Equivalence of (super-) renormalizable fermion-boson interactions and nonlinear fermion interactions

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Using well-defined cutoff theories it is shown that many (super-) renormalizable fermion-boson interactions are equivalent to nonlinear fermion interactions which are local when the cutoffs are removed. The reason is that the Dirac sea has antiscreening effects which generate long-range interactions from short-range interactions. Thus, for example, an Abelian current-current interaction is equivalent to quantum electrodynamics for $2 < d \leq 4$, where d is the space-time dimension. The eigenvalue condition on the four-fermion coupling constant corresponds to choosing the critical point of the transition to the phase in which Lorentz invariance is spontaneously broken. Thus the photon can be regarded as the Goldstone excitation of a spontaneously broken Lorentz symmetry. Also it is possible to regard fundamental interactions as due to self-interactions of a fermionic "matter" field.

I. INTRODUCTION

In this paper we will show that many (super-) renormalizable theories of bosons interacting with fermions can be regarded as theories involving only fermions with nonlinear interactions which are local in the limit of an infinite cutoff. For previous approaches to prove such equivalences see Refs. 1–10. The bosons are generated as collective excitations of the fermionic system. The "nonrenormalizable" nonlinear fermion interactions are tamed because[11,12] the coupling constants are infinitesimally small for large cutoffs. The approach of this paper is to establish the equivalence between cutoff theories. Divergences in the bare parameters as the cutoffs are removed are crucial for these equivalences.

In Sec. II the case of nonlinear scalar bilinear interactions of fermions is considered. The Dirac sea of these fermions has an antiscreening property, whereby a long-range interaction between physical fermions is generated out of a short-range interaction between the bare fermions. More simply, the fermion loop gives a *negative* divergent contribution to the (self-mass)^2 of the auxiliary field corresponding to the scalar bilinear. As a result a nonlinear fermion interaction with an infinitesimally small range for large cutoffs acquires a finite range due to quantum fluctuations.

Self-interactions of the bosonic collective excitations have a tendency to undo the antiscreening effects of the Dirac sea. That is to say, they give a positive (self-mass)^2 contribution. To have the equivalence with a local fermionic theory we require that the net bare (mass)^2 of the boson be positive and divergent with the cutoff. Hence the equivalence survives only if the cutoffs are removed in such a way that the cutoff associated with fermions is much larger than that associated with the scalars. An interesting example of this effect is provided by the supersymmetric theories.[13] Thus for $d=2$, there is no divergence in the bare mass of the bosons which would mean the equivalence cannot be valid. However, by considering cutoff theories in which supersymmetry is explicitly broken through the cutoffs in the kinetic energy, it is possible to realize the equivalence in this case also. Again it becomes necessary to keep the cutoff associated with the fermion much larger than that associated with the boson. For $d=4$, it is sufficient to have the common wave-function renormalization constant vanish with the cutoff.

In Sec. III, equivalence[3–5] of the current-current interactions with gauge theories for $2< d \leqslant 4$ is proved. The fact that the photon (self-mass)^2 is not zero but instead divergent and moreover negative, when the cutoff in the fermionic sector of quantum electrodynamics is not gauge invariant is crucial for this equivalence. The equivalence is not valid for $d=2$ because the self-mass is not divergent. On the other hand, the arguments require that the Thirring model defined via a certain regularization for $g=\pi$ be identical with the Schwinger model in the scale-invariant limit.[14] Such an equivalence is true with $g=2\pi/\sqrt{5}$, where g is Johnson's definition[15] of the Thirring coupling constant.

For $d=4$ there is an interesting possibility that QED defined via continuum regulators may not be meaningful in the infinite-cutoff limit, whereas the current-current interaction may make sense and reproduce the renormalized perturbation series of QED. This has to do with the Landau ghost problem[16] which suggests that the bare charge has to be pure imaginary. This corresponds to a negative sign of the current-current interaction (i.e., an attraction between like particles). This is allowed by the Hermiticity of the Hamiltonian in the current-current theory and moreover because of the Pauli exclusion principle the Hamiltonian may still be bounded from below. Boundedness is especially possible because the strength of the current-current interaction is infinitesimally small for large cutoffs.

In this proof of the equivalence, strength of the current-current interaction has been carefully adjusted to give a massless vector excitation. A stronger interaction would result in a tachyonic excitation driving the system to a phase in which the expectation value of the charge (-current) density is nonzero. In this phase Lorentz invariance is spontaneously broken.[3,17] To see this it is necessary to work with the cutoff theory. This is done in Sec. IV.

Thus the eigenvalue condition on the current interactions corresponds to choosing the critical point of the transition to the phase with a broken Lorentz symmetry. Therefore, the photon can be regarded as a Nambu-Goldstone excitation associated with a spontaneous breaking of Lorentz symmetry. Also, the choice of the critical point implies that the photon is simultaneously an e^+e^- bound state and a coherent excitation. These considerations are also valid for the non-Abelian current interactions.

All proofs of equivalences have involved just rewriting and reinterpreting the cutoff actions. It is to be suspected that the equivalence is also valid with different types of cutoff interactions especially for the case of gauge theories where the interactions are uniquely fixed by gauge invariance in the limit of an infinite cutoff. However, proving the equivalence cannot be easy in such cases. An illustration of the possibilities involved is given in the Appendix. It is shown how one may manufacture a kinetic energy for the photon from Abelian current-current interactions without hiding it in the range of nonlocality. For this, the four-fermion coupling constant is chosen to be infinitesimally away from the critical point for a finite cutoff. An infinitesimal breaking of local gauge invariance in the fermionic part of the action with the auxiliary photon field together with an infinitesimal (renormalized) photon mass can produce a finite adjustable kinetic energy for the photon. However, it has not been possible to demonstrate that this is the only effect and other unwanted interactions are not generated in higher orders.

In Sec. V a comparison of the present approach for proving the equivalences with previous approaches is made. The importance of working with well-defined actions with cutoff is emphasized. Because the equivalence demonstrated in this paper is valid without restrictions on the parameters for almost any fermion-boson theory, it would appear to be only a mathematical curiosity. However, it is possible that there are situations where a local fermionic theory has differences of physical significance from a local fermion-boson theory. These are pointed out in Sec. V.

II. EQUIVALENCE OF THE SCALAR-SPINOR INTERACTIONS AND NONLINEAR SPINOR INTERACTIONS

Throughout this paper we will work with the Euclidean formulation because, apart from the well-known advantages, it makes clear the signs to be associated with various parameters.

