# Gauge invariance and string interactions in a generalized theory of gravitation

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The gauge invariance of the Lagrangian in the nonsymmetric extension of general relativity is investigated. The skew parts of the nonsymmetric Hermitian $g_{\mu\nu}$, in the weak-field approximation, act as gauge potentials that correspond to the exchange of massless scalar mesons between one-dimensionally extended objects (strings) in space-time. For open strings a massive vector particle, associated with the torsion, is also exchanged between the end points of the strings.

## I. INTRODUCTION

In the following we shall present a new Lagrangian density with one-dimensionally extended sources, based on the nonsymmetric, Hermitian extension of general relativity. The present theory contains the earlier published versions[1,2] as special cases of a more general framework. In the weak-field approximation, we investigate the gauge structure of the Lagrangian density and we find that $g_{[\mu\nu]}$ is an antisymmetric potential with loop (string) sources. Such fields have been considered in the literature in connection with theories of gravitation[3] and string models[4-6] and also in supergravity.[7,8]

## II. THE LAGRANGIAN

We shall begin with a derivation of the Lagrangian including sources. The resulting Lagrangian and field equations will differ from previous derivations[1,2] in certain important respects. The basic notation will be the same as in Refs. 1 and 2.

We raise and lower indices by using the relation

$$g^{\mu\nu}g_{\sigma\nu}=g^{\nu\mu}g_{\nu\sigma}=\delta^{\mu}_{\sigma}\,. \tag{2.1}$$

A nonsymmetric affine connection $W^{\lambda}_{\mu\nu}$ is related to a (Hermitian) connection $\Gamma^{\lambda}_{\mu\nu}$ by the equation

$$W^{\lambda}_{\mu\nu}=\Gamma^{\lambda}_{\mu\nu}-\tfrac{2}{3}\delta^{\lambda}_{\mu}W_{\nu}\,, \tag{2.2}$$

where $W_{\nu}\equiv\frac{1}{2}(W^{\sigma}_{\nu\sigma}-W^{\sigma}_{\sigma\nu})=W^{\sigma}_{[\nu\sigma]}$ is a (pure imaginary) vector field. From (2.2) we have

$$\Gamma_{\mu}\equiv\Gamma^{\sigma}_{[\mu\sigma]}=0\,. \tag{2.3}$$

A Hermitian contracted curvature tensor can be formed,

$$R_{\mu\nu}(W)=W^{\beta}_{\mu\nu,\beta}-\tfrac{1}{2}(W^{\beta}_{\mu\beta,\nu}+W^{\beta}_{\nu\beta,\mu})-W^{\beta}_{\alpha\nu}W^{\alpha}_{\mu\beta}+W^{\beta}_{\alpha\beta}W^{\alpha}_{\mu\nu}\,. \tag{2.4}$$

By substituting (2.2) into (2.4), we get

$$R_{\mu\nu}(W)=R_{\mu\nu}(\Gamma)+\tfrac{2}{3}W_{[\mu,\nu]}\,, \tag{2.5}$$

where

$$W_{[\mu,\nu]}=\tfrac{1}{2}(W_{\mu,\nu}-W_{\nu,\mu})$$

and

$$R_{\mu\nu}(\Gamma)=\Gamma^{\beta}_{\mu\nu,\beta}-\tfrac{1}{2}(\Gamma^{\beta}_{(\mu\beta),\nu}+\Gamma^{\beta}_{(\nu\beta),\mu})-\Gamma^{\beta}_{\alpha\nu}\Gamma^{\alpha}_{\mu\beta}+\Gamma^{\beta}_{(\alpha\beta)}\Gamma^{\alpha}_{\mu\nu} \tag{2.6}$$

is a Hermitian tensor.

