Double equivalent-photon approximation including radiative corrections for photon-photon collision experiments without electron tagging

M. Defrise
Laboratorium voor Theoretische Natuurkunde, Vrije Universiteit Brussel, Brussels, Belgium

S. Ong and J. Silva
Laboratoire de Physique Théorique des Particules, Université de Picardie, Amiens, France

C. Carimalo
Laboratoire de Physique Corpusculaire, Collège de France, Paris, France
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It is shown that, in first approximation, radiative corrections in photon-photon collision experiments without electron tagging can be estimated by using a double equivalent-photon approximation of the same type as (but, of course, analytically much more complicated than) that used for computation of $\gamma\gamma$ cross sections without radiative corrections. It is obvious that, in such an approximation, radiative corrections—for a given beam energy E_0 and a given invariant mass M produced—become independent of the specific process $\gamma\gamma \to X$ considered. The invariant-mass spectrum, corrected for radiation, will be written in the general form $d\sigma^{\text{corr}}/dM = (1+\delta)d\sigma^0/dM$ where $d\sigma^0/dM$ is the uncorrected spectrum. Values obtained for δ at typical beam energies $E_0 = 1.5$, 3, 15, and 70 GeV, and for M/E_0 ranging between 0.1 and 0.6, are systematically of the order of less than $\pm 1\%$.

I. INTRODUCTION

Investigation of photon-photon collisions in electron-positron storage rings has started becoming an area of high-energy experimental physics. Last year, three experiments were published[1-3]; more of them are presently going on or being prepared at DCI (Orsay), PETRA, and PEP.

A priori, radiative corrections in such experiments are expected to be significant (four external electron lines in the $\gamma\gamma$ diagram). They cannot be derived, even roughly, from any standard formulas to be found in the literature. They may differ widely according to the experimental configuration considered, and in particular to the solution chosen as regards tagging of the outgoing electrons.[4]

Whereas radiative corrections in experiments with tagging at 0° were considered previously,[5] the study presented here, mainly based on the thesis of one of us (S.O.),[6] concerns those experiments where the final electrons remain undetected (notice that two[1,2] of the above-mentioned three experiments were of that type).

As usual, and more than usual, radiative corrections are of great complexity; therefore, we were led to introduce some simplifications. Limiting ourselves to terms of order not higher than α^5, we had to consider *a priori* the diagrams shown in Fig. 1 (leaving aside those which are eliminated by mass renormalization): the nonradiative diagram (a); diagrams (b) and (c), with virtual radiative corrections at one electron vertex (plus the symmetric diagrams obtained by exchange of the left- and right-hand electron); diagram (d), with real-photon emission at one electron vertex (plus the symmetric diagram); diagrams of the type of (e), with virtual radiative corrections connecting one electron vertex with the central vertex (plus the symmetric diagrams); diagrams of the type of (f), with real-photon emission from the central vertex; and diagram (g), with virtual radiative corrections connecting both electron vertices.

Diagrams (b), (c), (e), and (g) contribute, to order α^5, only through their interference with (a). In order to eliminate infrared divergences in the usual way, we shall consider the following groups of terms:

(A) The cross section of (d), in association with the interference between (a) and (b) + (c).

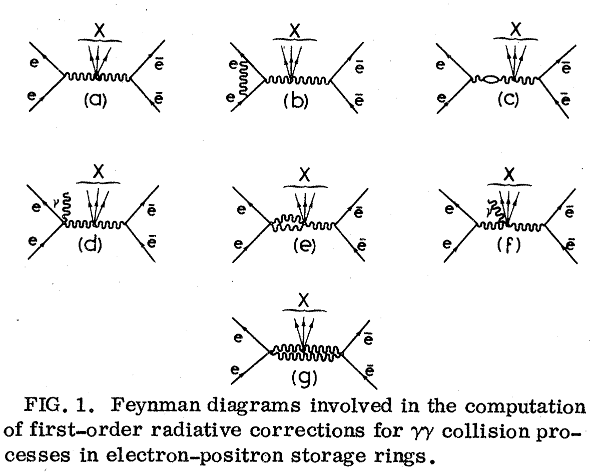

FIG. 1. Feynman diagrams involved in the computation of first-order radiative corrections for $\gamma\gamma$ collision processes in electron-positron storage rings.

