_1a_2;M}(x_1,x_2,x)\right] \xrightarrow[Q^2\to\infty]{} 1\,. \quad (4.5)$$

The functions as used by Chang and Hwa for pions seem at least consistent with the momentum-conservation bound; there is very little change in the curve calculated with their functions from $Q^2 = 10^3$ to $Q^2 = 10^6$ GeV^2, for instance (see Fig. 9).

B. Numerical results

We now direct our attention to numerical calculations with this formalism. Since the parameter Y_1 is arbitrary, we choose it to be equal to Y_0 for simplicity. One may now ask whether there is a value of Y_0 such that $D(x, Y_0) = 0$. If such a value can be found, we can compute all fragmentation functions in terms of the respective recombination functions, thus providing a link between the two sets of phenomenological parameters. More realistically, we can ask whether there is a value of Y_0 such that the recombination term alone can mimic the data on D functions in currently accessible regions of Q^2. This Y_0 would then serve as a simple one-parameter substitute for many unknown details of the confinement mechanisms. All the results presented here were computed using the values $\alpha_0 = 10$, $\Lambda^2 = 0.25$ GeV^2.

1. $Q\bar Q$ recombination

We begin by studying the simple recombination function of Sec. III. As "data" we take the Feynman-Field D functions at $Q^2 = 4$ GeV^2. As demonstrated in Fig. 3, adjustment of Y_0 and of C can yield agreement with the higher moments; however, the value of C required for this for positive pions is larger than the momentum conservation bound of Eq. (4.4). This is unfortunate, since of course one should have room for neutral pions and mesons of higher mass. Even if this course were adopted, additional Owens-type terms would have to be supplied to give correct values to the lower moments.

A different approach for fitting Y_0 would be to go to very large Q^2. If Q^2 is large, and if both $m+1$ and $n-1-m$ in Eq. (3.10) are large, then the Q^2 evolution of the $(p+n)$th moment of Eq. (3.2) is the same as that derived from the transposed Altarelli-Parisi equations given by Owens[13] for the evolution of the fragmentation $D_{M,i}$'s. We can see this by going to a basis in which the D_{ij} functions are diagonal and of the form $e^{y\lambda_m}$, with λ_m an eigenvalue of the Altarelli-Parisi set. In Eq. (3.5), the first term gives

$$D^{n+p}(Y) \sim \int_{Y_0}^{Y} dy\, e^{(Y-y)\lambda_{n+p}}\, e^{(y-Y_0)\lambda_p}\, e^{(y-Y_0)\lambda_n} \sim e^{(\lambda_p+\lambda_n)(Y-Y_0)} - e^{\lambda_{n+p}(Y-Y_0)}\,. \quad (4.6)$$

Examination of the leading eigenvalues of the Altarelli-Parisi equations shows that if both n and p are large, then $|\lambda_p+\lambda_n| > |\lambda_{n+p}|$, so the Y dependence is given by $\sim e^{Y\lambda_{n+p}}$, exactly as in Owen's equations. (The eigenvalues are negative. Also, Y_0 and Y are the values of y at opposite ends of the diagram, Fig. 1.)

One might thus concentrate on comparison at large Q^2 and large moment. This is especially reasonable for the Das-Hwa recombination function, which was believed by its inventors to be correct at large x (hence large moment). In Figs. 4(a) and 4(b) we show two large moments, $n=20$ and $n=27$, of the fragmentation function $D_{\pi^+,u}$, as a function of Q^2. The dashed curve labeled O-FF is calculated from the QCD evolution equation of Owens with the Field-Feynman empirical fit for $D^n_{\pi^+,u}$ as the initial value. The solid curves are calculated from Eqs. (2.3), (3.6), and (3.7) with the somewhat special, popular choice for R, Eqs. (3.1) and (3.8). The $Q_0{}^2$ values, as indicated on the curves, are 0.3, 0.5, and 1.5; clearly, the smaller $Q_0{}^2$ values reproduce the shape of the O-FF fragmentation function best in the range of Q^2 plotted.