We shall use geometrical units in which $G=c=1$. Our Lagrangian density is given by

$$\mathfrak{L}=\mathfrak{g}^{\mu\nu}R_{\mu\nu}(W)+\frac{g}{3e}\mathfrak{g}^{\mu\alpha}(W_{\mu}V_{\alpha}+W_{\alpha}V_{\mu})-\tfrac{1}{4}\mathfrak{H}^{[\mu\nu]}H_{[\mu\nu]}+L_m\,, \tag{2.7}$$

where we have used the notation $\mathfrak{x}_{\mu\nu}=\sqrt{-g}X_{\mu\nu}$. Moreover, $L_m$ is the Lagrangian density for the matter sources,

$$\frac{\partial L_m}{\partial g^{\mu\nu}}=-2\mathfrak{T}_{\mu\nu}\,, \tag{2.8}$$

where $\mathfrak{T}_{\mu\nu}$ is a nonsymmetric (Hermitian) generalized energy-momentum tensor. $V_{\mu}$ is a (pure imaginary) vector field and $H_{[\mu\nu]}$ is defined by

$$H_{[\mu\nu]}=V_{\nu+;\mu}-V_{\mu-;\nu}=V_{\nu,\mu}-V_{\mu,\nu}\,, \tag{2.9}$$

where we have used the Einstein + and − notation for covariant differentiation with respect to $\Gamma^{\lambda}_{\mu\nu}$.[1] We also define $H^{[\mu\nu]}=g^{\mu\alpha}g^{\nu\beta}H_{[\alpha\beta]}=-H^{[\nu\mu]}$. In our units $g$ is a dimensionless constant and $e$ is a constant with the dimensions of a length.

We can now use the Palatini method, varying $g$, $W$, and $V$ as independent field variables (such that $\delta g$, $\delta W$, and $\delta V$ vanish at the boundaries of integration). The $W$ variation gives

$$\mathfrak{g}^{\mu\nu}{}_{,\sigma}+\mathfrak{g}^{\rho\nu}W^{\mu}_{\rho\sigma}+\mathfrak{g}^{\mu\rho}W^{\nu}_{\sigma\rho}-\mathfrak{g}^{\mu\nu}W^{\rho}_{\sigma\rho}+\tfrac{2}{3}\delta^{\nu}_{\sigma}\mathfrak{g}^{\mu\rho}W^{\beta}_{[\rho\beta]}+\frac{g}{3e}(\mathfrak{g}^{(\nu\alpha)}V_{\alpha}\delta^{\mu}_{\sigma}-\mathfrak{g}^{(\mu\alpha)}V_{\alpha}\delta^{\nu}_{\sigma})=0\,. \tag{2.10}$$

Contracting over $\nu$ and $\sigma$ and antisymmetrizing gives the equation

$$g^{[\mu\nu]}{}_{,\nu}=\frac{g}{e}g^{(\mu\alpha)}V_\alpha\,. \tag{2.11}$$

The variation with respect to $g^{\mu\nu}$ gives

$$G_{\mu\nu}(W)=2T_{\mu\nu}-B_{\mu\nu}\,, \tag{2.12}$$

where $G_{\mu\nu}(W)$ is the generalized Einstein tensor

$$G_{\mu\nu}(W)=R_{\mu\nu}(W)-\tfrac{1}{2}g_{\mu\nu}R(W) \tag{2.13}$$

with $R_{\mu\nu}=g_{\mu\alpha}g_{\beta\nu}R^{\beta\alpha}$ and $R=g^{\mu\nu}R_{\mu\nu}$. Moreover, we have

$$B_{\mu\nu}=\frac{g}{3e}\left[W_\mu V_\nu+W_\nu V_\mu-\tfrac{1}{2}g_{\mu\nu}g^{\alpha\beta}(W_\alpha V_\beta+W_\beta V_\alpha)\right]+\tfrac{1}{2}E_{\mu\nu}\,, \tag{2.14}$$

where $B_{\mu\nu}$ is a Hermitian tensor and

$$E_{\mu\nu}=-g^{\alpha\beta}H_{[\nu\beta]}H_{[\mu\alpha]}+\tfrac{1}{4}g_{\mu\nu}H^{[\alpha\beta]}H_{[\alpha\beta]}\,. \tag{2.15}$$