(B) The cross section of diagrams of the type of (f), associated with interferences between (a) and diagrams with internal virtual radiative corrections at the central vertex (here we have not shown the latter ones).

(C) Interferences between (d) and (f), associated with interferences between (a) and (e).

(D) The interference between (d) and the corresponding symmetric diagram, associated with the interference between (a) and (g).

In our treatment, we shall keep only the group of terms (A), neglecting all other contributions. To do that, we shall use the following arguments.

(i) Corrections from the central vertex [i.e., the group of terms (B)] should be less significant than those originating from the electron lines, since in general particle masses at the central vertex are higher, and in addition particle energies are lower. Furthermore, we may treat radiative corrections of the central vertex separately, i.e., as included in the process $\gamma\gamma \to X$.

(ii) Since, in any experiment performed in a charge-symmetric way, the interference between contributions with $C=+1$ and -1, respectively (where C is the charge-conjugation number of the system X produced), would strictly be zero, the group of terms (C) would thus vanish.

(iii) The group of terms (D) may probably be neglected since there should be very little overlap in momentum space between (real or virtual) photons emitted by the left-hand and by the right-hand electron, respectively.

Thus sticking to the contribution (A) only, we shall consider two different ways of computing that contribution: In Sec. II, we introduce a single equivalent-photon approximation (EPA), i.e., we apply the EPA only at one electron vertex (that without radiative corrections). From that single EPA, we then derive (in Sec. III) a double equivalent-photon approximation including radiative corrections. In Sec. IV, numerical results obtained by using either the double or the single EPA (and also an exact calculation in a simplified case) are shown and compared, and conclusions are drawn. Some details of the calculation are given in an appendix.

II. RADIATIVE CORRECTIONS IN THE SINGLE EQUIVALENT-PHOTON APPROXIMATION

Since the equivalent-photon approximation is known to work well for high-energy electrons (leaving radiative corrections aside),[7] we shall apply it here at the right-hand electron vertex of diagrams (a)–(d) of Fig. 1, i.e., at the vertex without radiative corrections.

At a given beam energy E_0, the invariant-mass spectrum (calling M the invariant mass of the system X produced) corrected for radiative effects will be written as

$$d\sigma^{\mathrm{corr}}/dM = (1+\delta)d\sigma^0/dM\,, \tag{2.1}$$

where $d\sigma^0/dM$ is the uncorrected spectrum. The relative correction δ is thus defined, in our treatment, as

$$\delta = 2\,\frac{d\sigma^A/dM}{d\sigma^0/dM}\,, \tag{2.2}$$

where the factor 2 takes account of left-hand–right-hand symmetry, and $d\sigma^A/dM$ refers to the group of contributions A defined in Sec. I.

Applying the EPA at the right-hand vertex of diagrams (a)–(d) of Fig. 1, and calling ω the right-hand virtual photon's energy, one gets

$$\delta = 2\,\frac{\int N(\omega)d\omega\, d\bar\sigma^A(\omega,M)/dM}{\int N(\omega)d\omega\, d\bar\sigma^0(\omega,M)/dM}\,, \tag{2.3}$$

where the symbol $\bar\sigma$ is used to define cross sections for the reduced processes obtained by cutting off the right-hand vertex from diagrams (a)–(d); and where the equivalent-photon spectrum $N(\omega)$ is given by the usual formula[8]

$$N(\omega) = \frac{2\alpha}{\pi}\,\frac{1}{\omega}\left[\left(1-\frac{\omega}{E_0}+\frac{\omega^2}{2E_0{}^2}\right)\ln\frac{E_0}{m_e}-\frac{1}{2}\left(1-\frac{\omega}{E_0}\right)\right]. \tag{2.4}$$

Our task is thus reduced to computing the invariant-mass spectra obtained in the reaction between the left-hand electron and the right-hand photon (now treated as real and collinear with the beam axis) in diagrams (a)–(d) of Fig. 1. Such a reaction is a one-photon-exchange process, and we now may apply the factorization formula[9] established for any such process (Fig. 2):

$$\frac{d^3\sigma}{dt\,dW^2dW'^2} = \frac{1}{64\pi^3}\,\frac{1}{\Lambda(s,m^2,m'^2)t^2} \times[\bar\sigma_T\bar\sigma'_T(\zeta+2)+(\bar\sigma_T\bar\sigma'_L+\bar\sigma_L\bar\sigma'_T)\zeta+\bar\sigma_L\bar\sigma'_L(\zeta+1)]\,, \tag{2.5}$$