Consider the following regularized Euclidean action

$$S = -\tfrac{1}{2}(\partial_i\varphi)^2 - \tfrac{1}{2}\mu_c^2\varphi^2 + \bar\psi(-\not\partial_M - m_0 - g_0\varphi)\psi\ , \tag{2.1}$$

where $\not\partial_M$ corresponds to a regularization in the fermionic kinetic energy. For example,

$$\not\partial_M = \not\partial(1-\Box/M^2),\quad \Box = \partial_k\partial_k\ . \tag{2.2}$$

Here the Euclidean Dirac matrices γ_i obey the algebra

$$\{\gamma_i, \gamma_j\} = 2\delta_{ij},\quad \gamma_i^\dagger = \gamma_i \tag{2.3}$$

and products such as $\bar\psi\psi$ stand for an integral over space-time. All information regarding this theory can be obtained from the functional integral

$$Z = \int \mathfrak{D}\varphi\, \mathfrak{D}\psi\, \mathfrak{D}\bar\psi\, \exp(S + j\varphi + \bar\eta\psi + \bar\psi\eta)\ , \tag{2.4}$$

where j, η, and $\bar\eta$ are external sources for φ, $\bar\psi$, and ψ, respectively. Henceforth we will drop the external sources from our equations.

We will assume that the interaction term is normal ordered with respect to the fermion fields, thereby removing the divergent (as $M\to\infty$) contribution to the vacuum expectation value (VEV) of φ (Fig. 1). However φ acquires a nonzero, cutoff independent VEV from higher-order corrections (Fig. 2).

The only divergence as $M\to\infty$ in a perturbation expansion is in the $O(g_0^2)$ (self-mass)^2 contribution to the φ. This contribution is logarithmically divergent and *negative*. Thus by choosing

$$\mu_0^2 = cg_0^2\ln\left(\frac{M}{\mu}\right) + \mu^2,\quad c>0 \tag{2.5}$$

the limit $M\to\infty$ can be taken keeping all other parameters μ, g_0, m_0 (and the wave-function renormalization constants) finite.

We may rewrite Eq. (2.4) as

FIG. 1. Divergent contributions to the vacuum expectation value of φ.

$$Z=\int \mathfrak{D}\psi\,\mathfrak{D}\bar{\psi}\exp[\bar{\psi}(-\not{\partial}_M-m_0)\psi+G(\bar{\psi}\psi)\cdot(\bar{\psi}\psi)], \tag{2.6}$$

where

$$G=g_0^2/2\mu_0^2 \tag{2.7}$$

and

$$(\bar{\psi}\psi)\cdot(\bar{\psi}\psi)=\bar{\psi}\psi(1-\Box/\mu_0^2)^{-1}\bar{\psi}\psi\ . \tag{2.8}$$

Since μ_0^2 diverges with the cutoff, the exponent becomes local as $M\to\infty$. Thus the Yukawa interaction, Eq. (2.1), can be regarded as equivalent to a Fermi interaction. The boson is generated as a collective excitation. A careful tuning of the range of this four-fermion interaction is necessary to reach the required continuum limit. The equivalence is possible because the Dirac sea has an antiscreening effect resulting in a long-range interaction between the physical fermions from a short-range interaction between the bare fermions. Divergence in the self-mass in Yukawa theory is crucial to make the equivalent Fermi theory local.

If the Fermi interaction were strictly local even in the cutoff theory, we would not be able to generate the kinetic energy for the scalar. Such a theory can be regarded as a limiting case of the Yukawa theory in which the wave-function renormalization constant of the boson is zero[18] or the bare coupling constant is infinite. Such a limit is expected to make sense[11,14] and to correspond to a theory with anomalous scale dimensions. An example is provided by the Thirring model,[15] where G is finite, i.e., g_0 is divergent with the cutoff.

Consider the case $m_0=0$. Then we have a discrete symmetry $\varphi\to-\varphi$, $\psi\to\gamma_5\psi$. When $\mu^2<0$, where μ is the renormalized mass of the boson, this symmetry is spontaneously broken, φ acquires a nonzero vacuum expectation value and ψ acquires a nonzero mass. Thus $\mu=0$ corresponds to being at the critical point of the transition to the phase in which the discrete symmetry $\varphi\to-\varphi$ is spontaneously broken. μ in Eq. (2.5) is infinitesimally small compared with the unrenormalized mass μ_0. Thus, in the Fermi theory, the bare coupling constant is chosen to be infinitesimally close to the transition point to the phase in which the composite operator $\bar{\psi}\psi$ acquires a nonzero vacuum expectation value.

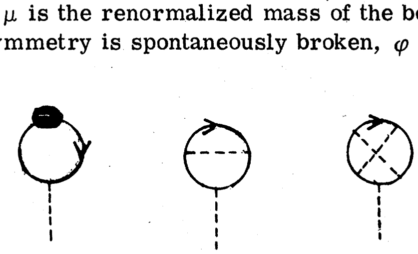

FIG. 2. Finite contributions to the vacuum expectation value of φ.

Let us now add a $\lambda_0\varphi^4$ self-interaction. There is an additional divergence in the (self-mass)^2 of φ, which can be isolated by just normal ordering $\lambda_0\varphi^4$ with respect to a reference mass, say μ. If we introduce a regularization characterized by Λ in the bosonic kinetic energy, $\partial_i\to\partial_{i(\Lambda)}$, the bare mass must be chosen to be (for large M and Λ)

$$\mu_0^2=cg_0^2\ln\frac{M}{m}-c_1\lambda\ln\frac{\Lambda}{\mu}+\mu^2\ ,\quad c,c_1>0\ . \tag{2.9}$$

As before we will consider an integration over φ to get an effective action involving only fermions. The result is the following. Consider the connected Green's functions of the normal ordered $\lambda_0\varphi^4$ theory with a mass μ_1 given by the transcendental equation

$$\mu_1^2=cg_0^2\ln\frac{M}{m}+\mu^2-c_1\lambda_0\ln\frac{\mu_1}{\mu}\ .$$

For a finite M, all these Green's functions have a limit as $\Lambda\to\infty$ when expanded in powers of λ_0. Now attach a factor $-g_0\bar{\psi}\psi$ for each external φ line.

As μ_1 diverges with M and occurs only in the denominators in the (finite) expressions for the Green's functions, it is clear that when $M\to\infty$ the effective fermionic action becomes local. However, for a finite M we have a complicated nonpolynomial, nonlocal action in $\bar{\psi}\psi$.