The variation with respect to $V^\mu$ gives

$$\mathfrak{H}^{[\mu\nu]}{}_{,\nu}=-\frac{2g}{3e}g^{(\mu\alpha)}W_\alpha\,. \tag{2.16}$$

If we introduce another Hermitian connection $\Lambda^\lambda_{\mu\nu}$ by the equation

$$\Lambda^\lambda_{\mu\nu}=\Gamma^\lambda_{\mu\nu}+D^\lambda_{\mu\nu}(V)\,, \tag{2.17}$$

where $D^\lambda_{\mu\nu}$ is defined by

$$g_{\rho\nu}D^\rho_{\mu\sigma}+g_{\mu\rho}D^\rho_{\sigma\nu}=-\frac{g}{3e}g^{(\rho\alpha)}V_\alpha(g_{\mu\sigma}g_{\rho\nu}-g_{\mu\rho}g_{\sigma\nu}+g_{\mu\nu}g_{[\sigma\rho]})\,, \tag{2.18}$$

then (2.10) can be written as a metrically compatible set of equations

$$g_{\mu+\nu-|\sigma}\equiv g_{\mu\nu,\sigma}-g_{\rho\nu}\Lambda^\rho_{\mu\sigma}-g_{\mu\rho}\Lambda^\rho_{\sigma\nu}=0\,, \tag{2.19}$$

where we have used (2.1) and (2.2). It can be shown that

$$\sqrt{-g}|_\sigma\equiv\sqrt{-g}_{,\sigma}-\sqrt{-g}\Lambda^\alpha_{(\sigma\alpha)}=0 \tag{2.20}$$

and

$$g^{[\mu\nu]}{}_{,\nu}=g^{(\mu\nu)}\Lambda_\nu\,, \tag{2.21}$$

where $\Lambda_\nu\equiv\Lambda^\alpha_{[\nu\alpha]}$. For a vector $B^\mu$ we have

$$B^\mu{}_{|\sigma}=B^\mu{}_{,\sigma}+B^\rho\Lambda^\sigma_{(\rho\sigma)}+B^\rho\Lambda_\rho\,. \tag{2.22}$$

Multiplying (2.22) by $\sqrt{-g}$ and using (2.20), we obtain by contracting (2.22) over $\mu$ and $\sigma$

$$\mathfrak{B}^\mu{}_{|\mu}=\mathfrak{B}^\mu{}_{,\mu}+\mathfrak{B}^\rho\Lambda_\rho\,. \tag{2.23}$$

If we choose $B^\mu$ to be a real vector and take into account the pure imaginary property of $\Lambda_\mu$, we get

$$\mathrm{Re}(\mathfrak{B}^\mu{}_{|\mu})=\mathfrak{B}^\mu{}_{,\mu}\,. \tag{2.24}$$

The variational principle yields the four generalized Bianchi identities

$$[g^{\alpha\nu}G_{\rho\nu}(\Gamma)+g^{\nu\alpha}G_{\nu\rho}(\Gamma)]_{,\alpha}+g^{\mu\nu}{}_{,\rho}\mathfrak{G}_{\mu\nu}(\Gamma)\equiv 0\,. \tag{2.25}$$

We also have the two additional identities

$$g^{[\mu\nu]}{}_{,\nu,\mu}=\frac{g}{e}(g^{(\mu\alpha)}V_\alpha)_{,\mu}\equiv 0 \tag{2.26}$$

and

$$\mathfrak{H}^{[\mu\nu]}{}_{,\nu,\mu}=-\frac{2g}{3e}(g^{(\mu\alpha)}W_\alpha)_{,\mu}\equiv 0\,. \tag{2.27}$$

We can write (2.7) as

$$\mathfrak{L}=g^{\mu\nu}(\Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\alpha\nu}-\Gamma^\alpha_{\mu\nu}\Gamma^\beta_{(\alpha\beta)})-\tfrac{2}{3}\left(g^{[\mu\nu]}{}_{,\nu}-\frac{g}{e}g^{(\mu\alpha)}V_\alpha\right)W_\mu-\tfrac{1}{4}\mathfrak{H}^{[\mu\nu]}H_{[\mu\nu]}-2g^{\mu\nu}T_{\mu\nu}+\mathfrak{U}^\alpha{}_{,\alpha}\,, \tag{2.28}$$

where $\mathfrak{U}^\alpha{}_{,\alpha}$ is a total divergence. We see that $W_\mu$ *acts as a Lagrange multiplier* that guarantees the four constraint equations (2.11).