where m, m' are the initial masses (here m is the electron mass, whereas m' is the real-photon mass, i.e., 0); W, W' are the invariant masses produced at both vertices (here $W'\equiv M$); s is the total c.m. energy squared (here $s=4\omega E_0+m_e{}^2$); t is the absolute value of the exchanged (here the left-hand) photon's four-momentum squared; $\bar\sigma_{T,L}$ and $\bar\sigma'_{T,L}$ are the virtual transverse and longitudinal cross sections (in a reduced, nondimensional form, i.e., eliminating the "flux factor"[10]) for the left-hand and right-hand vertex of Fig. 2 [corresponding to the left-hand and central vertex of Figs. 1(a)–1(d), the right-hand photon being

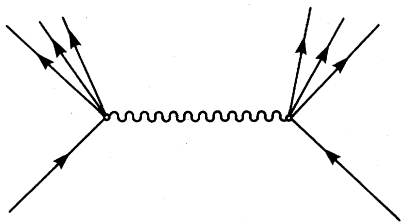

FIG. 2. General Feynman diagram for one-photon-exchange processes.

taken as real]; the Λ function is defined as $\Lambda(x,y,z)=x^2+y^2+z^2-2yz-2zx-2xy$ [here one has $\Lambda(s,m^2,m'^2)=16\omega^2E_0{}^2$]. Finally, ζ is a kinematic factor related to the Lorentz transformation performed, along the exchanged-photon line, from one vertex to the other in Fig. 2; it is called $\sinh^2\theta$ in Ref. 9, and its expression can be derived from Eqs. (2.21)–(2.23) given there.

We notice that, using formula (2.5), our task is further simplified, since all radiative corrections considered are now contained exclusively in the reduced cross section $\bar\sigma_{T,L}$ for the left-hand vertex. We shall now, as usual in radiative-correction computations, divide our calculation into two parts: (1) Contribution from virtual and "soft" photons. (2) Contribution from "hard" photons. The limit between soft and hard photons emitted at the left-hand vertex is of course artificial; it is set, actually, at an invariant mass W_0 which is larger than m_e only by an infinitesimal quantity.[11]

(1) Contribution from virtual and soft photons. Setting $W=m_e$ (left-hand vertex elastic), one gets from (2.5)

$$\frac{d^2\bar\sigma^{\rm corr}_{v+s}}{dM\,dt}=\frac{\alpha}{64\pi}\,\frac{M}{\omega^2E_0{}^2t^2}\{t(1+\delta_T)[\bar\sigma_T'(\zeta+2)+\bar\sigma_L'\zeta]+4m_e{}^2(1+\delta_L)[\bar\sigma_T'\zeta+\bar\sigma_L'(\zeta+1)]\}\,, \tag{2.6}$$

where terms containing δ_T, δ_L are due to the contributions called (A) in Sec. I. δ_T and δ_L can be expressed as

$$\delta_T=-\frac{\alpha}{\pi}\,(I+J+K+L)\,,\qquad \delta_L=-\frac{\alpha}{\pi}\,(I+J'+K+L)\,, \tag{2.7}$$

where the terms I and J (or J') are due to the interference between diagrams (a) and (b) of Fig. 1, whereas K is obtained from the interference between (a) and (c) (after charge renormalization), and L proceeds from the soft-photon contribution of diagram (d). All those terms are finite [an infrared divergence was canceled out between the interference of (a) and (b) on one hand, and the soft-photon contribution from (d) on the other hand]. One obtains

$$I=2+2(\ln\tau)\left[\frac{1+2\rho}{[\rho(1+\rho)]^{1/2}}\ln(1+\rho)^{1/2}-\frac{1}{2}\left(\frac{\rho}{1+\rho}\right)^{1/2}-\left(\frac{1+\rho}{\rho}\right)^{1/2}\right]+\frac{1+2\rho}{[\rho(1+\rho)]^{1/2}}\{\phi(-\tau\sqrt{\rho})-\phi(\sqrt{\rho}/\tau)+\tfrac{1}{2}[\phi(2\sqrt{\rho}/\tau)-\phi(-2\tau\sqrt{\rho})]\}\,, \tag{2.8}$$