One may expect to reach same local limit starting from a simple cutoff action of self-interacting fermions. We may replace an infinite number of vertices by only two. Consider

$$Z=\int \mathfrak{D}\psi\,\mathfrak{D}\bar{\psi}\exp[\bar{\psi}(-\not{\partial}_M-m_0)\psi+G(\bar{\psi}\psi)\cdot(\bar{\psi}\psi)-G_1(\bar{\psi}\psi)^4], \tag{2.10}$$

where $(\bar{\psi}\psi)$ is normal ordered. We will introduce an auxiliary scalar field φ in the standard fashion[8]:

$$Z=\int \mathfrak{D}\psi\,\mathfrak{D}\bar{\psi}\,\mathfrak{D}\varphi\exp[-\tfrac{1}{2}(\partial_\Lambda\varphi)^2-\tfrac{1}{2}\mu_0^2\varphi^2+\bar{\psi}(-\not{\partial}_M-m_0-g_0\varphi)\psi-G_1(\bar{\psi}\psi)^4]. \tag{2.11}$$

Now if we choose $G_1\propto\lambda_0[g_0\ln(M/m)]^{-4}$ the only

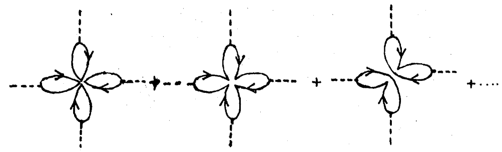

FIG. 3. Generating a $\lambda\varphi^4$ vertex from a $G_1(\bar\psi\psi)^4$ interaction. There are contributions from various ways of contracting the fermion fields.

effect of this new interaction will be to induce a $\lambda_0\varphi^4$ term in the limit $M\to\infty$ (Fig. 3). The induced $\lambda_0\varphi^4$ vertex is not normal ordered, so that μ_0 must be chosen as in Eq. (2.9). Hence to have an interaction that is local as M, $\Lambda\to\infty$ in Eq. (2.6) we require that the cutoff M associated with the fermion be much larger than that associated with the boson.

For $d=2$, there is an infinite class of local boson-fermion theories labeled by the strengths of φ^2, φ^3, $\varphi^4,\ldots$ couplings. Obviously we can hope to generate all these interactions only by choosing fermion interactions which are increasingly nonlinear. For $d>2$, we must consider $\lambda\varphi^4$ (for $d\leqslant 4$) and $\lambda'\varphi^6$ (for $d\leqslant 3$) couplings. It is clear that the above arguments go through.

In case of supersymmetric theories[13] more care is needed. There is no separate mass renormalization[19] so that $\mu_0{}^2\varphi^2$ term is of the same order as $(\partial_\Lambda\varphi)^2$ and it appears that it is not possible to get a local $(\bar\psi\psi)^2$ interaction. We will consider the Wess-Zumino model with a supersymmetric regularization[19] in terms of the physical fields only:

$$\begin{aligned}S=&-\tfrac12 Z\partial A(1+\xi\Box^2)\partial A-\tfrac12 Z\partial B(1+\xi\Box^2)\partial B\\&-\tfrac12 Z\bar\psi(1+\xi\Box^2)\partial\!\!\!/\psi-\tfrac12 Z^{-1}[mA+g(A^2-B^2)](1+\xi\Box^2)\\&\times[mA+g(A^2-B^2)]+\tfrac12 Z^{-1}(mB+g2AB)(1+\xi\Box^2)\\&\times(mB+g2AB)-\tfrac12 m\bar\psi\psi+g\bar\psi(A+\gamma_5B)\psi\,. \qquad (2.12)\end{aligned}$$

Consider $d=4$. Only Z has coefficients singular in ξ when written as a series in g. Hence we can have the bosonic mass terms to be much larger than the bosonic kinetic energy terms only if Z approaches zero with ξ. Even this cannot work for $d=2$, when Z has a cutoff-independent limit. In this case we will use two different regularizations M and Λ as before, thereby breaking the supersymmetry explicitly in the cutoff action. After normal ordering and ignoring the one-fermion-loop contribution to the self-mass of A and B, the limit M, Λ can be explicitly taken recovering the supersymmetry. Equivalently we may consider the cutoff action which is not normal ordered but which has in addition divergent (when M, $\Lambda\to\infty$) $\nu_0 A$ and $\delta M_0{}^2(A^2+B^2)$ terms, both of which would be zero with a supersymmetric regularization. With $M\gg\Lambda$, $\delta M_0{}^2$ will be positive divergent and we again get the equivalence with a nonlinear fermion theory. Vertices with the scalar bilinears involve the combination $(\bar\psi\psi+\nu_0/g)$.

III. EQUIVALANCE OF THE ABELIAN CURRENT-CURRENT INTERACTIONS AND QUANTUM ELECTRODYNAMICS

We first consider the case $d<4$. In this case, if one formally sets the self-mass contributions to the photon to zero, there are no divergences left in any order of the perturbation series. The electron self-energy is superficially divergent for $d\geqslant 3$, with a naive power counting, but by the Weisskopf theorem, it is convergent on doing angular integrations. Thus the theory can be defined via the functions integral:

$$\begin{aligned}Z&=\int \mathfrak{D}A_i\,\mathfrak{D}\psi\,\mathfrak{D}\bar\psi\exp[-\tfrac14 Z_3F_{ij}{}^2+Z_2\bar\psi\\&\qquad\times(-\partial\!\!\!/-m-ieA\!\!\!/)\psi+\delta m\bar\psi\psi]\\&=\int\mathfrak{D}A_i\,\exp[-\tfrac14 Z_3F_{ij}{}^2\\&\qquad+\mathrm{Tr}\ln'|1+ie(\partial\!\!\!/+m_1)^{-1}A\!\!\!/\,|\,]\,,\end{aligned}\qquad(3.1)$$

where $m_1=m-Z_2{}^{-1}\delta m$. All parameters entering in this equation are finite (cutoff independent). The prime on Tr ln means that it is defined to be gauge invariant, i.e., a term proportional to A^2 is ignored. It is assumed that a gauge is chosen in the usual manner. We have presumed that $Z_1=Z_2$ in writing this equation. Thus e is the renormalized charge.

We will now extract the term $-\frac14 c_2e^2F_{ij}{}^2$ from the Tr ln$'$ term corresponding to (finite) wave-function renormalization from the lowest-order vacuum polarization diagram, add this to the $Z_3F_{ij}{}^2$ term and rescale A_i to make the coefficient of $-\frac14 F_{ij}{}^2$ equal to unity. We get

$$Z=\int\mathfrak{D}A_i\exp[-\tfrac14F_{ij}{}^2+\mathrm{Tr}\ln''|1+ie_1(\partial\!\!\!/+m_1)^{-1}A\!\!\!/\,|\,]\,,\qquad(3.2)$$

where

$$e_1{}^2=\frac{e^2}{Z_3+c_2e^2}=\frac{e_0{}^2}{1+c_2e_0{}^2}\,.\qquad(3.3)$$

e_0 is the bare charge. Modified Feynman rules for Eq. (3.2) ignore $O(e^2)$ wave-function renormalization for the photon. With the assumptions that QED for $d<4$ makes sense for any value of $e^2>0$ and that $Z_3>0$ for any such e^2, Eq. (3.3) makes sense only for $0<e_1{}^2<c_2{}^{-1}$.