In the calculation of these solid curves, the coefficient $C_{u\bar{d}\to\pi^+}$ in Eq. (3.10) is set equal to unity. Comparing the solid and dashed curves, therefore, we note that the actual value of C is quite sensitive to the value of $Q_0{}^2$ chosen.

It is of interest to examine these same fragmentation functions for fixed Q^2 as a function of n or moment. In Fig. 5 we show values of $D^n_{M,i}$ at fixed $Q^2=10^6$ GeV^2 as a function of n for the fragmentation of the u quark into π^+ and the s quark into K^-. For this comparison we have made the nonzero $C_{a_1,a_2\to M}$ coefficients in Eq. (3.10) equal to unity. Calculations of curves as in Figs.

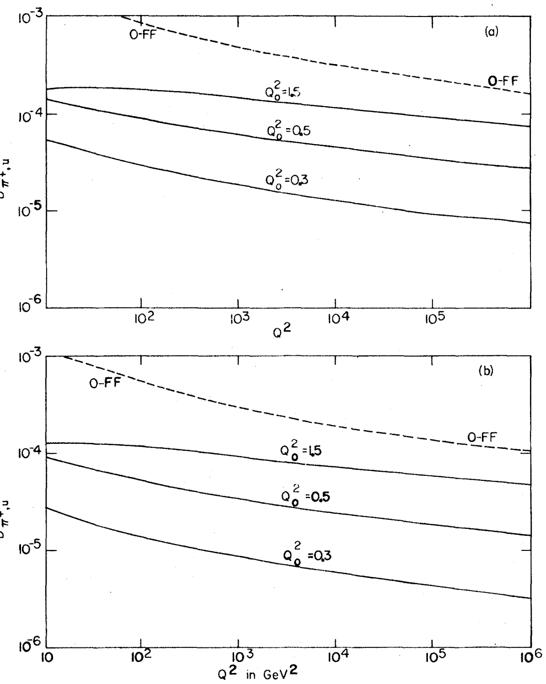

FIG. 4. Moments (a) $n=20$ and (b) $n=27$ of the fragmentation function for a u quark into π^+ mesons. The dashed curves are calculated from Owen's evolution equations with the Field-Feynman fit to e^+e^- annihilation and hadron leptoproduction data as the initializing condition. The solid curves show our results for the moments of the fragmentation function for u into π^+ calculated from Eqs. (2.8)–(2.12) of the text for $Q_0{}^2=0.3$, 0.5, and 1.5 GeV^2.

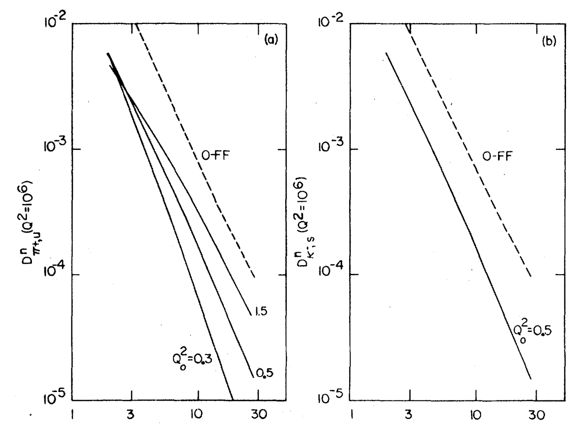

FIG. 5. Moments of fragmentation functions for (a) $u\to\pi^+$ and (b) $s\to K^-$ versus n, the moment number, for our calculation (solid line) compared with that (dashed line) of Owens and Feynman and Field. The dashed lines are calculated from Owen's evolution equations with the Field-Feynman fit of Ref. 12 for the initial (low-Q^2) condition. In (a) we show our calculation for $Q_0{}^2=0.3, 0.5$, and 1.5 GeV^2 to illustrate $Q_0{}^2$ dependence.