$$J=\frac{-1}{2[\rho(1+\rho)]^{1/2}}\ln\tau\,,\quad J'=-\rho J\,, \tag{2.9}$$

$$K=\frac{4}{3}\left[\left(1-\frac{1}{2\rho}\right)\left(\frac{1+\rho}{\rho}\right)^{1/2}\ln\tau+\frac{1}{2\rho}-\frac{5}{6}\right], \tag{2.10}$$

$$L=2+\left(2\ln\frac{W_0{}^2-m_e{}^2}{m_e{}^2}-1\right)\left(1-\frac{1+2\rho}{[\rho(1+\rho)]^{1/2}}\ln\tau\right)+\frac{1+2\rho}{4[\rho(1+\rho)]^{1/2}}\{\phi(-4\tau^2[\rho(1+\rho)]^{1/2})-\phi(4[\rho(1+\rho)]^{1/2}/\tau^2)\} \tag{2.11}$$

with

$$\rho=t/(4m_e{}^2)\,,\quad \tau=(1+\rho)^{1/2}+\sqrt{\rho}\,,$$

$$\phi(x)=\int_0^x-\frac{\ln|1-x|}{x}\,dx\,.$$

From Ref. 9, Eqs. (2.21)–(2.23), one gets

$$\zeta=\frac{4s(t_{\max}-t)(t-t_{\min})}{(t+4m_e{}^2)(M^2+t)^2} \tag{2.12}$$

with

$$t_{\max}=\frac{1}{2}\left(1-\frac{m_e{}^2}{s}\right)[s-M^2-m_e{}^2+\Lambda^{1/2}(s,M^2,m_e{}^2)]-\frac{m_e{}^2M^2}{s}\,, \tag{2.13}$$

$$t_{\min}=\frac{m_e{}^2M^4}{st_{\max}}\,. \tag{2.14}$$

As for $\bar\sigma_T'$ and $\bar\sigma_L'$, they depend of course on the specific $\gamma\gamma$ process considered.

(2) Contribution from hard photons. Here one gets from formula (2.5)

$$\frac{d^2\bar{\sigma}_{\text{hard}}}{dM\,dt}=\frac{\alpha}{64\pi}\frac{M}{\omega^2E_0{}^2t^2}\int_{W_0{}^2}^{W_{\max}{}^2}Y\,dW^2\,, \tag{2.15}$$

with

$$Y=\frac{1}{8\pi^2\alpha}[\bar{\sigma}_T\bar{\sigma}_T'(\zeta^h+2)+(\bar{\sigma}_T\bar{\sigma}_L'+\bar{\sigma}_L\bar{\sigma}_T')\zeta^h+\bar{\sigma}_L\bar{\sigma}_L'(\zeta^h+1)]\,. \tag{2.16}$$

The virtual Compton cross sections $\bar{\sigma}_{T,L}$ are given in the Appendix. As for ζ^h (where we use the subscript h for "hard"), it is given—according again to Ref. 9, Eqs. (2.21)–(2.23)—by the expression

$$\zeta^h=\frac{4st(t^h_{\max}-t)(t-t^h_{\min})}{\Lambda(W^2,m_e{}^2,-t)(M^2+t)^2}\,, \tag{2.17}$$

with

$$t^h_{\max}=\frac{1}{2}\left(1-\frac{m_e{}^2}{s}\right)[s-M^2-W^2+\Lambda^{1/2}(s,M^2,W^2)]-\frac{m_e{}^2M^2}{s}\,, \tag{2.18}$$

$$t^h_{\min}=\frac{M^2(W^2-m_e{}^2)}{t^h_{\max}}+\frac{m_e{}^2M^2(M^2-W^2+m_e{}^2)}{st^h_{\max}}\,. \tag{2.19}$$

As for the kinematic limit $W^2_{\max}$, it is given by[12]

$$W^2_{\max}=\frac{(st+m_e{}^2M^2)(s-M^2-m_e{}^2-t)}{(s-m_e{}^2)(M^2+t)}\,. \tag{2.20}$$

To get the total radiative correction, we now simply sum up both contributions (1) and (2).