We now consider current-current interactions with two cutoffs ξ and M:

$$Z=\int \mathfrak{D}\psi\,\mathfrak{D}\bar{\psi}\exp\{\bar{\psi}(-\not\partial_M-m_0)\psi -\tfrac{1}{2}e^2 j_i\,[\zeta^2\delta_{ij}-Z_3(\partial_k\partial_k\delta_{ij}-\partial_i\partial_j)]^{-1}j_j\}\,. \quad (3.4)$$

The currents have a nonlocal interaction of range $(Z_3/\zeta^2)^{1/2}$. We will introduce an auxiliary variable A_i in the standard fashion:

$$Z=\int \mathfrak{D}\psi\mathfrak{D}\bar{\psi}\,\mathfrak{D}A_i\,\exp[-\tfrac{1}{2}\zeta^2A^2-\tfrac{1}{4}Z_3F_{ij}^2+\bar{\psi}(-\not\partial_M-m_c-ie\not A)\psi]$$
$$=\int \mathfrak{D}A_i\exp(-\tfrac{1}{2}\zeta^2A^2-\tfrac{1}{4}Z_3F_{ij}^2+\mathrm{Tr}\ln|-\not\partial_M-m_0-ie\not A|)\,. \quad (3.5)$$

Since the regularization $\not\partial\to\not\partial_M$ in the fermionic part of the action is not gauge invariant, the (self-mass)^2 contribution to the photon from Tr ln term is not zero, but instead diverges with M for $d>2$. It is also negative. Hence we may choose ζ to cancel this contribution. Then we may formally take the limit $M\to\infty$ to recover Eq. (3.1). Since we have argued that this equation has a cutoff-independent meaning, taking the limit is justified, and the equivalence is proved.

If the cutoff action, Eq. (3.4), has a strictly local current-current interaction, we would get a formal equivalence with Eq. (3.2), but with $e_1^2=c_2^{-1}$. This corresponds to the limiting value $e_0^2\to\infty$ and therefore taking the limit $M\to\infty$ is justified only if the limit $e_0\to\infty$ is meaningful. It is expected that when properly taken, this limit is indeed meaningful[11] and corresponds to a scale-invariant theory with anomalous scale dimensions[14] for $2\leqslant d<4$ and a free field theory of photons and electrons for $d=4$.

Equivalence for the case $d=4$ is more complicated because there are additional divergences in QED. We will consider Pauli-Villars regularization for the fermion loops and a regularization via higher derivatives for the photon propagator:

$$Z=\int \mathfrak{D}A_i\,\mathfrak{D}\psi\,\mathfrak{D}\bar{\psi}\,(\mathfrak{D}\chi_a\,\mathfrak{D}\bar{\chi}_a)\times\exp\{-\tfrac{1}{4}Z_3(\Lambda,M)F_{ij}(1-\Lambda^{-2}\Box)F_{ij}+Z_2(\Lambda,M)\bar{\psi}(-\not\partial-m_1-ie\not A)\psi+\sum_a Z_2\bar{\chi}_a[(-\not\partial-M_i)b_i^{-1}-ie\not A]\chi_a\}\,. \quad (3.6)$$

Here the χ_a's are a set of wrong-statistics fermions. We have

$$Z=\int \mathfrak{D}A_i\exp[-\tfrac{1}{4}Z_3F_{ij}(1-\Lambda^{-2}\Box)F_{ij}+\mathrm{Tr}\ln|-\not\partial-M_i-ie\not A|-\mathrm{Tr}\ln|(-\partial-m_i)b_i^{-1}-ie\not A|]\,. \quad (3.7)$$

Power counting shows that the Pauli-Villars regularization is needed only for the lowest-order vacuum polarization diagram. A regularization in the photon propagator suffices for all fermion loops with more than two vertices. We will therefore separate out the photon wave-function renormalization contribution coming from the Tr ln term and absorb it in the $Z_3(\Lambda,M)$ term:

$$Z=\int\mathfrak{D}A_i\exp[-\tfrac{1}{4}z_3(\Lambda,M)F_{ij}(1-Z_3\Box/z_3\Lambda^2)F_{ij}+\mathrm{Tr}\ln''|-\not\partial-M_F-ie\not A|-\sum_i\mathrm{Tr}\ln''|(-\not\partial-M_i)b_i^{-1}-ie\not A|]\,, \quad (3.8)$$

$$Z_3(\Lambda,M)+c_2e^2\ln\frac{M}{m}=z_3(\Lambda,M)\,. \quad (3.9)$$

Here M represents a typical mass of the Pauli-Villars fields and c_2 is a positive number. Renormalized perturbation theory suggests that we can do away with the Pauli-Villars fields in (3.8). This means $z_3(\Lambda,M)$ has a limit $z_3(\Lambda)$ as $M\to\infty$. The modified Feynman rules are now to ignore the wave-function renormalization term from the lowest-order vacuum polarization diagram. All renormalization constants depend only on Λ.

We have run into basic contradictions. Equation (3.9) requires Z_3 to be negative and infinitely large in magnitude. This is contradictory to unitarity. Equivalently, the bare coupling constant $e_0(\Lambda,M)=ez_3^{-1/2}$ must be chosen to be purely imaginary. The Hamiltonian is no longer Hermitian.

This is the well-known Landau ghost problem.[16] Here with the special regularization chosen it appears that the problem persists to all orders, if interchangeability of the order of removing cutoffs is correct as renormalized perturbation theory suggests.

Thus QED defined with continuum regulators is possibly a self-contradictory theory. However, it is still possible that with a lattice regularization[20] there are no such problems. In this case M and Λ are intimately related and there are many-photon vertices which can significantly affect dependence of the renormalization constants on the lattice spacing. We will now argue there is a possibility that the current-current interaction can give the same renormalized perturbation series as QED and moreover may make sense nonperturbatively. It is possible to effectively get $Z_3<0$ from the current-current interaction without any contradictions. For this it is necessary to use higher derivatives in current-current interactions in addition to a nonlocal interaction,

$$j_ij_i\to j_i(1-\Box/\zeta^2)(1-\Box^\sigma/\Lambda^\sigma)^{-1}j_i\,. \quad (3.10)$$

We then have an effective interaction,

$$-\tfrac{1}{2}\zeta^2 A^2(p)\,\frac{1+p^6/\Lambda^6}{1+p^2/\zeta^2} - iej_i A_i \ . \qquad (3.11)$$

Now for canceling the photon self-mass term we require $\zeta^2 \propto M^2$. After removing this term we get

$$-\tfrac{1}{2}A^2(p)\,\frac{-p^2+\zeta^2 p^6/\Lambda^6}{1+p^2/\zeta^2} \ . \qquad (3.12)$$

If we now formally set Λ, $M\to\infty$ we get a kinetic energy term with a wrong sign, i.e., $Z_3 < 0$. However, the current-current theory is still meaningful.