4 and 5 were performed for several other Q_0^2 values in addition to 0.3, 0.5, and 1.5 GeV^2. On the basis of Fig. 4, comparison of curve shapes with O-FF suggests $Q_0^2 = 0.3-0.5$ is preferred. In Fig. 5(a), it appears that the curves labeled $Q_0^2 = 0.3$ and $Q_0^2 = 1.5$ are least like O-FF. From this moment dependence we clearly prefer $Q_0^2 \cong 0.5$. The two cases $u \to \pi^+$ and $s \to K^-$ in Figs. 5(a) and 5(b) from our jet-calculus recombination calculations are identical because of the assumed flavor independence of the propagators and vertices. The Owens-Feynman-Field fragmentation functions become very similar also at this large Q^2. (We use the "effective" fragmentation functions of Ref. 12, which include the consequences of vector meson creation and decay.) Notice that for large moments the two curves are nearly parallel; they differ by a factor of about 5.5 in the case of $u \to \pi^+$ and a factor of 4–5 in the case of $s \to K^-$. These obey the constraint $C \leq 6$ following from Eq. (3.11) and used by Das and Hwa; these factors are slightly higher than the value $C = 4$ adopted by Van Hove.[7]

Thus we see that in spite of the apparently *ad hoc* methods used by Das and Hwa to guess their recombination function, it in some sense passes our test of being consistent with the measured fragmentation functions given the following