We observe from (2.11) and (2.21) that

$$V_\mu=\frac{e}{g}\Lambda_\mu\,. \tag{2.29}$$

Thus the vector field $V_\mu$ is proportional to the vector torsion field associated with the $\Lambda$ connection. In the present theory the torsion is a propagating field.

When $V_\mu$ and $T_{\mu\nu}$ vanish, the field equations reduce to[9]

$$g_{\mu\nu,\sigma}-g_{\rho\nu}\Gamma^\rho_{\mu\sigma}-g_{\mu\rho}\Gamma^\rho_{\sigma\nu}=0\,, \tag{2.30}$$

$$g^{[\mu\nu]}{}_{,\nu}=0\,, \tag{2.31}$$

$$R_{\mu\nu}(\Gamma)=\tfrac{2}{3}W_{[\nu,\mu]}\,. \tag{2.32}$$

## III. WEAK-FIELD APPROXIMATION

In the weak-field approximation we have

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}\,, \tag{3.1}$$

where $|h_{\mu\nu}|\ll 1$ and $\eta_{\mu\nu}$ is the Minkowski metric tensor. We shall choose for convenience $x^4=ix^0$ so that $\eta_{\mu\nu}=-\delta_{\mu\nu}$. The first-order solution for $\Gamma^\lambda_{\mu\nu}$, obtained from (2.17)-(2.19), is given by[10]

$$\Gamma^\lambda_{\mu\nu}=-\tfrac{1}{2}(h_{\lambda\nu,\mu}+h_{\mu\lambda,\nu}-h_{\nu\mu,\lambda})-\frac{g}{3e}(\delta_{\lambda\nu}V_\mu-\delta_{\lambda\mu}V_\nu)\,. \tag{3.2}$$

We shall use the definition $g^{\mu\nu}=g^{\mu\alpha}g^{\beta\nu}g_{\beta\alpha}$ so that $h^{[\mu\nu]}=-\eta^{\mu\alpha}\eta^{\beta\nu}h_{[\beta\alpha]}=h_{[\mu\nu]}$.[2] The Lagrangian to second order is given by

$$L^{(2)}=L_{\mathrm{GR}}+L_s+\mathrm{TD}\,, \tag{3.3}$$

where $L_{\mathrm{GR}}$ is the second-order weak-field Lagrangian of general relativity with $h=h_{\alpha\alpha}$:

$$L_{GR}=-\tfrac{1}{4}h_{(\mu\nu)}\Box h_{(\mu\nu)}+\tfrac{1}{2}h_{(\mu\sigma),\sigma}h_{(\mu\nu),\nu}+\tfrac{1}{2}h_{,\mu}(h_{(\mu\alpha),\alpha}-\tfrac{1}{2}h_{,\mu})+2h_{(\mu\nu)}T_{(\mu\nu)}\,. \tag{3.4}$$