There is another technical problem with the case $d=4$. With a gauge-noninvariant regularization $\not\partial \to \not\partial_M$ the Tr ln term is not gauge invariant when $M\to\infty$ even after the term $\delta m^2 A^2$ is subtracted away. Thus, the photon-photon scattering amplitude is cutoff independent on doing an angular integration but not gauge invariant. It does not vanish for zero-frequency photons. Reason as usual is that a shift of integration variables in proving the Ward-Takahashi identity cannot be justified resulting in an anomaly.[21] Also the term quadratic in A and in momenta is neither finite when $M\to\infty$ nor is it transverse. A way to avoid this problem is to use just one Pauli-Villars regulator field of mass M instead of the regularization in the Dirac kinetic energy. Because there is just one regulator, the photon (self-mass)^2 in $O(e^2)$ is not zero but has the form $-e^2 c(M^2-m^2)$ where c is a positive constant. Therefore, ζ can be chosen to cancel this term. With this regulator, Tr ln$'$ is gauge invariant.

In case of the current-current interactions the equivalence is not valid for $d=2$. The reason is that the self-mass of the photon in $O(e^2)$ is not divergent as $M\to\infty$, but finite on carrying out angular integrations. Again there is an anomaly in the $W-T$ identity. An explicit calculation gives

$$\pi_{ij}(q^2) = -\frac{e^2}{2\pi}\left[\delta_{ij} + (\delta_{ij} - p_i p_j/p^2)\left(\frac{4m^2}{p^2 t}\ln\frac{t+1}{t-1} - 2\right)\right], \qquad (3.13)$$

where $t^2 = 1 + 4m^2/p^2$. If we ignore the nontransverse part, $\pi(q^2)\to 0$ as $q^2\to 0$ for $m\neq 0$ so that the self-mass is zero. Again ignoring the nontransverse part if we first take the limit $m\to 0$, ln term vanishes and we recover the well-known Schwinger value $\pi(q^2) = e^2/\pi$. On the other hand, the equation as it is gives $\pi_{ij}(0) = -e^2/(2\pi)\delta_{ij}$ which corresponds to a negative (mass)^2.

Let us consider the particular case of strictly local current-current interaction. For the particular value $e^2/\zeta^2 = \pi$ the functional integral is formally gauge invariant and as argued before, should be equivalent to the $e\to\infty$ limit of two-dimensional QED. When the electron mass is zero, this means that the Thirring model[15] for this value of the coupling constant should be identical with the scale-invariant limit[14] of Schwinger's model. In fact, for $g = 2\pi/\sqrt{5}$ where g is the Johnson definition[15] of the coupling constant, the scale dimension of the Thirring fermion is $\frac{1}{4}$, same as that in Schwinger's model in the scale-invariant limit.[14] With the same choice of the γ matrices, the fermionic Green's functions are also identical. Since the value of the Thirring coupling constant depends sensitively on the definition of the current,[15] we should not expect this g to be equal to π.

For $e^2/\zeta^2 \neq \pi$, the Thirring model as defined here with the unconventional cutoff does not have a conserved current and hence is of no interest. Only for the value $e^2/\xi^2 = \pi$ it coincides with the usual definitions.

It is now possible to establish the equivalence for non-Abelian current-current interactions for $2 < d \leq 4$. Now it is necessary to make use of many-current interactions to recover self-interactions of the gauge bosons as in the case of scalar-spinor theories. For $d=4$ in addition to one Pauli-Villars wrong-statistics fermion loops, it is necessary[22] to introduce negative-metric/wrong-statistics fermions to regularize one-loop-gauge-boson diagrams and a higher-order-*covariant* derivative to regularize many-loop gauge-boson diagrams. Thus the fermionic cutoff action involves an infinite number of many-current interactions. As with the $\lambda\varphi^4$ interaction in Sec. II, one may expect to have the equivalence with a simpler class of cutoff actions. This is further considered in the Appendix.

IV. PHOTON AS A NAMBU-GOLDSTONE BOSON: BJORKEN MECHANISM

In the previous section the current-current interaction strength $G = e^2/2\zeta^2$ was carefully adjusted to give a massless vector excitation. This choice actually corresponds to choosing the critical point of the transition to the phase in which Lorentz invariance is spontaneously broken. To see this one must first work with a finite cutoff. Consider the effective potential (Ref. 23) $v(\mathcal{A}^2)$ for the variable A_i in the functional integral Eq. (3.5). This measures the energy density in a state in which A_i is constrained to have a value $\mathcal{A}_i$. It is formally given by the diagrammatic expansion of Fig. 4. Here each external line is associated with a factor $\mathcal{A}_i$ and carries a zero momentum. There is a factor (-1) for each closed fermion loop. Because of Euclidean invariance v is a function of

$\mathcal{A}^2$ only and all diagrams with an odd number of external lines vanish. Because we do not have gauge invariance for a finite M, even the convergent diagrams do not vanish, in contrast with the case of quantum electrodynamics. It is easily seen that $v(\mathcal{A}^2)$ has the form $\zeta^2[\mathcal{A}^2+f(G\mathcal{A}^2)]$.

Let us assume that as G is varied the curvature of $v(\mathcal{A}^2)$ at the origin changes sign and the absolute minimum η_i smoothly changes away from $\mathcal{A}_i=0$. Then there is a continuous transition to a phase in which the expectation values of A_i and hence of the current $\bar{\psi}\gamma_i\psi$ is nonzero. In this phase the Euclidean invariance is spontaneously broken.[3,4,17]

Bjorken[3] has conjectured the mechanism which can do this. If $G<0$, i.e., there is an attractive force between like particles, then it may be favorable to have a nonzero charge (– current) density in the lowest energy state of the theory. However, we will show that in a certain approximation scheme (same as that considered by Blorken[3]), the transition takes place for the normal sign of G. The vacuum acting as a dielectric medium makes the static force between like particles attractive even when the bare coupling constant G is positive. More simply, this has to do with the fact that the (self-mass)^2 of the photon in the lowest order in QED with a gauge-noninvariant regularization is negative. In fact, this was crucial for the proof of equivalence with QED. It is possible that with more complicated cutoff actions the sign of G at the critical point is reversed, when $v(\mathcal{A}^2)$ is computed exactly.

If we assume the transition is continuous, G_c, the critical coupling, is obtained by setting the sum of all terms quadratic in $\mathcal{A}$ in Fig. 4 to zero. This gives an eigenvalue condition for G_c and corresponds to treating ζ as a mass counterterm to make the renormalized mass of A_i equal zero. Thus in the previous section we have really chosen the critical point of the transition to the phase with broken symmetry.