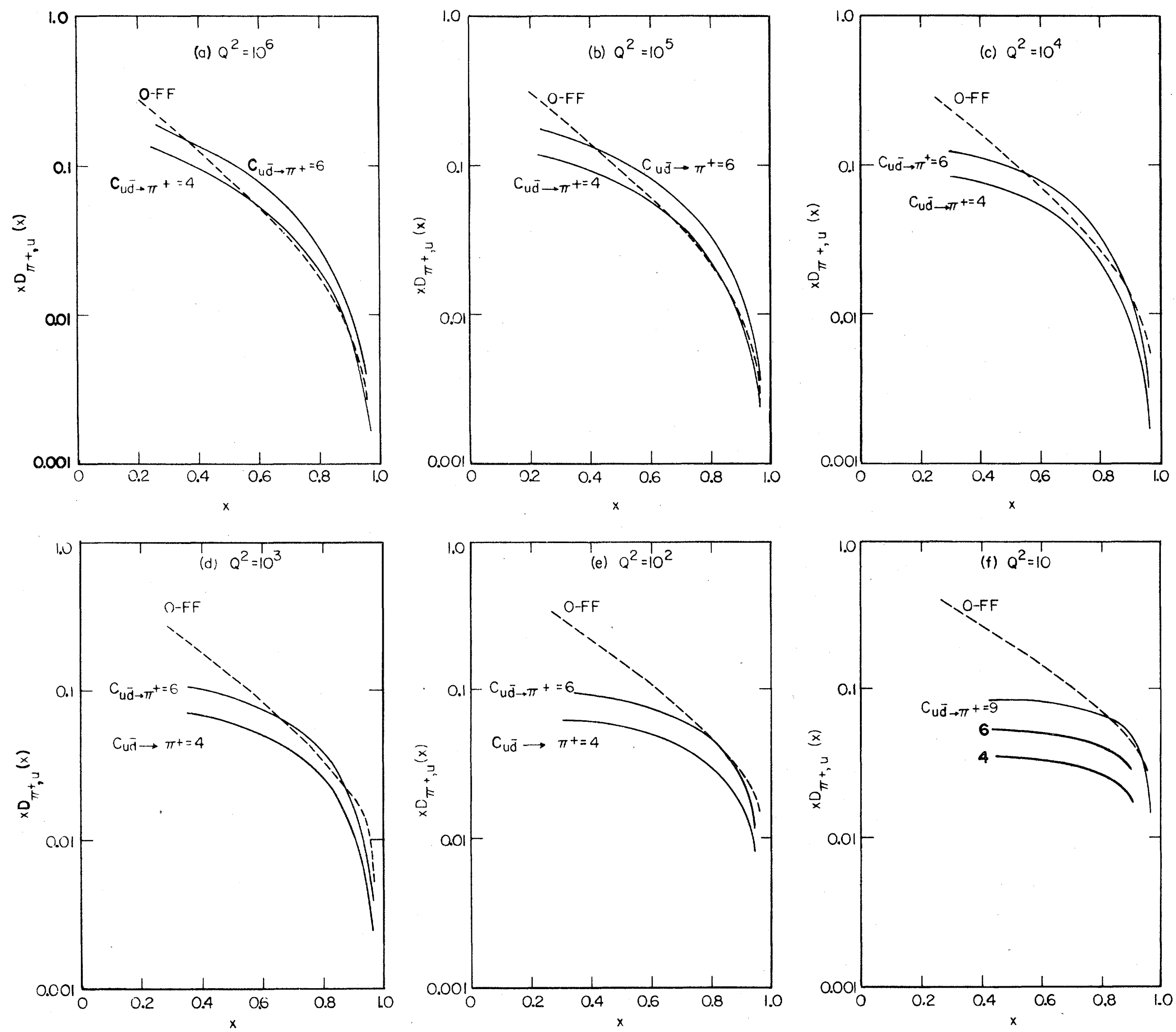

FIG. 6. Fragmentation functions times momentum fraction x vs x for $u \to \pi^+$. The dashed curves labeled O-FF are calculated for the various Q^2 = (a) 10^6, (b) 10^5, (c) 10^4, (d) 10^3, (e) 10^2, and (f) 10 GeV^2 from Owen's evolution equations with Field-Feynman initialization. The solid curves are calculated from Eqs. (2.8)–(2.12) for our jet-calculus recombination model with normalization of the recombination function fixed by the values $C_{u\bar{d} \to \pi^+} = 4$ or 6 as given in the literature (Ref. 17).

caveats:

(i) The two cases shown in Fig. 5 involve recombination of the main quark in the jet with a "created" quark; this is the case which recombination modelists would term "valence-sea" recombination. This is the major case addressed in their works and there is some uneasiness on their part about extending the model to other cases. The "sea-sea" recombination case follows from qq recombination discussed briefly in the next section.

(ii) We have demonstrated agreement in the region where the two approaches should have similar Q^2 dependence, namely very large Q^2 and large moment. The formula, Eq. (2.3), with the naive form for R in fact generates fragmentation functions which have rather different Q^2 dependence from that normally assumed at low Q^2.

However, at currently accessible values of Q^2, the recombination term alone grossly underestimates the fragmentation function at small x (hence at small moment); see Fig. 6. We conclude, therefore, that use of Eq. (2.5) with only $q\bar{q}$ recombination will require the addition of an extra Owens-type term, at least in the small moments. This then motivates the study of more complicated recombination functions.

2. Recombination of gluons into mesons

In a recent paper, Chang and Hwa[16] have discussed a fit to the $e^+e^- \to \pi^\pm X$ data using recombination techniques. They emphasize use of the Das-Hwa recombination function with $C=1$. One interesting feature of their calculation, which is not stressed in their paper, is the fact that almost all of the fragmentation function which they calculate comes from gluon-quark and gluon-gluon recombination rather than from quark-antiquark recombination. We show the breakdown here to demonstrate its relation to the results presented in the previous section.

As seen in Fig. 6, for $Q_0^2=0.5$ GeV^2 and $C=6$, the curve calculated from $q\bar{q}$ recombination alone falls below the data at $Q^2<100$ GeV^2 everywhere. Chang and Hwa use $Q_0=0.82$ GeV, which is reasonably close to our value, but they have $C=1$. Clearly, therefore, the contribution from $q\bar{q}$ recombination alone is very small and totally negligible at small x. In an attempt to capture the momentum in the gluons of the jet (there are many more gluons than antiquarks in the jet at this value of Q^2), they split the gluons into quark-antiquark pairs and then recombine these with the other available particles. As we show in Fig. 7, almost all of the function computed by them comes from this contribution. This is especially important at small x.