(3) Total radiative correction. The total radiative correction is now obtained as

$$\delta=2\frac{\int N(\omega)d\omega\int dt\{[d^2\bar{\sigma}^{\text{corr}}_{v+s}/(dM\,dt)]-[d^2\bar{\sigma}^0/(dM\,dt)]\}+\int N(\omega)d\omega\int dt[d^2\bar{\sigma}_{\text{hard}}/(dM\,dt)]}{\int N(\omega)d\omega\int dt[d^2\bar{\sigma}^0/(dM\,dt)]} \tag{2.21}$$

using Eqs. (2.4), (2.6), and (2.15), and noticing that $d^2\bar{\sigma}^0/(dM\,dt)$ is given by setting δ_T and δ_L equal to zero in (2.6). The limits of integration in the first term of the numerator and in the denominator of the right-hand member of Eq. (2.21) are $t_{\min}$, $t_{\max}$ given by (2.14) and (2.13); $\omega_{\min}=(M^2+2m_eM)/4E_0$; $\omega_{\max}=E_0-m_e$, whereas in the second term of the numerator they are only very slightly different, namely $t_{\min}$, $t_{\max}$ given by (2.19) and (2.18), setting $W=W_0$; $\omega_{\min}=[(M+W_0)^2-m_e{}^2]/4E_0$; $\omega_{\max}=E_0-m_e$.

III. RADIATIVE CORRECTIONS IN THE DOUBLE EQUIVALENT-PHOTON APPROXIMATION

The double equivalent-photon approximation is derived from the single EPA by making

$$\begin{aligned}\bar{\sigma}_T'(M^2,t)&\simeq\bar{\sigma}_T'(M^2,0)\equiv 2M^2\sigma_{\gamma\gamma}(M^2)\,,\\ \bar{\sigma}_L'(M^2,t)&\simeq\bar{\sigma}_L'(M^2,0)\equiv 0\,,\end{aligned} \tag{3.1}$$

where $\sigma_{\gamma\gamma}(M^2)$ is the cross section for both photons on shell. We know that such an approximation is justified for the nonradiative term.[7] The explanation is that $t_{\min}$, as given by (2.14), is extremely small, and that the t behavior of the differential cross section is $\sim dt/t$, i.e., t values close to $t_{\min}$ give a predominating contribution. Now the "soft+virtual" term has the same $t_{\min}$ and, as well, the same t behavior in the first approximation [see (2.6)]. Moreover, the "hard" term has also, practically, the same $t_{\min}$; and on the other hand, it results from Eqs. (2.15), (2.16), and (2.20) that its t behavior is, again, $\sim dt/t$. Therefore, we conclude that above approximation formula (3.1) may validly be used.

We thus get the total radiative correction as

$$\delta=2\frac{\int N(\omega)(d\omega/\omega^2)\int(dt/t^2)[t(\zeta+2)\delta_T+4m_e{}^2\zeta\delta_L]+\int N(\omega)(d\omega/\omega^2)\int(dt/t^2)\int\bar{Y}\,dW^2}{\int N(\omega)(d\omega/\omega^2)\int(dt/t^2)[t(\zeta+2)+4m_e{}^2\zeta]} \tag{3.2}$$

with

$$\bar{Y}=\frac{1}{8\pi^2\alpha}[(\zeta^h+2)\bar{\sigma}_T+\zeta^h\bar{\sigma}_L]\,. \tag{3.3}$$

The limits of integration are the same as before. We notice that δ has now become independent of $\sigma_{\gamma\gamma}$, i.e., of the specific $\gamma\gamma$ process considered.

IV. NUMERICAL RESULTS, CHECK OF THE DOUBLE EPA, AND CONCLUSIONS

In Table I, we show the total radiative correction δ in percent, computed numerically in different ways, i.e., (i) by using the double EPA (3.2), (ii) by using the single EPA (2.21) for the

TABLE I. Total radiative correction δ in percent, for various values of the beam energy E_0 and the ratio M/E_0 (where M is the invariant mass produced), for nontagging measurements of processes of the type $e\bar{e}\rightarrow e\bar{e}X$. $\delta_{\rm I}$: double equivalent-photon approximation (process-independent). $\delta_{\rm II}$: single equivalent-photon approximation, $X=\mu^+\mu^-$. $\delta_{\rm III}$: single equivalent-photon approximation, $X=\pi^+\pi^-$. $\delta_{\rm IV}$: exact calculation, $X=0^{++}$ state.