To understand how the collective field A_i propagates, it is necessary to compute the generating functional $Z(\mathcal{A}_i)$ of one-particle irreducible diagrams with only $\mathcal{A}_i$ external lines. This is again given by the diagrams of Fig. 4 except that the external lines can now carry nonzero momentum. In the symmetric phase, the propagation will be similar to that of a massive vector particle. In the cutoff model there is a conserved current

$$\partial_i^{(M)}=\left(1-\frac{\Box}{M^2}\right)j_i+\bar{\psi}\gamma_i\frac{\Box}{M^2}\psi+\frac{2}{M^2}\,\partial_k\psi\gamma_i\partial_k\psi$$
$$-\bar{\psi}\,\frac{\not\partial}{M^2}\overleftrightarrow{\partial}_i\psi \tag{4.1}$$

and the massive excitation will correspond to this rather than to $\bar{\psi}\gamma_i\psi$ which is equivalent to A_i.

Since $\partial_i j_i^{(M)}=0$, the excitation has only three polarization states (for $d=4$) and the scalar component decouples in the S matrix. In the broken symmetry phase, it is necessary to consider terms quadratic in a_i where $\mathcal{A}_i=\eta_i+a_i$. Since $dv(\eta^2)/d\eta_i=0$, we get for the quadratic term

$$\tfrac{1}{2}a_i(k)\{\alpha(k^2,\eta)[\delta_{ij}k^2+\beta(k^2,\eta)k_ik_j]$$
$$+4\eta_i\eta_jv''(\eta^2)\}a_j(k), \tag{4.2}$$

where v'' stands for a double derivative with respect to η^2. If for some reason $\beta(k^2,\eta)=-1$, this corresponds to a gauge-invariant theory with an axial gauge fixing, $\eta\cdot a=0$. The propagator then has exactly the same structure as that given by Bjorken.[3] For low-energy excitations ($E\ll M^{-1}$) there is an approximate local gauge invariance so that $\beta\simeq-1$ and we have a massless vector excitation. Bjorken[3] has argued that local gauge invariance and Lorentz (or Euclidean) invariance can be recovered in the continuum limit even in this phase. However, the divergent integrals involved have not been handled carefully and many ambiguities are involved. It is clear that if the charge (– current) density of the vacuum is nonzero, Lorentz invariance is not possible because such a vacuum responds differently to external charges in different frames of reference. It is therefore necessary to be infinitesimally close to the critical point of the transition.

We will now compute the effective potential in a one-loop approximation, i.e., only terms in the first line of Fig. 4 are retained. Other diagrams can be included perturbatively. This is essentially the Hartree-Fock approximation used by Bjorken[3] and others.[4] However, use of the effective potential approach has the added advantage that the true ground state is unambiguously fixed. The one-loop approximation also corresponds to considering the quantum fluctuations of only the fermions and looking for classical instabilities in the resulting effective action for $\mathcal{A}_i$. Thus from Eq. (3.5) we get

$$v_c(\mathcal{A}^2)=\tfrac{1}{2}\zeta^2\mathcal{A}^2-\frac{1}{\Omega}\,\mathrm{Tr}\ln|1+ie(\not\partial_M+m)^{-1}\not{\mathcal{A}}|\,, \tag{4.3}$$

where $\mathcal{A}_i$ has no coordinate dependence and Ω is the space-time volume. The extremum $\mathcal{A}_i=\eta_i$ is

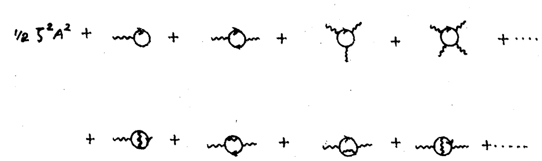

FIG. 4. Diagrammatic expansion of the effective potential for A_i.

given by

$$\zeta^2\eta_i = \int \frac{d^dk}{(2\pi)^d} \operatorname{tr}\left(\frac{1}{m - i\not{k}_M - ie\not{\eta}} ie\gamma_i\right) , \quad (4.4)$$

where tr is over the spinor variables only. Here $\not{k}_M = \not{k}(1 + k^2/M^2)$. By a rescaling $e\eta_i = Q_i$ we recover Bjorken's self-consistency condition.[3] The Tr ln term in Eq. (4.3) blows up at best logarithmically with $\mathfrak{a}$. Thus there is no possibility of making $v_c(\mathfrak{a}^2)$ unbounded for any sign of the current-current coupling $G = e^2/2\zeta^2$. However, when $G(>0)$ exceeds a critical value G_c, the absolute minimum of v_c shifts away from $\mathfrak{a}_i = 0$. In fact, when $M \to \infty$, all convergent diagrams (those with more than four external legs) vanish because of gauge invariance. The self-energy diagram diverges similar to M^{d-2} (for $d > 2$) and has a sign opposite to the $\zeta^2 A^2$ term for $G > 0$. Thus the critical value G_c vanishes similar to its canonical dimension if we hold the bare charge fixed. This can make a nonrenormalizable theory renormalizable because a power-counting argument suggests only a finite number of counterterms.

From Eq. (4.4) we get for $d = 4$,

$$Q^2 = 2\pi cG \int_0^\infty dk^2 k^2 \left(1 - \frac{k_M{}^2 + m^2 - Q^2}{2k_M{}^2 Q^2}\{(m^2 + k_M{}^2 + Q^2) - [(m^2 + k_M{}^2 + Q^2)^2 - 4k_M{}^2 Q^2]^{1/2}\}\right), \quad (4.5)$$

where

$$d^4k = (2\pi)^4 c\, dk^2 k^2 \sin^2\theta\, d\theta$$

and

$$k_M{}^2 = k^2(1 + k^2/M^2)^2 .$$

For $m = 0$ this becomes

$$Q^2 = 2\pi cG \int_0^\infty dk^2 k^2 \left[1 - \frac{k_M{}^2 - Q^2}{2k_M{}^2 Q^2}(|k_M{}^2 + Q^2| - |k_M{}^2 - Q^2|)\right] . \quad (4.6)$$

A perturbation expansion in Q is highly infared singular when $m = 0$ and this is reflected in the modulus sign entering in this equation. By integrating Eq. (4.4) with respect to e, we recover $v_c(\mathfrak{a}^2)$. We will do this for $m = 0$. We have

$$\frac{\partial v_c}{\partial e}(\mathfrak{a}^2; e^2) = -\frac{\pi c M^2}{e} \frac{x + \frac{5}{3}x^2 + \frac{5}{3}x^3 + \frac{4}{5}x^4}{(1+x)^2}, \quad (4.7)$$

with $v_c(\mathfrak{a}^2; 0) = \frac{1}{2}\tau^2\mathfrak{a}^2$. Therefore,

$$V_c(x) = \tfrac{1}{2}\pi c M^2[(2\pi cGM)^{-1}x(1+x)^2 + \tfrac{11}{10}x + \tfrac{12}{10}x^2 + \tfrac{54}{15} - 3\ln(1+x) - \tfrac{149}{30}(1+x)^{-1} + \tfrac{31}{30}(1+x)^{-2}]. \quad (4.8)$$