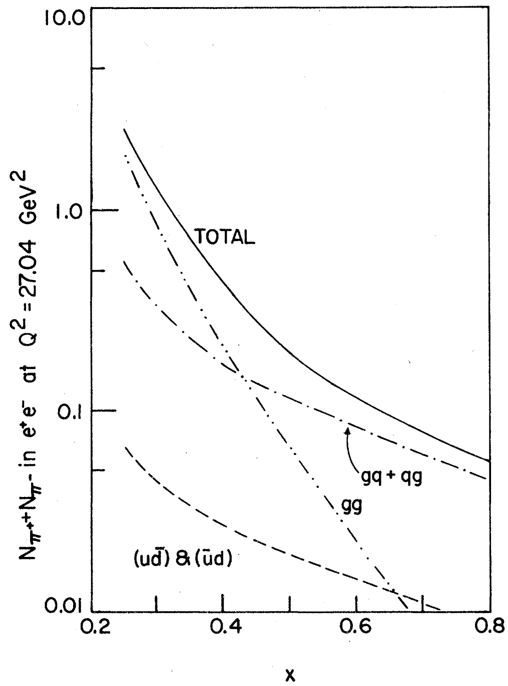

FIG. 7. Contributions to the x distribution of charged pions produced in $e^+e^- \to \pi^\pm X$ at $Q^2=27.04$ GeV^2. The dashed line shows the π^+ plus π^- distribution resulting from direct recombinations of u with $\bar{d}$ and $\bar{u}$ with d quarks. The dash-dot-dot curve depicts the π^+ plus π^- distribution resulting from two gluons in the jet, each of which are converted to $q\bar{q}$ pairs with recombination among these producing the pions. The dot-dash curve gives the contributions to π^+ plus π^- production from gluon-quark (or antiquark) and quark-gluon with gluon conversion to $q\bar{q}$ pairs followed by recombination to form pions.

Their success in fitting the data indicates that it may perhaps be possible to dispense with Owens-type terms in the fragmentation function if recombination of any two partons to make the meson under consideration is allowed.

V. SUMMARY AND CONCLUSIONS

We see that $q\bar{q}$ recombination alone, using the simple form for the recombination function popularized by Das and Hwa, cannot produce a large enough fragmentation function at low Q^2 to agree with the data. Such agreement can only be achieved by either (a) adding an Owens-type term to the recombination function, or (b) generalizing the recombination model to allow for gluon-quark and gluon-gluon recombination into mesons. This is not surprising, since the tests of the recombination function in strong interactions focused on mesons with large x. As we see in Fig. 6, simple $\bar{q}q$ recombination produces a shape at large x not unlike the O-FF fit to the data. The discrepancy,

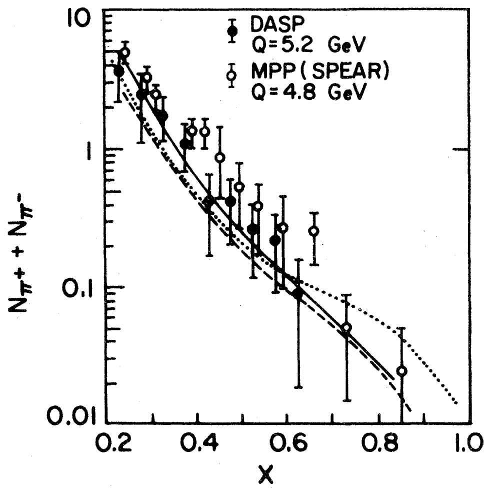

FIG. 8. The distribution of π^+ plus π^- versus x in $e^+e^- \to \pi^\pm X$ compared with SLAC and DESY data. The solid and dashed lines appear in Ref. 17. The prediction of our parametrization is the dotted curve shown.

as seen here and in Fig. 7, is much greater at small x than at larger x. If the normalization of the $q\bar{q}$ curve in Fig. 7 is adjusted to fit the large-x region, the small-x region still disagrees by close to an order of magnitude.