Moreover, $L_s$ is the part of the second-order Lagrangian pertaining to the skew field $h_{[\mu\nu]}$ and the torsion field $V_\mu$:

$$L_s=-\tfrac{1}{4}h_{[\mu\nu]}\Box h_{[\mu\nu]}+\tfrac{1}{2}h_{[\mu\sigma],\sigma}h_{[\mu\nu],\nu}-\tfrac{2}{3}h_{[\mu\nu],\nu}W_\mu-\frac{2g}{3e}h_{[\mu\nu]}V_{[\mu,\nu]}-\frac{2g}{3e}W_\mu V_\mu+\frac{g^2}{3e^2}V_\mu V_\mu-\tfrac{1}{4}H_{[\mu\nu]}H_{[\mu\nu]}+2h_{[\mu\nu]}T_{[\mu\nu]}\,, \tag{3.5}$$

and TD denotes a total divergence. The particle spectrum of the skew contribution has been analyzed previously[11] and found to be free of ghosts for the (complex) Hermitian theory but not for the real nonsymmetric theory. The additional new term $\frac{1}{4}H_{[\mu\nu]}H_{[\mu\nu]}$ in (3.5) will not generate ghosts in the physical particle spectrum, since it has the form of a Maxwell field contribution to the Lagrangian. Thus the Hermitian version of the theory possesses a unitary $S$ matrix.

## IV. GAUGE INVARIANCES OF THE LAGRANGIAN

Let us consider the situation when $V_\mu=T_{\mu\nu}=0$. We shall fix the auxiliary vector field $W_\mu$ by the condition[12]

$$W_\mu=\tfrac{3}{2}h_{[\mu\nu],\nu}\,. \tag{4.1}$$

Then (3.5) becomes

$$L_s=-\tfrac{1}{4}h_{[\mu\nu]}\Box h_{[\mu\nu]}-\tfrac{1}{2}h_{[\mu\sigma],\sigma}h_{[\mu\nu],\nu}\,. \tag{4.2}$$

The equations of motion that follow from (4.2) are

$$\Box h_{[\mu\nu]}+h_{[\nu\lambda],\lambda,\mu}+h_{[\lambda\mu],\nu,\lambda}=0\,. \tag{4.3}$$

We observe that (4.2) is invariant under the Abelian gauge transformation

$$h_{[\mu\nu]}\to h_{[\mu\nu]}+\lambda_{\mu,\nu}-\lambda_{\nu,\mu}\,. \tag{4.4}$$

The sources of $h_{[\mu\nu]}$ are closed strings.[4-6] The gauge transformation (4.4) is related to an infinitesimal displacement forming a loop $\oint\Lambda_\mu dx_\mu$ by Stokes's theorem. Thus the $h_{[\mu\nu]}$ act as gauge potentials and the gauge-invariant fields derived from the potentials are

$$F_{\mu\nu\lambda}=h_{[\mu\nu],\lambda}+h_{[\nu\lambda],\mu}+h_{[\lambda\mu],\nu}\,. \tag{4.5}$$

The Lagrangian $L_s$ can now be written in the manifestly gauge-invariant form

$$L_s=\tfrac{1}{12}F_{\mu\nu\lambda}F_{\mu\nu\lambda}\,. \tag{4.6}$$

The equations of motion

$$F_{\mu\nu\lambda,\mu}=0 \tag{4.7}$$

are equivalent to Eqs. (4.3). Let us impose the four gauge conditions

$$h_{[\mu\nu],\nu}=0 \tag{4.8}$$

which follow in the first-order from (2.11) when $V_\mu=0$. Then the equations of motion become

$$\Box h_{[\mu\nu]}=0\,. \tag{4.9}$$

It is well known[3-6] that $h_{[\mu\nu]}$ and $F_{\mu\nu\lambda}$ represent a scalar one degree of freedom. The coupling between two strings in the theory corresponds to the exchange of a scalar massless meson. If we define the dual field

$$^*F_\mu=\tfrac{1}{6}\epsilon_{\mu\alpha\beta\gamma}F_{\alpha\beta\gamma}=\epsilon_{\mu\alpha\beta\gamma}h_{[\alpha\beta],\gamma}\,, \tag{4.10}$$

then the equations of motion (4.7) or, alternatively,

$$^*F_{\mu,\mu}=0 \tag{4.11}$$

imply that $F_\mu=\phi_{,\mu}$ and the equations of motion reduce to the massless scalar wave equation

$$\Box\phi=0\,. \tag{4.12}$$

The quantization of the free-field Lagrangian has been considered by Kalb and Ramond[4] and by Townsend.[7] The renormalizability of one-loop diagrams for second-rank skew symmetric potentials coupled to pure Einstein gravity has been investigated by Sezgin and van Nieuwenhuizen.[8]