Here x is a monotonic function of $e\mathfrak{a}/M$:

$$x(1+x)^2 = e^2\mathfrak{a}^2/M^2 \quad (4.9)$$

and $V_c(x)$ is the function $v_c(\mathfrak{a}^2)$ written in terms of the variable x. The critical value G_c is

$$G_c M^2 = (2\pi c)^{-1} . \quad (4.10)$$

V. DISCUSSION AND IMPLICATIONS

We will now compare and contrast the approach of this paper with previous approaches[1–10] to prove such equivalences. All previous papers have employed a strictly local fermion interaction even for a finite cutoff. As a result two kinds of results have been claimed in the literature. The first[3–8] makes use of the divergence in the wave-function renormalization constant to say that all kinds of four-fermion interactions are renormalizable in $d = 4$. However, the proofs are extremely formal. In this paper divergence in the bare mass of the coherent excitation is made use of. This has been possible even in the case of current interactions leading to gauge theories because of the choice of a gauge-noninvariant cutoff. Equivalence is made possible even for $d < 4$ without restrictions on the renormalized parameters.

The other claim[10] has been that the four-fermion theories are equivalent to fermion-boson theories with restrictions to the renormalized parameters and therfore may be preferable to the later as a theory of fundamental interactions. This claim has been exhaustively verified[10] in the context of expansions about a mean field theory especially for $d = 2$. However, results of Refs. 11 and 12 suggest that although for $d < 4$ such equivalences are true and we get theories with anomalous dimensions, for $d = 4$ the only possible infinite cutoff limit may be a free theory of fermions and bosons. In the present approach, there is no restriction on the parameters of the equivalent fermion-boson theory. New parameters are hidden in the range and the strengths of the many-fermion interactions of the cutoff action. Although all these parameters vanish as the cutoff is removed, it is the way they vanish that sets the requires scales. In particular, because the range of interactions vanishes in the limit of an infinite cutoff the fermionic theory is local.

Thus in the approach of this paper, the Weinberg-Salam model or quantum chromodynamics could be rewritten as a local fermionic theory. Although the equivalence has not been explicitly demonstrated for a complicated system such as the Weinberg-Salam model in this paper, it should be noted that the crucial aspect for the equivalence is that the cutoff dependence in the bare mass of the bosons be stronger than the cutoff dependence in the wave-function renormalization constant. Renormalized perturbation theory gives quadratically divergent corrections to the self-mass and logarithmically divergent corrections to the wave-function renormalization. This taken with the deminstration for the $d=2$ in Secs. II and III can be taken as a strong indication of the equivalence in more complicated systems.

If almost any fermion-boson theory can be rewritten as a local fermionic theory, it would appear that there are no practical implications for this equivalence. One place where there can be an important difference is the case of quantum gravity which has not been meaningful in the context of renormalization theory. Once the graviton is not a fundamental field, it is not necessary to require just two derivatives in the action. Thus there is the possibility that there exists a simple fermionic action involving self-interactions via the energy-momentum tensor which has gravitons as coherent excitations[5] and a meaningful infinite cutoff limit. The equivalent fermion-graviton system may have the Einstein action as the effective action at energies much lower than the scale provided by the Planck length.

Another difference of physical significance arises if there is an intrinsic cutoff in nature, as for example that provided by the Planck length. In the approach of this paper, masslessness of the gauge bosons is due to the choice of the critical coupling. It is difficult to understand how such a special value of the coupling could have been chosen by nature. A different possibility is provided by the arguments made in the Appendix. Instead of the specific choice of the critical coupling, if a nearby coupling is chosen, there is still a possibility of having an effective local gauge invariance at low energies. However, at very high energies, the gauge invariance is not exact. Note that this possibility requires an infinitesimal, nonzero (renormalized) mass of the photon or an infinitesimal breaking of Lorentz invariance. There are stringent experimental bounds of both of these. In the context of these remarks reader's attention is drawn to Ref. 24.

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APPENDIX

In Sec. III the kinetic energy term for the photon was generated by carefully adjusting the range of the nonlocal current-current interaction. Is it possible to obtain the equivalence with QED by considering a cutoff current-current interaction which is local even for a finite cutoff? As an example we may consider the class of actions

$$s=\bar{\psi}(-\partial\!\!\!/_M - m)\psi - G(\bar{\psi}\gamma_i\psi)^2\,, \tag{A1}$$

where $\partial\!\!\!/_M = \partial\!\!\!/ \exp(-\Box/M^2)$. Introducing an auxiliary variable A_i it is seen that an explicit kinetic-energy term for the photon is missing. Previous attempts[2-9] at the equivalence have tried to generate this kinetic energy by using the divergences in the wave-function renormalization of the photon. The basic problem is that choosing the critical point makes ζ^2 [Eq. (3.5)] exactly proportional to e^2 so that A_i always appears in the combination eA_i. Thus there is no independent parameter associated with the scale of A_i.

However, for a finite M let us choose G close to but away from the critical point, on the side on which the symmetry is unbroken. Then we have a massive vector excitation with a renormalized mass μ that goes to zero as $M\to\infty$. Also for a finite M, the local gauge invariance of the fermionic part of the action, Eq. (3.5), is explicitly broken. As a result the longitudinal part $k_\mu k_\nu/\mu^2$ of the photon propagator gives a nonzero contribution to the photon self-energy. Let us presume that Ward's identity is broken to $O(1/M^2)$. By choosing $\mu^2=O(M^{-2})$ appropriately, we produce a finite and adjustable contribution to the transverse part of the self-energy. Thus the quantum fluctuations can generate a kinetic energy term for the photon.