M/E_0	$\delta_{\rm I}$	$\delta_{\rm II}$	$\delta_{\rm III}$	$\delta_{\rm IV}$
		$E_0=1.5$ GeV		
0.1	0.21	0.20	0.19	0.24
0.15	0.17	0.15	0.14	0.21
0.2	0.10	0.14	0.10	0.15
0.25	0.05	0.03	0.02	0.09
0.3	−0.04	−0.06	−0.05	0.01
0.35	−0.15	−0.13	−0.12	−0.07
0.4	−0.24	−0.23	−0.23	−0.16
0.45	−0.34	−0.31	−0.33	−0.25
0.5	−0.48	−0.40	−0.46	−0.35
0.55	−0.57	−0.46	−0.57	−0.45
0.6	−0.68	−0.63	−0.78	−0.55
		$E_0=4.5$ GeV		
0.1	0.36	0.45	0.45	0.40
0.15	0.32	0.30	0.28	0.36
0.2	0.20	0.13	0.09	0.30
0.25	0.15	0.00	0.08	0.22
0.3	0.12	−0.13	0.06	0.13
0.35	0.03	−0.21	−0.01	0.04
0.4	−0.13	−0.24	−0.09	−0.06
0.45	−0.17	−0.28	−0.18	−0.16
0.5	−0.32	−0.42	−0.34	−0.27
0.55	−0.44	−0.56	−0.47	−0.39
0.6	−0.52	−0.59	−0.54	−0.50
		$E_0=15$ GeV		
0.1	0.49	0.50	0.57	0.60
0.15	0.47	0.47	0.47	0.55
0.2	0.43	0.44	0.42	0.47
0.25	0.29	0.29	0.36	0.38
0.3	0.19	0.20	0.20	0.28
0.35	0.11	0.04	0.08	0.17
0.4	0.01	−0.05	0.05	0.06
0.45	−0.09	−0.08	−0.07	−0.06
0.5	−0.26	−0.28	−0.09	−0.18
0.55	−0.37	−0.36	−0.40	−0.31
0.6	−0.47	−0.49	−0.61	−0.44
		$E_0=70$ GeV		
0.1	0.84	0.81	0.85	0.86
0.15	0.74	0.73	0.83	0.79
0.2	0.61	0.51	0.70	0.70
0.25	0.55	0.44	0.62	0.59
0.3	0.39	0.35	0.44	0.47
0.35	0.33	0.19	0.31	0.34
0.4	0.19	0.15	0.18	0.21
0.45	0.11	0.07	0.01	0.08
0.5	−0.01	−0.05	−0.09	−0.07
0.55	−0.18	−0.18	−0.27	−0.21
0.6	−0.39	−0.32	−0.32	−0.36

specific processes $e\bar{e}\rightarrow e\bar{e}\mu^+\mu^-$ and $e\bar{e}\rightarrow e\bar{e}\pi^+\pi^-$, and (iii) through an exact calculation for a fictitious and particularly simple process, namely $e\bar{e}\rightarrow e\bar{e}X$, where X represents a scalar (0^{++}) continuum. In case (iii), formula (2.5) was applied as well, but now the reduced cross sections $\tilde{\sigma}'_{T,L}$ were those of the full process $\gamma^*\bar{e}\rightarrow X\bar{e}$ (where γ^* is the left-hand virtual photon) and were calculated without any approximation. Correspondingly, in formula (2.5), the definition of s, m', W', and ζ was changed as well (all those modifications are quite trivial). Let us also mention that, for the coupling between the $\gamma\gamma$ system and the 0^{++} state, we used the gauge-invariant tensor: $g_{\mu\nu}-k'_\nu k_\mu/(k\cdot k')$. We here considered various beam energies ranging between 1.5 and 70 GeV, and invariant masses ranging between 0.1 E_0 and 0.6 E_0.

Comparing numerical predictions of the process-independent double EPA with those of the single EPA and of the exact calculation for the specific processes considered, we see that the differences are insignificant. We thus conclude that the double EPA may be used with confidence for such calculations.

On the other hand, we notice that $|\delta|$ is less than 1% everywhere, i.e., the total radiative correction is systematically very close to zero. In other words, surprisingly enough, there is an almost perfect cancellation between the "virtual plus soft" contribution and the "hard" one.[13]

In conclusion, it appears that, in a very wide kinematic range, the total radiative correction may be neglected in $\gamma\gamma$ collision experiments without electron tagging.