## V. COUPLINGS BETWEEN CLOSED AND OPEN STRINGS

In the case of closed strings the torsion vector $V_\mu$ is zero, while $T_{[\mu\nu]}$ remains nonzero in the presence of matter. We can write the explicit dependence of $h_{[\mu\nu]}$ on the world sheet of string $a$ as[4]

$$h_{[\mu\nu]}=2ig_a\int d\sigma_{a\mu\nu}G(x-x_a)\,, \tag{5.1}$$

where

$$d\sigma_{a\mu\nu}=d\tau_a d\xi_a\sigma_{a\mu\nu} \tag{5.2}$$

with

$$\sigma_{a\mu\nu}=\frac{\partial x_\mu}{\partial\tau_a}\frac{\partial x_\nu}{\partial\xi_a}-\frac{\partial x_\mu}{\partial\xi_a}\frac{\partial x_\nu}{\partial\tau_a}. \tag{5.3}$$

Moreover, $G(x)$ is the retarded Green's function

$$G_R(x-x_a)=-\frac{1}{2\pi}\theta(x_0-x_{a0}(\tau,\xi))\times\delta([x-x_a(\tau,\xi)]^2)\,. \tag{5.4}$$

We treat the string as a one-dimensionally extended object which traces out a world sheet in spacetime, $x_\mu(\tau_a,\xi_a)$, where $\tau_a$ and $\xi_a$ are the invariant parameters needed to describe the world sheet.

It follows from (3.5) and (4.1) that

$$\Box h_{[\mu\nu]}=4T_{a[\mu\nu]}\,. \tag{5.5}$$

For closed strings we have

$$h_{[\mu\nu],\nu}=0 \tag{5.6}$$

and, according to (5.1) and (5.5),

$$T_{a[\mu\nu]}=\tfrac{1}{2}ig_a\int d\sigma_{a\mu\nu}\delta^4(y-x_a(\tau,\xi))\,. \tag{5.7}$$

Moreover $T_{a[\mu\nu]}$ is conserved:

$$T_{a[\mu\nu],\nu}=0\,. \tag{5.8}$$

The second-order Lagrangian in the $W_\mu$ gauge (4.1) is now

$$L_s=\tfrac{1}{12}F_{\mu\nu\lambda}F_{\mu\nu\lambda}+2h_{[\mu\nu]}T_{[\mu\nu]}\,, \tag{5.9}$$

which is explictly invariant under the gauge transformation (4.4) in view of (5.8).

For open strings the situation is different, since now the torsion $V_\mu$ is nonvanishing. We choose $V_\mu$ to be given by

$$V_\mu(y)=2ie_a\int_{\tau_i}^{\tau_f}d\tau\int_0^{\xi_f}d\xi D_{a\mu}G^*(y-x_a)\,, \tag{5.10}$$

where $G^*$ is the Green's function for open strings. The operator $D_{a\mu}$ for the $a$th string is

$$D_{a\mu}=\frac{dx_\mu}{d\xi_a}\frac{\partial}{\partial\tau_a}-\frac{dx_\mu}{d\tau_a}\frac{\partial}{\partial\xi_a}\,. \tag{5.11}$$

This operator has the property that

$$D_{a\mu}f(x)=\sigma_{a\mu\nu}\partial_{a\nu}f(x_a)\,. \tag{5.12}$$

Integration by parts in (5.10) gives

$$V_\mu(y)=-2ie_a\int_{\tau_i}^{\tau_f}d\tau\left[\frac{dx_\mu}{d\tau_a}(\tau,\xi)G^*(y-x_a(\tau,\xi))\right]_0^{\xi_f}\,. \tag{5.13}$$