To see this in perturbation theory is very difficult. One problem is that because the longitudinal term in the A_i propagator is not well behaved for large momenta the breaking of gauge invariance is not $O(M^{-2})$ but instead diverges with M. To avoid this problem it becomes necessary to use a nonlocal regularization for the currents. We will consider the cutoff action

$$S=-\tfrac{1}{2}\zeta^2A^2-\tfrac{1}{2}\mu^2A\left(1+\frac{\Box^2}{\Lambda^4}\right)A+\tfrac{1}{4}F_{ij}\frac{\Box}{\Lambda^2}F_{ij}+\bar{\psi}(-\not{\partial}_M-m-ie\not{A})\psi \tag{A2}$$

with $\not{\partial}_M$ as in Sec. III. Now the bare A_i propagator is $D_{ij,\Lambda}$ with

$$D_{ij,\Lambda}^{-1}=\mu^2(1+l^4/\Lambda^4)\delta_{ij}+\frac{l^2}{\Lambda^2}(l^2\delta_{ij}-l_il_j)\,. \tag{A3}$$

We will consider all processes of $O(e^4)$ involving only photons. These are given by the diagrams of Fig. 5. In Fig. 5(b) we can set $M\to\infty$ and we have the usual gauge-invariant contribution (for $d<4$). In Fig. 5(a) the internal photon propagator is

$$\frac{1}{l^4(\Lambda^{-2}+\mu^2\Lambda^{-4})+\mu^2}\left[\delta_{ij}+\frac{l_il_j}{\mu^2}\,\frac{l^2}{\Lambda^2(1+l^4/\Lambda^4)}\right]. \tag{A4}$$

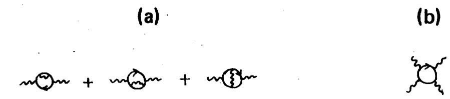

FIG. 5. All $O(e^4)$ diagrams with A_i external lines.

The longitudinal part is singular in μ. If we first take the limit $M\to\infty$ keeping μ finite, this longitudinal term would not contribute because it couples to a conserved current. This is the way the equivalence of the vector-meson theory, in the massless limit, with QED is proved. This proof shows the equivalence only at the level of gauge-invariant Green's functions. We will, however, take the $\mu\to 0$ limit differently.

For a finite M the l_il_j contributions are nonzero. An explicit calculation gives, for the self-energy,

$$\int\Bigg(\gamma_j\frac{1}{m+i(\not{k}+\not{p})}\,i\not{k}\,\frac{1}{m+i(\not{k}+\not{p})}\gamma_i\frac{1}{m+i\not{k}}\{k^2\}\{(k+p)^2\}\{(k+p)^2\}[\{(k+p+l)^2\}-\{(k+p-l)^2\}]$$
$$+\left[\gamma_j\frac{1}{m+i(\not{k}+\not{p}+\not{l})}\gamma_i\frac{1}{m+i\not{k}}-\gamma_j\frac{1}{m+i(\not{k}+\not{p})}\gamma_i\frac{1}{m+i\not{k}}\right]\{k^2\}\{(k+p)^2\}\{(k+l+p)^2\}[\{(k+p)^2\}-\{(k+l)^2\}]\Bigg)\,. \tag{A5}$$

Here $\int$ stands for

$$-\frac{2e^4}{\mu^2}\int\frac{d^dk}{(2\pi)^d}\frac{d^dl}{(2\pi)^d}\frac{1}{l^4(\Lambda^{-2}+\Lambda^{-4}\mu^2)+\mu^2}\,\frac{l^2}{\Lambda^2(1+l^4\Lambda^{-4})}\,\mathrm{tr} \tag{A6}$$

and $\{k^2\}$ stands for $(1+k^2M^{-2})^{-1}$. Retaining only terms of $O(M^{-2})$, we get

$$\frac{1}{M^2}\int\left[-2\gamma_j\frac{1}{m+i(\not{k}+\not{p})}\,i\not{l}\,\frac{1}{m+i(\not{k}+\not{p}+\not{l})}\gamma_i\frac{1}{m+i\not{k}}\right][2k\cdot(l-p)+l^2-p^2]. \tag{A7}$$

It is presumed that the terms independent of $\not{p}$ are removed by using ζ as a mass counterterm. Then it may be verified that the k integration behaves like d^dk/k^4 and hence convergent for $d<4$. For terms quadratic in p we get a contribution proportional to $e^4\Lambda^{2d-4}\mu^{-2}M^{-2}$. Thus by choosing $\mu\,\alpha\,e^2M^{-1}\Lambda^{d-2}$ and taking the local limit $M,\Lambda\to\infty$ in such a way that $M^{-1}\Lambda^{d-2}\to 0$, we get a finite and adjustable contribution to the photon self-energy of the form $\alpha\delta_{ij}p^2+\beta p_ip_j$, where α and β are constants of $O(e^0)$. This is effectively a kinetic energy term for the photon. In this limit, terms other than quadratic in p all vanish.

Notice that this choice of μ corresponds to an $O(e^4)$ contribution to ζ in addition to the $O(e^2)$ term coming from the eigenvalue condition. Therefore, we are indeed having an independent scale associated with A_i. Also this new contribution to the self-energy is not transverse. Hence the equivalence with QED can be only at the level of gauge-invariant quantities.

If we consider many-photon exchanges, Fig. 6, for a finite Λ and M there are anomalous contributions to many-photon scattering amplitudes. To any order, for a fixed Λ, the leading behavior in M remains similar to M^{-2}. Let us presume that the contributions to the wave-function renormalization sum to an expression of the form $e^2\Lambda^2M^{-2}\exp(e^2\Lambda^d\mu^{-2}m^{-d+2})$. Then by choosing $\mu\,\alpha\,e\Lambda^{d/2}(\ln e\Lambda/M)^{-1}$ and $M,\Lambda\to\infty$ such that $M^{-1}\exp(e\Lambda^{d/2})\to 0$ we generate the required kinetic energy. It is to be hoped that in the same limit, anomalous contributions to all other vertices vanish in the Abelian case but those for the three

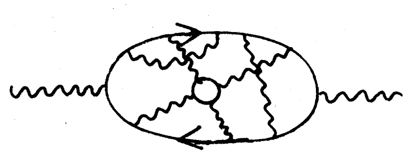

FIG. 6. Higher-order contributions to the photon self-energy.

and four gluon vertices survive in the non-Abelian case.

The singularity in μ [Eq. (1.8)] has been crucial for all our considerations. If we choose a regularization in which the "kinetic energy" term in Eq. (A3) is not transverse, this singularity is absent. However, as mentioned in Sec. IV, there is a conserved current in the cutoff theory and the residue of the propagator for this current must have a mass singularity. Hence the choice in Eq. (A3) is justified.

Equivalently one may work in the broken symmetry phase. Now the vacuum expectation value η_i of A_i [a measure of the charge (–current) density of the vacuum] is to be varied carefully. With $\alpha=-\beta=1$ in Eq. (4.2) we get for the propagator

$$\frac{1}{k^2}\left[\delta_{ij}-\frac{k_i\epsilon_j+\epsilon_i k_j}{k\cdot\epsilon}+\frac{k_i k_j}{(k\cdot\epsilon)^2}\right]+\frac{k_i k_j}{(k\cdot\epsilon)^2}\,\frac{1}{2}\,\frac{v''(0)}{\eta^2}\,, \tag{A8}$$

where ϵ is a unit vector. The last term is singular at $\eta=0$ and η here plays the role of μ in Eq. (A3).

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