A few additional remarks are in order.

(i) As already said in Sec. I, radiative corrections may strongly depend on the experimental conditions considered, and they may considerably differ from the figures here shown when a different solution is chosen as regards electron tagging (see Ref. 5).

(ii) A more realistic calculation would consist of taking account of acceptance cuts in the central detector; one of us (MD) is performing such a calculation.

(iii) Since the total first-order correction appears to be negligible under the conditions considered, there is no need for calculating higher-order corrections.

APPENDIX

We give here expressions for the virtual Compton cross sections:

$$\tilde{\sigma}_T=\frac{2\pi\alpha^2}{W^2}\left\{(W^2-m_e{}^2)\left[\frac{W^2+m_e{}^2+t}{W^2}+\frac{2W^2}{\Lambda^{1/2}}L+\frac{8m_e{}^2W^2}{\Lambda}\left(2-\frac{L}{\eta}\right)\right]\right.$$

$$\left.+2t\left[-\frac{4W^2}{W^2-m_e{}^2}-\frac{(W^2-m_e{}^2)(W^2+m_e{}^2+t)}{\Lambda}+\frac{2W^2}{\Lambda^{1/2}}L\left(\frac{W^2+m_e{}^2+t}{W^2-m_e{}^2}-\frac{m_e{}^2(W^2-m_e{}^2)}{\Lambda}\right)\right]\right\},$$

$$\tilde{\sigma}_L=\frac{8\pi\alpha^2 t}{\Lambda}\left\{-\frac{W^2+m_e{}^2+t}{W^2}(W^2-m_e{}^2)+\frac{2}{W^2(W^2-m_e{}^2)}[m_e{}^6-m_e{}^4(5W^2-t)-m_e{}^2(5W^4+4tW^2)+W^4(W^2+t)]\right.$$

$$\left.+\frac{2m_e{}^2L}{(W^2-m_e{}^2)\Lambda^{1/2}}[3m_e{}^4+2m_e{}^2(5W^2+3t)+3W^4+6W^2t+2t^2]-\frac{4m_e{}^2}{W^2-m_e{}^2}(2W^2+2m_e{}^2+t)\right\},$$

with

$$\Lambda\equiv\Lambda(W^2,m_e{}^2,-t),\quad \eta=\frac{\Lambda^{1/2}}{W^2+m_e{}^2+t},\quad L=\ln\frac{1+\eta}{1-\eta}.$$

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[8]See, for instance, P. Kessler, Acta Phys. Austriaca 41, 141 (1975).

[9]C. Carimalo, G. Cochard, P. Kessler, J. Parisi, and B. Roehner, Phys. Rev. D 10, 1561 (1974).

[10]That invariant "flux factor" in the denominator of a cross section [see, e.g., J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley, Reading, Mass., 1955), p. 167] is given by

$$4F=4[(p_1\cdot p_2)^2-m_1{}^2m_2{}^2]^{1/2}$$
$$=2\Lambda^{1/2}(s,m_1^2,m_2^2),$$

where the subscripts 1,2 refer to the incident particles of the process considered, and $s=(p_1+p_2)^2$. In the case of virtual photoproduction cross sections, the definition of that factor becomes arbitrary to the extent that a "mass" must be specified for the virtual photon. The prescription given by L. N. Hand [Phys. Rev. 129, 1834 (1963)] (choosing that mass to be zero) is only one possible option.

[11]Any dependence on that arbitrary cutoff W_0 must of course vanish from the total radiative correction (as we checked indeed in the numerical computations). Actually, W_0 may be eliminated from the analytical formulas, but that does not appear to make the computations easier.

[12]C. Carimalo, P. Kessler, and J. Parisi, Phys. Rev. D 14, 1819 (1976), formula (2.12).

[13]Former results showing positive and large values for δ, that were presented in Ref. 6, as well as in two unpublished reports (LPC 80-06 and LPTP 80-1), were wrong. It should be mentioned that a calculation of radiative corrections for electroproduction on nuclear targets under similar conditions (i.e., with the final electron undetected) was performed many years ago by P. K. Kuo and D. R. Yennie, Phys. Rev. 146, 1004 (1966). Here the total radiative correction was found to be, typically, of the order of minus a few percent. We are grateful to Professor Yennie for having drawn our attention to that work.