Then from (5.1) we have, replacing $G$ by $G^*$,

$$\begin{aligned}h_{[\mu\nu],\nu}&=-2ig_a\int_{\tau_i}^{\tau_f}d\tau\int_0^{\xi_f}d\xi D_{a\mu}G^*(y-x_a(\tau,\xi))\\&=2ig_a\int_{\tau_i}^{\tau_f}d\tau\left[\frac{dx_\mu}{d\tau_a}(\tau,\xi)G^*(y-x_a(\tau,\xi))\right]_0^{\xi_f}\end{aligned} \tag{5.14}$$

or, using (5.13),

$$h_{[\mu\nu],\nu}=-\frac{g_a}{e_a}V_\mu\,. \tag{5.15}$$

The solution of (5.5) for nonzero $V_\mu$ is such that (5.15) holds. This is consistent with the first-order result for (2.11). By the antisymmetry of $h_{[\mu\nu]}$ we see that

$$V_{\mu,\mu}=0\,. \tag{5.16}$$

The torsion field $V_\mu$ is generated by the end points of string $a$, each contributing an opposite "charge".

Let us now impose the gauge-fixing condition:

$$W_\mu=\tfrac{3}{2}\left(h_{[\mu\nu],\nu}-\frac{g}{3e}V_\mu\right)-6\Box^{-1}T_{[\mu\nu],\nu}\,. \tag{5.17}$$

The open string "current" $T_{[\mu\nu]}$ is given by

$$T_{[\mu\nu]}=\tfrac{1}{2}ig_a\int d\sigma_{a\mu\nu}\delta^4(y-x_a(\tau,\xi))+\frac{e}{2g}j_{[\nu,\mu]}\,, \tag{5.18}$$

where the first term in (5.18) is for *closed* strings only and $j_\mu(x)$ is given by

$$j_\mu=ie_a\int_{\tau_i}^{\tau_f}d\tau\left[\frac{dx_\mu(\tau,\xi)}{d\tau}\delta^4(y-x_a(\tau,\xi))\right]_0^{\xi_f}\,. \tag{5.19}$$

From (5.18) we have

$$T_{[\mu\nu],\nu}=-\frac{e}{4g}\Box j_\mu\,, \tag{5.20}$$

since $j_{\mu,\mu}=0$. The gauge-fixing condition (5.17) gives[12]

$$\begin{aligned}L_s=&\tfrac{1}{12}F_{\mu\nu\lambda}F_{\mu\nu\lambda}+\frac{2}{3}\frac{g^2}{e^2}V_\mu V_\mu-V_{[\mu,\nu]}V_{[\mu,\nu]}\\&+2h_{[\mu\nu]}(T_{[\mu\nu]}-2\Box^{-1}T_{[\mu\sigma],\sigma,\nu})\\&+\frac{4g}{e}\Box^{-1}T_{[\mu\sigma],\sigma}V_\mu\,.\end{aligned} \tag{5.21}$$

By substituting (5.18) and (5.20) into (5.21) we obtain

$$\begin{aligned}L_s=&\tfrac{1}{12}F_{\mu\nu\lambda}F_{\mu\nu\lambda}+\frac{2}{3}\frac{g^2}{e^2}V_\mu V_\mu-V_{[\mu,\nu]}V_{[\mu,\nu]}\\&+2h_{[\mu\nu]}T^c_{[\mu\nu]}-j_\mu V_\mu\,,\end{aligned} \tag{5.22}$$

where $T^c_{[\mu\nu]}$ refers to the closed-string contribution of $T_{[\mu\nu]}$ in (5.18) and $T^c_{[\mu\nu],\nu}=0$.

The Lagrangian (5.22) is manifestly gauge invariant under the gauge transformation (4.4). In Ref. 12 it was proved that the gauge-fixing condition (5.17) for the auxiliary field $W_\mu$ is a solution of the equations of motion. Kalb and Ramond[4] render their open-string Lagrangian invariant under (4.4) by adding compensating fields, leading to a massive pseudovector exchange between the ends of the string. We have chosen to generalize the open-string source $T_{[\mu\nu]}$ by Eq. (5.18), so that the current

$$j_{[\mu\nu]}=T_{[\mu\nu]}-\Box^{-1}(T_{[\mu\sigma],\nu,\sigma}-T_{[\nu\sigma],\sigma,\mu}) \tag{5.23}$$

satisfies explicitly $j_{[\mu\nu],\nu}=0$.