One can apparently fit the present data either by adding an Owens-type term or by allowing for more general recombinations. The Owens-type term could be read off from present data; its consequences at larger Q^2 are then uniquely predicted by Eq. (2.5). There are many possibilities for more general recombinations; the specific example given by Chang and Hwa can, as they point out, only by justified *a posteriori*. In Fig. 8 we recalculate their prescription using our parameters ($Q_0^2 = 0.5$ GeV^2, $\Lambda^2 = 0.25$ GeV^2) and compare with the large-x data of the Maryland-Pavia-Princeton (MPP) collaboration.[18]

The Q^2 evolution of the Chang-Hwa prescription is shown in Fig. 9. It is not unlike the behavior expected from the Owens evolution equations—the function rises at small x and drops at large x as the value of Q^2 increases. The "crossover" point, however, appears at a larger value of x than for the usual O-FF cases.

Since completing this work, we received a new paper by Chang and Hwa[19] in which they cut off the Q^2 integral in calculating the $D_{a;i_1i_2}$ function. Their rationale for this is to prevent the p_t^2 of the partons being recombined from getting very large. This has the remarkable result that for all Q^2 larger than their cutoff (chosen to be 30 GeV^2) the momentum into any particular final state *decreases* as Q^2 increases. This is somewhat different from the possibilities we have discussed

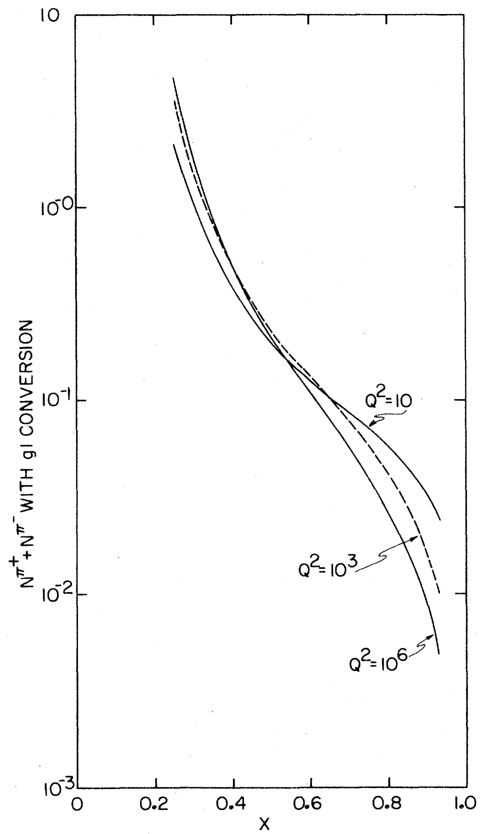

FIG. 9. The Q^2 evolution for the sum of positive and negative pions produced in e^+e^- annihilations calculated with our parameters when we use the prescription that all gluons in the jet are turned into $q\bar{q}$ pairs, either of which can be recombined.

here, in which the momentum in a particular final state tends to rise slowly toward an asymptotic value.

We see that the recombination model is compatible with present fragmentation functions, but that there are several different ways in which the model can be implemented. These tend to differ chiefly in their predicted Q^2 dependence. In many cases, the differences are too subtle to be detectable with the present experimental measurement errors. Further theoretical work would seem to be indicated, to reduce the possible number of models.

ACKNOWLEDGMENTS

The first two authors, L. M. J. and K. E. L., would like to thank their colleagues in the high-energy-theory program at Argonne National Laboratory, particularly Dr. D. Sivers and Dr. G. Thomas, for helping make participation in the Argonne Summer Visitor Program both profitable and enjoyable. This work was partially supported by the National Science Foundation under Grant No. NSF PH79-00272 and by the U. S. Department of Energy, under Contract No. W-7405-eng-82, Office of Basic Energy Sciences (HK-02-01-01).

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