The scale of the physical string constant $g'$, which has the dimensions of a mass, will be fixed by $g'=M_p g=\hbar^{1/2}g$, where $M_p$ is the Planck mass. From (5.22) we obtain the equations of motion of the $V_\mu$ field:

$$(\Box+\mu^2)V_\mu=j_\mu\ , \tag{5.24}$$

where $\mu$ is the inverse Compton wavelength

$$\mu\equiv\frac{m}{\hbar}=\left(\frac{4}{3}\right)^{1/2}\frac{g}{e}\ . \tag{5.25}$$

Here $m$ is the mass of the $V_\mu$ field. This result is consistent with (5.13) provided that $G^*(x)$ is the Green's function

$$G^*(x)=-\frac{1}{2\pi}\delta(x^2)-\theta(x^2)\frac{\mu}{4\pi x^2}J_1(\mu x^2)\ , \tag{5.26}$$

where $J_1(x)$ is the Bessel function of order 1. Thus, $G^*(x)$ obeys

$$(\Box+\mu^2)G^*(x)=\delta^4(x)\ . \tag{5.27}$$

By solving for $V_\mu$ in (5.24) we get

$$V_\mu=(\Box+\mu^2)^{-1}j_\mu\ . \tag{5.28}$$

Let us set $S_\mu=1/e\ j_\mu$, where $S_\mu$ is a conserved fermion-number current density, associated with the point sources on the ends of the string. Then in view of (5.15) we obtain in the low-energy limit $q^2\to 0$:

$$\frac{g}{e}V_4\sim\frac{g}{\mu^2}S_4\sim a^2S_4\ , \tag{5.29}$$

where $a$ is a fundamental length predicted to be

$$a=\frac{\sqrt{g}}{\mu}=\left(\frac{3}{4}\right)^{1/4}\left(\frac{e\hbar}{m}\right)^{1/2}\ . \tag{5.30}$$

In the low-energy limit, the Lagrangian $L_s$ goes over into the classical version considered in Ref. 2 with the additional contribution $-(e^2/g^2)\times a^4S_{[\mu,\nu]}S_{[\mu,\nu]}$. The terms such as $V_\mu{}^2$ now behave as contact interaction terms $V_\mu{}^2\sim S_\mu{}^2$. A three-string configuration could in the limit of infinitely short strings produce a point-like source with nonzero fermion number.

## VI. CONCLUDING REMARKS

We have found that the second-order Lagrangian of the nonsymmetric extension of general relativity has two fundamental gauge invariances. One is the gauge invariance of spacetime under the transformation

$$h_{(\mu\nu)}\to h_{(\mu\nu)}+\xi_{\mu,\nu}+\xi_{\nu,\mu}\ . \tag{6.1}$$

The other gauge invariance manifests itself under the gauge transformation (4.4) when the gauge of the auxiliary vector field $W_\mu$ is fixed. The helicity content of the meson exchanges between strings, including gravitation, is (2, 1, 0).

We are now able to understand more clearly why Einstein's interpretation[13] of $g_{[\mu\nu]}$ as Maxwell's electromagnetic field was incorrect and led to the apparent lack of success of his nonsymmetric extensions of general relativity. The gauge-invariance properties and the single physical degree of freedom of $g_{[\mu\nu]}$ cannot describe the Maxwell field $F_{\mu\nu}$. The rigorous Lagrangian describes a gauge theory of strings including gravity. It is interesting that the Lagrangian $L_s$ displays the same states as the dual resonance models.[4] On the other hand, it could possibly describe the confinement picture of quarks.[6] A supersymmetric extension in superspace of the nonsymmetric theory has been formulated by the author.[14,15]

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